On the effect of forecasting in peer-to-peer power networks with storage

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***Abstract*—The continuous integration of renewable energy sources into a power network has caused a paradigm shift in energy generation and distribution. The intermittent nature of renewable sources affects the prices at which energy can be sold or purchased. In addition, the network is subject to operational constraints, voltage limits at each node, rated capacities for the power electronic devices, current bounds for distribution lines; these constraints coupled with intermittent renewable injections may pose a threat to system stability and performance. We propose a distributed predictive controller to handle operational constraints while minimising generation costs, and an agent based market negotiation framework to obtain suitable pricing policies, agreed among participating agents, that explicitly considers availability of energy storage in its formulation. The controller handles the problem of coupled constraints using information exchanges with its neighbours to guarantee their satisfaction. We study the effect of different forecast accuracy have on the overall performance and market behaviours. We provide a convergence analysis for both the negotiation iterations, and its interaction with the predictive controller. Lastly, We assess the impact of the information availability with the aid of testing scenarios.**

***Index Terms*—Microgrids, Model Predictive Control, Multi Agent Systems, Smart Local Energy Systems**

I. Introduction

THE generation of energy has experienced a paradigm shift in terms of energy pricing and generation capabilities. The current landscape of the electricity market is marked by the ever growing presence of renewable energy sources and a push for a deregulation of the electric markets. Both of these streams entail a similar requirement: a decentralisation of operations which would grant more power to the participants in the network [1]. Decentralization of decision making within power networks has the added benefit of enhancing reliability and flexibility, *i.e.,* the systems involved are more responsive to local changes, and modifications do not require global redesigns. The challenges associated with this paradigm shift lie in the integration of pricing mechanisms and decision making at the control level allowing their stakeholders to engage with each other and perform energy transactions across the system [2].

The new grid has at its heart the Distributed Energy Resource (DER) which can be taken as an individual unit or agent. These units include among them renewable sources like Photovoltaic (PV), Wind Turbine (WT), or batteries; these devices introduce intermittent behaviour in terms of generation. To address the problem of adjusting demand and prices in face of this uncertainty, the authors in [3] propose a dynamic pricing mechanism based on demand response and feedback. Feedback methods have been used in pricing, see [4], in conjunction with automatic generation control. Similarly, [5] proposes a method for handling the volatile nature of renewable sources in a power network using economic equilibrium arguments. The approach taken in [6] considers coordination from the load perspective. The common denominator among these approaches is that their decision making elements can be considered agents. Multi Agent System (MAS) provide non-centralised solutions for market negotiation, in particular [7] and [8] provide a framework for energy dispatch of heterogeneous units with pricing using concepts of demand response. Furthermore, MAS can allow the exploration of peer-to-peer energy trading [9]. Peer-to-peer networks offer a way for its members to interact among them by performing energy transactions based on energy availability; these type of networks are spearheading the previously mentioned energy revolution [10]. Despite their emerging popularity, these networks present a challenge in terms of evaluation, see [11], since the metrics involved in measuring their performance can vary widely according to the services offered. In a nutshell, each node of such network behaves as a prosumer, *i.e.,* an entity capable of generating and consuming power. Each prosumer trades its energy surplus E_{sur} or deficit E_{def} with the grid at *a priori* given prices $c_{\text{buy}} > 0$ and $c_{\text{sell}} > 0$ which generally satisfy $c_{\text{sell}} \ll c_{\text{buy}}$ resulting in a negligible monetary gain, *i.e.,* $E_{\text{sur}}c_{\text{sell}} \ll E_{\text{def}}c_{\text{buy}}$ [12].

The decentralisation of the grid from a control perspective has been an active research field in the past years; the existing methods are departing from hierarchical approaches to a distributed scenario [13]. On the other hand, the problem of optimal Micro-Grid (MG) operation requires control laws to consider economic criteria such as generation costs, electricity prices as seen in [14], in addition a generalisation of the power sharing can be casted as an optimisation problem. The integration of optimal dispatch and generation has been studied in [15]. A practical way of implement optimal controllers is through receding horizon techniques. This type of controller has found as a natural way to handle supervisory controller tasks; for example in [16], the authors use distributed Model Predictive Control (MPC) controllers for DC and AC networks

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respectively where the nodes of the MG solve cooperatively a convex version of the power flow equation. In [17], [18], the authors used economic MPC arguments to control the power flow.

Surprisingly, the interplay of predictive controllers and pricing mechanisms has not been thoroughly explored in the literature. Furthermore, the effect of forecasts accuracy for both load consumption and renewable injection has not been studied in the control framework from a nominal point of view. This paper aims to bridge this gap by proposing a distributed predictive controller optimising a cost that is computed via a negotiation framework for a network with a meshed topology with nodes containing generation units and loads. This predictive controller handles both constraints on generation and coupled constraints in voltages and power flow; in addition, we include information on neighbouring voltages, as seen in [19], and renewable injections and load consumption forecasts. This idea of including information in a nominal setting has been explored in [20]; this type of approaches leverages on the properties of the Optimal Control Problem (OCP) to establish desired guarantees for recursive feasibility. The market part of the proposed system employs an agent based negotiation framework. All nodes forming the power network engage in negotiations to determine the pricing policy at which they trade their surpluses or deficits. Traditionally, each node can decide upon consumption to regulate prices [21], however this implies that each node can regulate consumption at will. In our setting, we turn to access to local storage devices to regulate consumption. Using batteries, however, has a drawback, these are dependent on their respective State of Charge (SoC) which implies that modifying consumption may not be feasible at all times. To overcome these difficulties, we propose a market mechanism that employs a pricing policy affecting the way batteries operate. A consequence of this approach is that we aim to shift the interest in choosing fixed prices along the range of available power to pricing policies which may more accurately reflect the abundance or scarcity of power in each node.

The contribution of this paper are the following:

- A distributed predictive controller which handles interactions between batteries, local loads, and renewable energy sources. The controller is subject to coupled voltage constraints and the proposed controller employs information about forecasts and neighbouring voltages to compute its control law. Section III-A describes the different components of the OCP.
- An agent based market negotiation framework where all network elements engage in bargaining to determine the price at which they will sell surplus or deficit of energy. Section III-B states the problem and relevant definitions of this negotiation problem.
- An analysis of the interplay between, predictive controller, agent based negotiation framework, and forecasting mechanisms in terms of convergence to game theoretic equilibrium concepts and recursive feasibility of predictive controllers.
- A set of testing scenarios were proposed to better understand and evaluate the impact of various forecasting approaches, pricing methods and system configurations on the value that an energy market and distributed control solution can offer.

Notation: For a given graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with nodes $\mathcal{V}$ and edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, the node-edge matrix $\mathcal{B} \in \mathbb{R}^{|\mathcal{E}|\times|\mathcal{V}|}$ characterises the relation between nodes and edges which for edge $e = (i, j) \in \mathcal{E}$ involving nodes i and j can be defined as $[\mathcal{B}]_{ei} = 1$ if node i is the source of $e \in \mathcal{E}$, and $[\mathcal{B}]_{ej} = -1$ if node j is its sink, and zero otherwise. The $2-$norm is denoted $|x| = \|x\|_2$. A *C-set* is a compact and convex set containing the origin; A *PC-set* is a C-set with the origin in its nonempty interior. For a given set $\mathcal{A} \subset \mathbb{R}^n$, and linear transformations $B \in \mathbb{R}^{m\times n}$ and $C \in \mathbb{R}^{n\times p}$, the image of $\mathcal{A}$ is by B is $B\mathcal{A} = \{Bx \colon x \in \mathcal{A}\} \subset \mathbb{R}^m$ and the preimage of $\mathcal{A}$ by C is $C^{-1}\mathcal{A} = \{x \colon Cx \in \mathcal{A}\} \subset \mathbb{R}^p$.

II. Problem setup and basic formulation

A. System description

Consider a connected directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ defining an electric network. The set of nodes $\mathcal{V}$ can be partitioned into two disjoint sets $\mathcal{V}_I$ and $\mathcal{V}_0$ which correspond to the set of renewable generators and the utility electric grid respectively. When $\mathcal{V}_0 = \emptyset$, the electric network can be considered islanded.

1) System Model: Each node in $i \in \mathcal{V}_I$ comprises DER sources interfaced via power converters and local loads, see Figure 1. The DER sources with variable input such as solar PV and WT operate with a maximum point tracking rationale which enables them to extract the maximum possible energy for given environmental conditions. The power dynamics in discrete time for each $i \in \mathcal{V}_I$ and $h \in \mathcal{H} := \{\text{wind}, \text{PV}\}$ are:

$$S^{+}_{h,i} = f_{h,i}(S_{h,i}, \Delta_{h,i}, w_{h,i}) \tag{1}$$

where $S_{h,i} = (P_{h,i}, Q_{h,i})$ denote active and reactive power, the subscript $^+$ denotes the successor state, $w_{h,i}$ is uncontrollable input power generated by either wind or solar sources, and $\Delta_{h,i}$ is a control input which ensures seamless transmission from the energy source to the electric network. The battery dynamics are described by the following difference algebraic system:

$$C_{b,i}(\text{SoC}^{+}_i - \text{SoC}_i) = -\frac{1}{R_{b,i}}(V(\text{SoC}_i) - V_{b,i}), \tag{2a}$$

$$V_{b,i}V(\text{SoC}_i) = \frac{1}{R_{b,i}}V^2_{b,i} + g_{b,i}(S_{b,i}), \tag{2b}$$

where the state is $(\text{SoC}_i, V_{b,i})$ are the SoC and battery DC voltage; $V(\text{SoC})$ is the battery output voltage that is SoC dependent; $R_{b,i}$ is the internal resistance; and $C_{b,i}$ is the battery capacity in $[\text{A h}]$. The nonlinear function $g_{b,i}(\cdot,\cdot)$ determines the power electronic steady-state behavior in terms of desired active and reactive desired power $S_{b,i} = (P_{b,i}, Q_{b,i})$ which we consider as inputs. The power electronic components from all renewable sources exhibit faster dynamic behavior, therefore we can consider each node operating in a *quasi* stationary operation [22]. The algebraic constraint in (2) yields a condition to ensure the existence of a solution to the DAE system which relates input power and SoC.

$$V(\text{SoC}_i)^2 \geq 4R_{b,i}P_{b,i} \tag{3}$$

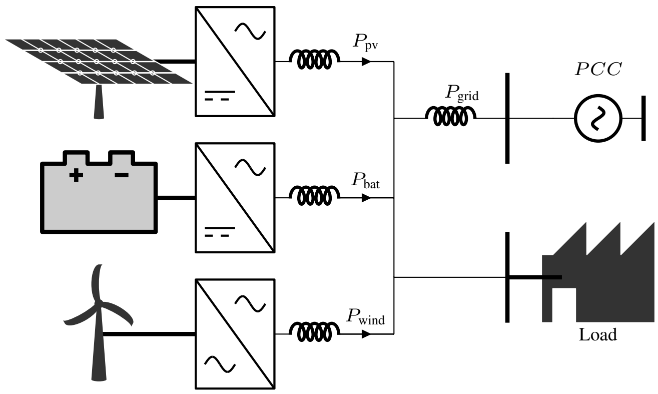

Fig. 1. Power sources comprising node i; local energy sources together with local loads are connected to PCC.

The overall state for each node can be summarised in $x_i = (\{S_{h,i}\}_{h\in\mathcal{H}}, SoC_i)$, with control inputs $u_i = (\{\Delta_{w,i}\}_h, S_{b,i})$. Each DER is subject to exogenous inputs $w_{g,i} = \{w_{h,i}\}_{h\in\mathcal{H}}$ corresponding to powers injected by renewable sources. In addition, local loads connected to node i draw an *a-priori* unknown active and reactive power $S_{l,i} = (P_{l,i}, Q_{l,i}) \in \mathbb{R}^2$; however, the system has access to preview information, *i.e.,* forecasts for loads and renewable power sources which satisfy the following Assumption:

Assumption 1 (Information available to the controller)**.** *1) The state $x_i(k)$ and exogenous input $w_i(k) = (\{w_{h,i}(k)\}_{h\in\mathcal{H}}, S_{l,i}(k))$ are known exactly at time k; future external inputs are not known exactly but satisfy $w_i(k+n) \in \mathbb{D}_i$ for $n \in \mathbb{N}$.*

2) At any time step k, a prediction, $\mathbf{d}_i = \{d_i(k)\}_{k\in\mathbb{N}_{0:N-1}}$, of N future exogenous inputs[1], over a finite horizon of time, is available.

Note that we do not assume anything about the accuracy of the predictions, and in fact will allow these to vary over time (this implicitly implies that previous predictions were not accurate). In practice, these sequences can be obtained in form of forecasts of loads and renewable sources. The states and inputs of each node are restricted to satisfy constraints $x_i \in \mathbb{X}_i$ and $u_i \in \mathbb{U}_i$ where

Assumption 2 (Constraints)**.** *For each $i \in \mathcal{V}_I$, the sets $\mathbb{X}_i$ and $\mathbb{D}_i$ are* C-sets. *The set $\mathbb{U}_i$ is a* PC-set.

The output of each node $y_i = (P_{o,i}, Q_{o,i})$ is given by its power balance equations

$$y_i = \sum_{h\in\mathcal{H}} S_{h,i} + S_{b,i} - S_{l,i}, \tag{4}$$

which following Assumption 2 is bounded for all time $k \in \mathbb{N}$.

2) Network model: The network topology is characterized by the set of edges $\mathcal{E}$ such that each $e \in \mathcal{E}$ defines the existence of a physical link between two nodes. The network topology allows us to define the set of neighbours of each $i \in \mathcal{V}$,

$$\mathcal{N}_i = \{j \in \mathcal{V} \colon (i,j) \in \mathcal{E}\}. \tag{5}$$

Similarly, we can define $\mathcal{E}_i \subseteq \mathcal{E}$ collecting all those edges emanating or terminating in $i \in \mathcal{V}$, *i.e.,*

$$\mathcal{E}_i = \{e \in \mathcal{E} \colon e = (i,j) \text{ or } e = (j,i),\ j \in \mathcal{V}\}.$$

For each $i \in \mathcal{V}$, the current delivered by node i is:

$$i_i = \frac{v_i}{Z_{ii}} + \sum_{(i,j)\in\mathcal{E}_i} \frac{v_i - v_j}{Z_{ij}}, \tag{6}$$

The admittance $Z_{ij}^{-1} = G_{ij} - jB_{ij} \in \mathbb{C}$ corresponds to the line connecting the i^{th} and j^{th} nodes. The current drawn from each node $i \in \mathcal{V}_I$ can be described in terms of $S_i = (P_i, Q_i)$, *i.e.,* active and reactive power, as[2]

$$i_i = h_i(v_i, S_i) = \frac{1}{|v_i|^2}\big(P_i v_i - Q_i \mathbb{J}_2 v_i\big) \tag{7}$$

Furthermore, the node-edge incidence matrix can be partitioned into $\mathcal{B} = [\mathcal{B}_0 \quad \mathcal{B}_I]$ corresponding to the utility grid and renewable nodes respectively. The current balance (6) for each $i \in \mathcal{V}_I$ can be rewritten as

$$h_I(v_I, S_I) - (Y_I + \mathcal{L}_I)v_I - \mathcal{B}_I^\top Y_E \mathcal{B}_0 v_0 = 0, \tag{8}$$

where $\mathcal{L}_I = \mathcal{B}_I^\top Y_E \mathcal{B}_I$ with $Y_E = \operatorname{diag}\{Z_{ij}^{-1} \colon (i,j) \in \mathcal{E}\}$ the admittance of each line and Y_I the shunt admittance of each node in $\mathcal{V}_I$. The vector $v_I = [v_i]_{i\in\mathcal{V}_I} \in \mathbb{R}^{2|\mathcal{V}_I|}$ collects all generator node voltages, similarly S_I captures the power injected by each node, and $h_I(v_I, S_I) = [h_i(v_i, S_i))]_{i\in\mathcal{V}_I} \in \mathbb{R}^{|\mathcal{V}_I|}$. The grid voltage is given by $v_0 \in \mathbb{R}^2$ and can be characterized by following relation:

$$-(Y_0 + \mathcal{L}_0)v_0 + Y_0 E_0 - \mathcal{B}_0^\top Y_E \mathcal{B}_I v_I = 0. \tag{9}$$

Similarly to the previous case, Y_0 is a local admittance, and $\mathcal{L}_0 = \mathcal{B}_0^\top Y_E \mathcal{B}_0$. The voltage E_0 is generated at the network connection point; the nature of this quantity varies according to the operation mode: stiff grid $E_0 = (220\sqrt{2}, 0)$, a weak grid when its magnitude and angle are power dependent, or $E_0 = (0,0)$ in case of an islanded system. The network states are given by node voltages $y = (v_0, v_I)$ lying in a constraint set $\mathbb{V} = \mathbb{R}^2 \times \prod_{i\in\mathcal{V}_I} \mathbb{V}_i$ where each set $\mathbb{V}_i$ with $i \in \mathcal{V}_I$ satisfies

Assumption 3. *The set $\mathbb{V}_i \subset \mathbb{R}^2$ is a* PC-set.

Similarly, if the currents flowing through the lines $i_E = Y_E \mathcal{B} y$ are also constrained to a PC-set $\mathbb{I}_E = \prod_{e\in\mathcal{E}} \mathbb{I}_e$, *i.e.,* bounds on each RMS current, these induce constraints on voltages by virtue of the algebraic relation $i_E = \mathcal{B}_0 v_0 + \mathcal{B}_I v_I$. Following (9), the closed form of these constraints are:

$$v_I \in \tilde{A}^{-1}(-\tilde{B}E_0 \oplus \mathbb{I}_E),$$

where $\tilde{A} = Y_E(\mathcal{B}_I - \mathcal{B}_0(Y_0 + \mathcal{L}_0)^{-1}\mathcal{B}_0^\top Y_E \mathcal{B}_I)$ and $\tilde{B} = Y_E \mathcal{B}_0 (Y_0 + \mathcal{L}_0)^{-1} Y_0$. The overall voltage constraint set is

$$\mathbb{V}_I = \tilde{A}^{-1}(-\tilde{B}E_0 \oplus \mathbb{I}_E) \cap \prod_{i\in\mathcal{V}_I} \mathbb{V}_i. \tag{10}$$

This set, by virtue of Assumption 2 is a PC-set.

[1] $\mathbb{N}_{a:b} = \{a, a+1, \dots, b-1, b\}$ for $a, b \in \mathbb{N}$ and $a < b$.

[2] The matrix $\mathbb{J}_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is a complex structure on $\mathbb{R}^2$. Any complex number $a + ib$ can be written as a 2×2 matrix or a vector in $\mathbb{R}^2$ as

$$a + ib \iff \begin{bmatrix} a \\ b \end{bmatrix} \iff \begin{bmatrix} a & -b \\ b & a \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

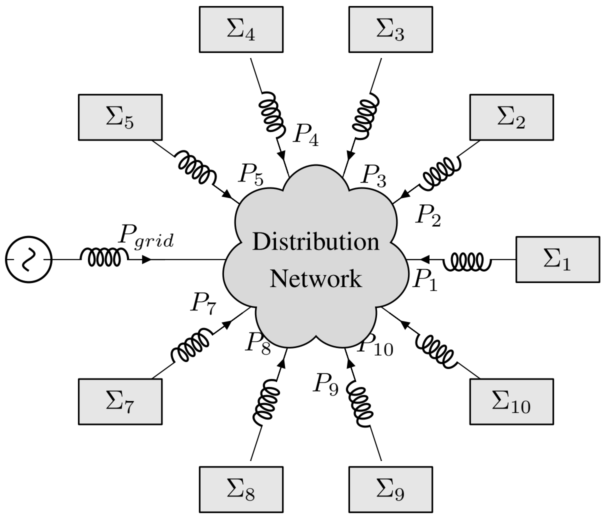

Fig. 2. Each unit is interfaced via inductive lines to a distribution network which may have a meshed topology.

3) Control Objective: The objective is twofold: find a suitable sequence of pairs (x, u) depending on external renewable injections $d(k)$ for $k \in \{0, 1, \dots\}$ that minimizes the infinite horizon criteria

$$J(x_0, v_0, d) = \sum_{k=0}^{\infty} \ell_k(x(k), u(k), d(k)) \tag{11}$$

where ℓ_k is a time-varying stage cost comprising generation costs. And second, to derive a suitable stage cost $\ell_k(\cdot, \cdot)$ that captures energy pricing mechanisms allowing for maximum profit at each node in terms of the exogenous inputs (load consumption and renewable injection) and available battery storage.

III. Peer-2-peer framework

A. OCP for energy system

To achieve our control objectives, consider the finite horizon criteria for each $i \in \mathcal{V}_I$ that employs exogenous predictions of Assumption 1:

$$J_i^N(\bar{x}_i, \mathbf{u}_i, \mathbf{d}_i) = \sum_{k=0}^{N-1} \underbrace{\gamma(u_i, x_i) + \lambda_{k,i}(y_i)}_{\ell_{k,i}(x_i,u_i,d_i)} \tag{12}$$

the stage cost $\ell_{i,k}(x_i, u_i, d_i)$ is composed by two terms capturing the cost of generating electricity from solar or aeolian energy given by $\gamma_i(\cdot)$, and a time-varying term $\lambda_{k,i}(\cdot)$ reflecting the price of purchasing or selling energy from and to the electric network. The former has usually a quadratic nature, see [23], with coefficients that are constant in case of PV panels and WTs and a function of the state of charge for the batteries. The latter function is determined from the market, see Section III-B. The performance criteria arguments are sequences of states $\mathbf{x}_i = \{x_i(0), \dots x_i(N)\}$ and controls $\mathbf{u}_i = \{u_i(0), \dots u_i(N-1)\}$. Both of these sequences depend on exogenous inputs $\mathbf{d}_i = \{d_i(0), \dots, d_i(N-1)\}$; each $d_i(k)$ comprises predicted renewable injections $\{w_{h,i}(k)\}_{h \in \mathcal{H}}$, load consumption $S_{l,i}(k)$, and neighboring voltages $\{v_j(k)\}_{j \in \mathcal{N}_i}$. The resulting optimal control problem for each node for $\bar{z}_i = (\bar{x}_i, \bar{v}_i)$ and available predictions $\mathbf{d}_i$ is

$$\mathbb{P}_i(\bar{z}_i, \mathbf{d}_i)\colon \min\{J_i^N(\bar{x}_i, \mathbf{u}_i, \mathbf{d}_i)\colon \mathbf{u}_i \in \mathcal{U}_i^N(\bar{z}_i, \mathbf{d}_i)\}. \tag{13}$$

The constraint set $\mathcal{U}_i^N(\bar{z}_i, \mathbf{d}_i)$ is defined by

$$(x_i(0), v_i(0)) = \bar{z}_i, \tag{14a}$$

$$x_{h,i}^+ = f_i(x_i, u_i, d_i), \tag{14b}$$

$$x_i \in \mathbb{X}_i, \quad u_i \in \mathbb{U}_i, \quad v_i \in \mathbb{V}_I(d_i), \tag{14c}$$

$$h_i(v_i, y_i) = \mathcal{L}_i v_i + \hat{\mathcal{L}}_i d_i. \tag{14d}$$

The prediction model (14b) differs from its counterpart (1)–(3) in the nature of the exogenous inputs used; the former employs sequences of forecasts while the later uses the "true" values. This optimization problem is subject to coupled constraints (14c) and (14d) with respect to network voltages. The set $\mathbb{V}_I(d_i)$ represents a "slice" of $\mathbb{V}_I$ corresponding to node i for given fixed values of neighboring voltages; The matrix $\hat{\mathcal{L}}_i$ maps d_i to the current balance for node i, *i.e.,* forming the i^{th} row of (8) representing the power flow for node i. The solution of $\mathbb{P}_i(\bar{z}_i, \mathbf{d}_i)$ is a sequence of optimal control inputs $\mathbf{u}_i^0$. Suppose at time k, each node i measures (x_i, v_i), exchanges voltage prediction sequences, obtains forecasts for renewable injections and load demands such that $\mathbf{d}_i(k)$ is available; then apply the first element of the optimal solution $\mathbf{u}_i^0$ of (13) to the system. At the next sampling time, we discard the existing sequence, measure the plant, and obtain new forecasts, then solve (13) with the updated information. This process is repeated *ad infinitum*. A standard Assumption on the stage cost for regularity purposes is

Assumption 4 (Positive definite stage cost)**.** *$\lambda_{i,k}\colon \mathbb{X}_i \times \mathbb{R}^2 \to \mathbb{R}$ and $\gamma_i\colon \mathbb{U}_i \times \mathbb{X}_i \to \mathbb{R}$ are, for each $i \in \mathcal{M}$ and $k \in \mathbb{N}$, continuous positive definite functions.*

B. Market negotiation

For this section, we propose an agent based approach to handle the market layer involving negotiations to choose adequate pricing policies. The market layer of our approach aims to handle power deficits and surpluses at each node $i \in \mathcal{V}_I$; a power surplus in one node may help alleviate deficits in other nodes. The time-varying component of the cost (12), $\lambda_{i,k}(\cdot)$, weighs this output power and is the tool used to interface both an energy trading scheme with the lower control levels. According to the sign of the output power, it is possible to partition $\mathcal{V}_I$ into two disjoint sets of sellers $\mathcal{V}_{I,S}$ when $y_i > 0$, and of buyers $\mathcal{V}_{I,B}$ for $y_i < 0$. In our approach, we propose the choice of a pricing policy for exporting and importing power as opposed to the traditional linear pricing used in the literature [24], [25]. This results in a negotiation framework where the participating nodes decide upon a policy which provides a mechanism to take predicted battery storage levels into account.

This negotiation framework can be cast a game, with a leader when the network is connected, or leaderless when operating in islanded mode. The set of players is given by the nodes $\mathcal{V}_I \cup \mathcal{V}_0$; the set of actions for each $i \in \mathcal{V}_I$ is the set of functions $\mathcal{A}_i = \{\lambda_i \in \mathcal{L}_2[\mathbb{Y}_i, \mathbb{R}]\colon \lambda_i(y_i) \geq 0\}$ with $\mathbb{Y}_i$ the output constraint set which by Assumption 1 and 2 is compact. The action set for $\mathcal{V}_0$ is given by $\mathcal{A}_0 = \{(\lambda_{0,s}, \lambda_{0,b}) \in \mathbb{R}^2 \colon \eta_s \leq \lambda_{0,s} < b_{0,b} \leq \eta_b\}$ which represents selling and buying prices for each network node that are upper and lower bounded by $\eta_s > 0$ and $\eta_b > 0$ respectively. The total average network revenue is[3]

$$R = \sum_{i\in\mathcal{V}_I} \frac{1}{\mu(\mathbb{Y}_i)} \int_{Y_i} \lambda_i(\sigma) d\mu \tag{15}$$

where each pricing policy is averaged over a set $Y_i = (y_i + S_b^{\max}[-\text{SoC}, 1-\text{SoC}]) \cap \mathbb{Y}_i$ reflecting the available power with respect to current storage levels SoC and μ is the Lebesgue measure for $\mathbb{Y}_i$. On the other hand, the power purchased by each $i \in \mathcal{V}_I$ acting as a buyer at a given time is

$$\xi_i(\lambda_i, \lambda_{-i}, y) = \frac{\lambda_i(-\sigma(-y_i))}{\sum_{j\in\mathcal{N}_i^{\text{com}}\cup\{i\}} \lambda_j(-\sigma(-y_j))} \sum_{j\in\mathcal{N}_i^{\text{com}}\cup\{i\}} \sigma(y_j) \tag{16}$$

which depends on the neighbouring available power $y = (y_1, \ldots, y_{|\mathcal{V}_I|})$; it is worth stressing that the only information needed is that of the communication neighbours. Each node behaves as a prosumer and it has attached to it a utility functional $r_i \colon \mathcal{A}_i \times \mathcal{A}_{-i} \to \mathbb{R}$, which also depends on current available predictions on both storage and output power, defined as

$$\begin{aligned} r_i(\lambda_i, \lambda_{-i}) = \sum_{k=0}^{N-1} \Bigg(& \lambda_{0,b}\xi_i(\lambda_i, \lambda_{-i}, y(k)) \\ & - \lambda_i(\xi_i(\lambda_i, \lambda_{-i}, y(k))) \\ & + \log(1 + \gamma_i(\lambda_{0,s}, b_i^{\max})b_i) \\ & + \gamma_i R(k) \frac{\sigma(y_i(k))}{1 + \sum_{i\in\mathcal{V}_i} \sigma(y_i(k))} \Bigg). \end{aligned} \tag{17}$$

This utility functional measures revenue and satisfaction of each node with its current pricing policy, see [21] and [24]. There are two prominent parts: the first two terms represent the cost of purchasing with respect to the price set by the utility grid $\lambda_{0,b} > 0$ which the agent seeks to minimise; the remaining terms correspond to the advantages of selling available power,the effect of available storage, and the trade-off between increasing prices and loss of revenue. This trade-off is characterised by the gain γ_i which depends on selling prices $\lambda_{0,s}$ and the maximum allowable price $b_i^{\max}$. The function $\sigma(\cdot)$ is a key component of this cost, depending on the output power sign, two terms will vanish implying that each node is either maximising profit or minimising costs but never both; this synergizes with our piece-wise definition of pricing policy. This formulation avoids unwanted saddle or conservative behaviour when optimising. This approach naturally partitions the set $\mathcal{V}_I$ into buyers and sellers.

The set $\mathcal{A}_{-i} \triangleq \prod_{j\in\mathcal{N}_i^{\text{com}}} \mathcal{A}_j$ collects the actions of the neighbours in the communication network which are used to compute the total revenue $R(k)$ known by the utility grid agent, characterises the negotiation framework communication properties, and is defined as an unweighted graph characterised by a set of edges $\mathcal{E}^{\text{com}} \subset \mathcal{V} \times \mathcal{V}$ which generates a set of neighbours $\mathcal{N}_i^{\text{com}} \subset \mathcal{V}$ similar to (5). The particularity of this network is that for all $i \in \mathcal{V}_I$, the utility grid, if present, satisfies $0 \in \mathcal{N}_i$ implying each negotiating agent can communicate with the utility grid. In the following we describe the proposed framework for the grid connected and islanded cases: In the former case $V_0 \neq \emptyset$, and the utility grid solves the following optimisation problem:

$$\mathbb{P}_0^{\text{com}} \colon \max_{\lambda_0\in\mathcal{A}_0} \{r_0(\lambda_0, \lambda_{-0}) \colon \lambda_{-0} \in \prod_{j\in\mathcal{N}_0} \mathcal{R}_j(\lambda_{-j})\} \tag{18}$$

where the best reply map associated with λ_{-j} is $\mathcal{R}_j(\lambda_{-j}) = \{\lambda_j \in \mathcal{A}_j \colon r_j(\lambda_j, \lambda_{-j}) \leq r_j(\tilde{\lambda_j}, \lambda_{-j}),\ \forall \tilde{\lambda}_j \in \mathcal{A}_j\}$. The associated utility functional is

$$\begin{aligned} r_0(\lambda_0, \lambda_{-0}) = & \lambda_{0,s} \sum_{i\in\mathcal{V}_I} \sigma(y_i) + \lambda_{0,b} \sum_{i\in\mathcal{V}_I} \sigma(-y_i) \\ & - \sum_{i\in\mathcal{V}_I} \lambda_i(\sigma(y_i)) \end{aligned}$$

In this way, the utility grid agent sets the price according to the best response of the network members given by $\mathcal{V}_I$. Similarly, for each $i \in \mathcal{V}_I$ the corresponding optimisation problem is

$$\mathbb{P}_i^{\text{com}}(\lambda_{-j}) \colon \max_{\lambda_i\in\mathcal{A}_i} r_i(\lambda_i, \lambda_{-i}) \tag{19}$$

For the islanded case, $\mathcal{V}_0 = \emptyset$ and the negotiation framework occurs only by solving (19) for each $i \in \mathcal{V}_I$. An important feature of the presented approach and its relation to peer-to-peer networks lies in the way revenue of energy transactions is obtained. After the agents are split into groups of buyers and sellers, the total revenue gained from a transaction is shared among the agents selling power. A method to establish direct contracts or bilateral negotiation schemes is the subject of ongoing research.

Tractable reformulation: The negotiation framework as stated in the previous section may be prohibitively difficult to solve. The action spaces for each $i \in \mathcal{V}_I$ are infinite dimensional spaces, and to exacerbate the problem, the optimisation problem (18) has equilibrium constraints. We propose a method to simplify this problem to make it computationally tractable for both utility and network elements. Our first step towards this goal is to invoke the following:

Assumption 5. *Each node updates its pricing policy with a period $k_n \in \mathbb{N}$; the utility grid updates prices every $Hk_n \in \mathbb{N}$ for some $H \gg 0$.*

This Assumption may refer to the common practice of setting a day ahead price based on existing consumption and is common in leader follower games as mentioned in [26]. Furthermore, Assumption 5 ensures that the utility grid agent to be able to react only to the best replies from each network element. Another potential bottleneck, from an implementation point of view, is that of the action space infinite dimensionality $\mathcal{A}_i \subseteq \mathcal{L}_2[\mathbb{Y}_i, \mathbb{R}]$. To overcome this hurdle, we propose to reduce the action space to a class of parameterised functions. The starting point is the traditional linear pricing policies, *i.e.,* $-\lambda_{0,b}y_i$ and $\lambda_{0,s}y_i$ for purchase and sale respectively. From Assumption 2, the output power is constrained to a bounded set. We seek a piece-wise smooth such that $\lambda_i(-y_i^{\max}) = \lambda_{0,b}y_i^{\max}$,

[3]The function $\sigma(x) = \frac{xe^{\alpha x}}{e^{\alpha x}+1}$ for a fixed $\alpha > 0$ is a smooth approximation of $\max(0, x)$. As $\alpha \to \infty$, $\sigma(\cdot) \to \max(0, \cdot)$.

$\lambda_i(0) = 0$, and $\lambda_i(y_i^{\max}) = \lambda_{0,b} y_i^{\max}$. Clearly, it is always possible to find a quadratic function fitting the positive part, and another fitting the negative one. The parameter we introduce is the deviation from a linear pricing: a way to measure this is to consider the area in between curves such that

$$\begin{aligned} b_i &= \int_0^{y_i^{\max}} \lambda_{0,s} y_i dy - \int_0^{y_i^{\max}} \lambda_i dy \\ &= \int_{-y_i^{\max}}^{0} -\lambda_{0,b} y_i dy - \int_{-y_i^{\max}}^{0} \lambda_i dy. \end{aligned}$$

Solving the above conditions yield the desired parameterised piece-wise smooth convex parameterisation. The missing ingredient is to satisfy Assumption 4, This can be done by suitably constraining the available values for $b_i \in [0, b_i^{\max}]$ such that $\lambda_i(y_i, b_i^{\max}) \geq 0$ for all $y_i \in \mathbb{Y}_i$. This allows us to assign a correspondence between real positive numbers and $\mathcal{A}_i$ such that $b_i \mapsto \lambda_i(\cdot)$. Injectivity of this map follows naturally from construction; surjectivity, however, is not ensured since the image of the real numbers is not $\mathcal{A}_i$ but only a strict subset. This discrepancy is because of the inequality condition used to parameterise desired positive definite functions. In this way and owing to the continuity of $b \mapsto \lambda_i(\cdot)$, the problem of optimising over function spaces is reduced to optimising over $\mathbb{R}^{|\mathcal{V}_I|}$.

Following Assumption 5, the game can be played in two stages. The initialisation part corresponds to the utility grid agent choosing $\lambda_0 = (\eta_s, \eta_b)$. Then sequentially, following a best reply type updating, each node agent updates its desired pricing policy until they reach an equilibrium. This negotiation process occurs every k_n steps and takes into account the availability of both power forecasts for renewable sources and storage predictions at the given sampling time; the utility grid updates their prices $H * k_n$ steps for $H \geq 0$. The utility grid updates its decision variables in response to optimal pricing profiles obtained by network agents. The negotiation between network and utility grid agents occurs on top of the control layer described in Section III-A.

IV. Stability and Convergence Analysis

In this section, we analyse the theoretical properties of the proposed pricing approach.We divide our analysis into two main parts: market negotiation convergence, and recursive feasibility. We finish this Section with remarks on how both market and control layers interact.

A. Negotiation Convergence

In this Section, we aim to make precise the notion of how all $i \in \mathcal{V}_I$ play the game defining their pricing policy. We pay close attention to the convergence properties of this game in the sense of Stackelberg, in the grid connected case, and Nash, in the islanded one, equilibrium. We begin this part of the analysis by defining equilibrium concepts that will be used:

Definition 1 (Nash equilibrium)**.** *An action profile* $\lambda^0 = (\lambda_1^0, \dots, \lambda_{|\mathcal{V}_I|}^0)$ *is said to be a* Nash equilibrium *of the game* $(\mathcal{V}_I, \{\mathcal{A}_i\}_{i\in\mathcal{V}_I}, \{r_i\}_{i\in\mathcal{V}_I})$ *if, for all* $i \in \mathcal{V}_I$,

$$r_i(\lambda_i^0, \lambda_{-i}^0) = \min_{\lambda_i \in \mathcal{A}_i} r_i(\lambda_i, \lambda_{-i}^0). \tag{20}$$

The set of Nash equilibrium points for $\mathcal{V}_I$ parameterised by $\lambda_0 \in \mathcal{A}_0$ are $\mathbb{NE}(\lambda_0) \subset \prod_{i\in\mathcal{V}_I} \mathcal{A}_i$, this set represents the best network response to the prices set by the utility grid. This concept leads us to the other equilibrium concept we leverage on:

Definition 2 (Stackelberg equilibrium)**.** *An action profile* $a^* = (a_0^*, \dots, a_M^*)$ *is said to be an* Stackelberg equilibrium *of the 1 leader,* $M-$*follower game* $(\mathcal{V}, \{\mathcal{A}_i\}_{i\in\mathcal{V}}, \{r_i\}_{i\in\mathcal{V}})$ *if for all* $i \in \mathcal{V}$

$$\sup_{\lambda_{-0}\in\mathbb{NE}(\lambda_0^0)} r_i(\lambda_0^0, \lambda_{-0}) \leq \sup_{\lambda_{-0}\in\mathbb{NE}(\lambda_0)} r_i(\lambda_0, \lambda_{-0}) \tag{21}$$

The Stackelberg equilibrium complements that of the Nash equilibrium and essentially leads to an optimal response from the utility grid side in response to the best possible actions from the network side. Following Assumption 5, it is possible to solve these two problems independently with the caveat that it leads to a problem with equilibrium constraints, see [27] for an in-depth study of this type of problems.

Given the utility functionals r_i naturally partition the set of nodes, it is possible without loss of generality to choose the buyer nodes to play first. Owing to the parameterisation of the pricing policy, $b_i \mapsto \lambda_i(\cdot)$, is convex by construction.

Our first result considers the case when $y_i < 0$, *i.e.,* node i is purchasing power.

Lemma 1. *Suppose* $y_i < 0$*, then the purchased power in* (16) *is a concave function with respect* $b_i \in [0, b_i^{\max}]$.

Proof. By construction, $\forall i \in \mathcal{V}_I$ $\lambda_i(y_i, b_i) \geq 0$ and since the parameterisation is a linear problem, the policy is linear with respect to the parameter. Indeed, the conditions defined in Section IV-A for the negative part of the policy $\lambda_i(y_i) = a_2 y_i^2 + a_1 y_i$ are

$$\begin{aligned} a_2(y_i^{\max})^2 - a_1 y_i^{\max} &= \lambda_{0,b} y_i^{\max} \\ \frac{1}{2}\lambda_{0,b}(y_i^{\max})^2 + \frac{1}{3}a_2(y_i^{\max})^3 + \frac{1}{2}a_1(y_i^{\max})^2 &= b_i \end{aligned}$$

This yields a policy

$$\lambda_i(y_i, b_i) = \frac{6b_i}{(y_i^{\max})^3} y_i^2 + \frac{6b_i - \lambda_{0,b}(y_i^{\max})^2}{(y_i^{\max})^2} y_i.$$

This implies that the policy is concave on b_i. To prove the purchased power concavity, it is enough to check its derivatives. A direct calculation shows that

$$\frac{\partial^2 \xi_i}{\partial b_i^2} = -2\frac{\sum_l \sigma(y_l) \sum_{l\neq i} \lambda_l(y_l, b_l) \left(\frac{\partial \lambda_i}{\partial b_i}\right)^2}{\left(\sum_{j\in\mathcal{N}_i^{\text{com}}\cup\{i\}} \lambda_j(y_j)\right)^3} < 0$$

is negative definite. □

Lemma 2. *Suppose* $b_i \in [0, b_i^{\max}]$ *for all* $i \in \mathcal{V}_I$*, then* $r_i(\lambda_i, \lambda_{-i})$ *is a concave function with respect to* b_i *and* b_{-i}.

Proof. The proof proceeds by cases: $i)$ $y_i \geq 0$. In this case, the first two terms of $r_i(\cdot, \cdot)$ vanish. The total revenue is by construction a linear function of b_i and b_{-i} and the logarithmic term is concave with respect to its domain. The resulting function which is a sum of concave functions is therefore concave.

ii) $y_i < 0$. In this case only the first two terms of $r_i(\cdot,\cdot)$ contribute, the later vanish. Following Lemma 1, the purchased energy is a concave function of b_i. The argument follows, *mutatis mutandis*, that of Lemma 1 to obtain the negativity of the second derivative within the range $[0, b_i^{\max}]$. □

These two Lemmas lead to our first result, the proof of which follows from an application of [28, Theorem 4.3]

Theorem 3 (Network nash equilibrium). *The game* $(\mathcal{V}_I, \{\mathcal{A}_i\}_{i\in\mathcal{V}_I}, \{r_i\}_{i\in\mathcal{V}_I})$ *admits a* Nash equilibrium.

Theorem 3 implies that all network nodes operate optimally with respect to the values given by the utility grid. The move played by the grid agent satisfies $\lambda_0^0 = \arg\max r_0(\lambda_0, \lambda_{-0})$ which is by construction a linear problem over a compact set. A consequence of this fact is the existence of a Stackelberg equilibrium between network and utility grid.

B. Recursive feasibility

From Assumption 1 and 3, the exogenous information available to the controller is contained within a PC-set $d_i(k) \in \mathbb{D}_i$ for all $k \geq 0$. The set $\mathcal{D}_i^N = \prod_{k=0}^{N-1} \mathbb{D}_i$ contains all sequences of length N. The challenge arises as a consequence of the receding horizon implementation of the control action: if $\mathbb{P}_i(z_i, \mathbf{d}_i)$ is feasible and yields a solution sequence $\mathbf{u}_i^0(z_i, \mathbf{d}_i)$, then its first element, which defines an implicit control law $\kappa_i^N(z_i, \mathbf{d}_i)) = u_i^0(0; z_i, \mathbf{d}_i)$, is applied to the system resulting in the state evolution $z^+ = (x_i^+, v_i^+)$ obtained as a solution of the difference algebraic model (1), (2), and (6). At this sampling instant a new sequence of information is available to the controller, *i.e.,* $\mathbf{d}_i^+ \in \mathcal{D}_i^N$, and the problem to be solved is now $\mathbb{P}_i(z_i^+, \mathbf{d}_i^+)$. In this Section, we seek an answer to the question: What are the conditions necessary to ensure $\mathbb{P}_i(z_i^+, \mathbf{d}_i^+)$ has a solution when $\mathbf{d}_i$ changes (perhaps arbitrarily) to $\mathbf{d}_i^+$?

We begin by defining *recursive feasibility*, and some useful terminology in the analysis:

Definition 3 (Recursive feasibility). *For each $i \in \mathcal{V}$, the OCP $\mathbb{P}_i(z_i, \mathbf{d}_i)$ is said to be* recursively feasible *if $\mathcal{U}_i^N(z_i, \mathbf{d}_i) \neq \emptyset$, then for a successor state $z_i^+ = (x_i^+, v_i^+)$, and $\mathbf{d}_i^+ \in \mathcal{D}_i^N$, the constraint set $\mathcal{U}_i^N(z_i^+, \mathbf{d}_i^+) \neq \emptyset$.*

For a disturbance sequence at time k, $\mathbf{d}_i = \{d_i(0), \ldots, d_i(N-1)\}$, its associated $\tilde{k}^{\text{th}}$ tail for time $k+1$ is $\tilde{d}_{\tilde{k}}(\mathbf{d}_i) = \{d_i(1) \ldots, d_i(\tilde{k}-1), d_i(\tilde{k}-1), d_i(\tilde{k}), \ldots, d_i(N-1)\}$. It is clear that both $\mathbf{d}_i$ and $\tilde{d}(\mathbf{d}_i)$ belong to the set $\mathbb{D}_i$; the notion of the tail allows us to quantify the change in information that a controller is subject to, for two sequences $\mathbf{d}, \mathbf{e} \in \mathcal{D}_i^N$, the distance $\rho(\mathbf{d}, \mathbf{e}) = |\tilde{d}(\mathbf{d}) - \mathbf{e}|$ is a metric on the sequence space $\mathcal{D}_i^N$.

A system is said to be locally controllable at a point $z_0 \in Z_0$ if for every $\varepsilon > 0$, $H \in \mathbb{N}$ and $\bar{z}$ such that $|z - z_0| \leq \varepsilon$, there exists a finite sequence of controls $\{u(0), \ldots, u(H-1)\}$ such that its solution satisfies $|z(k) - z_0| < \varepsilon$ for all $j \in \mathbb{N}_{0:H-1}$ with $z(0) = z_0$ and $z(H) = \bar{z}$. The set of feasible states is $\mathcal{Z}_i^N(\mathbf{d}_i) = \{z_i \colon \mathcal{U}_i^N(z_i, \mathbf{d}_i) \neq \emptyset\}$ defines the region in the state space such that the OCP is feasible. The first of our results is concerned when the exogenous information is unchanging, *i.e.,* the future information is taken to be the tail of the initial sequence.

Theorem 4. *Let $\mathbf{d}_i \in \mathcal{D}_i^N$ and suppose each node is locally controllable with respect to $\mathbf{d}_i$. If $\mathbf{d}_i^+ = \tilde{d}_{\tilde{k}}(\mathbf{d}_i)$, then $(x_i, v_i) \in \mathcal{Z}_i^N(\mathbf{d}_i)$ implies $(x_i^+, v_i^+) \in \mathcal{Z}_i^N(\mathbf{d}_i^+)$.*

Proof. Given $(x_i, v_i) \in \mathcal{Z}_i^N(\mathbf{d}_i)$, then an optimal sequence of control actions $\mathbf{u}_i^0(z_i, \mathbf{d}_i) = \{u_i^0(0; z_i, \mathbf{d}_i), u_i^0(1; z_i, \mathbf{d}_i), \ldots, u_i^0(N-1; z_i, \mathbf{d}_i)\}$ exists and generates a sequence of states and voltages $\mathbf{x}_i^0 = \{x_i^0(0), \ldots, x_i^0(N)\}$ and $\mathbf{v}_i^0 = \{v_i^0(0), \ldots, v_i^0(N-1)\}$ for a given information $\mathbf{d}_i \in \mathcal{D}_i^N$. Now, we construct a sequence of control actions $\tilde{\mathbf{u}}_i = \{\tilde{u}(0), \ldots, \tilde{u}(N-1)\} \in \mathcal{U}_i^N(z_i^+, \mathbf{d}_i^+)$. Using the definition of a disturbance tail, the first $\tilde{k}$ elements of $\mathbf{d}_i^+$ satisfy $\mathbf{d}_i^+(k) = \mathbf{d}_i(k+1)$ implying that $\tilde{\mathbf{u}}_i(k) = u_i^0(k+1; z_i, \mathbf{d}_i)$ for all $k \in \{0, \ldots, \tilde{k}-1\}$. The state evolution is governed by a set of difference algebraic equations $F_i(z_i, u_i, d_i, z_i^+) = 0$ composed of $\mathcal{C}^1$ dynamics (1)– (3) and (8). Using the implicit function theorem, it is possible to locally define a function $\xi_V(\cdot,\cdot,\cdot)$ such that $z_i^+ = \xi_V(z_i, u_i, d_i)$ and ensure the existence of neighbourhoods V and Z such that for $(z_i, u_i, d_i) \in V$, $(z_i, u_i, d_i, \xi_V(z_i, u_i, d_i)) \in Z$ and $F_i(z_i, u_i, d_i, \xi_V(z_i, u_i, d_i)) = 0$. Consider the initial state to be $\tilde{z}_i(0) = (x_i^0(1), v_i^0(1))$, the subsequent states, following ξ_V, satisfy $\tilde{z}_i(k) = (x_i^0(k+1), v_i^0(k+1))$ for all $k \in \{0, \ldots, \tilde{k}\}$. The next element satisfies $\tilde{z}_i(\tilde{k}+1) = \xi_V(\tilde{z}_i(\tilde{k}), \tilde{u}_i, d_i(\tilde{k}))$ for some $\tilde{u}_i \in \mathbb{U}_i$. Since each node is locally controllable with respect to $\mathbf{d}_i$, there exists a control action $\tilde{u}_i \in \mathbb{U}_i$ such that $\xi_V(z_i, \tilde{u}_i, d_i) \in \mathbb{X}_i \times \mathbb{V}_I(d_i)$. Similarly, there exists a control law $\hat{u} \in \mathbb{U}_i$ such that $\tilde{z}_i(\tilde{k}+2) = z_i^0(\tilde{k}+2) = \xi_V(z_i(\tilde{k}+1), \hat{u}, d_i(\tilde{k}+1))$. The resulting sequence satisfies

$$\begin{aligned}\tilde{\mathbf{u}}_i =& \{u_i^0(1), \ldots, u_i^0(\tilde{k}), \tilde{u}_i, \\ & \hat{u}_i, u_i^0(\tilde{k}+2), \ldots, u_i^0(N-1)\} \in \mathcal{U}(z_i^+, \mathbf{d}_i^+).\end{aligned}$$

□

Once recursive feasibility of the tail is achieved, we allow the information to vary (perhaps arbitrarily). To study this case, we turn to the continuity properties of both value function and constraints which depend on exogenous inputs and initial states. The set

$$\Gamma_i^N = \{(z_i, \mathbf{d}_i) \colon z_i \in \mathcal{Z}_i^N(\mathbf{d}_i),\ \mathbf{d}_i \in \mathcal{D}_i^N\}$$

is the graph gr $\mathcal{Z}_i^N$ of the set-valued map corresponding to the feasible set $\mathcal{Z}_i^N \colon \mathcal{D}_i^N \to 2^{\mathbb{X}_i}$.

Definition 4 (Upper semicontinuity for set-valued maps). *A set $\Phi \colon U \to 2^X$ is upper semicontinous at $\xi_0 \in U$ if for an open neighbourhood $V_U \subset U$ of ξ_0, then for all $\xi \in V_U$ $\Phi(\xi) \subset V_X$ for an open neighbourhood $V_X \subset X$.*

Proposition 1. *Suppose Assumption 1– 4 hold and the dynamics are continuous. Then the set valued map $\mathcal{Z}_I^N(\cdot)$ is upper semicontinuous.*

Proof. The constraints given by (14), by Assumptions 2 and 3 together with the continuity of each node's dynamics, have a structure $\mathcal{U}_i^N(\bar{z}_i, \mathbf{d}_i) = \{\mathbf{u}_i : G_i(\mathbf{u}_i, \bar{z}_i, \mathbf{d}_i) \in \mathbb{K}\}$ for a fixed compact set $\mathbb{K}$ and a continuous function $G(\cdot,\cdot,\cdot)$. This implies that the graph of $\mathcal{U}_i^N(\cdot,\cdot)$ is a compact set; since the underlying space is finite dimensional, there is a compact neighbourhood V_Φ such that gr $\mathcal{U}_i^N \subset V_\Phi$. If the set Γ_i^N is closed and $\mathbb{X}_i \times \mathbb{V}_I(\mathbf{d}_i)$ is compact then by [29, Lemma 4.3] our result follows. Consider a converging sequence $\{\bar{z}_{i,k}, \mathbf{d}_{i,k}\} \subset \Gamma_i^N$; to prove our result, we need to ensure that the limit point $(\bar{z}_i, \mathbf{d}_i)$ belongs to Γ_i^N. Using the definition of Γ_i^N, there exists $\mathbf{u}_{i,k}$ such that $G_i(\mathbf{u}_{i,k}, \bar{z}_{i,k}, \mathbf{d}_{i,k}) \in \mathbb{K}$ which by continuity of $G_i(\cdot,\cdot,\cdot)$ yields $\mathbf{u}_{i,k} \to \bar{\mathbf{u}}_i$ and $G_i(\bar{z}_i, \bar{\mathbf{u}}, \mathbf{d}_i) \in \mathcal{K}$. The closedness of Γ_i^N follows. □

The feasible map $\mathcal{Z}_i^N(\cdot)$ is the domain of the constraint map $\mathcal{U}_i^N(\cdot)$. The proof of proposition 1 shows the close relation between the set defining constraints and the feasible region; moreover, this results implicitly states that for small deviation in the exogenous inputs $\mathbf{d}_i$, the resulting optimisation problems associated to the feasible sets have a solution.

The following result builds on the previous ones:

Proposition 2 (Value function continuity)**.** *Suppose Assumption 1– 4 hold continuous dynamics, and the set of optimisers of $\mathbb{P}_i^N(\bar{z}_i, \mathbf{d}_i)$ is compact. Then the value function*

$$\nu_i^{N,0}(\bar{z}, \mathbf{d}_i) = \min\{J_i^N(\bar{x}_i, \mathbf{u}_i, \mathbf{d}_i) : \mathbf{u}_i^N \in \mathcal{U}_i^N(\bar{z}_i, \mathbf{d}_i)\} \quad (22)$$

is continuous.

Proof. The proof is an adaptation of [29, Proposition 4.4] to our setting. The continuity of the value function depends on the upper semicontiuity of the constraint set, which holds by Proposition 1, and the existence of neighbourhoods of the set of optimisers of (13), which is guaranteed since these form a compact set by assumption. The compactness of implies that there exists a finite open covering of $\{(\bar{z}_i, \mathbf{d}_i)\} \times \mathcal{S}(\bar{z}_i, \mathbf{d}_i) \subset V_Z \times V_U$, where $\mathcal{S}(\bar{z}_i, \mathbf{d}_i) = \arg\min\{J_i^N(\bar{x}_i, \mathbf{u}_i, \mathbf{d}_i) : \mathbf{u}_i^N \in \mathcal{U}_i^N(\bar{z}_i, \mathbf{d}_i)\}$. In this neighbourhood, the "almost" optimal points that satisfy $J_i(\tilde{x}_i, \mathbf{u}_i, \tilde{\mathbf{d}}_i) \le \nu_i^{N,0}(\bar{z}_i, \mathbf{d}_i) + \varepsilon$ for all $(\tilde{z}, \tilde{\mathbf{d}}_i, \mathbf{u}_i) \in V_Z \times V_U$. Since the neighbourhood V_U contains an optimal point which belongs to a closed set, the intersection $V_U \cap \mathcal{U}_i^N(\tilde{z}_i, \tilde{\mathbf{d}}_i) \neq \emptyset$ which yields: $\nu_i^{N,0}(\tilde{z}_i, \tilde{\mathbf{d}}_i) \le \nu_i^{N,0}(\bar{z}_i, \mathbf{d}_i) + \varepsilon$.

On the other hand, $\nu_i^{N,0}(\bar{z}_i, \mathbf{d}_i) - \varepsilon \le J_i(\bar{x}_i, \mathbf{u}_i, \mathbf{d}_i)$ holds for all $\mathbf{u}_i \in \mathcal{U}_i^N(\bar{z}_i, \mathbf{d}_i)$. From the proof of Proposition 1, the set $\mathcal{U}_i^N(\cdot,\cdot)$ is upper semicontinuous and there exists neighbourhoods $V_{U'}$ and $V_{Z'}$ such that for all $(\tilde{z}_i, \tilde{\mathbf{d}}_i) \in V_{Z'}$ and $\mathbf{u}_i \in V_{U'} \cap \mathcal{U}_i^N(\tilde{z}_i, \tilde{\mathbf{d}}_i)$, $\nu_i^{N,0}(\bar{z}_i, \mathbf{d}_i) - \varepsilon \le J_i(\tilde{x}_i, \tilde{\mathbf{u}}_i, \tilde{\mathbf{d}}_i)$ holds. Since $\varepsilon > 0$ is arbitrary, then $\nu_i^{N,0}(\bar{z}_i, \mathbf{d}_i) - \varepsilon \le \nu_i^{N,0}(\tilde{z}_i, \tilde{\mathbf{d}}_i)$. Continuity of $\nu_i^0(\cdot)$ follows. □

The continuity of the value function is a crucial property for our objective. Consider a subset $\Omega_{i,\beta} = \{(z_i, \mathbf{d}_i) \in \Gamma_i^N : \nu_i^{N,0}((z_i, \mathbf{d}_i)) \le \beta\}$ for $\beta > 0$.

Assumption 6. *The exogenous input sequence evolves as $\mathbf{d}_i^+ = \tilde{d}_{\tilde{k}}(\mathbf{d}_i) + \Delta\mathbf{d}_i$ where $\Delta\mathbf{d}_i = \mathbf{d}^+ - \tilde{d}_{\tilde{k}}(\mathbf{d}_i) \in \Delta\mathcal{D}_i^N$. The set $\Delta\mathcal{D}_i^N$ is chosen such that[4] $\lambda_i = \mathrm{diam}_{\mathcal{D}_i^N} \Delta\mathcal{D}_i^N$ satisfies*

$$\lambda_i \le \sigma_{\nu,i}^{-1}((id - \gamma_i)(\alpha_i))$$

where γ_i is a $\mathcal{K}-$function, and $\alpha_i > 0$ such that $\Omega_{i,\alpha} \subset \Gamma_i^N$.

The overall dynamics for both states and disturbance satisfy

$$F_i(z_i, \kappa_i^N(z_i, \mathbf{d}_i), d_i, z_i^+) = 0 \quad (23a)$$

$$\mathbf{d}_i^+ \in \tilde{d}_{\tilde{k}}(\mathbf{d}_i) + \Delta\mathcal{D}_i^N \quad (23b)$$

We claim that a subset $\Omega_\rho \subset \Omega_\beta$ is a positive invariant set. The implications of this assertion is that the evolution of (23) is contained within one of this level sets. This is a nonlinear generalisation to the one presented in [20].

Theorem 5. *Suppose Assumptions 1– 4, 6 hold and for all $i \in \mathcal{V}_I$ $(z_i(0)\mathbf{d}_i(0)) \in \Omega_{i,\beta} \subset \Gamma_i^N$ for some $\beta \ge \alpha$. The set $\Omega_{i,\beta}$ is positively invariant for the composite system* (23)*.*

Proof. Consider $(z_i, \mathbf{d}_i) \in \Omega_{i,\beta}$, from Theorem 4, the optimisation problem remains feasible when the available exogenous sequence assumes the $\tilde{k}^{\text{th}}-$tail. Following classical results from the MPC literature, feasibility of the optimisation problem implies stability. A consequence of this is that the value function is a Lyapunov function, *i.e.,* $\nu_i^{N,0}(z_i^+, \tilde{d}_{\tilde{k}}(\mathbf{d}_i)) \le \nu_i^{N,0}(z_i, \mathbf{d}_i) - \theta_{3,i}(|z_i|)$ and $\theta_{1,i}(|z_i|) \le \nu_i^{N,0}(z_i, \mathbf{d}_i) \le \theta_{2,i}(|z_i|)$ for some $\theta_{3,i}, \theta_{2,i}, \theta_{1,i} \in \mathcal{K}$. On the other hand, the continuity of $\nu_i^{N,0}$ over a compact set implies that is uniformly continuous on that set. From [30, Lemma 1], there exists a $\mathcal{K}_\infty-$function $\alpha\sigma_{\nu,i}$ such that

$$\nu_i^{N,0}(z_i^+, \mathbf{d}_i^+) \le \nu_i^{N,0}(z_i, \mathbf{d}_i) - \theta_i(|z_i|) + \sigma_{\nu,i}(|\mathbf{d}_i^+ - \tilde{d}_{\tilde{k}}(\mathbf{d}_i)|)$$

Using Assumption 6, we obtain

$$\begin{aligned}\nu_i^{N,0}(z_i^+, \mathbf{d}_i^+) &\le \nu_i^{N,0}(z_i, \mathbf{d}_i) - \theta_{3,i}(|z_i|) + (\rho_i - \gamma_i)(\alpha_i)\\ &\le (id - \theta_{3,i} \circ \theta_{2,i}^{-1})\nu_i^{N,0}(z_i, \mathbf{d}_i) + (\rho_i - \gamma_i)(\beta_i)\\ &\le (id - \theta_{3,i} \circ \theta_{2,i}^{-1})(\beta_i) + (\rho_i - \gamma_i)(\beta_i)\end{aligned}$$

Taking $\gamma_i = id - \theta_{3,i} \circ \theta_{2,i}^{-1}$ yields $\nu_i^{N,0}(z_i^+, \mathbf{d}_i^+) \le \beta_i$ which implies that $(z_i^+, \mathbf{d}_i^+) \in \Omega_{i,\beta}$ □

The main assertion of this section is a consequence of the above theorem

Corollary 1 (Recursive Feasibility)**.** *If $z_i(0) \in \{z_i : (z_i, \mathbf{d}_i) \in \Omega_{i,\beta}\} \subset \mathcal{Z}_i^N(\mathbf{d}_i)$, then $z_i(k) \in \mathcal{Z}_i^N(\mathbf{d}_i(k))$ provided the exogenous inputs update rate is limited.*

Recursive feasibility is obtained then as a consequence of the stability properties of the predictive controllers. The negotiation framework modifies the cost according to the change in exogenous inputs, a variation in load demand or renewable injection will result in a potential update in the pricing policy. If Assumption 6 holds, then the controller can handle variations in its cost criteria. On the other hand, the sources of potentially large variations arise from sudden load demands; renewable injections can be kept within prescribed variation using the available inputs Δ_i and local storage.

V. Testing and evaluation environment

A. Physical Deployment and Testing Environment

The digital extension of the DER unit was designed with the intention of integrating intelligent control and market

[4]For a set $\mathcal{A} \subset \mathcal{B}$, its diameter with respect to $\mathcal{B}$ is $\mathrm{diam}_{\mathcal{B}}\mathcal{A} = \max\{|x - y| : x, y \in \mathcal{B}, x - y \in \mathcal{A}\}$.

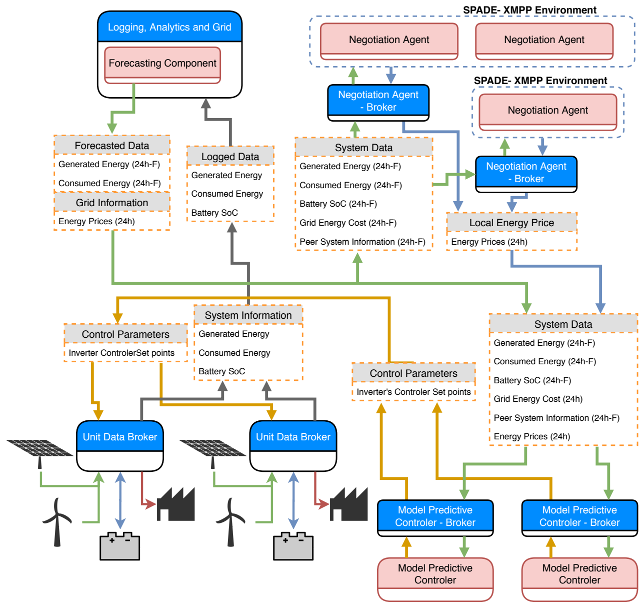

Fig. 3. Data Flow Diagram of the Extended DER Components

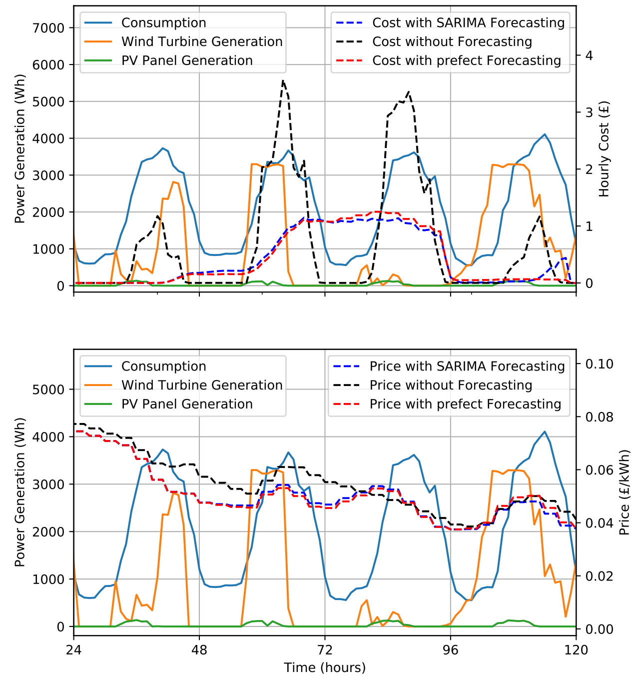

Fig. 4. The effect of forecasting on the price of a Single DER

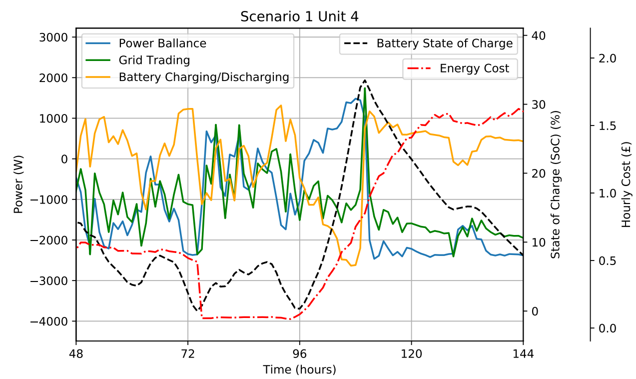

Fig. 5. The behaviour of the price of a single DER based on its Energy Balance, Battery SOC and Peers

negotiation into the existing system. The requirements of the systems and its extension needs were evaluated based on the framework published in [31]. The technical requirements and implementation details of the digital extension are shown in [32]. The extension was built by reviewing the requirements of each field against the existing state of the art. To bridge the protocol and technology gap a messaging system and broker based solution is proposed that translates messages from the various systems.

An overview of the data that is being passed from one digital component to another can be seen in Fig. 3. This deployment showcases a fully decentralised implementation where each component can be deployed in various location depending on system specifications. This allows local markets and fully functioning systems to be made available to remote regions of a sparse DERs and scale from a few units to hundreds in which case the messaging system can be expanded in a clustered deployment.

B. Date Sources and Scenario Generation

The scenarios are generated so that they cover a wide range of DER configurations. The primary parameter that is iterated through is the time of year which varies between $[Summer, Spring - Autumn, Summer]$. This influences the week selection in the PV panels and sets the week for the consumption and wind turbine as well. The size of the panels varies between $[15.4m^2, 45.6m^2, 85.2m^2]$. The wind turbine selects a value in the time of the year that matches one of the 4 identified patterns in the data. The battery sizes vary between $[5kWh, 13.5kWh, 25kWh]$. The number of units in each scenario vary between $[3, 6, 9]$.

C. Scenario Evaluation and Results

The variation of the prices and cost that an individual received based on the type of forecast that was used can be seen in Fig. 4.

The behaviour of the DER and it's energy price variation with respect to its peers and it's battery SOC can be seen in Fig. 5.

An overview of how the Cost of Energy Changed for each unit in all the scenarios as compared to the their generation and consumption balance can be seen in Fig. 6. The three different markers denote the results from the the three types of runs where the green hexagon represents a perfect forecasting; the blue square represents the SARIMA based forecasting and the red triangle is the naive forecasting scenario.[Details Pending Full Data for Plots].

We conclude this section illustrating the convergence properties of the market framework.

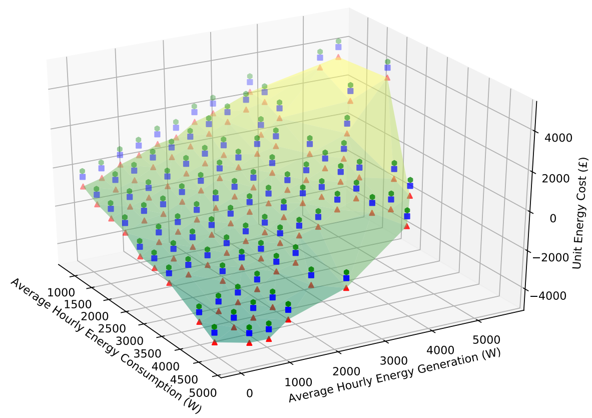

Fig. 6. Relation between the Energy Generation, Consumption and the Cost of energy for Each Unit

M	Number of Rounds		Average Utility	
	Mean	SD	Mean	SD
2	1.257	0.786	0.750	0.326
3	2.037	1.026	0.747	0.306
4	2.916	1.176	0.785	0.247
5	3.679	1.392	0.796	0.244
6	4.561	1.487	0.822	0.219
7	5.519	1.552	0.835	0.193
8	6.452	1.575	0.841	0.189
9	7.290	1.808	0.866	0.164
10	8.101	1.773	0.879	0.147

TABLE I
PERFORMANCE OF AGENT NEGOTIATION WITH DIFFERENT NUMBER OF AGENTS

VI. Conclusions and Future Work

We proposed a distributed predictive controller capable of handling coupled constraints which optimises generation costs. These costs are obtained via a negotiation framework based on a MAS. The subject of the negotiation was done over policies rather than fixed prices such that it is possible to contemplate multiple scenarios given the availability of storage. Both control and market layers are subject to exogenous inputs dictating the interaction of the system with its environment. A rigorous analysis of the properties, existence of game theoretic equilibrium points for the market layer and recursive feasibility for the control layer, was given. The controller is proven to be recursively feasible in presence of time-varying information, both voltages form neighbouring nodes and forecasts. We have developed a testing and evaluation environment to assess the performance of our controller. We explored the effects of varying forecast accuracy in the controller performance and the pricing policies. The proposed system is a scalable solution to the problem of pricing and control.

VII. Acknowledgements

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