

# A practical shear wall layout optimization framework for the design of high-rise buildings

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**Abstract:** In the design of high-rise buildings, the lateral structural system that resists seismic and wind actions often dominates the overall design, and as a result, the optimum design of the lateral structural system is important for material efficiency. This paper aims to present a new design methodology that relies on an extended Evolutionary Structural Optimization (ESO) method to optimize the shear wall layout for the design of high-rise buildings. By discretizing all the shear walls of the ground structure into wall elements, different shear wall layouts were described by a binary matrix where ‘0’ and ‘1’ indicate the absence or presence of a wall element respectively. The proposed framework borrowed the intuitive concept of the ESO method, where the less stressed wall elements in the structure were gradually removed but the highly stressed wall elements were reserved. The conceptual design method was combined with the optimization algorithm to control the optimization process and to reduce structural eccentricity considering architecture, strength and serviceability constraints. Then, the structural weight could be automatically minimized under multiple loading cases and design constraints. By integrating the commercially available ETABS, a dedicated optimization software with an independent interface was developed and details were also presented in this paper for practical software development. The proposed framework was used to optimize the layouts of different shear wall structures, and case studies showed that the proposed framework could optimize shear wall structures with a material saving of up to 14% compared to the conventional design, which proved the feasibility and effectiveness of the framework. The paper can therefore be referenced by other researchers and software engineers to further explore the layout optimization of shear wall structures and to reduce material usage for a more sustainable design.

**Keywords:** high-rise buildings; shear wall; multiple loading cases; ESO; conceptual design; layout optimization.

## 1 Introduction

Complex high-rise buildings are one of the most visually striking trends in contemporary architecture and can bring obvious social and economic benefits. A number of high-rise buildings with attractive designs and eye-catching shapes have been erected, as shown in Fig. 1. In the design of high-rise buildings, the layout of the shear wall often has a large impact on the lateral stiffness of the structure. However, the traditional design method is very inefficient, where designers repeatedly modify the layout and check the structural component capacity and deformations of the buildings using structural design software. It is therefore difficult to obtain the optimal structure layout, which not only results in a large amount of material waste but may also cause unsafe designs due to unreasonable shear wall layouts.



(a) Burj Khalifa Tower, Dubai, UAE



(b) Taipei 101 Tower, Taipei, China

Fig. 1. Project cases of modern high-rise buildings

1        With the development of modern computational capacity, it is possible to use the theory of mechanics and  
2 optimization algorithms to automatically optimize the size and layout of structures, and there are several methods  
3 for this purpose. The main approach to structural optimization has been mathematical programming [1-3], which is  
4 highly applicable and efficient for convex optimization problems with continuous variables. However, the shear  
5 wall layout optimization problem is a discrete problem and, due to the relationship between design variables and  
6 constraints, it is nonlinear and nonconvex. So, mathematical programming algorithms may converge to local  
7 optima but may not be able to find the globally optimum solution. Gradient-based approaches have been tested for  
8 many structural optimization problems [4-7]. They are usually computationally efficient for most optimization  
9 problems. However, the current design of high-rise buildings involves a large number of design formulas that are  
10 nonlinear and the solvers of design software are generally not open to the public. Gradient-based approaches, which  
11 require solving the derivative of the objective and constraint functions, are unsuitable for problems with implicit  
12 objective and constraint functions, i.e. black-box problems. Indeed, there has been a method of total enumeration  
13 for discrete optimization problems, but the current computational technologies are not capable of performing an  
14 exhaustive search in the feasible region for the design optimization of large-scale building structures. The heuristic  
15 approaches, based on the stochastic process, provide a means of finding near-optimal solutions with a reasonable  
16 number of iterations and usually do not require a continuous or differentiable objective function. Therefore, they  
17 have a wide range of applications in structural optimization and many other fields [8-11]. Nevertheless, the main  
18 limitation of heuristic algorithms is that, although they do not require analyses of all possible solutions as the  
19 method of total enumeration does, they still require a large population per generation and the optimization time may  
20 be unacceptable for problems whose individual evaluations are very time-consuming. Evolutionary Structural  
21 Optimization [12] was originally developed upon a simple criterion by gradually removing lowly stressed materials  
22 so that the materials could be used in the highly stressed areas to the greatest extent and their mechanical properties

1 could be fully exploited. In the optimization process, the algorithm pays more attention to the stress of each element  
2 than to the specific forms of the objective and constraint functions. Although there is no rigorous theory to prove  
3 that the results are optimal, many optimization examples have demonstrated the effectiveness of the ESO method  
4 [13-16]. Compared to heuristic algorithms, the ESO method requires fewer structural analyses and is more efficient  
5 because of the introduction of stress as an optimality criterion. The stressed-based optimization criterion of the ESO  
6 method is effective, but the application of the ESO method is still limited to the optimization of 3D continua and  
7 trusses. An additional problem is that the optimized structures are generally irregular and can not meet the  
8 construction requirements of shear wall structures.

9 In the research field of topology optimization, there is a lot of relevant literature. Zhang et al. [17,18] adopted  
10 the ground structure method to study both continuum and truss topology optimization and determined the addition  
11 and deletion of components and nodes on the basis of a heuristic algorithm or the component strain from the initial  
12 structure with all possible components laid out. Guo et al. [19] proposed a new computational framework for  
13 structural topology optimization based on the concept of moving morphable components, which can incorporate  
14 more geometry and mechanical information into topology optimization directly and therefore render the solution  
15 process more flexible. Zuo and Saitou [20] used an ordered multi-material SIMP interpolation to solve  
16 multi-material topology optimization problems without introducing any new variables. Emmendoerfer et al. [21]  
17 developed a level set-based topology optimization procedure to design compliant mechanisms subject to local stress  
18 constraints. Most of the algorithms are elegant in terms of mathematical derivation and are applied to the design of  
19 mechanical components that are small scale and continuum. For large scale optimization of high-rise buildings,  
20 more research is required on applicability.

21 A great deal of the existing literature in high-rise building optimization focuses on only the size optimization  
22 [22-24] and the topology optimization [25-30] of the beams and columns, while the more important layout  
23 optimization of shear walls is rarely addressed. Currently, a common approach to layout optimization of shear wall  
24 structures is to choose the best from several feasible schemes [31-34]. The available alternatives are limited, so it is  
25 impossible to obtain the optimal layout of shear walls and it is inefficient. Hofmeyer et al. [35] found that the  
26 hierarchic structural layout could be generated by the robust 3D topology optimization approach, providing a more  
27 constructible structural design. Nevertheless, the shear walls in the optimized structure were irregular and could not  
28 meet the current code-stipulated design constraints and construction requirements, so there is still much work to do  
29 for practical application. Zhang and Mueller [36] developed a novel approach based on the ground structure and a  
30 modified Evolutionary Algorithm for automatically generating diverse shear wall layouts which took both

1 architectural and structural goals into consideration. One of the major drawbacks was that the structures were  
2 evaluated by simplified formulas but without Finite Element (FE) analysis. The results could only be used for the  
3 conceptual design stage and should be analyzed and detailed by structural engineers in the later design process.  
4 Mathieu [37] worked on an optimization framework using linearized modal analysis and a modified Genetic  
5 Algorithm (GA) to design the shear wall layout of a building under wind and seismic load cases. But only a  
6 five-story building was investigated, and for high-rise buildings, it was poor in optimization efficiency. Gan et al.  
7 [38] incorporated parametric modelling to identify the relationship between different geometrical entities, and the  
8 complex form of building geometries could be systematically manipulated. On this basis, a hybrid method based on  
9 the Genetic Algorithm and Optimality Criteria (OC) was proposed to optimize concrete building structures by first  
10 establishing the optimal structural topology and then optimizing individual member sizes. Talatahari and Rabiei [39]  
11 presented the Quantum Charged System Search (QCSS) algorithm as a new optimization method to improve the  
12 convergence capability of the original Charged System Search, and it was used for shear wall layout optimization  
13 of tall buildings considering both structural and architectural requirements. Lou et al. [40] combined a data-driven  
14 Tabu Search (TS) algorithm with the conceptual design method to reduce structural FE analyses and improve the  
15 convergence process in the shear wall layout optimization. Almost all of the above methods of shear wall layout  
16 optimization were based on heuristic algorithms, such as the GA, the QCSS, and the TS. Even if great  
17 improvements have been made to accelerate the convergence process, heuristic algorithms still require thousands of  
18 individual evaluations. If each evaluation is fully calculated by the structural analysis software, it will be  
19 computationally inefficient.

20 Shear wall layout optimization is a complicated topology optimization problem whose objective function is  
21 characterized by high nonlinearity and multiple peaks. Furthermore, the objective and constraint functions are  
22 implicit, making it a typical black-box optimization problem. Meanwhile, different constraints under multiple  
23 loading cases need to be considered. In light of this fact, this paper aims to develop a practical shear wall layout  
24 optimization framework for the design of high-rise buildings. The overall optimization objective is to minimize the  
25 amount of structural weight, subject to a series of code-stipulated constraints under multiple loading cases. First,  
26 the shear walls of the ground structure are discretized into wall elements of a certain length and a binary matrix is  
27 used to describe the topology of the shear walls. Thus, a mathematical model of the shear wall layout optimization  
28 based on the shear wall coding matrix has been established. Based on the intuitive concept of the ESO method, the  
29 lowly stressed wall elements in the structure are gradually removed under constraints. In addition, the identification  
30 technologies for horizontal labels and vertical labels have been developed and applied to the processing of

1 structural symmetry and vertical continuity, so that the optimization results can meet the geometry design  
2 requirements. Combined with the conceptual design method, the eccentricity of buildings is eliminated by dividing  
3 the structural plane into multiple areas and adopting a corresponding shear wall removal procedure. With regard to  
4 the demand for optimization under multiple loading cases, a case-by-case optimization method and a direct  
5 multi-case optimization method are presented, and two equivalent stress values for the direct optimization method  
6 are given. The optimization results by using these methods are compared in subsequent case studies. Finally, a  
7 dedicated optimization software with an independent interface is compiled in C++ using ETABS to perform  
8 structural analyses. The interaction between the developed software and ETABS is provided via the Open  
9 Application Programming Interface (OAPI). The proposed method was used to optimize the layouts of shear wall  
10 structural systems with different building layouts, different sizes of wall elements, and different loading scenarios.  
11 The direct multi-case optimization method was also compared against the general Genetic Algorithm to verify its  
12 applicability and effectiveness.

## 13 **2 Shear wall layout optimization framework**

### 14 **2.1 Pre-processing**

15 Shear walls can not be randomly positioned in the plane of a building. Their positions are often constrained by  
16 the architectural layout of buildings, such as doors and windows. The area where shear walls can be laid out is  
17 called the ground structure of shear walls. Before the optimization procedure in this paper, the user is required to  
18 determine the ground structure (design domain) of shear walls according to the architectural plan requirements. Fig.  
19 2 shows the ground structure of the shear walls of an architectural plan. The structure full of shear walls in the  
20 ground structure is called the full structure, which is used as the initial structure for optimization in this paper.

21 Similar to Finite Element meshing, the shear walls in the ground structure are discretized into wall elements of  
22 a specified length. For ease of description, '1' indicates that a shear wall is laid out at the wall element and '0'  
23 indicates that there is no shear wall laid out at the wall element. In this way, the shear wall layout can be  
24 represented by a matrix of binary codes. Shear walls can be approximately discretized into wall elements of any  
25 specified length, and the discretization process will be carried out automatically by the program. The discretization  
26 of shear wall elements follows the following procedure:

- 27 1) For any undivided shear wall, initialize its division number  $d$  with  $d = 1$ .
- 28 2) If the following equation is satisfied,

$$\left| \frac{L}{d} - l \right| \leq \left| \frac{L}{d+1} - l \right| \quad (1)$$

1 the shear wall is divided evenly into  $d$  wall elements; otherwise, let  $d = d + 1$  and re-execute step 2). In the equation,  
 2  $L$  is the original length of the shear wall and  $l$  is the length of the wall elements to be divided into.

3 3) Perform 1) and 2) for all initial shear walls to complete the shear wall discretization.

4 As an example, Fig. 3 shows the ground structure after the program completes the shear wall discretization,  
 5 where  $l = 1$ .

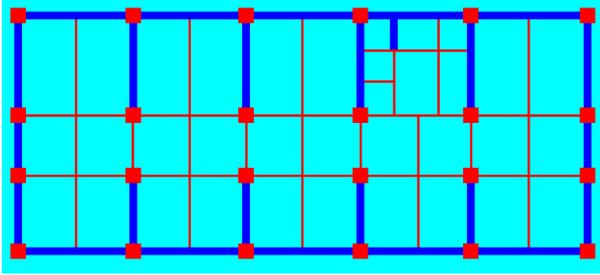


Fig. 2. Ground structure of shear walls

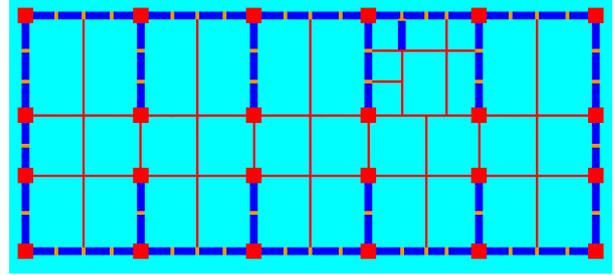


Fig. 3. Discretization of shear walls

## 6 2.2 Definition of the shear wall layout optimization problem

7 The layout optimization of shear walls is a discrete variable optimization problem under multiple loading  
 8 cases and complex constraints. Some of the optimization design constraints are inherent structural characteristics  
 9 that are related to geometry rationality and do not change with different loading cases, but optimization constraints  
 10 also exist under specific cases while they are not considered under other cases. In this paper, according to the  
 11 Chinese building design code of JGJ 3-2010 [41], the loading cases, corresponding constraints and their limits are  
 12 concluded and shown in Table 1.

13

Table 1. Constraints and limits of different loading cases

Constraints	Limit	DEAD, LIVE <sup>(8)</sup>	EX, EY, EX+, EY+, EX-, EY-, EXM, EYM <sup>(9)</sup>	StaticEX, StaticEY, StaticEX+, StaticEY+, StaticEX-, StaticEY- <sup>(10)</sup>	WIND_X+, WIND_Y+, WIND_X-, WIND_Y- <sup>(11)</sup>
PR <sup>(1)</sup>	$\leq 0.9$	√	√	√	√
SSR_X, SSR_Y <sup>(2)</sup>	$\geq 0.9/1.1/1.5^{(12)}$	√	√	√	√
FSAO_X, FSAO_Y <sup>(3)</sup>	$\geq 3$	√	√	√	√

RGR_X,	$\geq 1.4$	√	√	√	√
RGR_Y <sup>(4)</sup>					
Angle_X,	$\leq 1/1000$	×	√	×	√
Angle_Y <sup>(5)</sup>					
DR_X,	$\leq 1.5$	×	×	√	√
DR_Y <sup>(6)</sup>					
IDR_X,	$\leq 1.5$	×	×	√	√
IDR_Y <sup>(7)</sup>					

(1) PR denotes the period ratio and it represents the ratio of the maximum torsional cycle to the maximum translational cycle ( $T_i/T_1$ ), which is used to ensure the torsional stiffness of structures.

(2) SSR\_X and SSR\_Y represent the story stiffness ratio in the X and Y directions, respectively. The story stiffness ratio is the ratio of the current story stiffness to the upper story stiffness to prevent sudden changes in story stiffness.

(3) FSAO\_X and FSAO\_Y represent the factor of safety against overturning in the X and Y directions respectively, which is the ratio of the anti-overturning moment to the overturning moment and is used to prevent the structure from overturning under external loads.

(4) RGR\_X and RGR\_Y denote the rigidity-gravity ratio in the X and Y directions respectively, which refers to the ratio of the lateral stiffness of the structure to the design value of the gravity load and is used to verify the overall stability of the structural system.

(5) Angle\_X and Angle\_Y respectively denote the story drift in the X and Y directions, and the story drift refers to the ratio of the maximum horizontal displacement between floors to the floor height ( $\Delta u/h$ ) subject to wind or frequent earthquake loads calculated by the elastic method, which is a macro index to ensure the translational stiffness of high-rise structures.

(6) DR\_X and DR\_Y denote the displacement ratio in the X and Y directions respectively. It is the ratio of the maximum horizontal displacement to the average displacement under the rigid floor assumption, reflecting the torsional effect of the structure.

(7) IDR\_X and IDR\_Y denote the interlayer displacement ratio in the X and Y directions respectively. Similar to the displacement ratio, it is the ratio of the maximum interlayer displacement to the average interlayer displacement under the rigid floor assumption and is a macro index to control structural torsion.

(8) DEAD and LIVE are dead load and live load.

1 <sup>(9)</sup> EX and EY represent seismic loads in the X and Y directions; EX+, EY+, EX- and EY- represent seismic loads  
 2 with accidental eccentricity; EXM and EYM represent seismic loads in the most unfavorable seismic directions.

3 <sup>(10)</sup> They denote the specified horizontal forces corresponding to the seismic loads.

4 <sup>(11)</sup> They denote wind loads in different directions.

5 <sup>(12)</sup> It is 0.9 in general, 1.1 when the height of this floor is 1.5 times higher than that of the upper floor, and 1.5 when  
 6 this floor is the bottom embedded floor.

7 Considering that there are positions that need to be laid out with shear walls in the structure, such as the  
 8 stairwells, the corresponding wall element codes are frozen at '1' in the optimization, which will not change in the  
 9 iterations. After the discretization of shear walls, the mathematical model of the shear wall layout optimization can  
 10 be stated as Fig. 4.

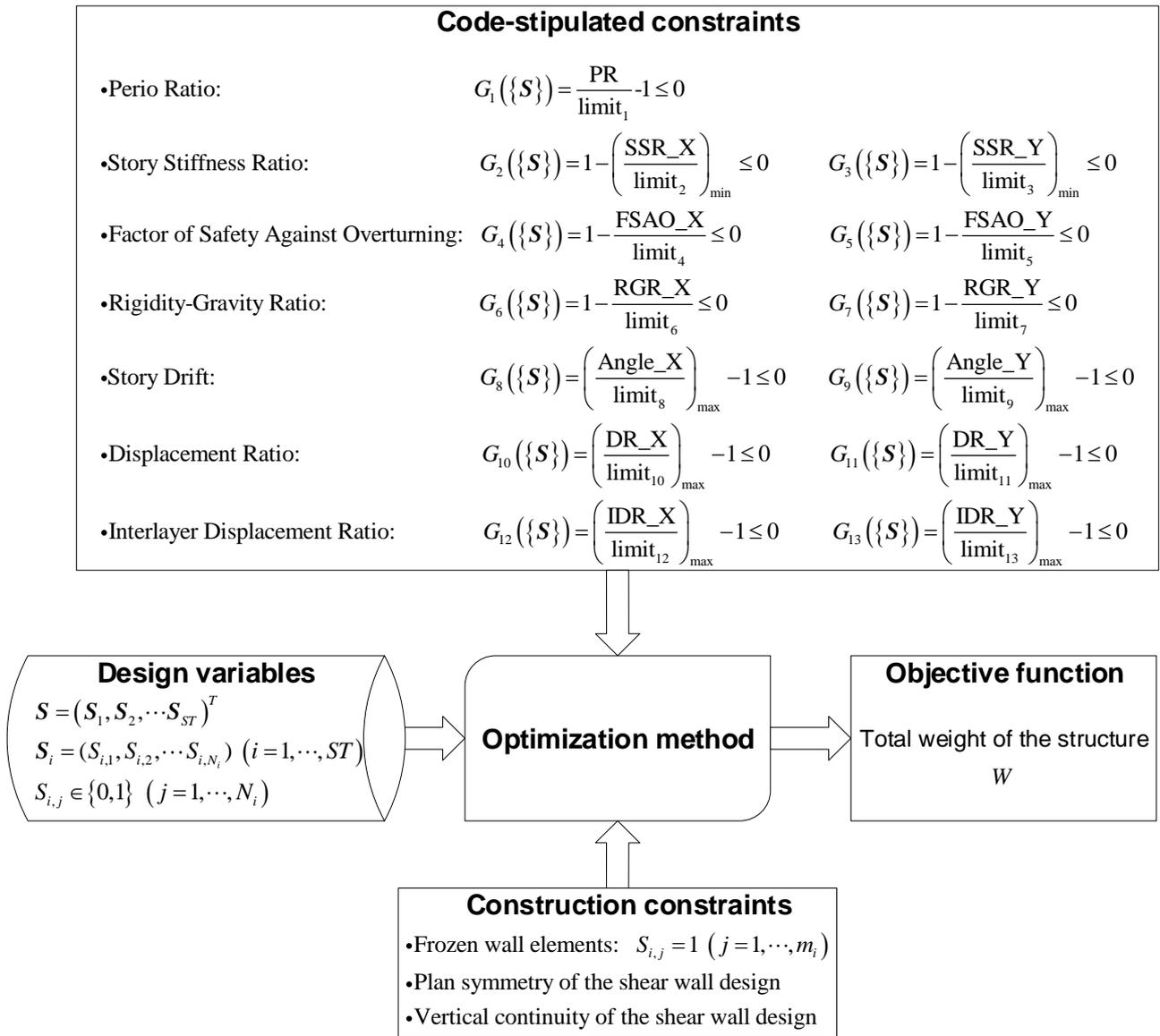


Fig. 4. Optimization model of shear wall layout

1 In Fig. 4,  $S$  denotes the topological form of the shear wall.  $i$  and  $j$  are the floor number and the wall element  
 2 number respectively.  $ST$  denotes the total number of stories and  $N_i$  denotes the total number of wall elements on the  
 3  $i$ -th floor.  $m_i$  denotes the number of frozen wall elements on the  $i$ -th floor.  $W$  is the total weight of the structure. The  
 4 code-stipulated constraints in Table 1 are standardized for ease of description.  $( )_{\max}$  and  $( )_{\min}$  denote the maximum  
 5 and minimum value of all the floors and all the loading scenarios, respectively. The plan symmetry and vertical  
 6 continuity of the shear wall design will be described in detail in the subsequent sections.

## 7 **2.3 General methodology**

### 8 **2.3.1 Extended Evolutionary Structural Optimization**

9 Evolutionary Structural Optimization is a numerical optimization method that has been developed to solve 3D  
 10 continuum optimization problems. It is based on the general idea that by gradually removing invalid or inefficient  
 11 materials from the structure, the remaining structure will gradually tend to be optimized. This paper borrows the  
 12 intuitive idea of the ESO method and extends the ESO method to the layout optimization of shear walls. The  
 13 von-Mises stress of a shear wall element under applied loads is considered as the contribution of the shear wall to  
 14 the load resistance. Generally, different from 3D continuum optimization, the shape of shear walls is normally  
 15 regular. In the optimization procedure, a wall element after discretization in Section 2.1 is taken as a basic deletion  
 16 element, and the maximum Mises stress of the four nodes of a wall element is taken as the stress of the element, as  
 17 shown in the following equation.

$$\sigma_{i,j}^{case} = \max(\sigma_{i,j \text{ A}}^{case}, \sigma_{i,j \text{ B}}^{case}, \sigma_{i,j \text{ C}}^{case}, \sigma_{i,j \text{ D}}^{case}) \quad (i = 1, \dots, ST; j = 1, \dots, N_i) \quad (2)$$

18 where  $\sigma$  is the Mises stress,  $case$  denotes the loading case, and A, B, C, and D denote the four nodes of the wall  
 19 element respectively.

20 There are generally standard floors in high-rise buildings, and the structural layout of each floor from the same  
 21 standard floor is the same. Therefore, in the optimization process, each floor from the same standard floor adopts  
 22 the same addition and deletion method so that the results still have the original standard floor settings. Starting from  
 23 the initial ground structure, the initial number of wall elements to be eliminated for each floor is determined, and  
 24 the wall elements on each floor are gradually removed until the constraints in Section 2.2 are not satisfied. Then the  
 25 deletion procedure is revoked and the number of wall elements to be eliminated is reduced according to constraint  
 26 violations. After reduction of the deletion number, the wall element removal is then executed again. This stage of  
 27 optimization ends when the total number of wall elements to be eliminated for all floors is 0.

1 **2.3.2 Plane partition optimization based on conceptual design**

2 The center of mass tends to coincide with the center of stiffness as much as possible in a well-designed and  
 3 high-performance structural layout to reduce the torsional effect of the structure under lateral loads. To reduce the  
 4 structural eccentricity, a coordinate system is established with the center of mass as the origin, and the current  
 5 structure is divided into nine types based on the area in which the center of stiffness falls, as shown in Fig. 5, where  
 6 the positions of the dotted lines are determined by  $15\% \times 0.7 = 10.5\%$ . 15% is the maximum acceptable  
 7 eccentricity of high-rise building structures designated by structural designers based on engineering experience, and  
 8 0.7 is the guarantee factor. For different types of structural eccentricity, different shear wall removal criteria are  
 9 adopted, and the shear wall elements in specific areas are more likely to be removed in each iteration. The priority  
 10 removal areas for shear walls of different types of structural eccentricity are shown in Table 2. For example, if the  
 11 center of stiffness falls into area 1, as shown in Fig. 5, the type of structural eccentricity is 1. Therefore, the lowly  
 12 stressed wall elements in the first quadrant ( $x > 0, y > 0$ ) will be removed in priority to move the center of stiffness  
 13 towards the center of mass. Then structural analysis will be carried out and the type of structural eccentricity will be  
 14 updated.

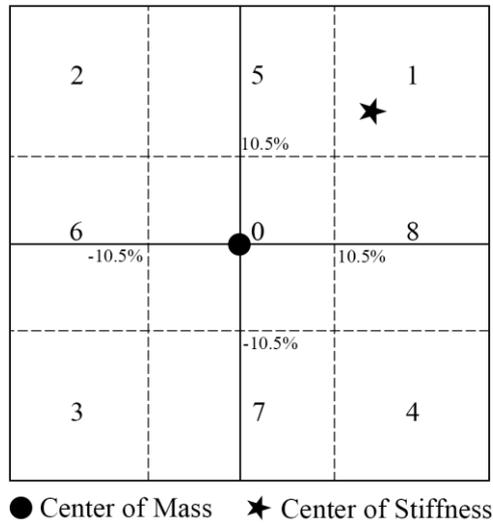


Fig. 5. Types of structural eccentricity

15

Table 2. Types of structural eccentricity and priority removal areas for shear walls

Types of eccentricity	Eccentricity in X	Eccentricity in Y	Priority removal areas
0	$[-10.5\%, 10.5\%]$	$[-10.5\%, 10.5\%]$	No limit
1	$(10.5\%, +\infty)$	$(10.5\%, +\infty)$	$x > 0, y > 0$
2	$(-\infty, -10.5\%)$	$(10.5\%, +\infty)$	$x < 0, y > 0$

3	$(-\infty, -10.5\%)$	$(-\infty, -10.5\%)$	$x < 0, y < 0$
4	$(10.5\%, +\infty)$	$(-\infty, -10.5\%)$	$x > 0, y < 0$
5	$[-10.5\%, 10.5\%]$	$(10.5\%, +\infty)$	$y > 0$
6	$(-\infty, -10.5\%)$	$[-10.5\%, 10.5\%]$	$x < 0$
7	$[-10.5\%, 10.5\%]$	$(-\infty, -10.5\%)$	$y < 0$
8	$(10.5\%, +\infty)$	$[-10.5\%, 10.5\%]$	$x > 0$

1

### 2 2.3.3 Plan symmetry of the shear wall design

3 For a structure with obvious symmetry, the layout of the shear walls should also be symmetrical. However, in  
4 the general optimization process, the structure tends to have an unsymmetrical shear wall layout. This is because  
5 the stresses in the corresponding wall elements of the symmetrical structure calculated by the structural analysis  
6 software may be a little different or the number of wall elements removed in a single iteration may not match the  
7 structural symmetry types, failing to maintain the symmetry of the geometry.

8 Therefore, a symmetry identification technology and horizontal labels have been developed to process plan  
9 symmetry. Through symmetry identification, symmetrical wall elements on the same floor of a symmetrical  
10 structure are given the same horizontal label. Subsequently, the wall elements with the same label are  
11 symmetrically processed to make the optimization results satisfy the symmetry requirement. The process of  
12 symmetry identification and horizontal label assignment is as follows:

13 1) Initialize the horizontal labels  $LL_{i,j}$  with the following equation.

$$LL_{i,j} = j \quad (i = 1, \dots, ST; j = 1, \dots, N_i) \quad (3)$$

14 2) For any floor  $i$ , search for the maximum and minimum X coordinates ( $x_{\max}$  and  $x_{\min}$ ) of all the nodes in the  
15 plane. The assumed symmetry axis of the floor in the X direction ( $X = x_{\text{mid}}$ ) can be obtained by the following  
16 equation.

$$x_{\text{mid}} = \frac{x_{\max} + x_{\min}}{2} \quad (4)$$

17 This floor is symmetric about  $X = x_{\text{mid}}$  if, for any wall element  $j$  in the plane, a wall element  $k$  satisfying the  
18 following equation can be found.

$$\begin{cases} x_j + x_k = 2 \cdot x_{\text{mid}} \\ y_j = y_k \end{cases} \quad (5)$$

19 where  $(x_j, y_j)$  and  $(x_k, y_k)$  are the centers of wall elements  $j$  and  $k$ , respectively.

1           3) Perform step 2) for all floors. If all floors have the same symmetry axis in the X direction, the structure has  
 2 symmetry in the X direction. Then let the horizontal labels of the symmetrical wall elements be equal to each other.  
 3 In other words, if Equation (5) is satisfied, then let  $LL_{i,k} = LL_{i,j}$ .

4           4) For any floor  $i$ , search for the maximum and minimum Y coordinates ( $y_{\max}$  and  $y_{\min}$ ) of all the nodes in the  
 5 plane. The assumed symmetry axis of the floor in the Y direction ( $Y = y_{\text{mid}}$ ) can be obtained by the following  
 6 equation.

$$y_{\text{mid}} = \frac{y_{\max} + y_{\min}}{2} \quad (6)$$

7 This floor is symmetric about  $Y = y_{\text{mid}}$  if, for any wall element  $j$  in the plane, a wall element  $k$  satisfying the  
 8 following equation can be found.

$$\begin{cases} x_j = x_k \\ y_j + y_k = 2 \cdot y_{\text{mid}} \end{cases} \quad (7)$$

9           5) Perform step 4) for all floors. If all floors have the same symmetry axis in the Y direction, the structure has  
 10 symmetry in the Y direction. Then let the horizontal labels of the symmetrical wall elements be equal to each other.

11           6) For any floor  $i$ , assuming that the structure is centrosymmetric, then the center of symmetry is  $(x_{\text{mid}}, y_{\text{mid}})$ . If  
 12 for any wall element  $j$  in the plane, a wall element  $k$  can be found to satisfy the following equation, this floor is  
 13 centrosymmetric about  $(x_{\text{mid}}, y_{\text{mid}})$ .

$$\begin{cases} x_j + x_k = 2 \cdot x_{\text{mid}} \\ y_j + y_k = 2 \cdot y_{\text{mid}} \end{cases} \quad (8)$$

14           7) Perform step 6) for all floors. If all floors are centrosymmetric about the same symmetry center, the  
 15 structure is centrosymmetric. Then let the horizontal labels of the centrosymmetric wall elements be equal to each  
 16 other.

17 In addition to shear walls, a strictly symmetrical structure requires symmetry of other components as well as  
 18 loads. The identification method is similar and will not be repeated in this paper.

19 After plan symmetry identification, the horizontal labels have been obtained and can be used in the  
 20 optimization procedure. In each iteration, the stresses of wall elements with the same horizontal label are averaged  
 21 by the following equation.

$$\bar{\sigma}_{i,j}^{\text{case}} = \frac{\sigma_{i,j_1}^{\text{case}} + \cdots + \sigma_{i,j_{\text{num}}}^{\text{case}}}{\text{num}} \quad (9)$$

$$\text{Let } \sigma_{i,j_1}^{\text{case}} = \bar{\sigma}_{i,j}^{\text{case}}, \cdots, \sigma_{i,j_{\text{num}}}^{\text{case}} = \bar{\sigma}_{i,j}^{\text{case}}$$

22 where  $\text{num}$  denotes the number of wall elements that have the same horizontal label as the wall element  $j_1$

1 (including  $j_1$ ) on the floor  $i$ . Except for the wall elements that lie exactly at the axis of symmetry,  $num = 1$  means  
 2 that the structure has no symmetry property,  $num = 2$  means that the structure has a certain symmetry property, and  
 3  $num = 4$  means that the structure has three symmetry properties at the same time.

4 For the stress difference caused by the structural analysis software, this calculation error can be eliminated by  
 5 calculating the average stress. For symmetrical wall elements that can not be removed in a single iteration, the  
 6 average stress will be smaller since the stresses of the removed wall elements are set to be 0, and the remaining wall  
 7 elements can be removed preferentially in the next iteration, which enables the structure to maintain its initial  
 8 symmetry.

### 9 2.3.4 Vertical continuity of the shear wall design

10 In the optimization, the wall elements on different standard floors are removed independently. As a result,  
 11 there may be a shear wall on one floor but no shear wall in the corresponding position on the lower floor, which is  
 12 called vertically discontinuous, as shown in Fig. 6. Such structures are not good for improving structural  
 13 performance and non-compliance with architectural design and construction requirements. To process vertical  
 14 discontinuity, a vertical continuity identification technology and vertical labels have been developed.

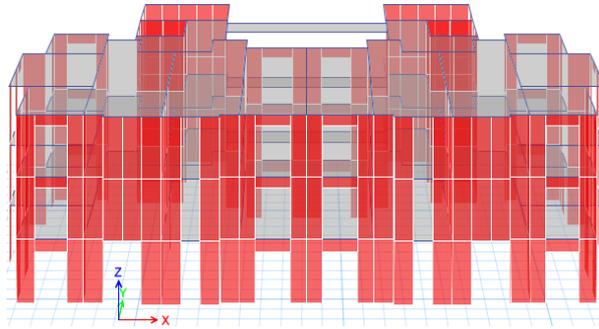


Fig. 6. Vertical discontinuity of shear walls (local structure)

15 Through vertical continuity identification, the vertical corresponding wall elements on different floors are  
 16 given the same vertical label. After subsequent processing, the structure can meet the vertical continuity  
 17 requirements. The process of vertical continuity identification and vertical label assignment is as follows:

18 1) Initialize the vertical labels  $LV_{i,j}$  with the following equation.

$$LV_{i,j} = j \quad (i = ST; j = 1, \dots, N_{ST}) \quad (10)$$

19 So, the current maximum label value satisfies  $LV_{\max} = N_{ST}$ .

20 2) Let  $i = i - 1$ . If for the wall element  $j$  ( $j = 1, \dots, N_i$ ) of the current floor  $i$ , a wall element  $k$  ( $k = 1, \dots, N_{i+1}$ )  
 21 on the floor  $i + 1$  satisfying the following equation can be found,

$$\begin{cases} (y_{k,start} - y_{k,end}) \cdot (x_j - x_{k,end}) = (y_j - y_{k,end}) \cdot (x_{k,start} - x_{k,end}) \\ \min(x_{k,start}, x_{k,end}) \leq x_j \leq \max(x_{k,start}, x_{k,end}) \\ \min(y_{k,start}, y_{k,end}) \leq y_j \leq \max(y_{k,start}, y_{k,end}) \end{cases} \quad (11)$$

1 then  $j$  is the vertical corresponding wall element of  $k$  and its vertical label is assigned by  $LV_{i,j} = LV_{i+1,k}$ ; otherwise,  
 2 let  $LV_{i,j} = LV_{\max} + 1$  and update  $LV_{\max}$  by  $LV_{\max} = LV_{\max} + 1$ . In the equation,  $(x_{k,start}, y_{k,start})$  and  $(x_{k,end}, y_{k,end})$   
 3 denote the start and end points of the wall element  $k$ , respectively.

4 3) Repeat step 2) until  $i = 1$  to complete the assignment of vertical labels to all wall elements.

5 After a stage of optimization ends and the total number of wall elements to be eliminated for all floors is 0,  
 6 vertical continuity processing is carried out using the vertical labels obtained above. From the sub-top floor to the  
 7 bottom, the wall elements are rearranged into vertical discontinuous positions. The specific process is as follows:

8 1) Initialize  $i$  with  $i = ST - 1$ .

9 2) For the wall element  $j$  on the floor  $i$  and the wall element  $k$  on the floor  $i + 1$ , if the following equation is  
 10 satisfied,

$$\begin{cases} LV_{i,j} = LV_{i+1,k} \\ S_{i,j} = 0 \\ S_{i+1,k} = 1 \end{cases} \quad (12)$$

11 let  $S_{i,j} = 1$ . After all the wall elements of this floor are processed, let  $i = i - 1$ .

12 3) Repeat step 2) until  $i = 1$  to complete the vertical continuity processing for all floors.

13 The vertical continuity processing leads to an increase in the structural weight, and the obtained structure  
 14 generally has more surplus in the constraints. This structure is used as the initial structure to be re-optimized in a  
 15 new stage of optimization until the increased structural weight due to the vertical continuity processing is less than  
 16 the convergence criteria  $E$  and then the optimization is completed.

## 17 2.4 Layout optimization method under multiple loading cases

18 In the shear wall layout optimization of high-rise buildings, different constraints under multiple loading cases  
 19 need to be considered, as shown in Table 1, and the stress of a wall element under different loading cases is also  
 20 varied, which makes it difficult to directly apply the optimization method of shear walls under a single loading case  
 21 presented in Section 2.3. In this section, two applicable multi-case optimization methods are proposed based on the  
 22 general methodology and will be compared in subsequent case studies.

### 23 2.4.1 Case-by-case optimization method

24 For multiple loading cases, a simple way to expand the single-case optimization method is to sort multiple

1 loading cases in order of priority and optimize the structure under the most unfavorable loading case first using the  
2 single-case optimization method. Then all the present wall elements in the results at the end of the optimization are  
3 frozen. If the current structure can satisfy the constraints under the next loading case, the next case will be skipped;  
4 otherwise, the structure will be optimized again from the ground structure under the next loading case. Since the  
5 previous wall elements have been frozen, they will not be removed in the subsequent optimization process and the  
6 obtained structure will satisfy the constraints under the previous loading cases. After traversing all the loading cases,  
7 the final results can satisfy the constraints under all these cases. Moreover, because the structure is optimized under  
8 the most unfavorable loading case first, most of the subsequent loading cases will be skipped directly without  
9 optimization. The overall process of the case-by-case optimization method is as follows:

10 1) Pre-process and get the number of loading cases  $K$ ; identify plan symmetry and vertical continuity; assign  
11 horizontal labels and vertical labels. Initialize  $k$  with  $k = 1$ .

12 2) Structural analysis is then carried out with ETABS for the current structure under the loading case  $k$ .

13 3) The stresses of wall elements with the same horizontal label are averaged by Equation (9) and a defined  
14 number of lowly stressed wall elements in the priority removal areas of different floors are removed, based on the  
15 structural eccentricity type of the current structure.

16 4) The obtained structure is then reanalysed with ETABS.

17 5) Repeat steps 3) to 4) until the constraints under the loading case  $k$  are not satisfied. Then the deletion  
18 procedure is revoked and the number of wall elements to be eliminated is reduced according to constraint violations.  
19 After reduction of the deletion number, steps 3) to 4) are then executed again.

20 6) As the iteration progresses, the total number of wall elements to be eliminated decreases. When it becomes  
21 0, this stage of optimization ends and vertical continuity processing in Section 2.3.4 is carried out. The obtained  
22 structure is used as the initial structure to be re-optimized in a new stage of optimization from step 3). The  
23 optimization under the loading case  $k$  ends when the material increment due to the vertical continuity processing is  
24 less than the convergence criteria  $E$ .

25 7) Let  $k = k + 1$ . The present wall elements in the current structure are frozen and the current structure is tested  
26 to see if it satisfies the constraints under the loading case  $k$ . If it is satisfied, the loading case  $k$  is skipped and step 7)  
27 is repeated; otherwise, the current structure is refilled with wall elements and is re-optimized from step 2).

28 8) The whole multi-case optimization ends when  $k > K$ .

29 The above process is also drawn into a flow chart, shown in Fig. 7.

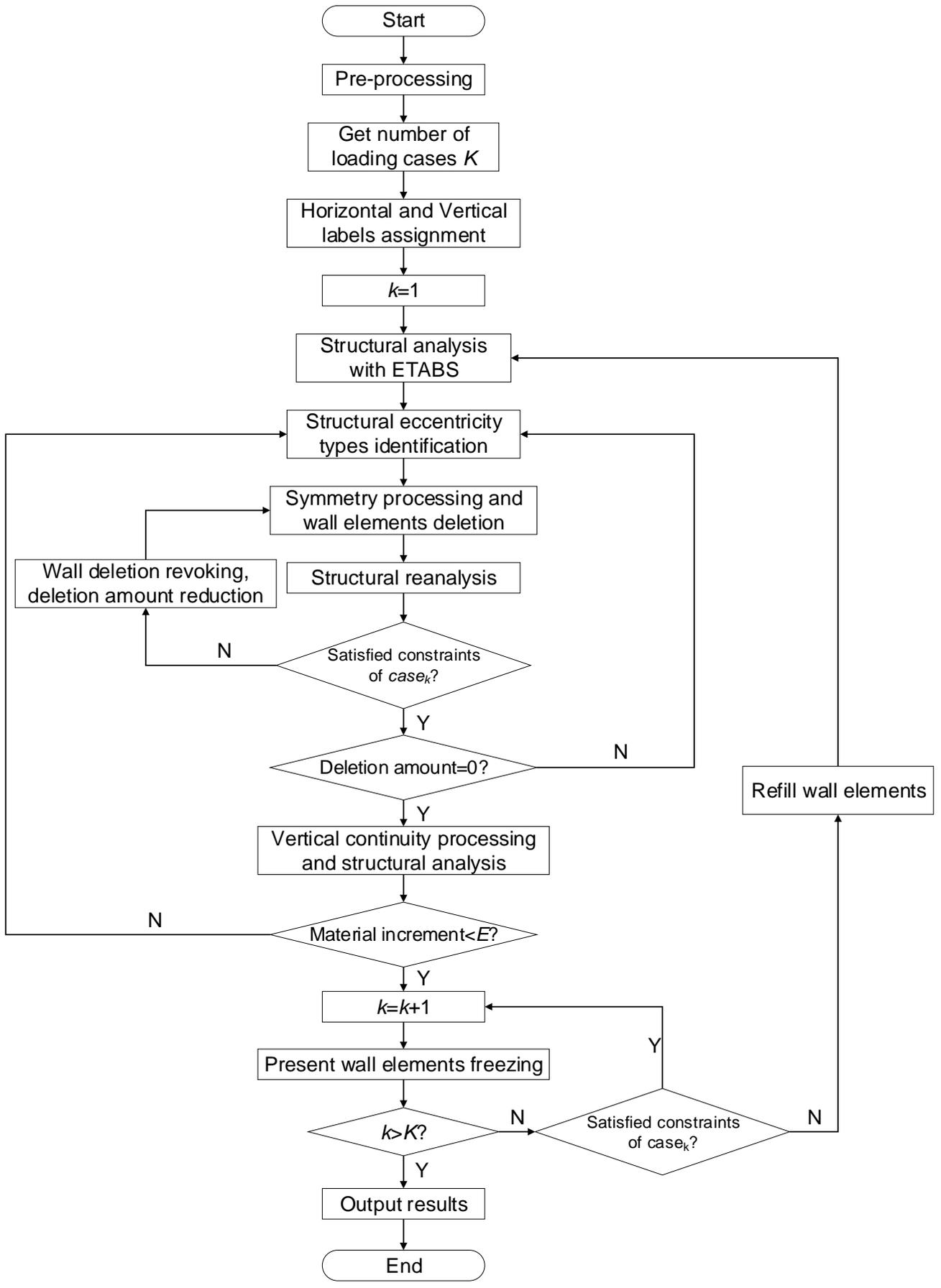


Fig. 7. Flow chart of the case-by-case optimization method

## 1 2.4.2 Direct multi-case optimization method

2 The idea of the case-by-case optimization method is simple, but its obvious disadvantage is that with the  
 3 increase of load cases, the shear walls of the optimized structure increase gradually, and there may be a large  
 4 surplus for the constraints under the first loading case in the final results. Therefore, a direct multi-case  
 5 optimization method is proposed in this section. By combining the wall element stresses under different loading  
 6 cases into equivalent stresses, the shear wall layout optimization under multiple loading cases is converted into a  
 7 single-case optimization problem and then the general methodology in Section 2.3 is applicable. Two ways of stress  
 8 combination are presented in this paper:

- 9 • The first technique tried in this paper is the maximum equivalent stress, which is the maximum stress of a  
 10 wall element under all the loading cases and is obtained by the following equation:

$$\sigma_{i,j}^{\max} = \max(\sigma_{i,j}^{\text{case}_1}, \dots, \sigma_{i,j}^{\text{case}_K}) \quad (i = 1, \dots, ST; j = 1, \dots, N_i) \quad (13)$$

11 where  $K$  is the total number of loading cases considered.

- 12 • The second technique is to use the equivalent stress, which is a dimensionless comprehensive value of a  
 13 wall element under all the loading cases and is obtained by the following equation:

$$\sigma_{i,j}^{\text{coe}} = \prod_{\text{case}=\text{case}_1}^{\text{case}_K} \frac{\sigma_{i,j}^{\text{case}}}{\max(\sigma_{i,1}^{\text{case}}, \dots, \sigma_{i,N_i}^{\text{case}})} \quad (i = 1, \dots, ST; j = 1, \dots, N_i) \quad (14)$$

14 The comprehensive equivalent stress can reflect the comprehensive contribution of a wall element to all the  
 15 loading cases. Furthermore, different coefficients can be adopted for different loading cases to reflect their  
 16 importance, as shown in the following equation:

$$\sigma_{i,j}^{\text{coe}} = \prod_{\text{case}=\text{case}_1}^{\text{case}_K} \frac{\gamma^{\text{case}} \cdot \sigma_{i,j}^{\text{case}}}{\max(\sigma_{i,1}^{\text{case}}, \dots, \sigma_{i,N_i}^{\text{case}})} \quad (i = 1, \dots, ST; j = 1, \dots, N_i) \quad (15)$$

17 where  $\gamma^{\text{case}}$  ( $0 \leq \gamma^{\text{case}} \leq 1$ ) is the importance factor of  $\text{case}$ , and when it adopts 1 for each loading case, Equation (15)  
 18 degenerates into Equation (14). The overall process of the direct multi-case optimization method is as follows:

19 1) Pre-process and get the number of loading cases  $K$ ; identify plan symmetry and vertical continuity; assign  
 20 horizontal labels and vertical labels.

21 2) Structural analysis is then carried out with ETABS for the current structure under all the loading cases and  
 22 equivalent stresses for all the wall elements are calculated by Equation (13) or Equation (14).

23 3) The equivalent stresses of wall elements with the same horizontal label are averaged by Equation (9) and a  
 24 defined number of lowly stressed wall elements in the priority removal areas of different floors are removed, based

1 on the structural eccentricity type of the current structure.

2 4) The obtained structure is then reanalysed with ETABS and equivalent stresses are calculated.

3 5) Repeat steps 3) to 4) until the constraints under a certain loading case are not satisfied. Then the deletion  
4 procedure is revoked and the number of wall elements to be eliminated is reduced according to constraint violations.  
5 After reduction of the deletion number, steps 3) to 4) are then executed again.

6 6) As the iteration progresses, the total number of wall elements to be eliminated decreases. When it becomes  
7 0, this stage of optimization ends and vertical continuity processing in Section 2.3.4 is carried out. The obtained  
8 structure is used as the initial structure to be re-optimized in a new stage of optimization from step 3). The whole  
9 multi-case optimization ends when the material increment due to the vertical continuity processing is less than the  
10 convergence criteria  $E$ .

11 The process of the direct multi-case optimization method is also drawn into a flow chart, shown in Fig. 8.

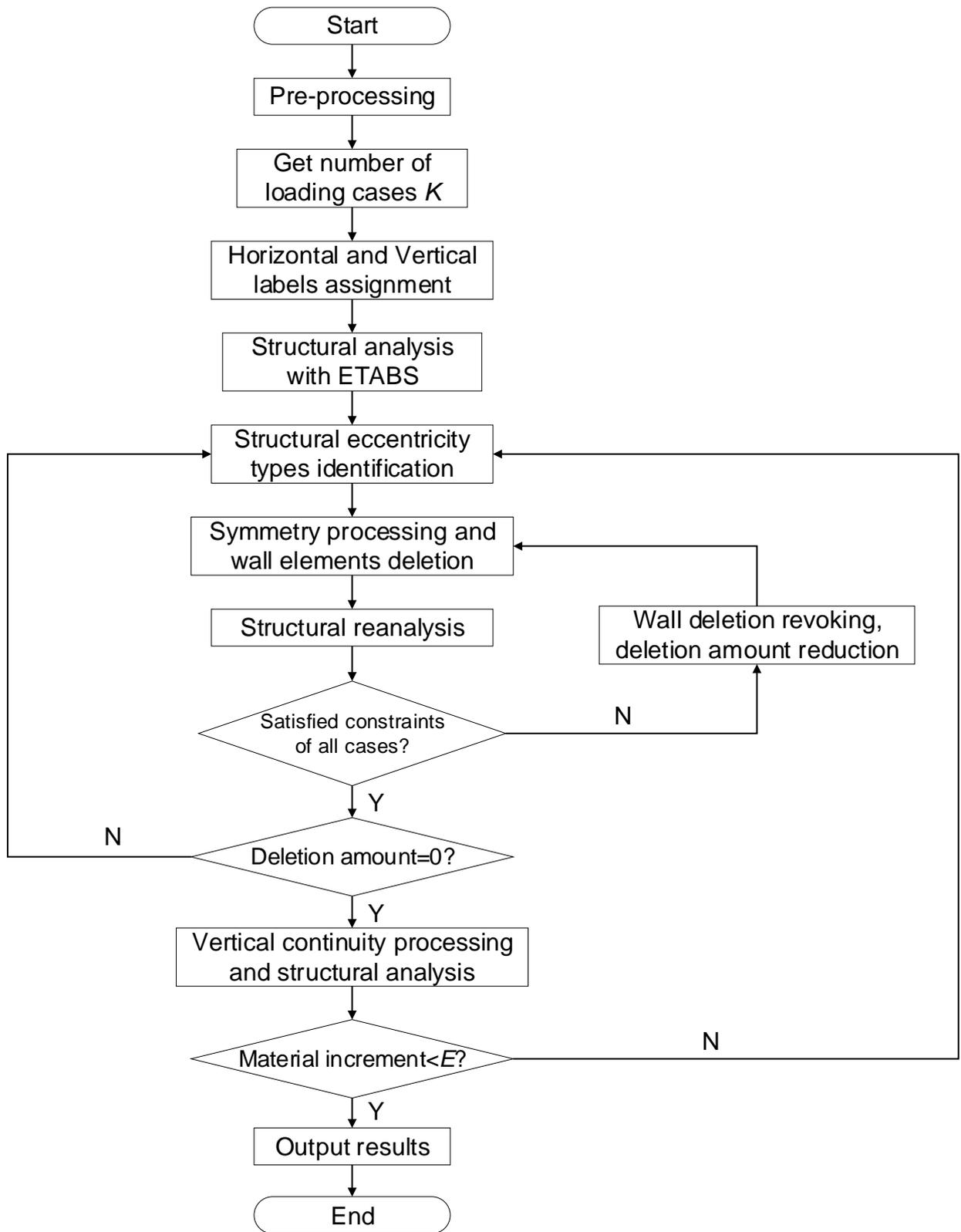


Fig. 8. Flow chart of the direct optimization method under multiple loading cases

### 1 3 Software development

2 Based on the above research, a closed-source optimization software is being developed by the authors using  
3 C++ and the IDE of Microsoft Visual Studio 2015. The GUI framework is based on MFC (Microsoft Foundation  
4 Classes). A screenshot of the current version of the GUI is shown in Fig. 9.

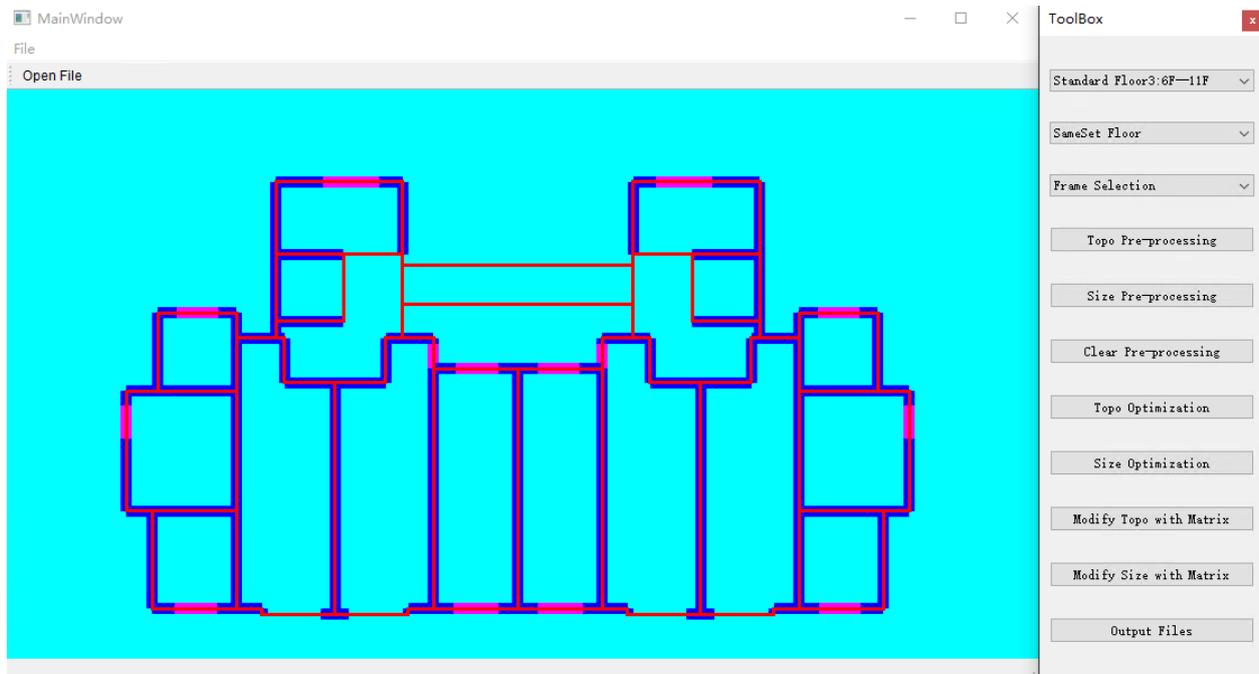


Fig. 9. GUI of the optimization software

5 The framework of the software is shown in Fig. 10. The software for shear wall layout optimization can be  
6 divided into four modules: the pre-processing module, the optimization module, the structural analysis module, and  
7 the data storage and processing module. The pre-processing module has an independent interface where users can  
8 perform the determination of the ground structure, discretization of shear walls, and element freezing. The  
9 optimization module is the core module of the software. It continuously outputs shear wall coding matrices and  
10 transmits them to the structural analysis module, from which it obtains the analysis results. The structural analysis  
11 module uses the ETABS library to perform structural analysis and the interaction between the developed software  
12 and ETABS is provided via the Open Application Programming Interface. As soon as the structural analysis module  
13 receives the shear wall coding matrix transmitted by the optimization module, it modifies the model according to  
14 the matrix, performs structural analyses via ETABS, and sends the results back to the optimization module. The  
15 data storage and processing module can record the optimization process and export it in a fixed format for  
16 subsequent processing.

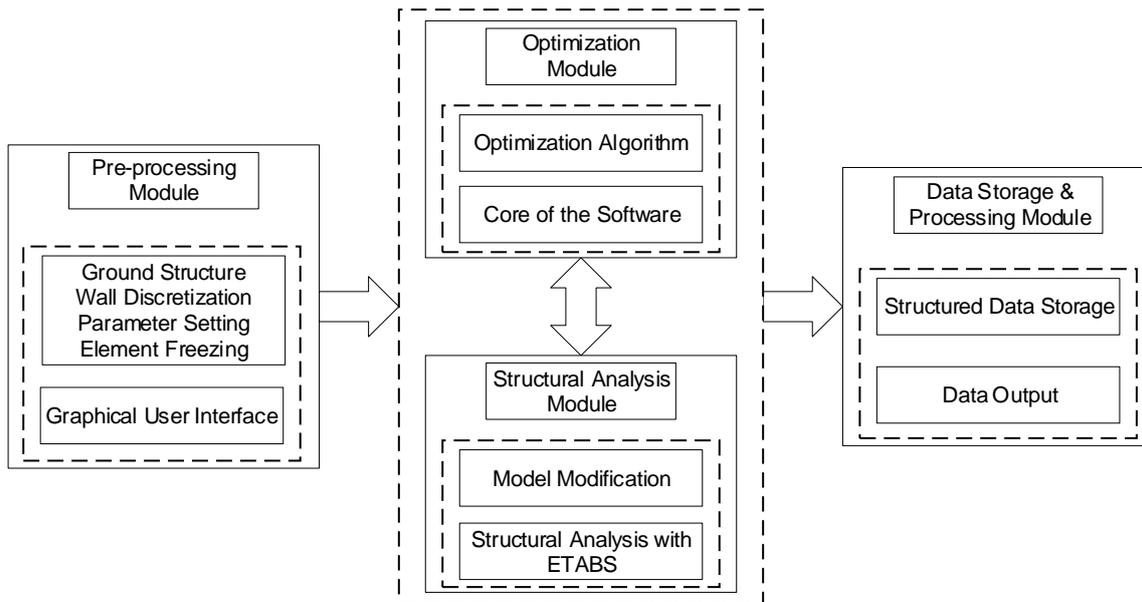


Fig. 10. Framework of the optimization software

## 1 4 Case studies

2 The three models used in this section were all modeled in ETABS. The beams and columns were modeled by  
 3 beam elements, and the shear walls were modeled by shell elements with both in-plane and out-of-plane stiffness.  
 4 The period of the building was obtained by modal analysis using ETABS and varied as the optimization proceeded.  
 5 The damping ratio of all the models is 0.05. According to the Chinese design code of JGJ 3-2010 [41], given the  
 6 basic wind pressure, the wind load on the building could be automatically calculated by ETABS based on the  
 7 exposure width, the shape coefficient, the building height, etc. The wind load was applied in a static manner. The  
 8 seismic response of high-rise buildings is generally calculated using the response spectrum method, and it is only  
 9 for particularly irregular or important high-rise buildings that supplementary calculations using time-history  
 10 analysis with multiple seismic waves are required. Therefore, the response spectrum method was used for seismic  
 11 response calculation in this paper. Given the seismic intensity and the damping ratio, the design response spectrum  
 12 could be obtained and then used for structural response calculation with the modal analysis results. The typical  
 13 dimensions of the components, the basic wind pressure, and the seismic intensity of each model will be given in the  
 14 subsequent sections.

### 15 4.1 Case-by-case optimization – model-1

16 A symmetrical shear wall building structure (model-1) with 30 floors and 86.97 m in height was created from a  
 17 real high-rise residential building design project. The approximate dimensions of the building plan are  $27.9 \times 15.5$   
 18 m. The strength classes of concrete for constructing the main lateral force-resisting components are C40 for the

1 1-5th floors, C35 for the 6-11th floors, and C30 for the 12-30th floors. The typical widths of shear walls are 180  
 2 mm, 200 mm, and 250 mm. The typical breadths and depths of beams are  $180 \times 400$  mm,  $200 \times 400$  mm, and  $200 \times$   
 3  $500$  mm. The information about typical loading actions can be seen in Table 3. The structural engineers proposed in  
 4 advance a shear wall structural design according to the floor plan prearranged by the architects, shown in Fig. 11.  
 5 The maximum story drift of the design under all the loading cases is 1/996 in the X direction, appearing on the 13th  
 6 floor, and the maximum story drift is 1/990 in the Y direction, appearing on the 20th floor. The period ratio of the  
 7 structure is 0.60 and the structural weight is 79806.2 kN, which meets the engineering requirements.

8  
 Table 3. Loading values used for model-1

Dead Load (D)	Live Load (L)	Basic wind pressure (W)	Seismic intensity (SI)
1.9 kN/m <sup>2</sup>	2.0 kN/m <sup>2</sup>	0.36 kN/m <sup>2</sup>	7 (0.15 g)

9

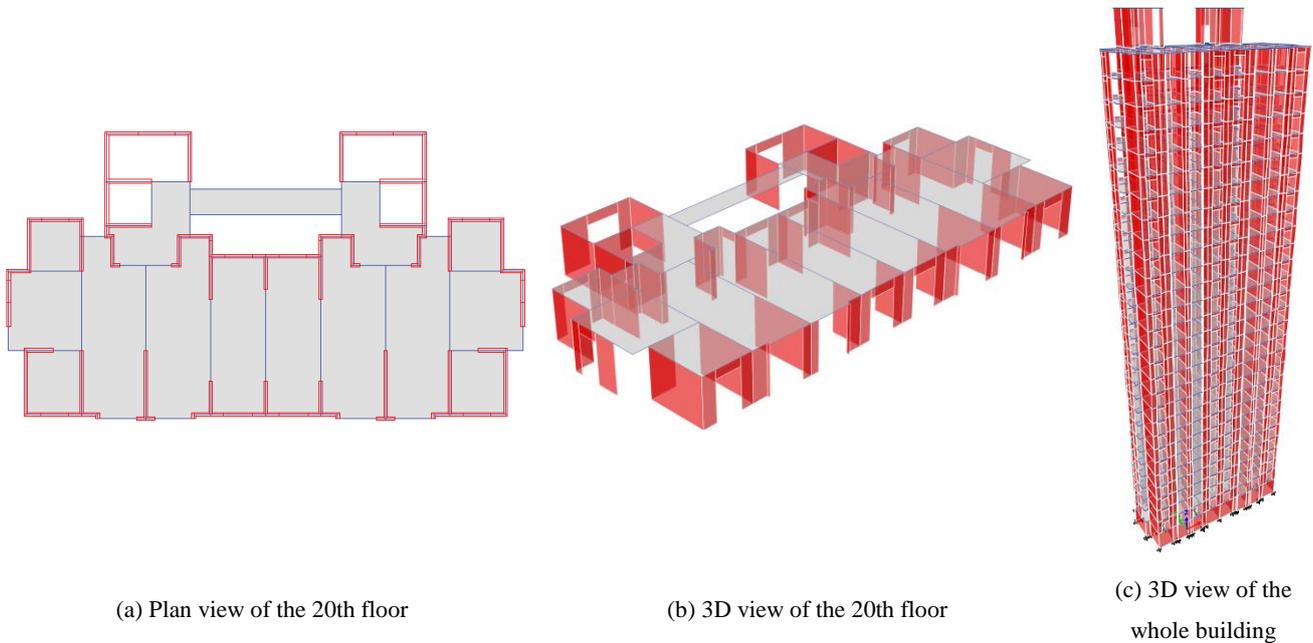


Fig. 11. The design of the structural layout of model-1 by engineers

10

11 To optimize the high-rise building, the pre-processing operation was carried out first and the shear walls in the  
 12 ground structure were divided into wall elements with a length of about 1 m. The ground structure and  
 13 discretization of shear walls for model-1 are shown in Fig. 12. This structure full of shear walls is considered as the  
 14 initial design solution, whose structural weight is 89246.3 kN. After the discretization of shear walls, the wall  
 15 elements of the elevator room in the north were frozen, and symmetry identification and continuity identification  
 16 were carried out for the structure. During the optimization process, different standard floors are optimized  
 17 simultaneously and independently, but the structural layouts of the floors from the same standard floor are the same.  
 18 In each iteration of the optimization process, only one structural analysis using ETABS is required.

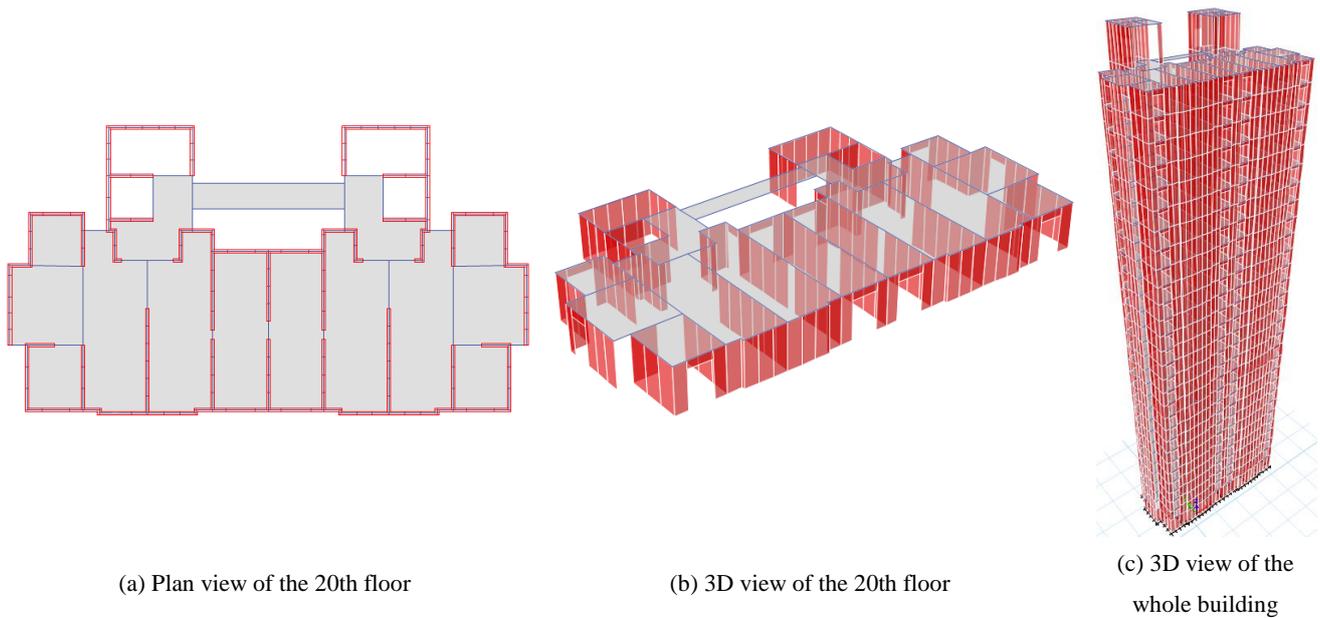
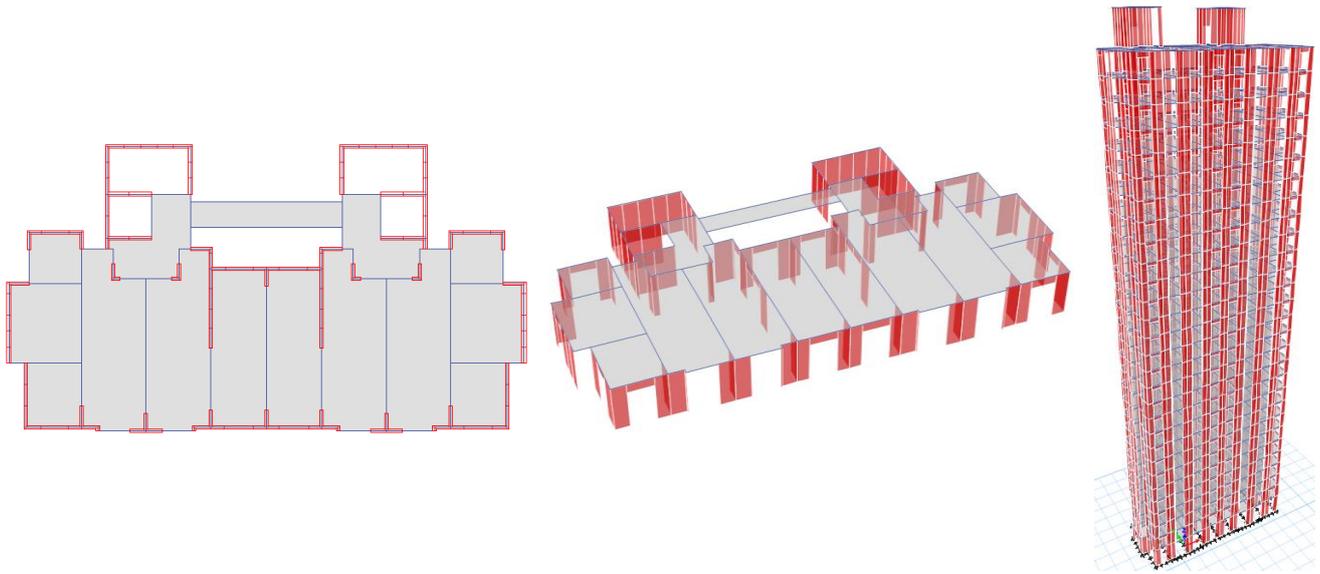


Fig. 12. Ground structure and discretization (wall element length = 1 m) of the shear walls for model-1

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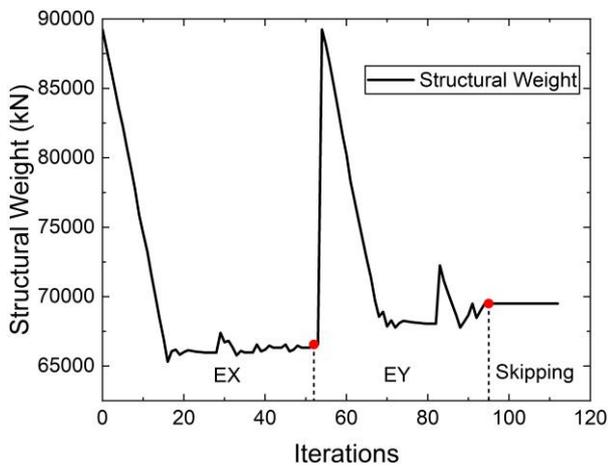
The results obtained from the case-by-case optimization procedure in Section 2.4.1 are shown in Fig. 13. Since the most time-consuming part of the optimization process is the structural analysis in ETABS, the number of structural FE analyses by ETABS when the solution converges is considered as an index to measure the computational efficiency. According to the figure, in the case-by-case optimization procedure, the optimization under the loading case EX is finished after 52 iterations and the optimization under EY is completed after 95 iterations. During the 96-111th iterations, the remaining load cases are skipped after checking calculations as the constraints under the remaining cases are satisfied. Finally, all the constraints are verified and the total number of Finite Element iterations at the time of convergence is 112. The structural weight of the optimized structure is 69499.9 kN, which is 12.9% less compared to the weight of the structure designed by the engineers, as shown in Fig. 11, and 22.1% less than that of the ground structure. In addition, the optimized structure can satisfy the constraints under all the loading cases and the eccentricity of the optimized structure in the Y direction is reduced to 9.3% compared to the value of 28.2% in the initial design solution (Fig. 12). Due to the plan symmetry of the structure in the X direction, the eccentricities in the X direction are both small before and after the optimization. The vertical continuity processing was carried out under all the loading cases in the optimization. Therefore, the shear walls of the final structure are continuous in the vertical direction.



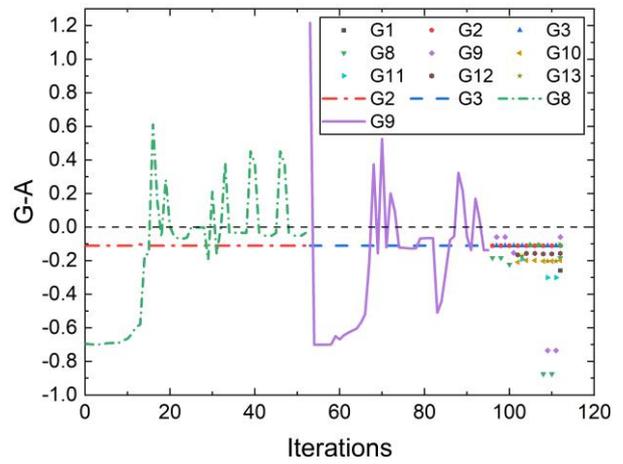
(a) Plan view of the 20th floor

(b) 3D view of the 20th floor

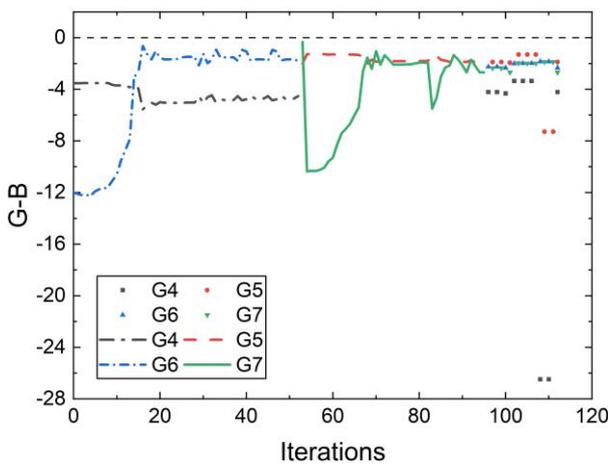
(c) 3D view of the whole building



(d) Structural weight



(e) Partial constraints - A



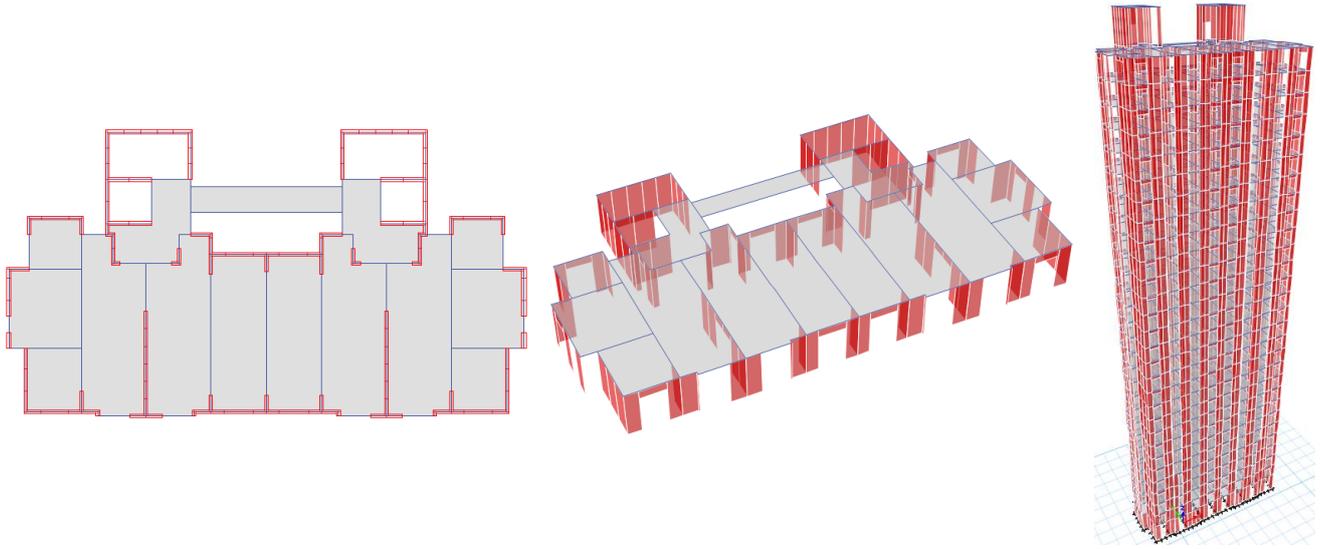
(f) Partial constraints - B

Fig. 13. Optimization results of case-by-case optimization method for model-1 (wall element length = 1m)

1 **4.2 Direct optimization of model-1 under multiple loading cases**

2 To compare the results obtained from the case-by-case optimization method with the results obtained from the  
3 direct multi-case optimization method, the model in Section 4.1 was optimized using the maximum equivalent  
4 stress method and the comprehensive equivalent stress method in Section 2.4.2, respectively. The optimization  
5 results are shown in Fig. 14 and Fig. 15.

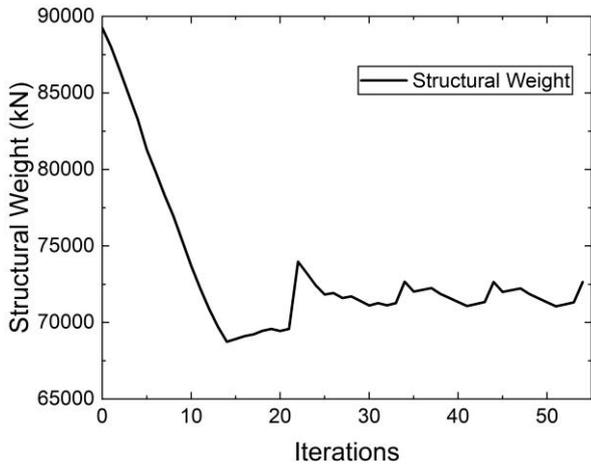
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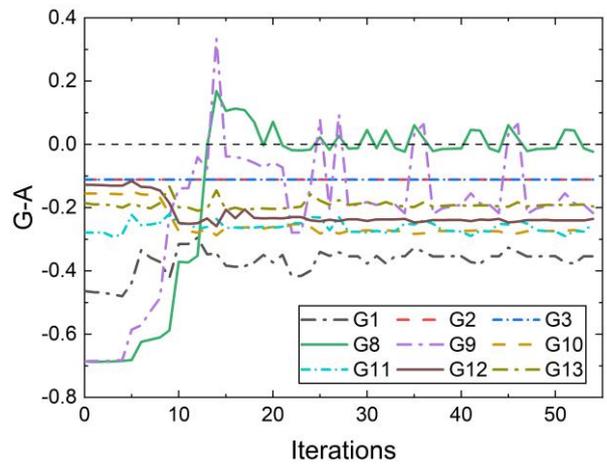
(a) Plan view of the 20th floor

(b) 3D view of the 20th floor

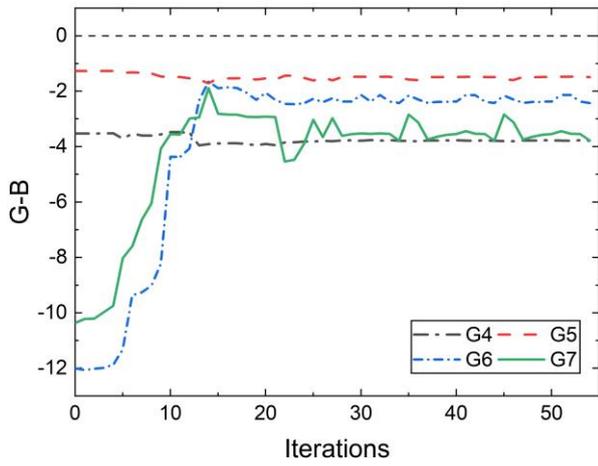
(c) 3D view of the whole building



(d) Structural weight



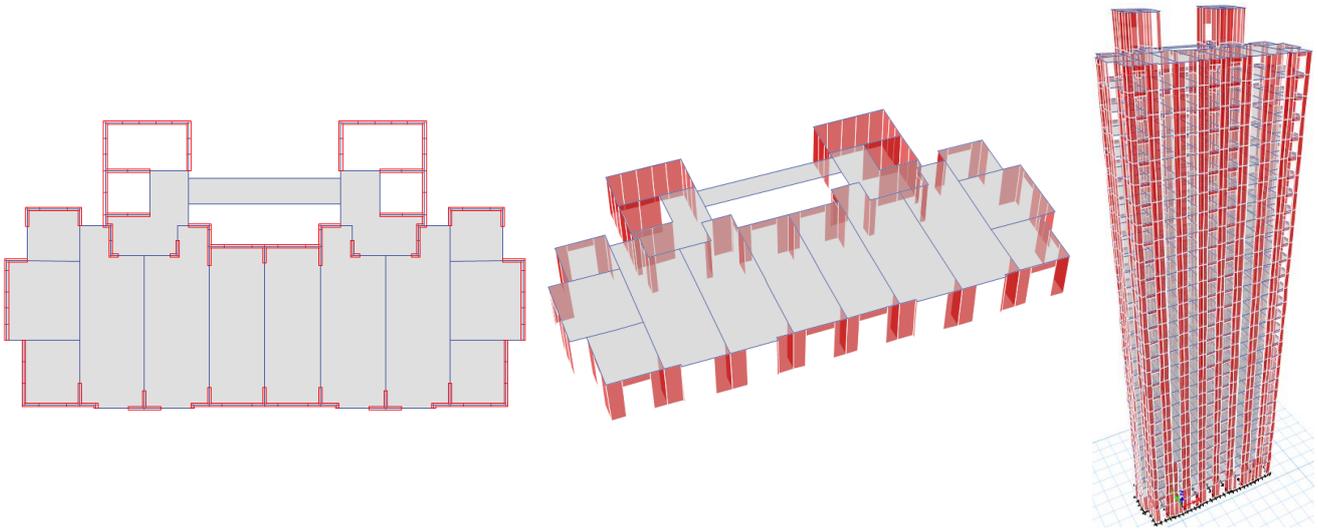
(e) Partial constraints - A



(f) Partial constraints - B

Fig. 14. Optimization results of maximum equivalent stress method for model-1 (wall element length = 1m)

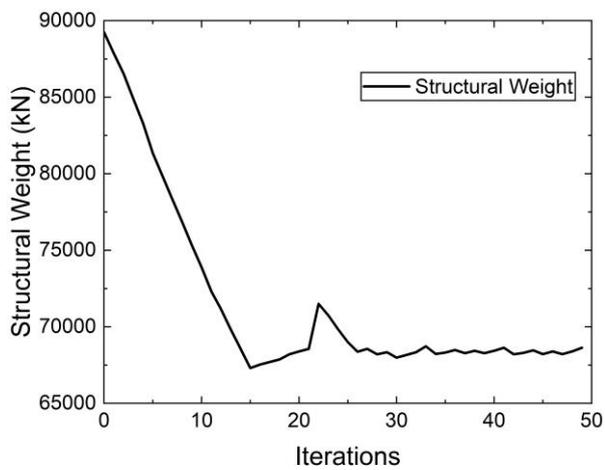
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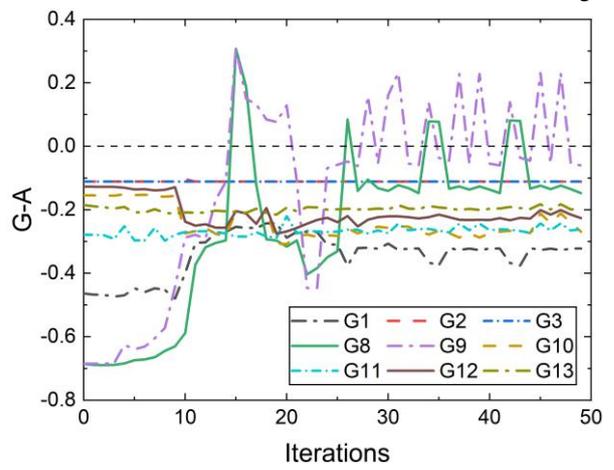
(a) Plan view of the 20th floor

(b) 3D view of the 20th floor

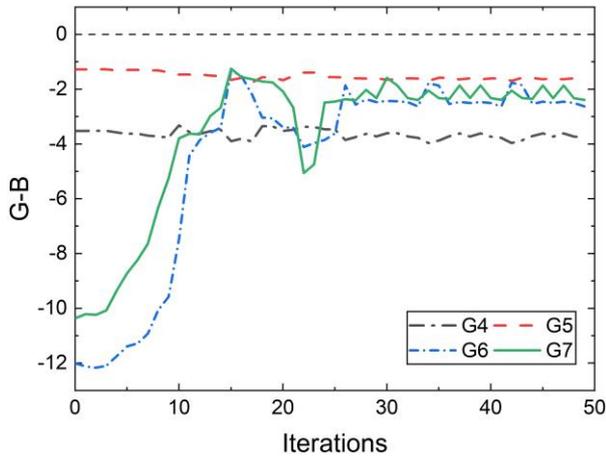
(c) 3D view of the whole building



(d) Structural weight



(e) Partial constraints - A



(f) Partial constraints - B

Fig. 15. Optimization results of comprehensive equivalent stress method for model-1 (wall element length = 1m)

1

2 As shown in the figure, the weight of the structure optimized using the maximum equivalent stress  
 3 optimization method converges after 54 iterations, and the optimal structural weight is 72646.7 kN. The  
 4 optimization results can satisfy the constraints under all the loading cases, and the eccentricity of the structure in  
 5 the Y direction is reduced to 2.5%. Smaller eccentricity generally means smaller torsional effect. The total number  
 6 of structural iterations at the time of convergence is almost halved compared to the case-by-case optimization  
 7 method, but the structural weight is not as good as the results presented in Section 4.1, indicating that the maximum  
 8 equivalent stress can not fully reflect the contribution of a wall element to all the loading cases. The weight of the  
 9 structure optimized using the comprehensive equivalent stress optimization method converges after 49 iterations  
 10 with an optimal structural weight of 68632.6 kN, which is better than the results in Section 4.1. The number of  
 11 structural calculations at the time of convergence is reduced by 55.4% compared to the case-by-case optimization  
 12 method. The optimization results satisfy all the constraints with a Y-directional eccentricity of 8.9% and a structural  
 13 natural vibration period of 2.27 s. Generally, based on engineering design experience, a suitable natural vibration  
 14 period for shear wall structures is  $0.05ST-0.08ST$  [42] ( $ST$  denotes the total number of stories), which is 1.5 s-2.4 s  
 15 for model-1, and the optimization results are well coincident within the range. It is demonstrated that the direct  
 16 multi-case optimization method with comprehensive equivalent stress has obtained the best structural performance  
 17 and will be used in subsequent case studies.

### 18 4.3 Effect of discrete element length of a shear wall

19 To study the sensitivity of the approach to the discretization process, the shear walls were discretized at  
 20 lengths of 2-3 m and 0.5 m, apart from the discrete length of 1 m presented in Section 4.1. The ground structure and  
 21 discretization of shear walls with lengths of 2-3 m and 0.5 m are shown in Fig. 16 and Fig. 17, respectively. They

1 were both optimized using the direct multi-case optimization method with comprehensive equivalent stress.  
2

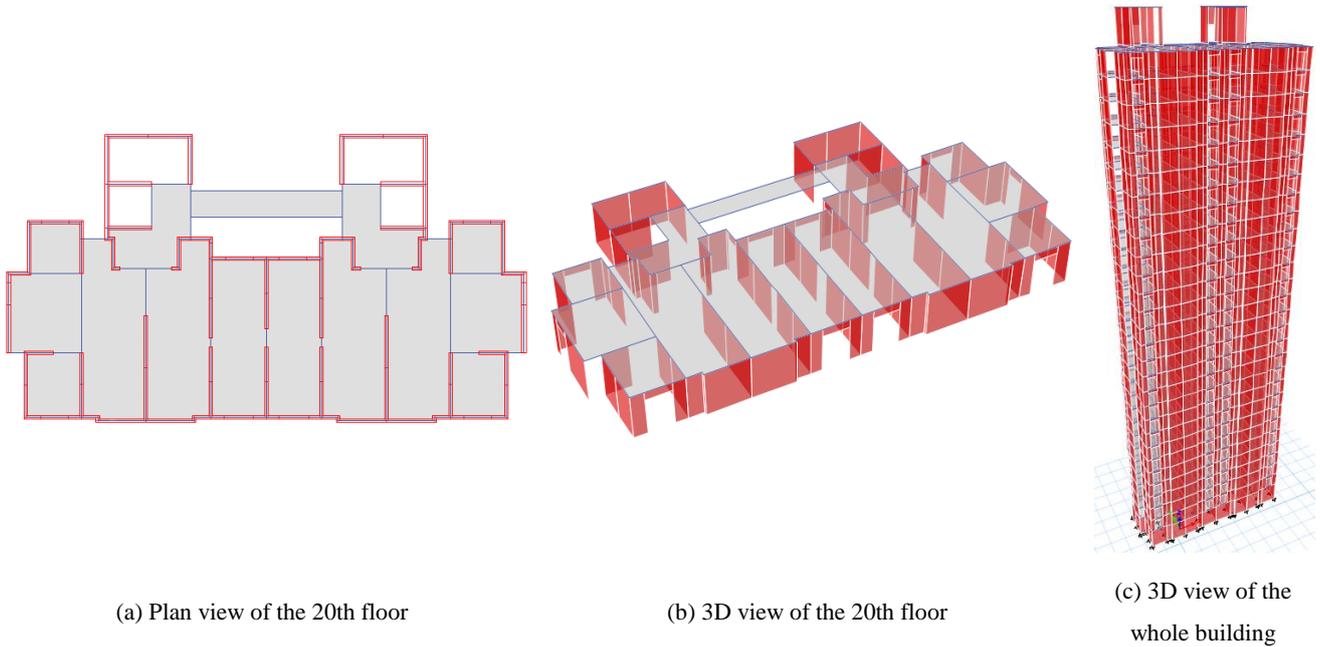


Fig. 16. Ground structure and discretization (wall element length = 2-3 m) of the shear walls for model-1

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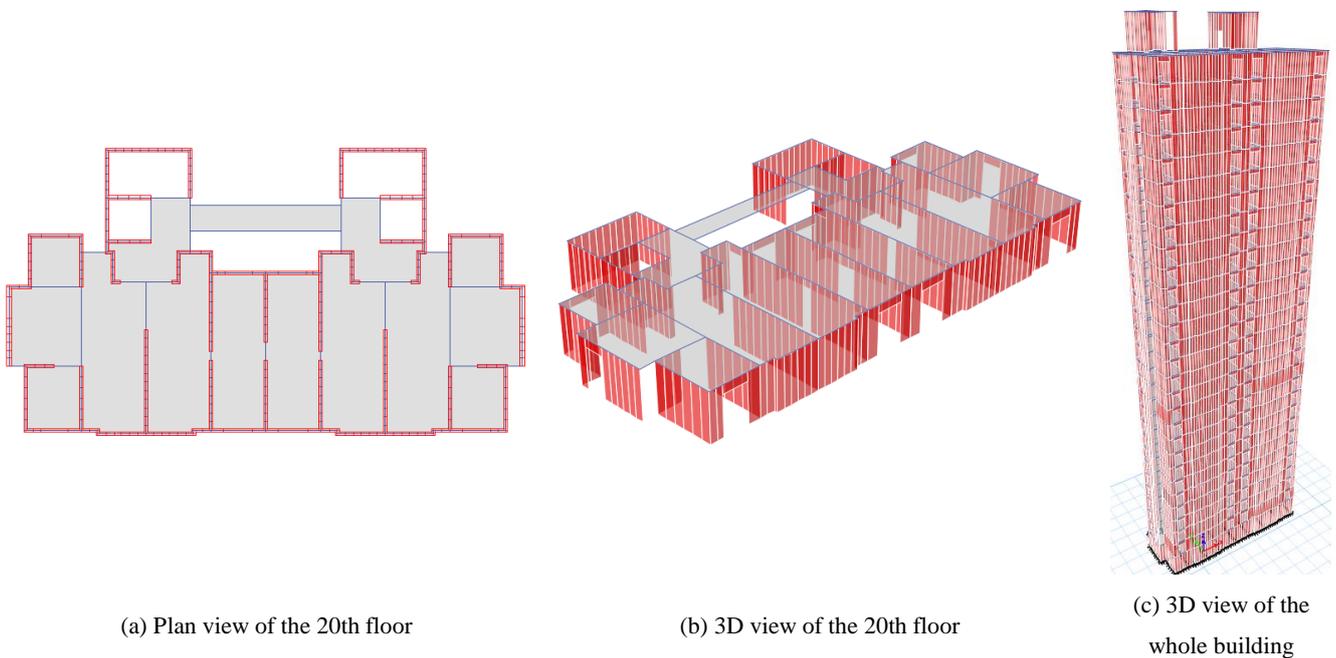


Fig. 17. Ground structure and discretization (wall element length = 0.5 m) of the shear walls for model-1

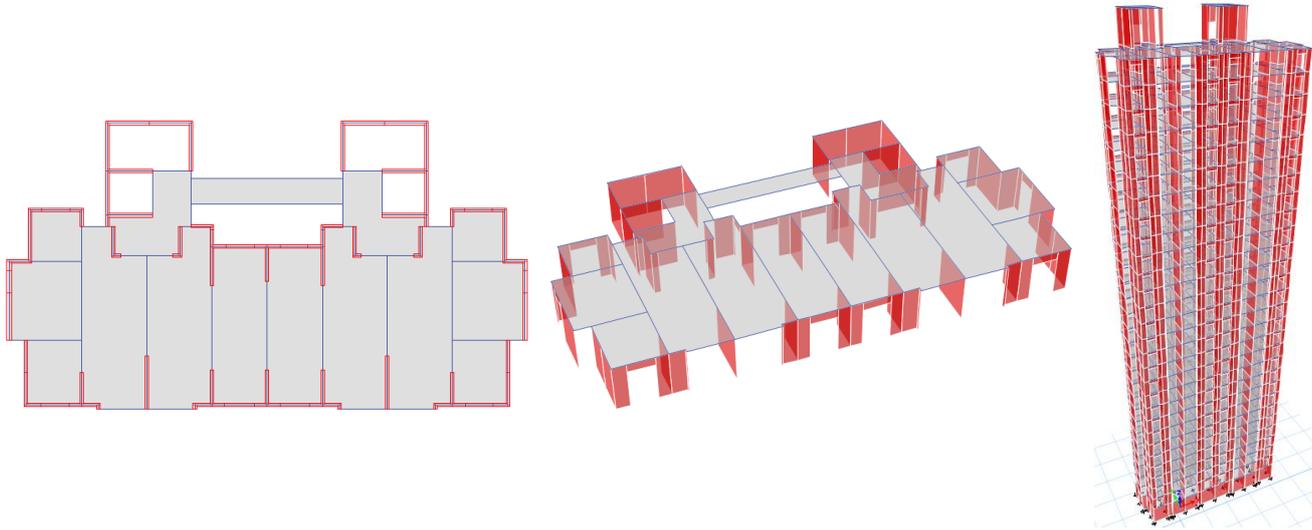
4

5 The optimization results are shown in Fig. 18 and Fig. 19. When the shear walls are discretized at a length of  
6 2-3 m, the structural weight converges to an optimized value after 16 iterations. The structural weight of the  
7 optimized structure is 73946 kN. However, when the shear walls are discretized into wall elements with a length of  
8 0.5 m, the structural weight converges after 78 iterations, 59.2% more than the iteration number of 49 used when

1 the element length equals 1 m. The structural weight of the optimized structure is 66670.9 kN, only 2.9% better  
 2 than the optimization results with an element length of 1 m.

3 The optimization of shear wall layout is an optimum design problem with discrete variables. As the more  
 4 finely the shear walls are discretized, the smaller structural weight can be obtained subjected to the same design  
 5 constraints. This is because, with smaller wall element lengths, the distribution of shear walls can be more flexible  
 6 in localized regions of the floor plan. However, the computational cost in the optimization process is subjected to a  
 7 sharp increase with refined element mesh. Moreover, a small division of a shear wall may cause short-leg shear  
 8 walls, which are of low efficiency in structural performance and construction, to be more easily generated. A shear  
 9 wall element with a large value of length may not be appropriate to search for the optimum solutions as there is less  
 10 flexibility in the optimization process. Therefore, the discrete length of 1 m is more feasible for shear wall layout  
 11 optimization.

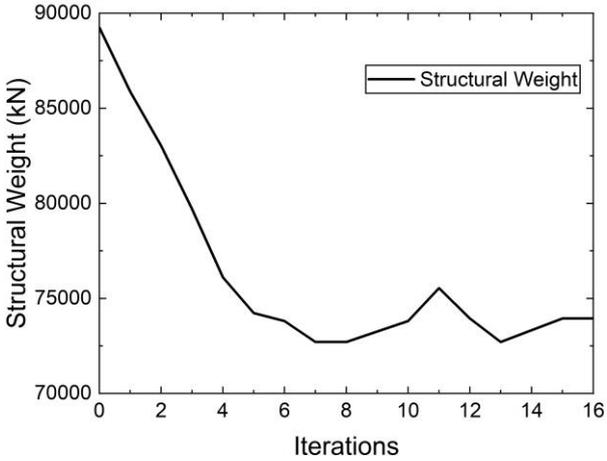
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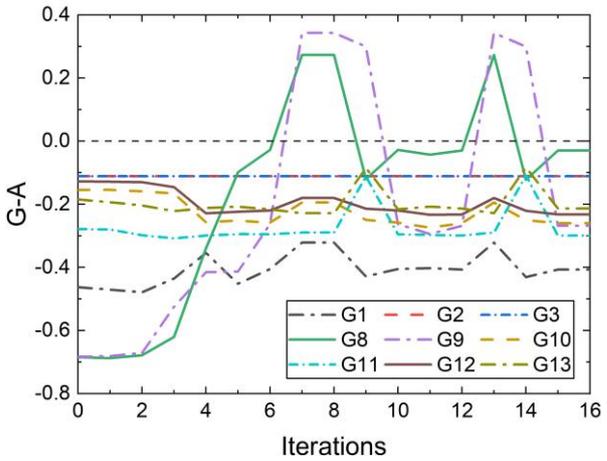
(a) Plan view of the 20th floor

(b) 3D view of the 20th floor

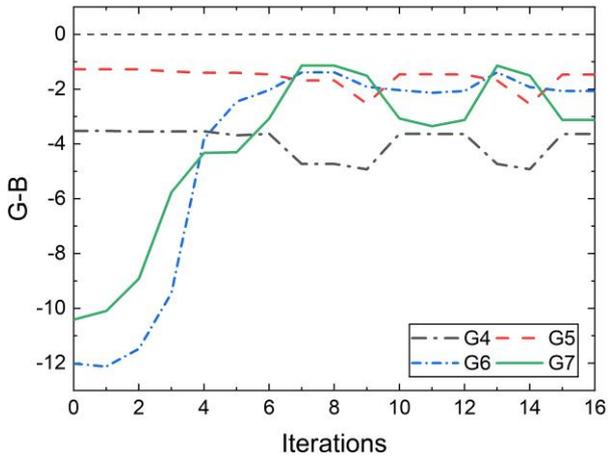
(c) 3D view of the whole building



(d) Structural weight



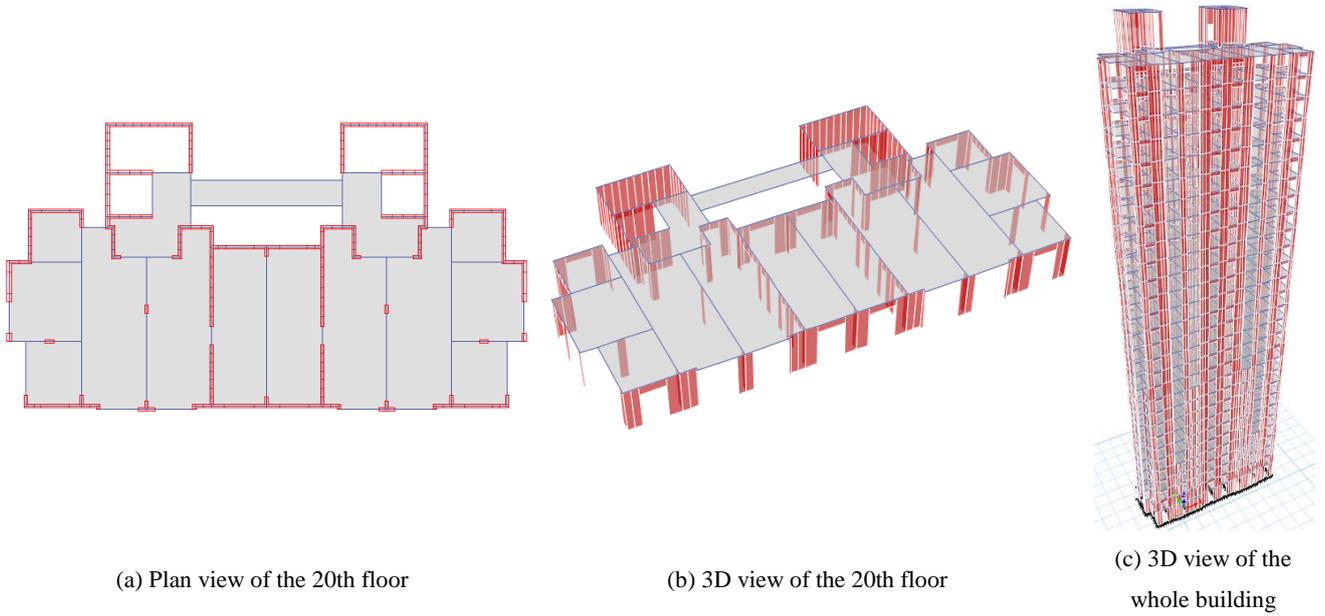
(e) Partial constraints - A



(f) Partial constraints - B

Fig. 18. Optimization results of comprehensive equivalent stress method for model-1 (wall element length = 2-3 m)

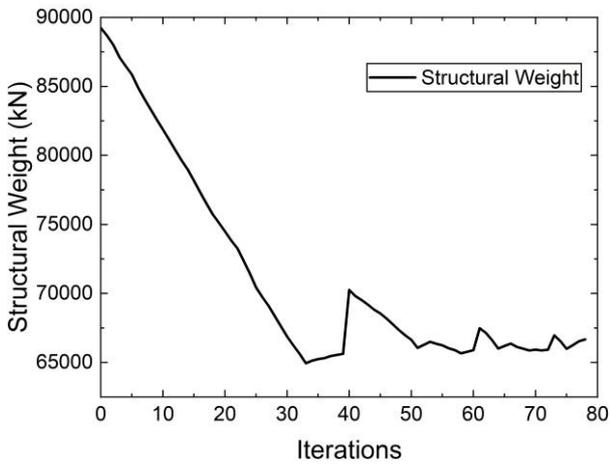
1



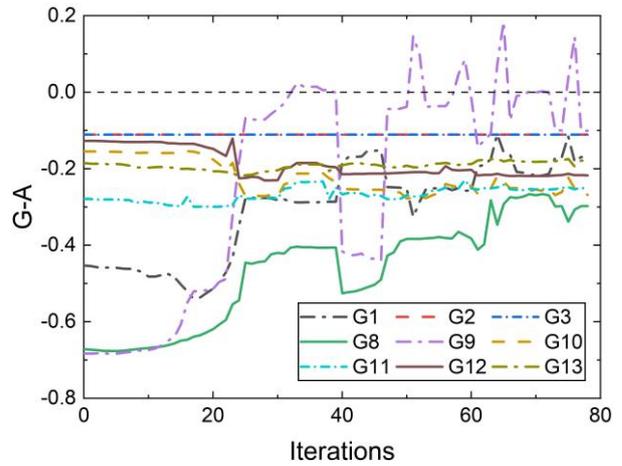
(a) Plan view of the 20th floor

(b) 3D view of the 20th floor

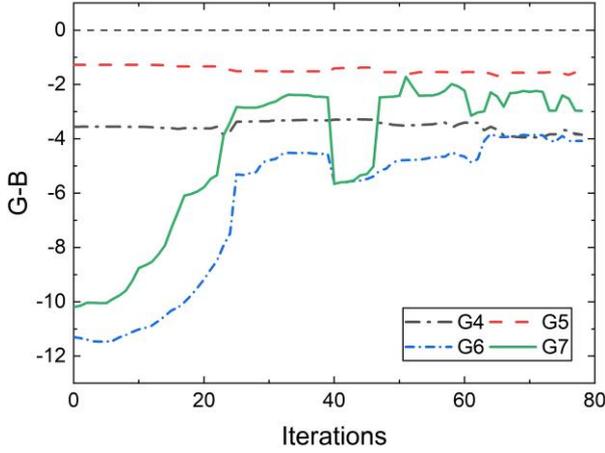
(c) 3D view of the whole building



(d) Structural weight



(e) Partial constraints - A



(f) Partial constraints - B

Fig. 19. Optimization results of comprehensive equivalent stress method for model-1 (wall element length = 0.5 m)

#### 4.4 Comparison with results from the general Genetic Algorithm

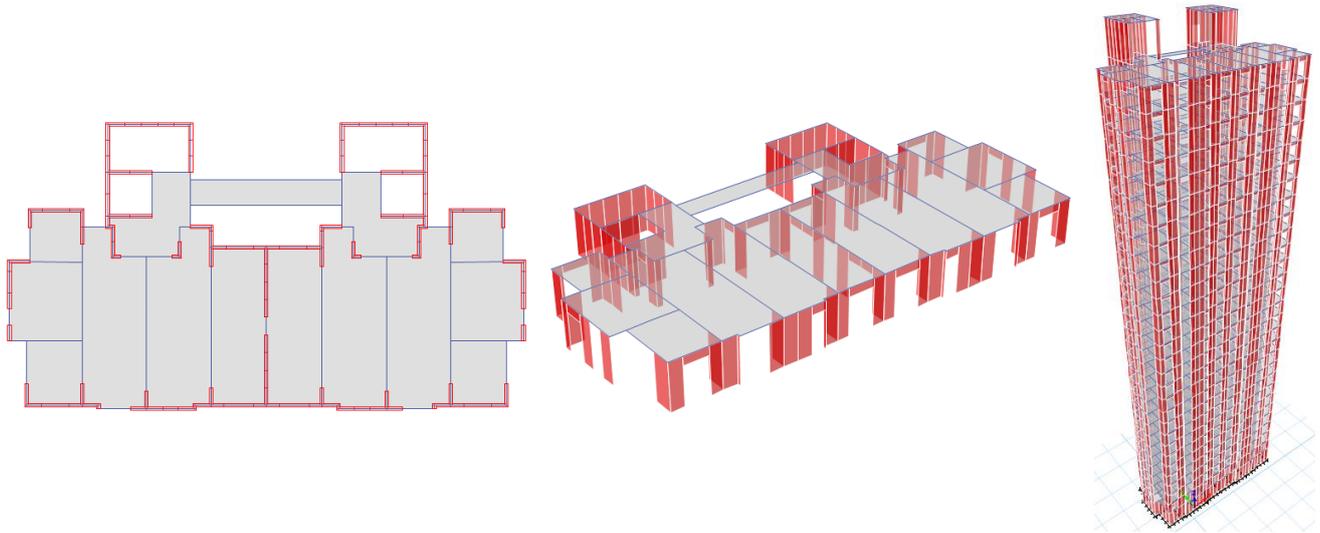
The Genetic Algorithm is a stochastic search heuristic that models the natural selection process and reproduces different structural topologies to derive the fittest design option, and it has been widely used to solve various optimization problems [38]. In this section, the general Genetic Algorithm is adopted to optimize the shear wall layout to compare with those results presented in Section 4.2. The pre-processing operation is the same and the parameter settings for GA are shown in Table 4. To simplify the Genetic Algorithm optimization process, the same shear wall layout was used for floors 2-29, with the same wall thickness and concrete strength as the initial design presented in Section 4.1.

Table 4. Parameters for the general Genetic Algorithm (model-1, wall element length = 1 m)

Generation size $N_{size}$	Maximum number of generations $N_{gen}$	Crossover rate $r_{cross}$	Mutation rate $r_{mut}$
1000	15	0.8	0.1

The optimization results of the general Genetic Algorithm are shown in Fig. 20. It is shown that the structural weight converges after 7 generations and the total number of structural calculations at convergence is around 7000, which took nearly half a month since each structural analysis of model-1 took about two minutes. The structural weight of the optimized building structure is 69225.7 kN, which is 0.86% inferior to the results using the direct multi-case optimization method with comprehensive equivalent stress in Section 4.2 due to the restriction of the same shear wall layout for floors 2-29. Moreover, because the shear wall layout optimization problem is a discrete variable optimization problem with exponential complexity, the computational cost of using a general Genetic Algorithm is very large, even if the problem is simplified. In this paper, based on an extended Evolution Structural Optimization method and conceptual design, the complex combinatorial optimization problem is transformed into a structural stress analysis problem, and the problem complexity is significantly reduced. In terms of model-1, the

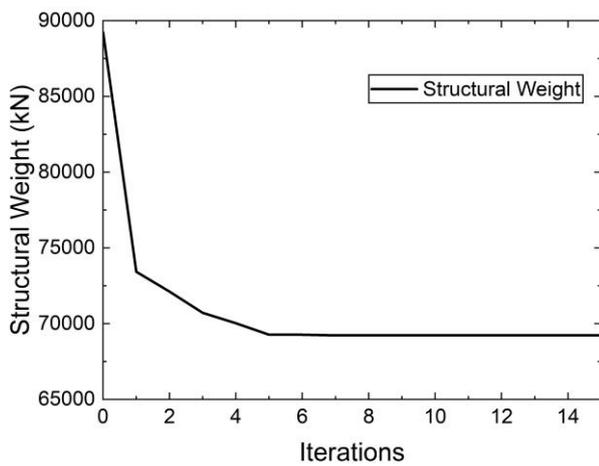
- 1 number of structural analyses of the direct multi-case optimization method with comprehensive equivalent stress is
- 2 only 1/143 of that of the general Genetic Algorithm, demonstrating its high efficiency.



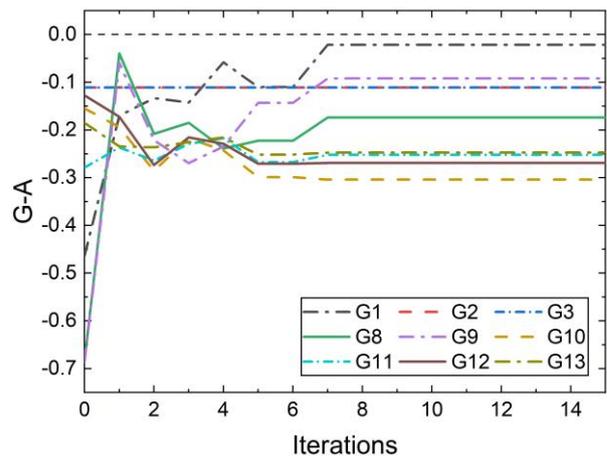
(a) Plan view of the 20th floor

(b) 3D view of the 20th floor

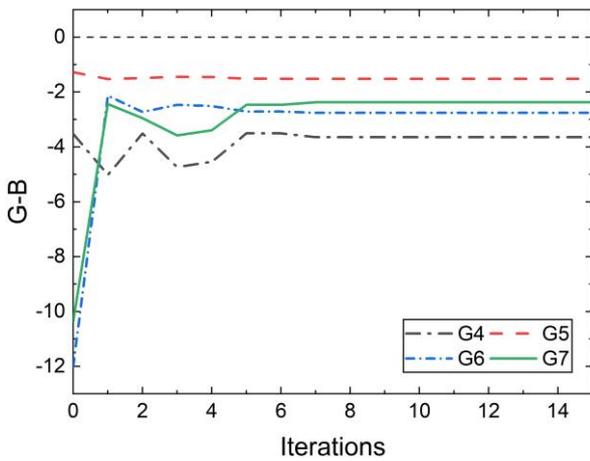
(c) 3D view of the whole building



(d) Structural weight



(e) Partial constraints - A



(f) Partial constraints - B

Fig. 20. Optimization results of the general Genetic Algorithm for model-1 (wall element length = 1 m)

## 4.5 Optimization of model-2 and model-3

In this section, two more shear wall building structures with different building layouts and different loading scenarios are investigated to demonstrate the applicability and efficiency of the proposed methodology.

Model-2 is a shear-wall residential building with 35 floors and is 106 m in height. The dimensions of the building plan are about  $50.5 \times 16.4$  m. The grades of concrete for the main lateral force-resisting components are C50 for the 1-20th floors, C45 for the 21-27th floors, and C40 for the 28-35th floors. The typical widths of shear walls are 200 mm, 300 mm, and 500 mm. The typical breadths and depths of beams are  $200 \times 400$  mm,  $200 \times 500$  mm, and  $200 \times 800$  mm. The typical breadths and depths of columns are  $400 \times 400$  mm,  $500 \times 1000$  mm, and  $500 \times 1200$  mm. The typical values and loading types are shown in Table 5.

Table 5. Loading values used for model-2

Dead Load (D)	Live Load (L)	Basic wind pressure (W)	Seismic intensity (SI)
1.3 kN/m <sup>2</sup>	2.0 kN/m <sup>2</sup>	0.3 kN/m <sup>2</sup>	7 (0.15 g)

10

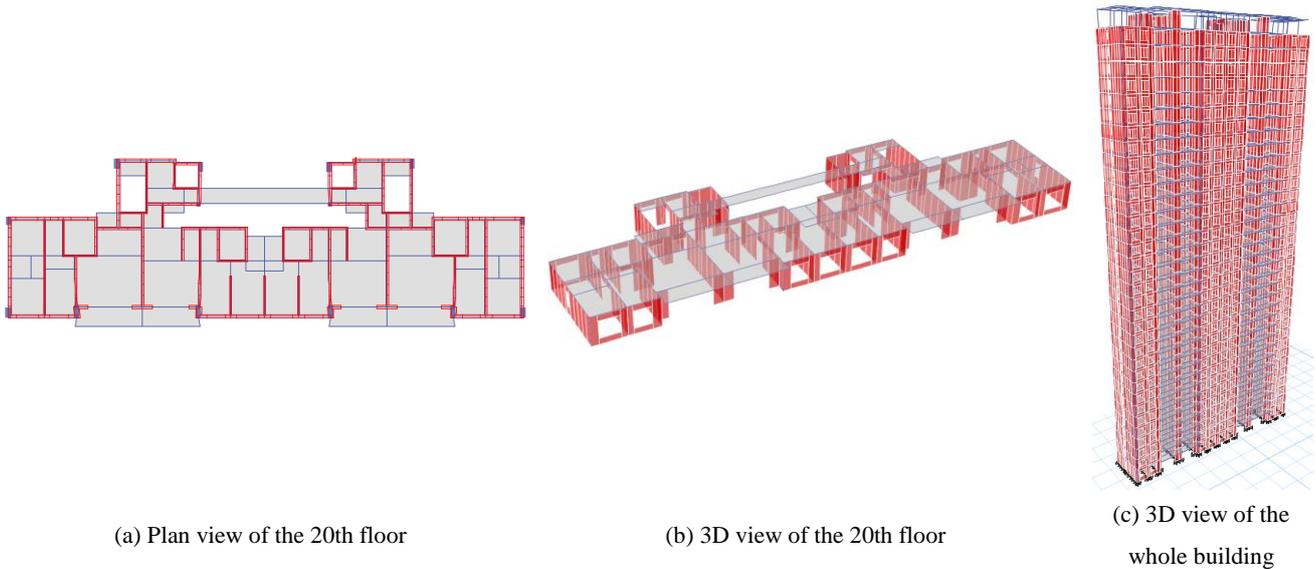


Fig. 21. Ground structure and discretization (wall element length = 1 m) of the shear walls for model-2

During the pre-processing, the shear walls in the ground structure were divided into wall elements with a length of about 1 m. The ground structure and discretization of shear walls for model-2 are shown in Fig. 21. The maximum story drift of all floors under both wind and earthquake load cases is  $1/2259$  in the X direction, appearing on the 16th floor, and the maximum story drift is  $1/1441$  in the Y direction, appearing on the 28th floor. The period ratio is 0.64 and the original structural weight is 248490 kN. It can be seen that the initial solution is a little far from the limits on drift ( $1/1000$ ) and period (0.9), indicating the initial layout is too conservative. The optimization

1 results of model-2 using the direct multi-case optimization method with comprehensive equivalent stress are shown  
 2 in Fig. 22.

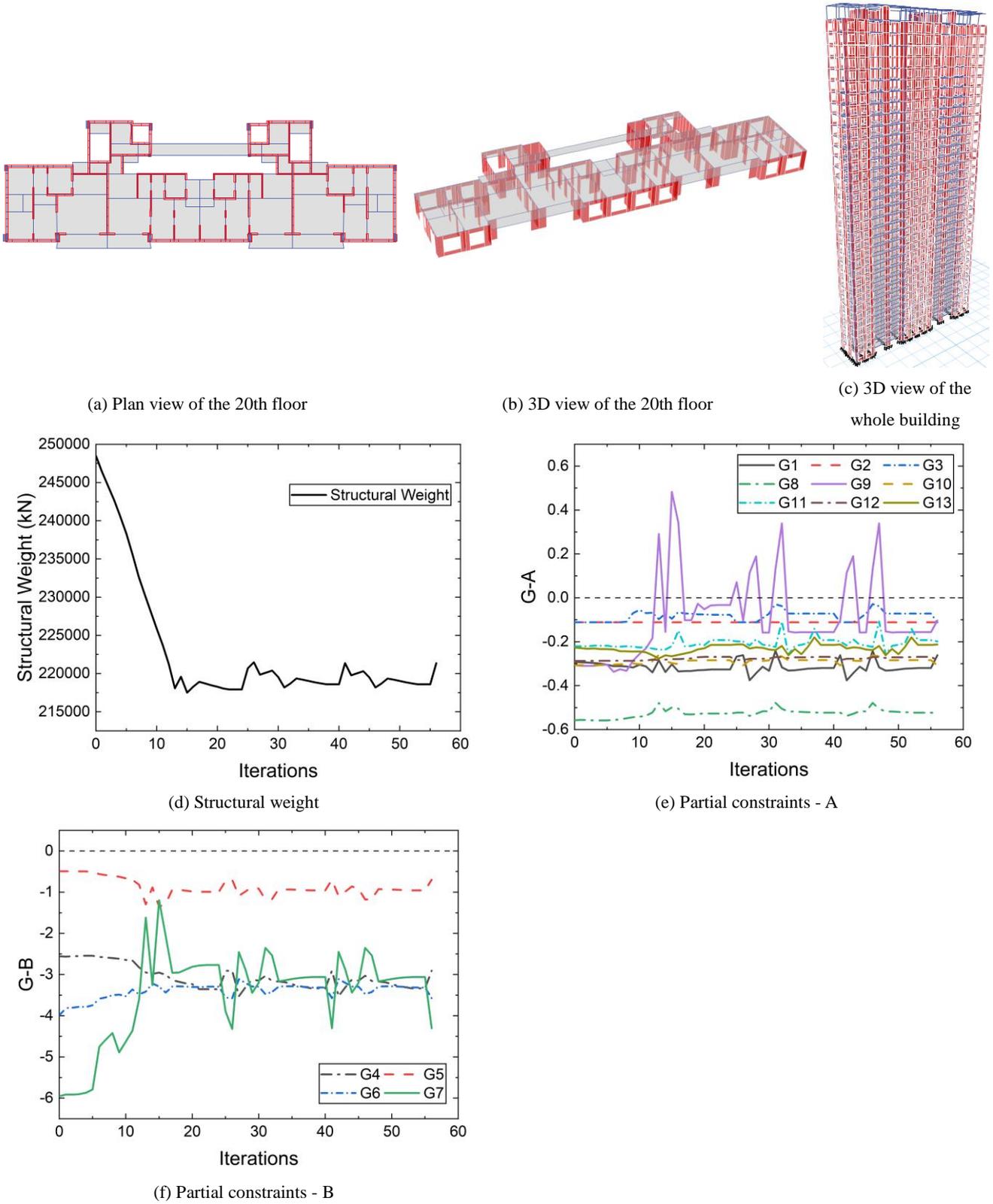


Fig. 22. Optimization results of model-2 (wall element length = 1 m)

3

1 According to Fig. 22, the structural weight gradually decreases and converges after 56 iterations. The structural  
 2 weight of the optimized structure is 221367 kN, which is 10.9% less compared to the weight of the initial solution  
 3 and the story drift in the Y direction is closer to the limit. Due to the large length-width ratio of the structure in its  
 4 plan, the story drift in the X direction is not sensitive to the shear wall layout, and the optimization regarding story  
 5 drift is mainly controlled by the story drift in the Y direction. The optimization results can satisfy all the design  
 6 constraints, with eccentricities of 0.01% in the X direction and 5.7% in the Y direction. The natural vibration period  
 7 is 2.06 s, which is well coincident with the recommended natural period of 1.75 s-2.8 s [42].

8 Model-3 is a real case shear wall hotel with 30 floors and is 101.4 m in height. The dimensions of the building  
 9 plan are about 52.8 × 22.4 m. The grades of concrete in the optimization process for the main lateral force-resisting  
 10 components are C60 for the 1-7th floors, C55 for the 8-11th floors, C50 for the 12-22nd floors, C45 for the 23-27th  
 11 floors, and C40 for the 28-30th floors. The typical widths of shear walls are 200 mm, 250 mm, and 300 mm. The  
 12 typical breadths and depths of beams are 200 × 600 mm, 250 × 600 mm, and 400 × 800 mm. The typical breadths  
 13 and depths of columns are 600 × 600 mm and 600 × 700 mm. The information about typically applied actions is  
 14 shown in Table 6.

Table 6. Loading values used for model-3

Dead Load (D)	Live Load (L)	Basic wind pressure (W)	Seismic intensity (SI)
1.3 kN/m <sup>2</sup>	2.0 kN/m <sup>2</sup>	0.3 kN/m <sup>2</sup>	7 (0.15 g)

16

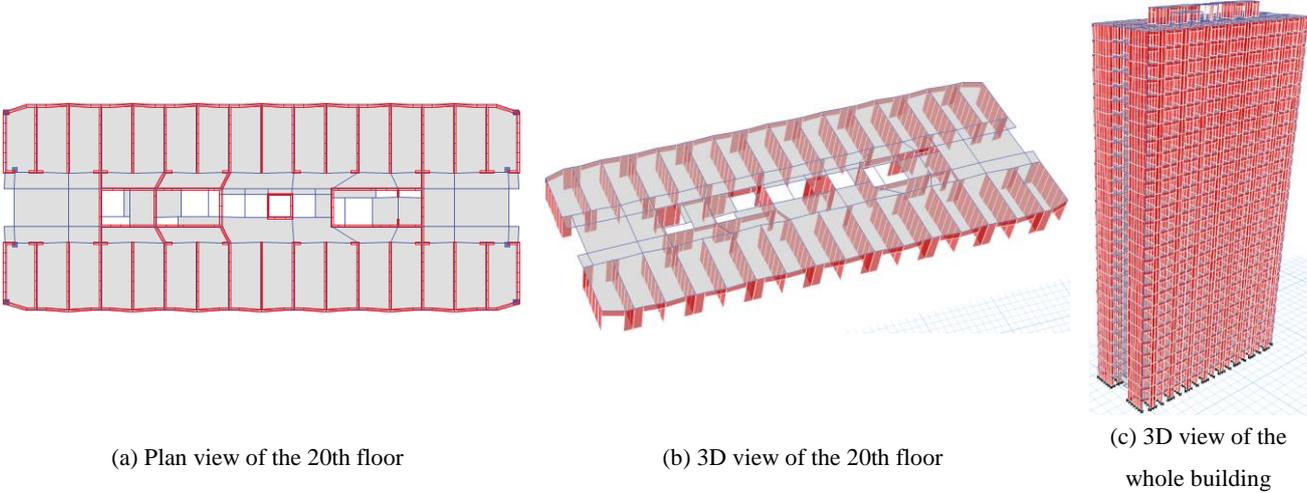
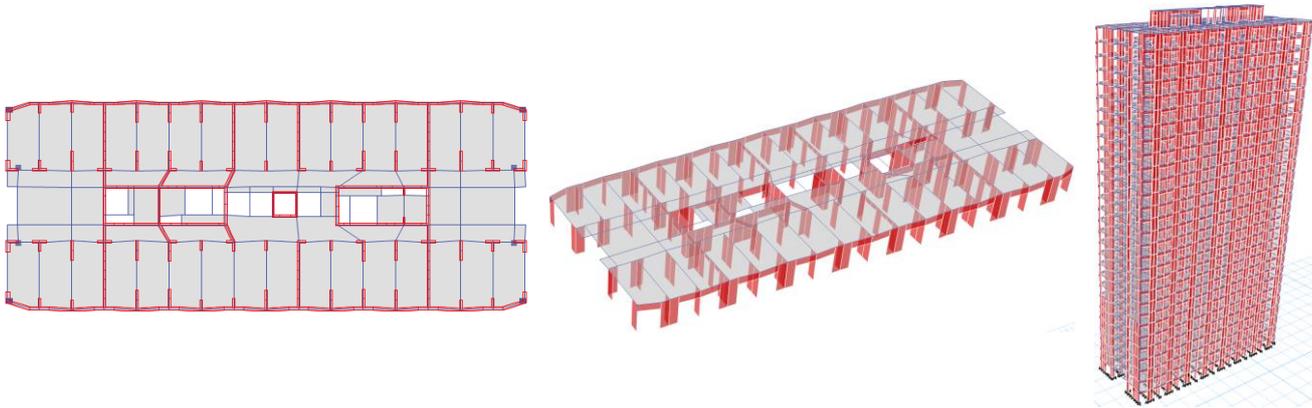


Fig. 23. Ground structure and discretization (wall element length = 1 m) of the shear walls for model-3

17

1 Unlike model-1 and model-2, model-3 is strictly an unsymmetrical structure. The shear walls in the ground  
 2 structure were also divided into wall elements with a length of about 1 m, and the ground structure and  
 3 discretization of shear walls are shown in Fig. 23. Its maximum story drift of all floors is 1/2281 in the X direction,  
 4 which happened on the 23rd floor, and the maximum story drift is 1/1885 in the Y direction, which happened on the  
 5 18th floor. The period ratio is 0.83 and the initial structural weight is 346785 kN. The drift of the initial solution is  
 6 also far from the limits in both X and Y directions.

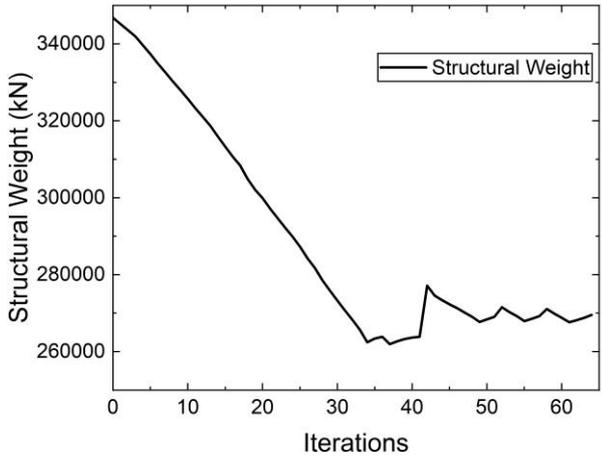
7 Fig. 24 shows the optimization results of model-3 using the direct multi-case optimization method with  
 8 comprehensive equivalent stress. As shown in Fig. 24, the structural weight converges after 64 iterations. The  
 9 structural weight of the optimized structure is 269566 kN, which is 22.3% less than that of the initial solution. The  
 10 story drift in the Y direction becomes closer to the limit, but there is almost no change in the story drift in the X  
 11 direction due to the large length-width ratio of the structure. The optimization results can also satisfy all the design  
 12 constraints, with an eccentricity of 0.92% in the X direction and 0.46% in the Y direction. The natural vibration  
 13 period is 1.63 s, which is within the scope of the recommended value of 1.5 s-2.4 s [42].



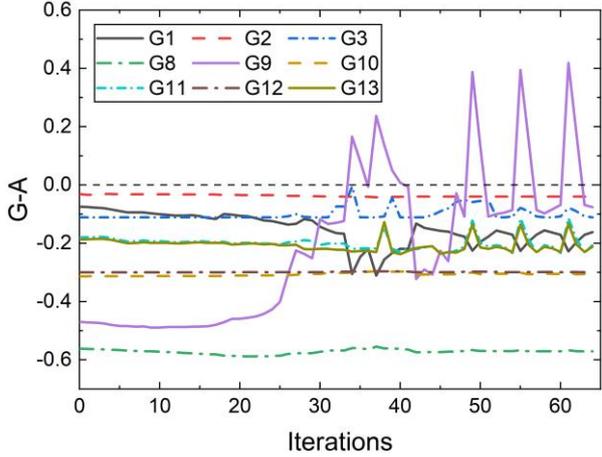
(a) Plan view of the 20th floor

(b) 3D view of the 20th floor

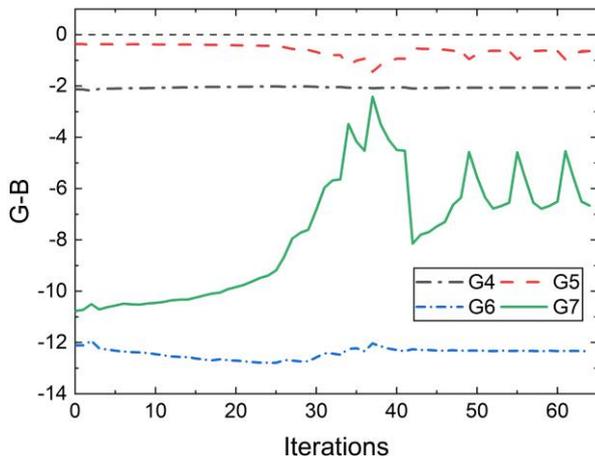
(c) 3D view of the whole building



(d) Structural weight



(e) Partial constraints - A



(f) Partial constraints - B

Fig. 24. Optimization results of model-3 (wall element length = 1 m)

## 1 5 Conclusions

### 2 5.1 Summary of contributions

3 This paper presents a new design methodology for shear wall layout optimization of high-rise buildings based  
 4 on an extended ESO method and conceptual design. The shear wall layout optimization problem was first  
 5 transformed into an optimization model with discrete variables and structural design/architecture constraints. Based  
 6 on the defined optimization model, an improved ESO strategy and two multi-case optimization methods were  
 7 proposed to optimize the shear wall layout under multiple loading cases, subject to a series of code-stipulated  
 8 constraints and other conceptual design requirements. Through building plan symmetry and vertical continuity  
 9 processing techniques, the corresponding geometry rationalization was carried out and the requirements for plan  
 10 symmetry and vertical continuity were achieved. The proposed method was developed into an optimization  
 11 software using C++ by integrating the IDE of Microsoft Visual Studio 2015 and the commercially available  
 12 software ETABS for structural analysis, which will be well accepted by engineering practice.

13 In the case studies of the shear wall building structures, the results indicate that the direct multi-case  
 14 optimization method with comprehensive equivalent stress is the best method to optimize the shear wall layout. The  
 15 proposed method is also examined by optimizing several shear wall structures with different plan layouts, different  
 16 sizes of wall elements, and different loading scenarios. It is shown that using the proposed optimization method can  
 17 achieve good structural performance with reduced structural weight by up to 14.0%, regardless of the building  
 18 layouts and the loading scenarios. It is also found that the optimization results could be influenced by the discrete  
 19 element length of shear walls, and 1 m is recommended as a feasible discrete element length considering both  
 20 computational efficiency and effectiveness. Compared to other methods, such as the Genetic Algorithm, the

1 algorithm presented in this paper has great advantages in computational efficiency, which is practical for engineers  
2 to automatically and conceptually optimize shear wall buildings.

### 3 **5.2 Future work**

4 The work presented in this paper focuses on shear wall layout optimization under multiple loading cases and  
5 design constraints. However, the variation in dimensions for shear wall thickness along the height is ignored.  
6 Generally, compared to the variation in wall thickness, the layout of shear walls is more complex and has a greater  
7 impact on the lateral structural system. While the optimization results may be better if both the layouts of shear  
8 walls and the dimensions of the components (e.g., walls, beams, and columns) are considered in the optimization,  
9 there is no suitable optimization method other than heuristic algorithms due to the complexity of the hybrid  
10 problem. Nevertheless, their time-consuming individual evaluations have greatly hindered the application of  
11 heuristic algorithms in practical engineering. In this paper, the shear wall layout is considered preferentially and the  
12 wall thicknesses remain the same as that of the initial design. Future work can achieve a better design by addressing  
13 both the structural topology and the distribution of member sizes simultaneously, or by first exploring the optimal  
14 structural topology and then determining the distribution of member sizes.

15 Since the design response spectrum is the comprehensive response of a series of earthquakes, the seismic  
16 response of high-rise buildings is generally calculated using the response spectrum method, and it is only for  
17 particularly irregular or important high-rise buildings that supplementary calculations using time-history analysis  
18 with multiple seismic waves are required. Nonetheless, the same method can be used to optimize the shear wall  
19 building structures subjected to several earthquakes, and the structural weight of the optimized structure may be  
20 slightly increased compared to the results considering a single earthquake.

21 Further, the optimization process has not considered the failure consequences. Buildings with different  
22 importance levels have different requirements for loads, reliability, and seismic intensity. With increasing severity  
23 of consequences, the target reliability increases, resulting in the design of stronger, more ductile structures. The  
24 current code adopts the Coefficient for Importance of a Structure in the design of buildings, and the optimal layout  
25 may vary according to the importance levels. Future work can optimize the overall cost/benefit of buildings by  
26 considering economic, human and environmental consequences.

27 Moreover, only the main conceptual design constraints under design specifications are considered in this paper,  
28 without considering the limit states of the components and the failure modes of the structure under rare earthquakes.  
29 However, once the conceptual layout of shear walls is selected, the building structures can be analyzed and detailed  
30 more precisely by structural engineers in later stages of design. Thus, future work can improve the methodology by

1 considering various structural types and materials, and optimizing high-rise buildings under limit states to achieve  
2 the default failure mode.

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6 financial support.

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