An IGA-BEM solver for Lifting Flows around Wings

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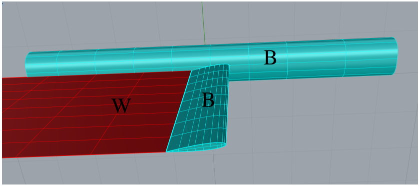
September 20 2019

Overview

Introduction



$S = S_B \cup S_W$ - B:Body, W:Wake

Aircraft Vortex Wake

The Problem

Velocity Potential

$$\mathbf{V}_t = \mathbf{v} + \mathbf{V}$$

$$\mathbf{V}_t = \left(\frac{\partial \Phi_t}{\partial x}, \frac{\partial \Phi_t}{\partial y}, \frac{\partial \Phi_t}{\partial z}\right)$$

$$\Phi_t = \phi + \mathbf{V} \cdot \mathbf{x}$$

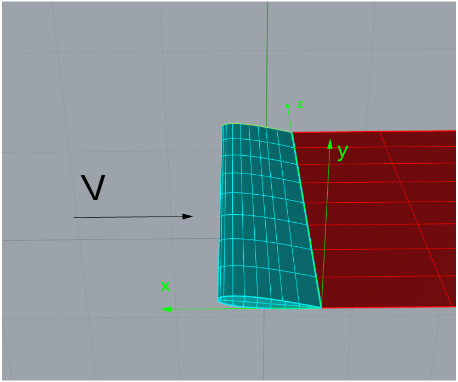
$$\nabla^2 \phi = 0$$

Boundary Conditions

$$\frac{\partial \phi}{\partial n} = -\mathbf{V} \cdot \mathbf{n}, \ S_B$$

$$p_u - p_l = 0, \ S_W$$

$$\frac{\partial \phi_u}{\partial n} - \frac{\partial \phi_l}{\partial n} = 0, \ S_W$$



Boundary Integral Equation (BIE)

2^{nd} Green's identity & kinematic bc's yield:

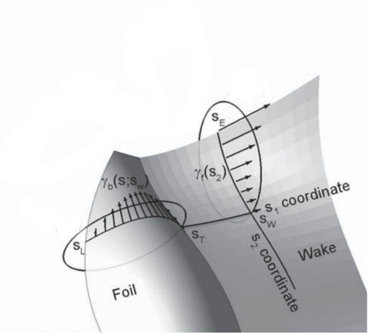
$$2\pi\phi(\mathbf{P}) - \int_{S_B} \phi(\mathbf{Q}) \frac{\partial G(\mathbf{P},\mathbf{Q})}{\partial n} dS - \int_{S_W} \Delta\phi(\mathbf{Q}) \frac{\partial G(\mathbf{P},\mathbf{Q})}{\partial n} dS$$
$$= \int_{S_B} (\mathbf{V}_\infty \cdot \mathbf{n}) G(\mathbf{P},\mathbf{Q}) dS, \quad \mathbf{P} \in S_H \cup S_B \setminus TE,$$

- 3D Laplace basic singularity: $G(\mathbf{P},\mathbf{Q}) = \frac{1}{4\pi} r^{-1}(\mathbf{P},\mathbf{Q})$
- r: distance between $\mathbf{P}$ and $\mathbf{Q}$
- $\phi(\mathbf{P})$: potential on boundary surface
- $\Delta\phi = \phi_u - \phi_l$: potential jump on wake
- $TE = S_B \cap S_W$: wing trailing edge from which wake emanates

Kutta condition

$|\nabla\phi(\mathbf{P})| < \infty, \quad \mathbf{P} \in TE$

Wake Conditions



Potential Jump on Wake

- $\Delta\phi(\mathbf{Q})|_{S_W} = \Delta\phi(s_1, s_2)|_{S_W} = \Delta\phi(s_1, s_2)|_{S_W \cap TE} := \widehat{\Delta\phi}(s_2)$
- $\widehat{\Delta\phi}(s_2) = \Delta\phi(\xi|_{S_B \cap TE}, \eta)$

Isogeometric Boundary Element Method

Body represented by N_P bicubic T-spline surfaces (multi-patch):
$S_{B_I} = \cup_{e=1}^{n_{B_I}} S_{e,B_I},\ I = 1, \dots, N_P$

Wake represented by a single T-spline surface: $S_W = \cup_{e=1}^{n_W} S_{e,W}$:

$$\tilde{\mathbf{x}}^{e,m}(\tilde{\xi}) = \sum_{\alpha=1}^{n_\alpha} \mathbf{d}_\alpha^{e,m} R_\alpha^{e,m}(\tilde{\xi}) \quad m = B_I, W.$$

$$\phi(\mathbf{P}^I) = \sum_{i=1}^{n_{A,I}} \phi_i^I \tilde{R}_i^I(\mathbf{P}^I), \quad \mathbf{P}^I \in S_{e,B_I}$$

Isogeometric Boundary Element Method

The potential jump $\Delta\phi(\mathbf{Q})$ on the trailing edge of the wing is approximated by using the basis of the trailing edge curve

$$\Delta\phi(\mathbf{Q}) = \Delta\phi(\xi|_{S_B \cap TE}, \eta) = \sum_{i=1}^{n_W} \Delta\phi_i R_i^1(\xi_{TE}, \eta)$$

Discrete BIE

$$2\pi \sum_{i=1}^{n_{A,l}} \phi_i^l R_i^l(\mathbf{P}_j^l) - \sum_{q=1}^{N_P} \sum_{i=1}^{n_{A,q}} \phi_i^q \sum_{e=1}^{n_{e,q}} \int_{S_e} R_i^q(\mathbf{Q}) \frac{\partial G(\mathbf{P}_j^l, \mathbf{Q})}{\partial n(\mathbf{Q})} dS(\mathbf{Q})$$

$$- \sum_{i=1}^{n_W} \Delta\phi_i \int_{\eta_1}^{\eta_2} R_i^1(\xi_{TE}, \eta) \int_{\xi_{TE,w}}^{\xi_\infty} \frac{\partial G(\mathbf{P}_j^l, \mathbf{Q})}{\partial n(\mathbf{Q})} dS(\mathbf{Q})$$

$$= \sum_{q=1}^{N_P} \sum_{e=1}^{n_{e,q}} \int_{S_e} \mathbf{V}_\infty \cdot \mathbf{n}(\mathbf{Q}) G(\mathbf{P}_j^l, \mathbf{Q}) dS(\mathbf{Q})$$

$$j = 1, \ldots, n_{A,l}, \quad l = 1, \ldots, N_P$$

- $\mathbf{P}_j$: Collocation points based on the generalisation of Greville abscissae (Scott et al. 2013).

IGA-Enhanced Kutta Condition

Zero-pressure jump along the trailing edge (Kutta Condition) on n_W points:

$$\Delta p(\xi_{TE,u}, \xi_{TE,l}, \eta_j) = 0, \quad j = 1, ..., n_W$$

leads to quadratic system:

$$\mathcal{P}_{j(l)}(\Phi) = 0,$$
$$j(l) = 1, \dots, n_{A,l} \quad l = 1, \dots, N_P + 1, \quad j(N_P + 1) = 1, \dots, n_W$$
$$\Phi = (\phi_1^1, \dots, \phi_{n_{A,1}}, \dots, \phi_1^{N_P}, \dots, \phi_{n_{A,N_P}}^{N_P}, \Delta\phi_1, \dots, \Delta\phi_{n_W})^T$$

solved numerically by applying an iterative Newton-Raphson scheme.

Morino Kutta Condition

$$\Delta\phi(\xi_{TE},\eta) = \sum_{i=1}^{n_A} \phi_i^{TE}((\tilde{R}_i(\xi_{u,TE},\eta) - \tilde{R}_i(\xi_{l,TE},\eta))$$

- Potential jump equal to difference between upper and lower potential values on TE
- Essentially a 2D condition
- Used as N-R initial approximation

Morino Kutta Condition

Linear system:

$$2\pi \sum_{i=1}^{n_{A,l}} \phi_i^{l,0} R_i^l(\mathbf{P}_j^l) - \sum_{q=1}^{N_P} \sum_{i=1}^{n_{A,q}} \phi_i^{q,0} \sum_{e=1}^{n_{e,q}} \int_{S_e} R_i^q(\mathbf{Q}) \frac{\partial G(\mathbf{P}_j^l, \mathbf{Q})}{\partial n(\mathbf{Q})} dS(\mathbf{Q})$$

$$- \sum_{i=1}^{n_A} \phi_i^{TE,0} \int_{\eta_1}^{\eta_2} (\tilde{R}_i(\xi_{u,TE}, \eta) - \tilde{R}_i(\xi_{l,TE}, \eta)) \int_{\xi_{TE,w}}^{\xi_\infty} \frac{\partial G(\mathbf{P}_j^l, \mathbf{Q})}{\partial n(\mathbf{Q})} dS(\mathbf{Q})$$

$$= \sum_{q=1}^{N_P} \sum_{e=1}^{n_{e,q}} \int_{S_e} \mathbf{V}_\infty \cdot \mathbf{n}(\mathbf{Q}) G(\mathbf{P}_j^l, \mathbf{Q}) dS(\mathbf{Q})$$

$$j = 1, \dots, n_{A,l}, \quad l = 1, \dots, N_P$$

Morino Kutta Condition

Then the unknown $\Delta\phi_i^0$s can be found by solving the following linear system:

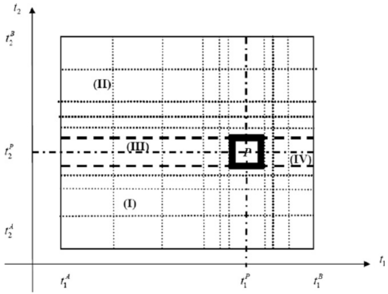
$$\sum_{i=1}^{n_W} \Delta\phi_i^0 R_i(\xi_{TE}, \eta_j) = \sum_{i=1}^{n_A} \phi_i^0 (\tilde{R}_i(\xi_{u,TE}, \eta_j) - \tilde{R}_i(\xi_{l,TE}, \eta_j)) \qquad j = 1, ..., n_W$$

and the zero-th iteration solution vector is given by:

$$\Phi^0 = (\phi_1^{1,0}, \ldots, \phi_{n_{A,1}}^{1,0}, \ldots, \phi_1^{N_P,0}, \ldots, \phi_{n_{A,N_P}}^{N_P,0}, \Delta\phi_1^0, \ldots, \Delta\phi_{n_W}^0)^T$$

Evaluation of Singular Integrals

- Far field case: $\mathbf{P}_j$ and $\mathbf{Q}$ not on the same element and $d(\mathbf{P}_j, \tilde{\mathbf{x}}^e) > 2diag(\mathbf{x}^e) \rightarrow$ Gauss - Kronrod
- Near field case: $\mathbf{P}_j$ and $\mathbf{Q}$ not on the same element and $d(\mathbf{P}_j, \tilde{\mathbf{x}}^e) < 2diag(\mathbf{x}^e) \rightarrow$ Telles transformation
- In field case: $\mathbf{P}_j$ and $\mathbf{Q}$ on the same element $\rightarrow$ ϵ region cut-off



Singularity cut-off (Belibassakis et al. 2013)

Evaluation of Singular Integrals

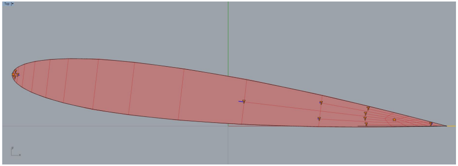
Linear transformation of variables:

$$\int_{t_{2A}}^{t_{2B}} \int_{t_{1A}}^{t_{1B}} \mathbf{f}(t_1, t_2) dt_1 dt_2 = \int_{-1}^{1} \int_{-1}^{1} \mathbf{f}_*(\xi_1, \xi_2) d\xi_1 d\xi_2, \quad \mathbf{f}_* = | \, d\mathbf{t}/d\xi \, | \, \mathbf{f}$$

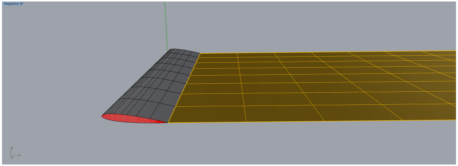
Non-linear Transformation (Telles and Oliveira 1994):
$\xi_k(s_k) = \alpha_k s_k^3 + b_k s_k^2 + c_k s_k + d_k, \quad k = 1, 2$ leads to:

$$\int_{-1}^{1} \int_{-1}^{1} \mathbf{f}_*(\xi_1(s_1), \xi_2(s_2))(9\alpha_1\alpha_2(s_1 - s_1^P)^2(s_2 - s_2^P)^2) ds_1 ds_2$$

Application - Rectangular Wing

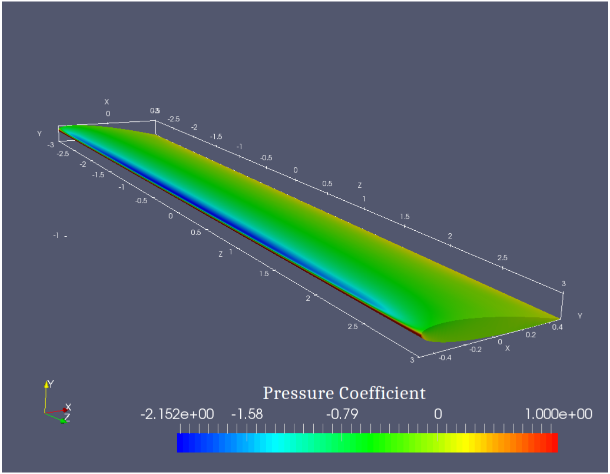


NACA 0012 profile - 6.75°

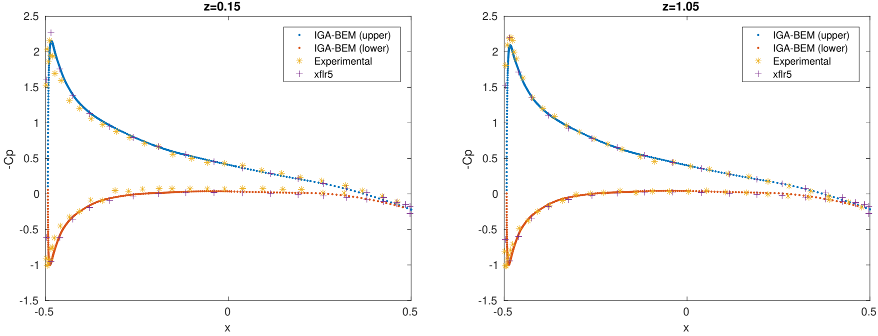


Wing (308 DoFs), wake (128 DoFs) and cap (86 DoFs) - 6meter span

Application - Rectangular Wing



Sectional Pressure Coefficients



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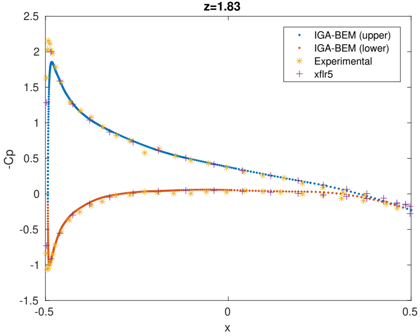

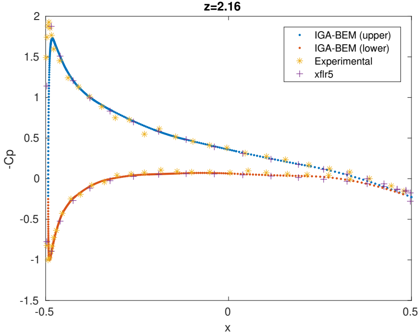

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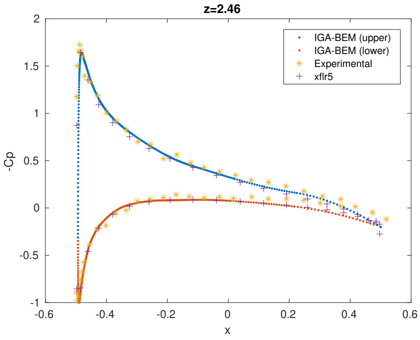

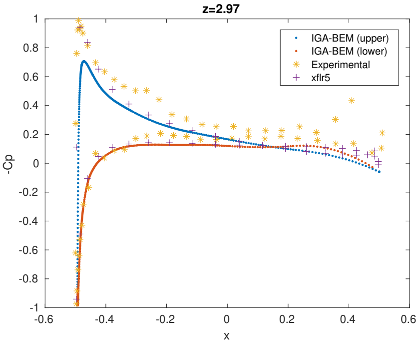

Conclusions and Future work

Conclusions:

- Good agreement with experimental results along most of the span
- Significant deviation very close to the tip
- Kutta condtion satisfied all along the span

Future Work:

- Investigation of tip behaviour
- Tests with complex wing geometries (cambered etc.) and/or fuselage

Acknowledgements

- the European Unions Horizon 2020 research and innovation programme under the Marie Skodowska-Curie ITN grant ARCADES (agreement no 675789) and
- the Nazarbayev University Grant No. 090118FD5328: "Shape Optimization of Lift- and Thrust-generating surfaces with the aid of IsoGeometric Analysis (SOLTIGA)"

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