# PROBABILISTIC SHIP CORROSION WASTAGE MODEL WITH BAYESIAN INFERENCE

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## ABSTRACT

Corrosion wastage is one of the critical problems for the ship structures and prediction of corrosion depth is essential to monitor and maintain the ageing parts. This study targets to propose a probabilistic method to predict the corrosion depth considering the uncertainties, potentially induced by measurement. To achieve this goal, the probabilistic distributions of parameters were employed to the conventional corrosion wastage model. Then Bayesian inference was introduced to update the obtained probabilistic model using inspection data. Firstly, one of the nonlinear corrosion wastage models was selected for a fundamental model and the parameters of the model and variance of error term were assumed as random variables. Hence, the number of corrosion wastage models corresponding to sets of random variables and their prior joint probabilities were obtained. At the second stage, likelihoods of each corrosion model were calculated using the corrosion field data and the error distributions which were originated from the variance of error term. Bayesian inference was then applied and the updated joint probability, called posterior joint probability, was obtained. Finally, the corrosion depth distribution over time was calculated based on the posterior joint probability and the reliability of the corrosion depth was evaluated.

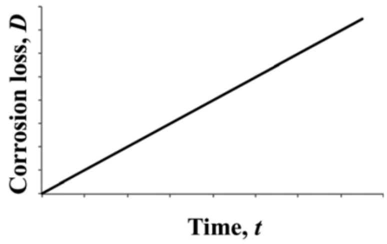
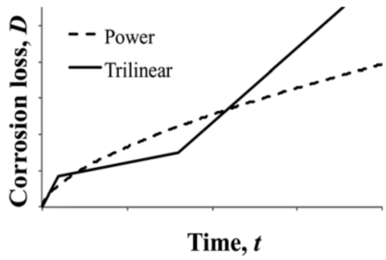
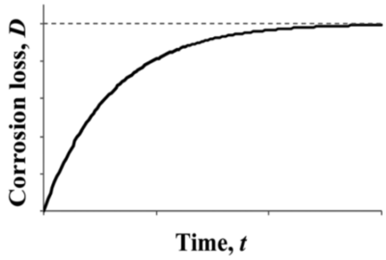
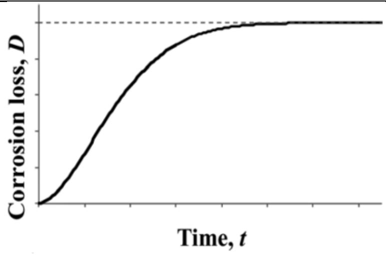
KEYWORDS: *Ship corrosion, Ship corrosion wastage model, Bayesian inference, Joint probability, Likelihood, Reliability assessment*

## 1. Introduction

In our modern industries, steel is the most preferred metal for the constructional purposes because of its mechanical properties, relatively low cost, high strength and ease of fabrication. However, one of the fatal disadvantages is that it corrodes easily in the marine environment. Due to this characteristic, if the steel structural members are not properly protected, they corrode and lose their strength in a short time. For this reason, the models to predict the corrosion depth have been considered very important in terms of monitoring the structural members and deciding the time point for the next inspection(Paik et al., 2003, Yamamoto, N. & Kobayashi, H., 2005).

Previous studies for the corrosion problems led to the relationship between the corrosion depth and time. Those models varied from the simple linear models to the more complex models. Southwell et al. (1979) suggested a linear and bilinear model to predict the corrosion depth over time. However, a lot of studies and experimental evidence showed that the nonlinear models are more appropriate to predict the corrosion depth. For this reason, Melchers (1998) proposed a steady-state trilinear and power model for the corrosion depth and the process is divided into four stages: initial corrosion; diffusion controlled; aerobic activity; and anaerobic activity phase. Soares and Garbatov (1999) suggested a nonlinear model with three major phases. Firstly, it is assumed that no corrosion occurs because of the protective coating on the structural members. In the second phase, the coating has a problem for its function and the corrosion wastage begins to grow nonlinearly over time. Finally, corrosion depth reaches a certain level and the degradation comes to a halt because of corrosion products. Qin and Cui (2003) also proposed a similar nonlinear model with the one from Soares and Garbatov (1999). The whole corrosion process consists of three major stages: no corrosion because of the coating, corrosion accelerating and corrosion decelerating. Table 1 shows various corrosion wastage models after the protective coating breakdown.

**Table 1 Deterministic corrosion wastage models after the protective coating breakdown**

| Model | Parameter | Graph |
|---|---|---|
| Linear model<br>$d(t) = R\,t$<br>Southwell et al. (1979) | $t$ : time<br>$R$ : Constant corrosion rate |  |
| Trilinear and power model<br>$d(t) = \begin{cases} c_1\,t & (0 < t < T_1) \\ b_2 + c_2\,t & (T_1 < t < T_2) \\ b_3 + c_3\,t & (T_2 < t < T_3) \end{cases}$<br>$d(t) = c\,t^a$<br>Melchers (1998) | $b_2, b_3, c_1, c_2, c_3,$<br>$a, c$ : Empirical model parameters |  |
| Nonlinear model<br>$d(t) = d_\infty\,(1 - e^{-\frac{t}{\tau_t}})$<br>Soares and Garbatov (1999) | $\tau_t$ : Scale parameter<br>$d_\infty$ : Long-term loss |  |
| Nonlinear model<br>$d(t) = d_\infty\,(1 - e^{\left(-\frac{t}{\tau_t}\right)^{\beta}})$<br>Qin and Cui (2003) | $\tau_t$ : Scale parameter<br>$d_\infty$ : Long-term loss<br>$\beta$ : Shape parameter |  |

However, these conventional corrosion wastage models were usually simplified and the possibility of other candidate models could be neglected when selecting a representative model. Moreover, the deterministic model cannot show the reliability of the predicted corrosion depth. For this reason, the probabilistic approach has been introduced to the corrosion model. Qin and Cui (2003) defined the parameters of several corrosion models as random variables and determined the mean and standard deviation of them by minimizing the error function for the sampled data. Then the probabilistic corrosion depth was obtained and the reliability analysis for the ultimate strength of a plate element was conducted with net thickness approach. Garbatov et al. (2007) suggested the probabilistic corrosion model using data from American Bureau of Shipping. The corrosion depth was measured on the deck plates of ballast tank and cargo tank of tankers over 32 years and the sample number was 1168 and 4665 respectively. Then the mean and standard deviation by year was calculated and the author applied the Lognormal distribution to show the corrosion depth stochastically. Guo et al (2008) assessed the time-varying ultimate strength of deck plate for aging tankers based on a semi-probabilistic model. The study applied several distributions to fit corrosion depth data such as Gamma, Lognormal and especially Weibull distribution. Then, it suggested the proper inspection interval based on corrosion depth distribution and time-varying normal stress. Paik (2012) also studied the probabilistic approach to the corrosion degradation. In the study, the corrosion depth by year was shown with Weibull distribution using the data measured over 27 years from seawater ballast tank. These models are helpful because they can show the corrosion degradation stochastically in terms of the uncertainty. Moreover, when the inspection data from the target ships which have similar routes and operational environments with them is not sufficient, they could be used as reference data. However, there was a limitation for applying the reference models directly because the condition could not be exactly the same. Hence, more reliable corrosion models were needed to be created and Bayesian inference was applied to achieve this goal. Bayesian inference is the best suited for the case that a certain probabilistic model needs to be modified based upon the newly obtained measurement data, and popularly used in many engineering applications(Guida and Penta, 2009, DNV-GL, 2015, Babuska et al., 2016, Liu et al., 2017).

Zhang(2014) proposed probabilistic corrosion model of pipeline in the direction of wall thickness. The model has three parameters representing the average growth rate, corrosion initiation time and a scale parameter and all the parameters were assumed as unknown. Then the hierarchical Bayesian inference was used with the imperfect inline inspection(ILI) data and the posterior distributions of the parameters were estimated. The comparison between the predicted depth and field-measured depth showed that the corrosion model could predict the corrosion depth reasonably well. On the other hand, Bayesian inference was also used for the corrosion degradation of the ship structural members. Luque et al. (2017)

proposed the probabilistic method in terms of the spatial characteristics using field data measured from floor plates of four tankers which were between 14 and 18 years old. In this research, data was classified with parameters, such as mean and standard deviation of corrosion rate, considering the spatial characteristics and operational environment. Then Bayesian inference was applied to update the parameter values and it was possible to estimate points with a high corrosion rate and their vulnerabilities in the future.

This research proposes a method combining a nonlinear model and probabilistic approach with Bayesian inference numerically. The novelty of this research is that it offered a more reliable corrosion model to predict the corrosion depth. Fig. 1 shows the overall procedure of this research. First, a nonlinear corrosion wastage models from Soares and Garbatov (1999) was selected as a fundamental model. Then it was assumed the parameters of the model and variance of error distribution were random variable. Hence, a number of corrosion wastage models were obtained from the parameter sets and each model had the joint probability of parameter set. At the second stage, likelihoods of the models were calculated based on the corrosion data from Garbatov et al. (2007) and error distribution. Bayesian inference was then introduced to update the distributions of parameters and reliability of the structural member for the corrosion reduction was evaluated.

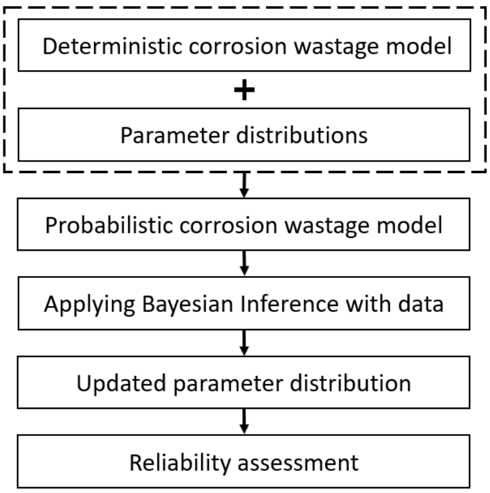



**Fig. 1 Flow chart of the main stages**

## 2. Theoretical background

### 2.1. Deterministic ship corrosion wastage model

A lot of researchers have studied corrosion wastage models to predict its degradation. Among the models, it is seen that several authors chose or modified the model suggested by Soares and Garbatov (1999). The model shows the corrosion depth over time with a nonlinear function and it has four stages in detail as shown in Fig. 2. The first stage (O'O) is considered that corrosion wastage does not happen because of the protective coating. In the second stage (OB), it is assumed that the failure of the protective coating occurs and the corrosion degradation starts nonlinearly. This phase is considered that the structural members experience the fast decrement of their thickness because there is no factor preventing the corrosive reaction. The third stage (BC) shows the corrosion depth grows slowly and the last stage (t>C) the corrosion degradation reaches at a limited level.

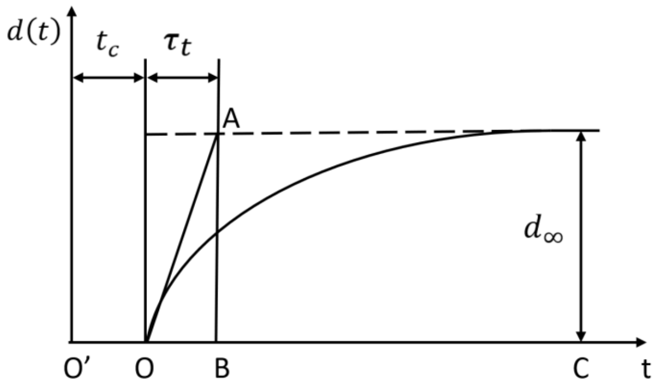



**Fig. 2 Corrosion depth with a function of time (Soares and Garbatov, 1999)**

$$d(t) = d_\infty(1 - e^{-(t-t_c)/\tau_t}), \quad t > t_c$$
$$d(t) = 0, \quad t < t_c \tag{1}$$

Equation (1) shows the corrosion depth over time of the model, where $d(t)$ is the corrosion depth at time $t$ and $d_\infty$ is the long-term corrosion wastage which corresponds to the last stage. $t_c$ is the coating life regarding the first stage. $\tau_t$ is the transition time and it is deeply related to the second stage, which may be calculated as equation (2). where $\alpha$ is the angle between OA and OB in Fig. 2.

$$\tau_t = \frac{d_\infty}{tan\ \alpha} \tag{2}$$

## 2.2. Probabilistic prediction model

Deterministic prediction model is not suitable when the factors affecting the process is not exactly revealed. Hence, the probabilistic approach considering the uncertainties is often used to solve these kinds of problems. One of the methods for making probabilistic model is applying the distributed model parameters and error distribution to the deterministic model. In this way, the model parameters and the variance of error term are defined as random variables. Then the models which are originated from the different parameter sets are made numerically. With applying this approach, each model has the joint probability of random variables and the probabilistic results can be obtained after random sampling.

## 2.3. Bayesian inference

Probabilistic inference is largely divided into two types as Frequentist inference and Bayesian inference. Frequentist inference is a statistical method that makes results from sample data while focusing on the frequency or proportion. In other words, frequentist defines probability as the frequency in which the event occurs from a long-term perspective. However, it might not be practical in the problems where knowledge of the governing law is incomplete or non-existent and experiments are not realistic or too expensive. Bayesian inference defines probability as the degree of belief for the events. This belief is then verified and modified through the results of the subsequent attempts or data. This inference establishes a relationship between prior and posterior beliefs as new information is obtained. Equation (3) shows the basic form of Bayesian inference. H and D represents the hypothesis and the new piece of information such as data respectively. Therefore, $P(H)$ corresponds to the prior belief for the hypothesis and $P(H|D)$ is the posterior belief for the hypothesis after considering the data. On the other hand, $P(H|D)$ is the likelihood of the hypothesis and $P(D)$ is the prior probability of data acting as a normalizing factor.

$$P(H|D) = \frac{P(D|H) \cdot P(H)}{P(D)} \tag{3}$$

The important factor for updating the initial belief is the Likelihood. Likelihood is not a probability, but proportional property to the probability. Its role relates to showing the extent how well the hypothesis can represent the data. Equation (4) is the basic form about calculating likelihood of hypothesis using data. In this equation, $n$ means the number of data sets.

$$P(D \mid H) = \prod_{i=1}^{n} P(D_i \mid H) \tag{4}$$

Likelihood of each prediction model composed with the distributed model parameter set, $\beta$, could be calculated using the error distribution and data $(x_i, y_i)$. For the corrosion model introduced in 2.1, for example, $\beta$ can be represented by the set of $d_\infty$ and $\tau_t$ when coating life is assumed as a constant value based on the empirical knowledge. Equation (5) shows the way of calculating likelihood of a random variable set, $(\beta, \sigma^2)$, with Gaussian error distribution and Fig. 3 shows the example in more detail. In this equation and figure, $\mu_i$ means the predicted value from the prediction model obtained from $\beta$ and $x_i$. The results are then used in Bayesian inference by multiplying to prior distribution to obtain the posterior. Equation (6) shows the more detailed representation of equation (3) for this case(Bishop and Tipping, 2003, Yuen, 2010).

$$L(y \mid \mathrm{x}, \beta, \sigma^2) = \prod_{i=1}^{n} P(y_i \mid \mathrm{x_i}, \beta, \sigma^2) = (2\pi\sigma^2)^{-\frac{n}{2}} \exp\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i - \mu_i)^2\} \tag{5}$$

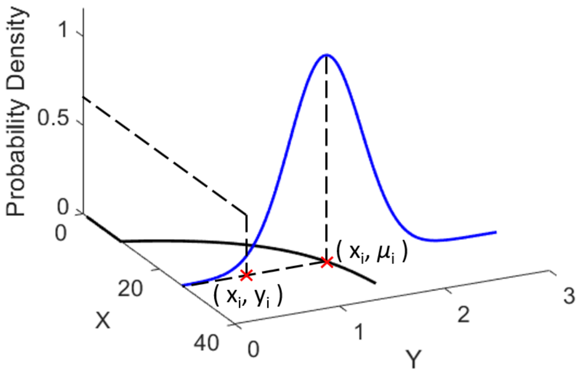



**Fig. 3 Calculating the likelihood of the model using data and error distribution**

$$P(\beta, \sigma^2 \mid x, y) = \frac{L(y \mid \mathrm{x}, \beta, \sigma^2) \cdot P(\beta, \sigma^2)}{P(x, y)} \tag{6}$$

On the other hand, the explained Bayesian inference targets to obtain the posterior distributions of parameters so far. The predicted value from the distribution of parameters could be obtained by applying random sampling based on Monte Carlo simulation.

## 2.4. Random sampling of data out of model parameters

When the parameter sets are consisted of the model parameters and standard deviation of error distribution which follows zero-mean Gaussian distribution, the probability distribution of the predicted values for a certain time point, in this particular case corrosion depth, can be obtained by random sampling method based on Monte Carlo simulation as shown in Equation (7). In this equation, j represents the j$^{th}$ of the random sampling. $P_j$ means a randomly sampled parameters based on their spaces and probability densities, for example, $d_\infty$, $\tau_t$ and standard deviation of error term, $\sigma$ for the corrosion model suggested in section 2.1. $D_j$ means a randomly sampled data, for example corrosion depth, based on $P_j$. In which, $\mu_j$ means the predicted depth from the model parameter set, $\beta_j$, without considering error term and the sampling is implemented with respect to normal($\mu_j$ , $\sigma_j^2$). This is two consecutive random sampling connected together and each sampling depends on the probability distributions of $P$ and $D$. Finally, the probability distribution of the predictive data can be obtained after a large number of sampling trials(Bishop and Tipping, 2003; Yao et al., 2018).

$$\begin{array}{lcl} 1^{st}\ Random\ sampling & P_1(\beta_1,\sigma_1) \ \rightarrow & D_1 \sim Normal\ (\mu_1\ ,\sigma_1^2) \\ 2^{nd}\ Random\ sampling & P_2(\beta_2,\sigma_2) \ \rightarrow & D_2 \sim Normal\ (\mu_2\ ,\sigma_2^2) \\ & \vdots & \\ j^{th}\ Random\ sampling & P_j(\beta_j,\sigma_j) \ \rightarrow & D_j \sim Normal\ (\mu_j\ ,\sigma_j^2) \end{array} \tag{7}$$

# 3. Application of the proposed method to predict corrosion depth

In this research, the data from Garbatov et al. (2007) was used to update the distribution of the prior parameter sets using Bayesian inference and also to validate the probabilistic results. Bayesian inference was applied once at the time instance of 24 year using the data up to that time instance and the probability distributions of model parameters were updated. For the validation of inferred probabilistic results, the statistical characteristics of probability distribution of corrosion depth between the measured data and prediction were compared at the time instance of 25 and 30 years. Comparison was also conducted for the time instances before 24 years, such as 15 and 20 years, to check the validity of inferred results for the past data as well.

### 3.1. Corrosion field data

The data from Garbatov et al. (2007) were measured on deck plates of ballast tanks of oil. The number of whole data includes 1168 measurements and the original thickness varied from 13.5mm to 35mm. Table 2 shows the characteristics of data by the several time points and Fig. 4 shows the mean corrosion depth with nonlinear model suggested by Garbatov et al. (2007). For the nonlinear model, the long-term wastage, $d_{\infty}$, was selected as the maximum mean corrosion depth within the observed time. The coating life, $t_c$, and the transition time, $\tau_t$ were defined after performing a least squares method using a quasi-Newton algorithm.

**Table 2 Corrosion depth on deck plates of ballast tanks in oil tanker (Garbatov et. al., 2007)**

| Time (year) | Mean Corrosion Depth (mm) | Standard Deviation | Time (year) | Mean Corrosion Depth (mm) | Standard Deviation |
|---|---|---|---|---|---|
| 12 | 0.12 | 0.12 | 21 | 0.68 | 0.58 |
| 14 | 0.38 | 0.31 | 22 | 0.8 | 0.59 |
| 15 | 0.52 | 0.04 | 23 | 0.21 | 0.11 |
| 16 | 0.6 | 0.49 | 24 | 1.02 | 0.57 |
| 17 | 0.3 | 0.25 | 25 | 1.02 | 0.48 |
| 18 | 0.9 | 0.59 | 26 | 1.08 | 0.23 |
| 19 | 0.9 | 0.69 | 32 | 1.85 | 0.67 |
| 20 | 1.01 | 0.77 | | | |

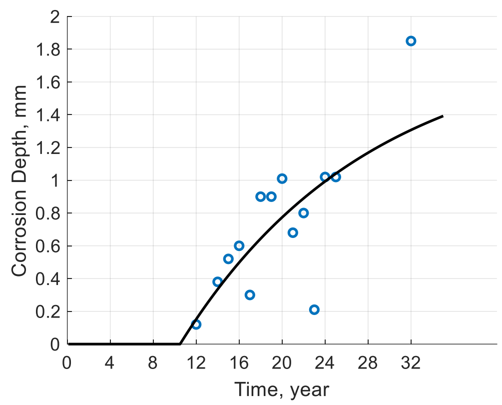



**Fig. 4 Mean corrosion depth with nonlinear model**

On the other hand, considering the period and condition of the real inspection, 5 numbers of corrosion depth which follow the characteristic of the reference data were randomly sampled at certain time points for the application. The time points were selected as 12, 16, 20, 24th year and the sampled data are shown in Fig. 5.

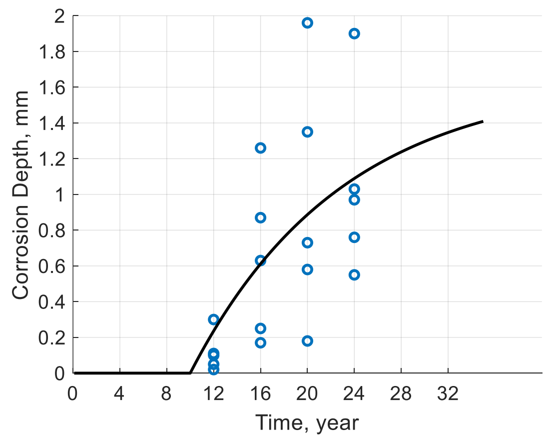



**Fig. 5 Sampled corrosion depth with nonlinear model**

## 3.2. Probabilistic model with prior distributions of parameters

In this research, the nonlinear model from Soares and Garbatov(1999) was used as the fundamental model. However, the long-term corrosion wastage, $d_\infty$, could not be estimated before gaining the whole data. Hence, $d_\infty$ and $\tau_t$ were defined as the model parameters which should be estimated. On the other hand, Coating life, $t_c$, was selected as fixed value since it could be estimated approximately after the initial inspection. Then the distributions for the two random variables, $d_\infty$ and $\tau_t$, were introduced to change the deterministic model to probabilistic model and their relationship was assumed as being independent. On the other hand, assuming the prior distribution is very important part in Bayesian inference. If the definition of the prior distribution is not proper or insufficient to show the parameter's characteristics, the results could not be reliable. With this reason, only informative data was used to define the distribution of parameter in this study. Table 3 shows the information of the prior parameters. For the initial distribution of $d_\infty$, normal distribution was applied by using the information from Paik et al. (2004) and its space was defined as [0.1, 5] after excluding the spaces which have lower probability density or negative values. Hence, the shape of the distribution of $d_\infty$ is partially normal distribution. The initial distribution of $\tau_t$ was assumed as the uniform distribution due to weak information and its space was defined as [5, 45] which is wide enough. The life of protective coating was defined 10 years based on the result from Garbatov et al. (2007). For the numerical calculation, 50 values for $\tau_t$ and 50 values for $d_\infty$ were evenly selected within the defined space of the initial distributions of parameters, and hence 50 × 50 number of corrosion models was obtained. On the other hand, the error term which follows Gaussian distribution was employed to reflect the uncertainty of each corrosion model and the variance, $\sigma^2$, was defined as random variable. Its distribution was assumed as the uniform distribution and 50 number of variances were selected within even interval in the space, [0.1 1]. Fig. 6 shows the example of the corrosion models and Fig. 7 shows the joint marginal distribution of $d_\infty$ and $\tau_t$ with respect to the standard deviation of error distribution.

**Table 3 Information of the prior parameters**

| $d_\infty$ (mm) | $\tau_t$ (year) | Variance of error distribution | $t_c$ (year) |
|---|---|---|---|
| Normal (1.36, $0.725^2$) | Uniform (5,45) | Uniform (0.1, 1) | 10 |

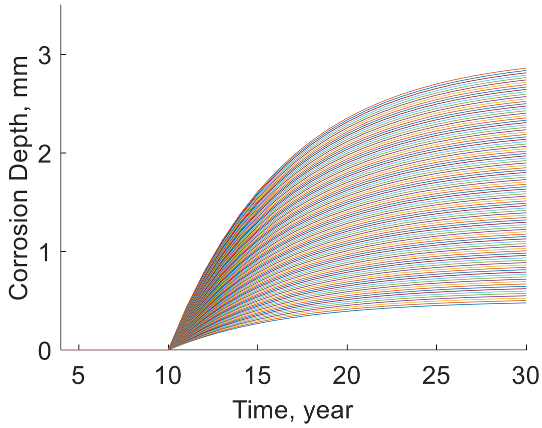



**Fig. 6 Example of corrosion wastage models**

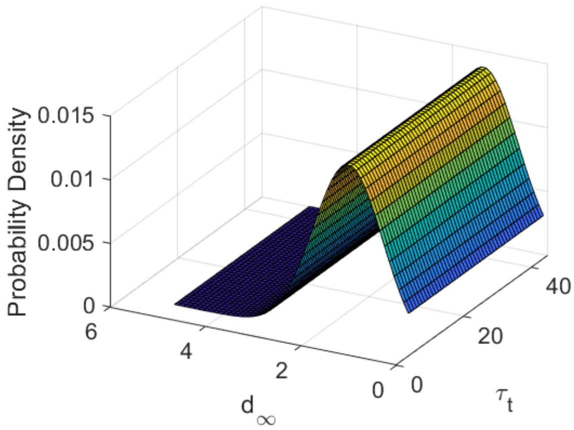
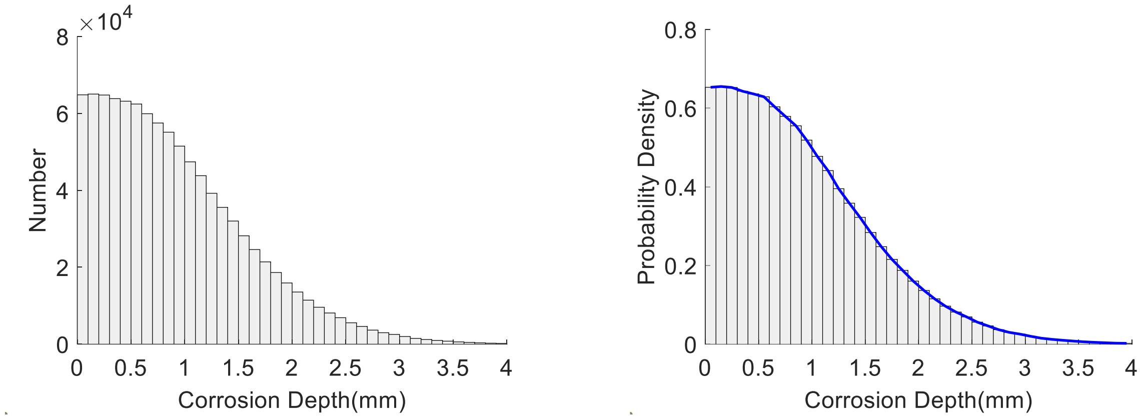



**Fig. 7 Prior joint probability of model parameters**

## 3.3. Prior corrosion depth distribution

Each parameter set consisted of model parameters and variance of error term has the triple joint probability. Hence, the prior corrosion depth distribution could be numerically obtained after performing the random sampling described in equation (7). Fig. 8 (a) and Fig. 8 (b) shows the histogram and probability density for the predicted corrosion depth at year 26 which is a close point with year 24 based on the prior information. In this calculation, the number of sampling events was defined large enough as 1,000,000. Fig. 9 shows the prior corrosion depth distribution at the entire time.




(a) Histogram (b) Probability density

**Fig. 8 Prior corrosion depth distribution at the year 26**

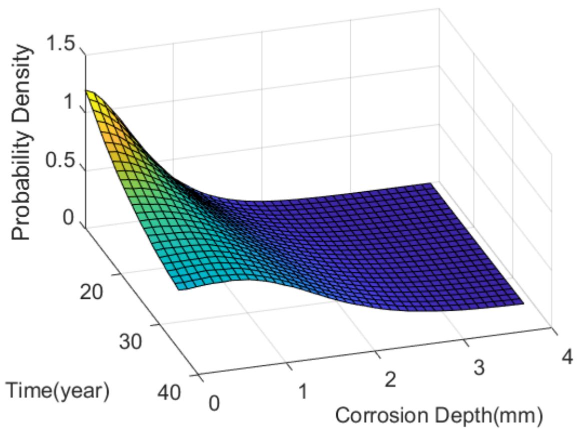
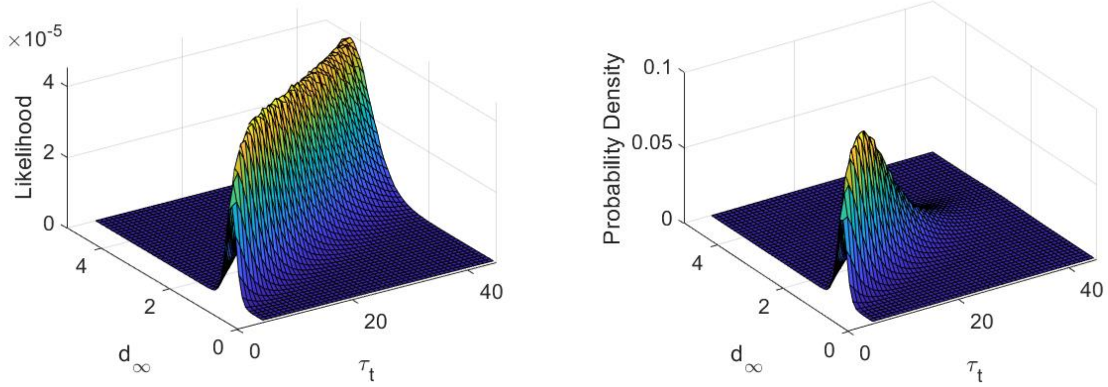



**Fig. 9 Prior corrosion depth distribution at the entire time**

### 3.4. Application of Bayesian inference with the field data

Firstly, the likelihood of each model was calculated based on the equation (5). Fig. 10 (a) shows the likelihood in the view of model parameters, $d_\infty$ and $\tau_t$. In which, the concept of marginal distribution with respect to the variance of error term was applied. It was because all the 3 parameters could not be expressed with the value of likelihood in 3D space simultaneously. On the other hand, the unlikely parameter sets, for example $d_\infty$>4 and $\tau_t$>40, had also high likelihood according to the calculation results. It was because the fundamental model was nonlinear function, and hence the distribution was modified after applying Bayesian inference with prior information. Fig. 10 (b) shows the posterior in the view of model parameters after applying marginal distribution for the variance of error term.

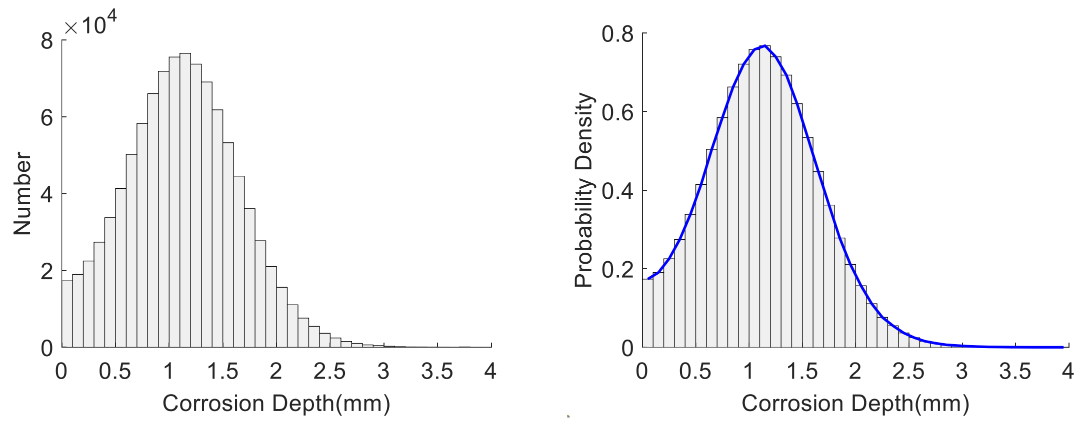



(a) Likelihood (b) Posterior distribution

**Fig. 10 Likelihood and Posterior distribution in the view of model parameters**

### 3.5. Updated corrosion depth distribution

With the updated distributions for the model parameters and variance of the error term, the corrosion depth distribution at the entire year was calculated with sample number, 1,000,000. Fig. 11 (a) and Fig. 11 (b) shows the histogram and the probability density for the predicted corrosion depth at the year 26 which is close time point with 24 years. Fig. 12 shows the corrosion depth distribution at the entire time.




(a) Histogram (b) Probability density

**Fig. 11 Posterior corrosion depth distribution at the year 26**

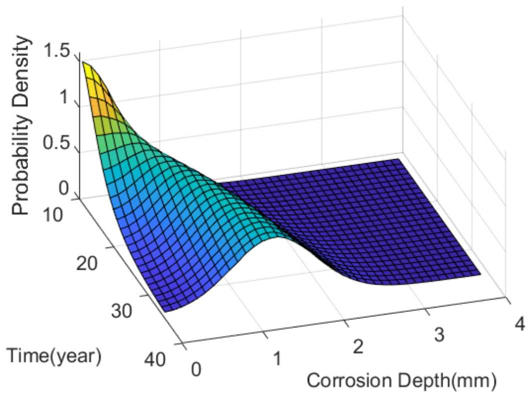
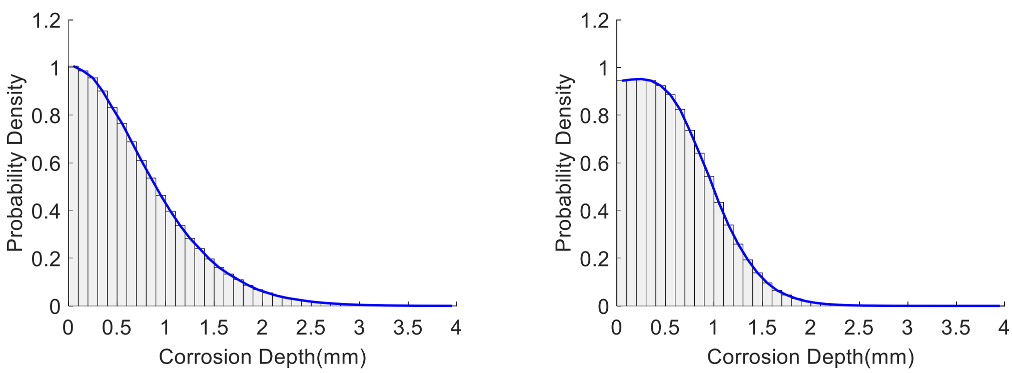


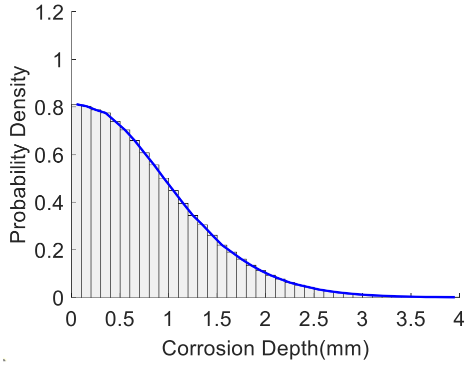

**Fig. 12 Posterior corrosion depth distribution at the entire time**

Fig. 13 to Fig. 16 show the overall change of the corrosion depth distributions at several years including the time points before 24 years. In the figures, the left and right one indicates the prior distribution and the posterior distribution respectively. The shape of distribution became wider in both distributions as the interested time point increases because the difference of the predicted depth from the corrosion model increases. On the other hand, the posterior corrosion depth distribution was evaluated to better represent the form and statistical characteristics of verification data compared to the prior corrosion depth distribution and the results were similar to the reference, Garbatov et al. (2007). Table 4 shows the comparison of the statistical characteristics of several corrosion depth distribution. As shown in Table 4, the standard deviation of the posterior distribution decreased compared to the value of the prior distribution. These changes mean that the triple joint probabilities of random variable sets which were more reliable and represents the data well were increased after applying Bayesian inference with data.

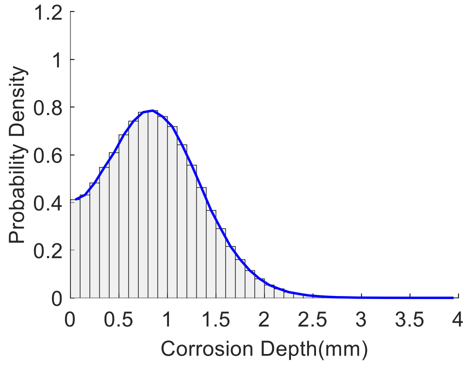



(a) Prior corrosion depth distribution (b) Posterior corrosion depth distribution

**Fig. 13 Corrosion depth distribution at the year 15**

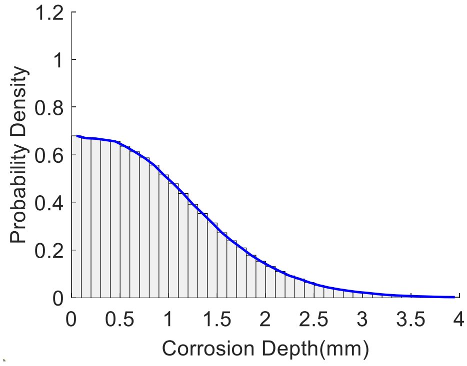



(a) Prior corrosion depth distribution

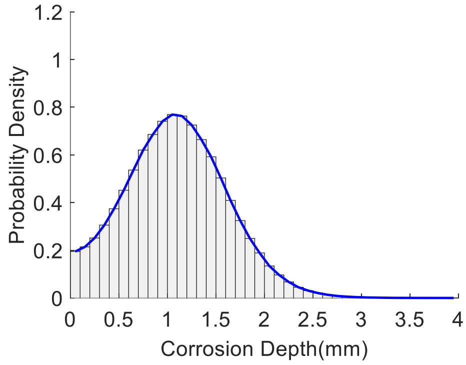



(b) Posterior corrosion depth distribution

**Fig. 14 Corrosion depth distribution at the year 20**

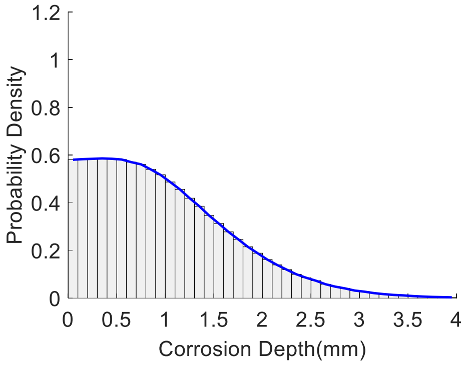



(a) Prior corrosion depth distribution

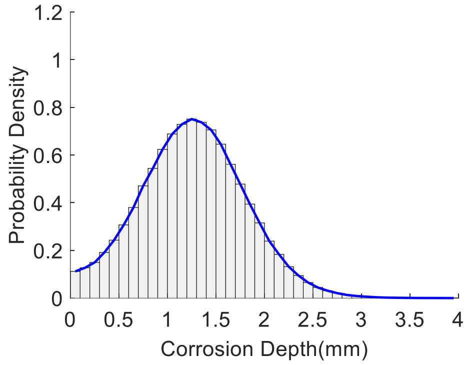



(b) Posterior corrosion depth distribution

**Fig. 15 Corrosion depth distribution at the year 25**




(a) Prior corrosion depth distribution




(b) Posterior corrosion depth distribution

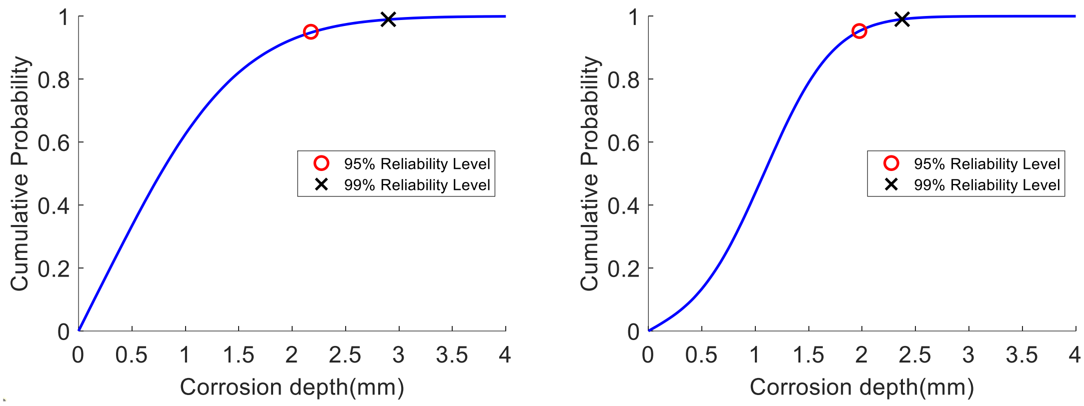
**Fig. 16 Corrosion depth distribution at the year 30**

**Table 4 Comparison of the statistical characteristics**

| Time(years) | | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|
| **Reference** | **Mean** | 0.48 | 0.79 | 1.08 | 1.25 |
| | **Variance** | $0.33^2$ | $0.45^2$ | $0.54^2$ | $0.60^2$ |
| **Posterior** | **Mean** | 0.59 | 0.87 | 1.09 | 1.26 |
| | **Variance** | $0.41^2$ | $0.48^2$ | $0.52^2$ | $0.54^2$ |
| **Prior** | **Mean** | 0.66 | 0.79 | 0.90 | 0.99 |
| | **Variance** | $0.53^2$ | $0.61^2$ | $0.67^2$ | $0.72^2$ |

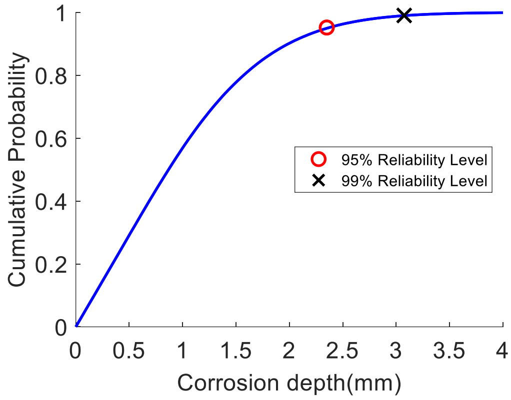
### 3.6. Reliability assessment for the predicted corrosion depth

Measuring reliability for the structural member is critical to validate the structural stability. Hence, the predicted corrosion depth corresponding to 95% and 99% of reliability level was estimated. Fig. 17 and Fig. 18 show the cumulative distribution for the corrosion depth with 95% and 99% of reliability level at the future time points than the used data. The left and right one indicates the results from the prior distribution and the posterior distribution respectively. Table 5 shows the values of the expected depth in terms of each reliability level. The predicted depth decreased in the posterior distribution at each time point and the change can be regarded as a result of applying corrosion data with Bayesian inference. With comparing these informative results with the permissible depth from the reference data or rules, the corrosion degradation could be monitored and maintained properly.

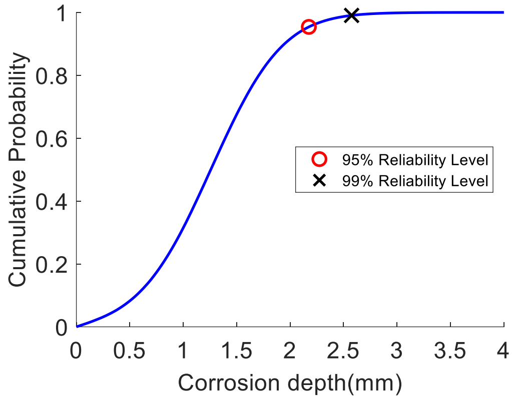



(a) Prior cumulative distribution (b) Posterior cumulative distribution

**Fig. 17 Reliability assessment for the result at the year 25**




(a) Prior cumulative distribution




(b) Posterior cumulative distribution

**Fig. 18 Reliability assessment for the result at the year 30**

**Table 5 Corrosion depth(mm) at 95% and 99% reliability level**

| **Time (year)** | | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|
| **95%** | **Prior** | 1.70 | 1.98 | 2.18 | 2.35 |
| | **Posterior** | 1.38 | 1.73 | 1.98 | 2.18 |
| **99%** | **Prior** | 2.33 | 2.65 | 2.9 | 3.08 |
| | **Posterior** | 1.78 | 2.13 | 2.38 | 2.58 |

## 4. Conclusion

This research introduced the probabilistic approach for the corrosion prediction model to consider the uncertainties of the corrosion problems. The key findings of this research can be summarized as below.

- There are several deterministic corrosion wastage models and the efficiency of the nonlinear models was verified from a lot of studies and experiments. However, the conventional model cannot reflect the uncertainties of the corrosion problem which could be induced by various environmental factors.
- To address the uncertainty issue related to the corrosion prediction, a probabilistic model was proposed in this study. Parameters introduced in the corrosion wastage model were treated as random

variables and the probability distribution of these random variables were updated when measurement data were obtained. This process was realized by applying Bayesian inference technique.

- The corrosion depth distribution was obtained from the sampling simulation and the predicted corrosion depth corresponding to 95% and 99% reliability level was calculated. When comparing the depth from the prior and posterior result, the predicted depth from posterior results decreased at each time point and the change can be regarded as a result of applying corrosion data with Bayesian inference. The results could be used for health monitoring or selecting the next periodic inspection time.

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