**HYSTERESIS AND UNEMPLOYMENT:
A PRELIMINARY INVESTIGATION**

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ABSTRACT:

This paper points out what hysteresis is using a simple model of market entry and exit. A procedure for calculating hysteresis indices for economic time series is outlined. Some preliminary results are presented to assess the explanatory power of hysteresis variables with regard to the equilibrium rate of unemployment in the UK.
We find that both natural and "unnatural" variables enter a cointegrating vector for UK unemployment 1959-1996. The natural variable is the replacement ratio. The "unnatural" variables are the hysteresis index of the exchange rate; and hysteresis indices for the real oil price and the real interest rate.

JEL Classification: E31, C52, E24.

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The natural rate hypothesis applies the axiom of monetary neutrality to the equilibrium rate of unemployment. This means that monetary shocks can affect the actual rate of unemployment but not the equilibrium rate, which is determined by a set of real variables, z, which represent the endemic "structural characteristics of labour and commodity markets" (Friedman 1968, p8). The models which embody the natural rate hypothesis (see Cross, Darby and Ireland 1997a for a brief survey) tend to have the stronger implication that only sustained real shocks can generate sustained changes in the equilibrium rate of unemployment. In a "battle of the mark -ups" model, for example, an increase in the z index of mark-up pressures arising from unemployment benefits, minimum wages and so on, followed by a fall in the z index back to its previous value, would see the natural rate return to the *status quo ante*. Similarly, in a "structural" model of the Phelps (1994) type shocks to real variables such as relative oil prices, real interest rates or exchange rates that are subsequently reversed would generate only a temporary perturbation in the time path for equilibrium unemployment.

The question is then one of whether the natural rate decomposition of variables that can and cannot change the equilibrium rate of unemployment is evidentially coherent. For European countries it is by no means clear that natural rate variables can explain the ratchets, predominantly but not exclusively, upwards in unemployment rates since the 1970s (see Blanchard 1990, Bean 1994). The present paper focuses on the UK experience. Here some investigators have failed to find cointegration between unemployment and natural rate variables (Darby and Wren-Lewis 1993, for example), with other studies (Westaway 1996, for example) finding cointegration only when "unnatural" variables such as unemployment

The present paper outlines an alternative approach to the explanation of equilibrium unemployment. Instead of ruling out nominal variables, or temporary shocks to real or nominal variables, we allow aspects of the past profile of such variables to help determine the equilibrium rate of unemployment. The analytical innovation is to introduce *hysteresis* into the processes underlying the determination of equilibrium unemployment. This is done by specifying that economic agents respond in a *non-linear* way to shocks; and by respecting the *heterogeneity* of economic agents by allowing them to respond differently to aggregate shocks. The key implications are that the economic system displays *remanence*, in that the application and removal of a shock will not be accompanied by a return to the *status quo ante*; and that the equilibrium rate of unemployment contains a *selective memory* of past shocks, retaining only the *non-dominated extremum values* of the shocks experienced.

This analysis of hysteresis produces some sharp contrasts with treatments of "hysteresis" effects elsewhere in the economics literature. In the *"hysteresis as persistence"* usage shocks can produce only persistence in the deviations of actual unemployment from unperturbed or *homeostatic* natural rate equilibria; whereas the presence of hysteresis actually means that shocks can change unemployment equilibria. In the *"unit root hysteresis"* usage the equilibrium rate of unemployment is a palimpsest bearing the marks of all past shocks, with a positive shock followed by a negative shock of equal size leaving no net effect; whereas hysteresis actually implies a selective, erasable memory of shocks, and remanence, in that positive and negative shocks of equal size do not cancel each other.

The paper is organised as follows. Section I outlines a simple model of hysteresis based on firms having two separate triggers for market entry and exit (see Dixit and Pindyck 1994). Only active firms employ labour, so the entry-exit decisions determine employment. The possible implications for the equilibrium rate of unemployment are sketched. Section II outlines the steps involved in calculating a hysteresis memory index, and presents a programme for calculating such hysteresis indices for economic time series. Section III presents some tentative, preliminary econometric results which are designed to investigate the explanatory power of hysteresis variables with regard to UK unemployment.

I. HYSTERESIS

The term hysteresis comes from the Greek "to be late, or come behind". The term was first coined for application to scientific explanation by the physicist Ewing (1881) to refer to effects (in terms of magnetisation) that remain after the initial cause (the application of a magnetising force) is removed. Such effects have subsequently been discovered or invoked in relation to a wide array of physical, biological and social phenomena. A general account of hysteresis as a systems property has been provided in Krasnosel'skii and Pokrovskii (1989). The key elements required to produce hysteresis are some form of *non-linearity* in the way the elements in a system respond to shocks; and *heterogeneity* in the elements and therefore in their responses to shocks.

The key implications of hysteresis are *remanence*, in that the application and reversal of a shock will not be followed by a return to the *status quo ante*; and a *selective memory*, in dominated extremum values being wiped (see Cross 1993 for a general account of hysteresis in economic systems).

Standard economic analysis assumes that economic equilibria are *homeostatic*, in that the reversal or removal of a temporary shock will be accompanied by a return to the initial equilibrium. The issue of hysteresis raises the question of whether this assumption holds in economic systems. Marshall (1890, p.425-426) thought that this assumption was likely to be violated in actual market processes, citing the effects of the shock to the supply of cotton during the American Civil War as an example. At a more aggregate level Keynes (1934) answered the question "are economic systems self-adjusting?" in the negative. If temporary shocks can have permanent effects economic equilibria become characterised by *heterostasis*, there now being a range of possible equilibrium values, with the actual equilibrium realised being determined by the temporary shocks experienced.

Hysteresis thus involves stronger properties than those conveyed by the use of the term to describe persistence or zero/unit roots. In the persistence case the natural rate equilibrium is unchanged by shocks affecting actual unemployment, whereas hysteresis implies that each new extremum value of the shocks experienced will lead to a new unemployment equilibrium. In the zero/unit root case all the shocks experienced shape the equilibrium, whereas hysteresis involves only the non-dominated extremum values of the shocks counting in the equilibrium selection process.

In the policy literature the key distinction is usually perceived as being between structural

> " ...economic analysis generally distinguishes between the actual unemployment rate prevailing at any time, and the "natural" (or "structural") unemployment rate (OECD 1994 Pt.1, p.66).

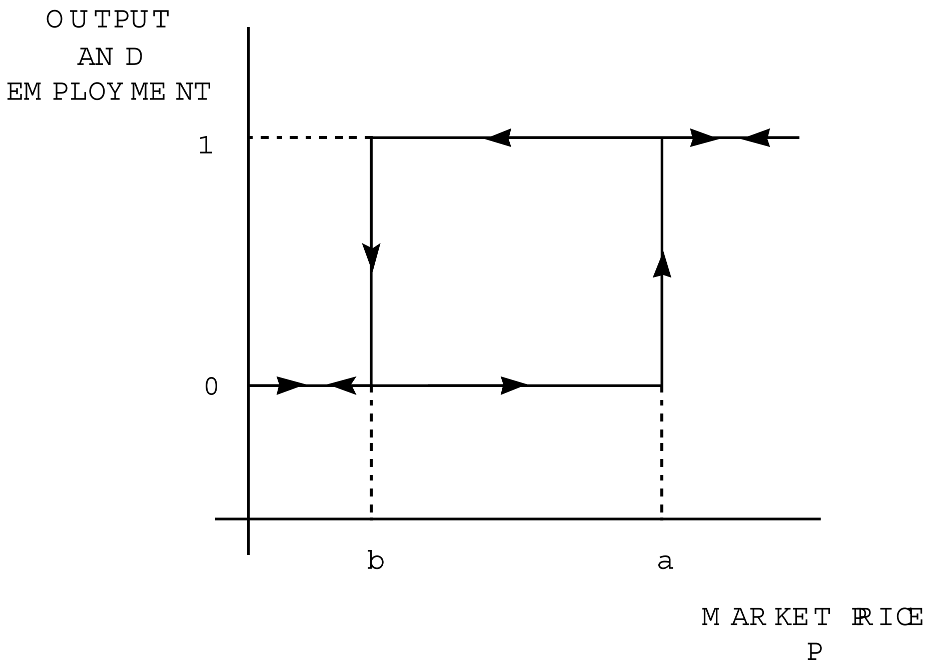
The presence of hysteresis implies that temporary shocks can change the structural dynamics which help determine equilibrium unemployment (see Amable, Henry, Lordon and Topol 1995). Thus, in contrast to the natural rate hypothesis, the shocks associated with the peaks and troughs of actual unemployment are themselves part of the process determining equilibrium unemployment.

An Illustrative Model

The simplest form of *non-linearity* in the Krasnosel'skii and Pokrovskii (1989) hysteresis analysis is the piecewise linear case analysed in Mayergoyz (1991). This framework is well suited to micro foundations based on discontinuous adjustment (Cross 1994). The presence of fixed costs of adjustment in the form of, for example, the sunk costs associated with investment or market entry (Dixit and Pindyck 1994), including entry into export markets (Amable, Henry, Lordon and Topol 1994), implies the existence of separate triggers for upward and downward adjustment. The following exposition is based on Piscitelli, Grinfield, Lamba and Cross (1996), which analyses market entry and exit under sunk costs in a Dixit-Pindyck framework.

The market has M potential suppliers. The number of active firms is N. When active, that is in the market, each firm produces one unit of output and employs one unit of labour. When out of the market firms produce zero output and employ zero units of labour. Each firm faces sunk costs of market entry, the i-th firm requiring a market price of $p \geq a$ to induce entry and a price of $p \leq b$ to induce exit. Figure 1 illustrates the two switching points. In the range $b<p<a$ the firm will either be active or inactive depending on its previously acquired propensity, which turns on whether this range has been approached from above or below.

FIGURE 1: SWITCHING POINTS



Heterogeneity is introduced by allowing the a and b switching points to differ between firms:

> "...different firms have different technologies or managerial abilities ...historical accidents may leave different firms with stocks of capital that are differently situated relative to their action thresholds ...then they will have different action thresholds..."
>
> (Dixit and Pindyck 1994, p.421)

Each firm is thus identified by a pair of a and b switching points which define its hysteron or hysteresis operator F_{ab} which maps from shocks to prices into output and employment.

The market price is specified as:

$$p_t = x_t f(q_{t-1}) \tag{1}$$

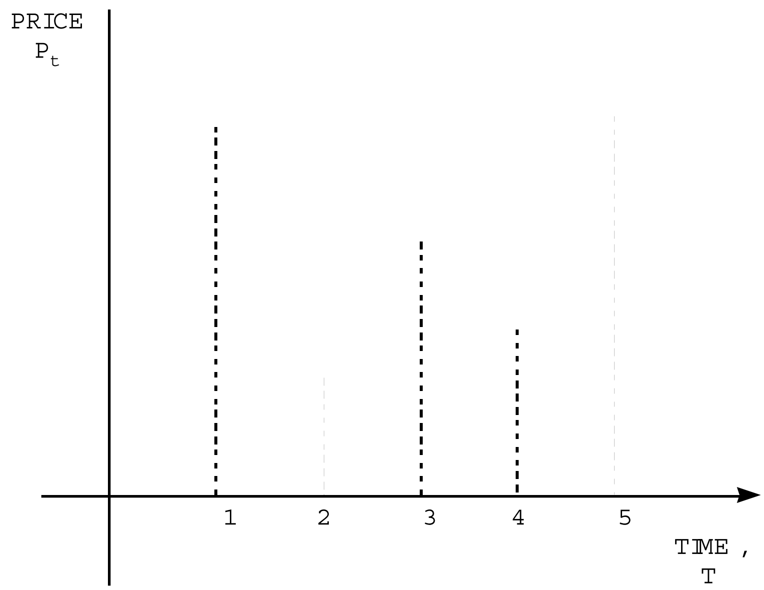
where x is an aggregate shock, to interest rates or exchange rates for example, faced by all firms, and f(q) is the deterministic component of the inverse demand function, with $q_{t-1} \cong N_{t-1}/M$. The dynamics of (1) turn on how p_{t+1} determines q_{t+1}, which can be written:

$$q_{t+1} = \frac{1}{M} \sum_{i=1}^{M} F_{ab}\left[x_{t+1} f\left(q_t\right)\right] \tag{2}$$

In a shockless economy, i.e. where x is constant, it can be shown that every initial condition converges to a fixed point or a two-period solution in which q swings between two points (Piscitelli, Grinfield, Lamba and Cross 1996, p.3). The interesting question is what happens in an economy with shocks.

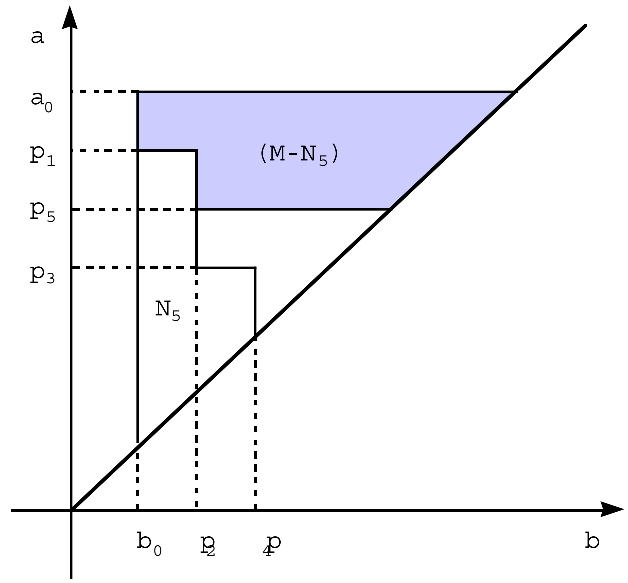
Consider the effects of a sequence of aggregate shocks that generates the price fluctuations, illustrated in Figure 2. As the price rises to p_1 firms with $a \leq p$ enter the market and employ labour, as the price falls to p_2 firms with $b \geq p_2$ leave the market and cease to employ labour, and so on. The division between active and inactive firms is illustrated in Figure 3, which uses the Mayergoyz (1991) half-plane diagram in which each firm is represented by its (a, b) switching characteristics. The (a_0, b_0) vertex of the triangle is determined by boundary conditions. The distribution of firms within the triangle can be seen as depending, inter alia, on the "structural characteristics of labour and commodity markets", such as the "z" variables stressed in the natural rate literature. Thus a more favourable set of "z" characteristics would tend to shift the (a, b) values south-westwards in the triangle, so yielding higher activity and employment levels for any given set of aggregate shocks.

FIGURE 2: A SEQUENCE OF SHOCKS



Referring to Figure 3, the rise in price to p_1 serves to create a horizontal partition between the N_1 active firms below the p_1 line, and the $(M-N_1)$ inactive firms above the line. The subsequent p_2, p_3 and p_4 shocks then trace out a staircase partition between N_4 and $(M-N_4)$, the coordinates being (p_1, p_2), (p_3, p_2) and (p_3, p_4). This illustrates the memory property of systems with hysteresis: only the extremum values of the shocks experienced count. The wiping-out property can be seen by considering the effect of the rise in price to p_5, illustrated in Figure 2. This dominates the previous local maximum price at p_3 and so wipes the effect of this dominated extremum value from the memory bank. Thus the coordinates of the staircase partition between N and (M-N), (p_3, p_2) and (p_3, p_4) are removed from the memory. This leaves the new staircase partition between the unhatched area in Figure 3, N_5, and the hatched area, $(M-N_5)$. Thus the memory of systems with hysteresis is selective: only the non-dominated extremum values of the shocks experienced retain an effect.

FIGURE 3: STAIRCASE PARTITION BETWEEN ACTIVE AND INACTIVE FIRMS

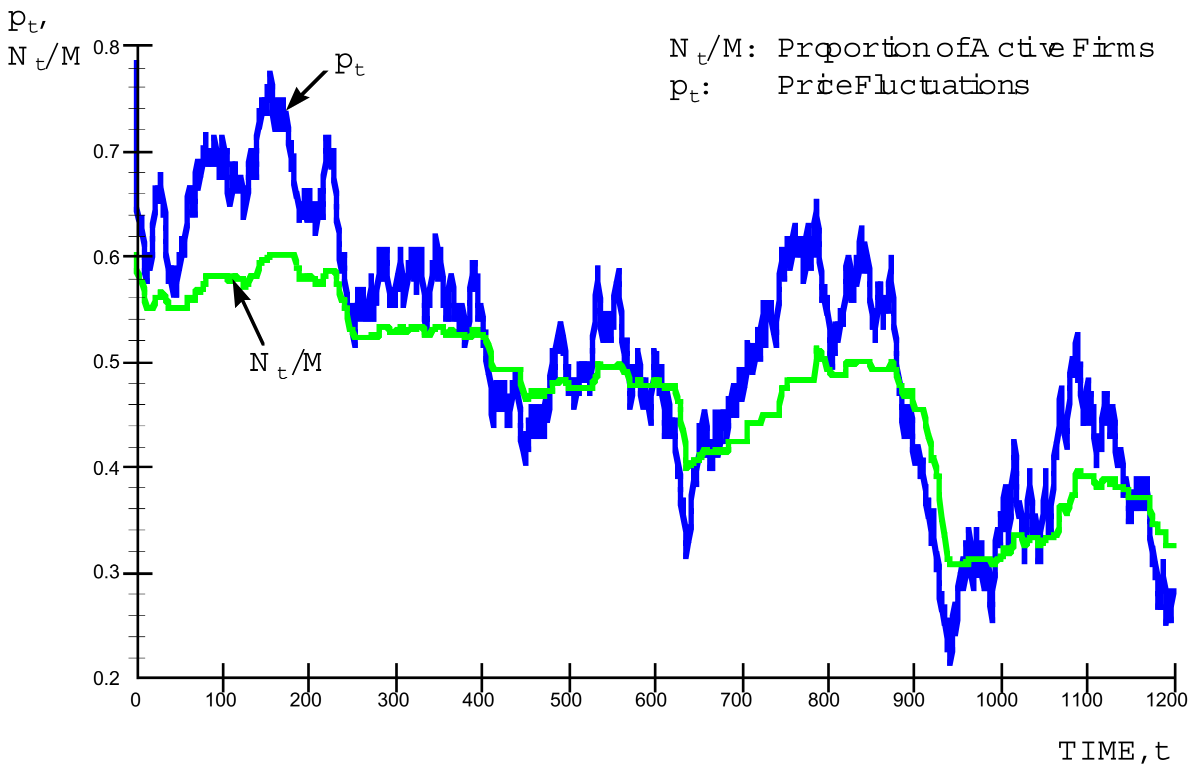


N ≡ ACTIVE FIRMS (M - N) ≡ INACTIVE FIRMS

The contrast with the unit root characterisation of the memory properties of time series is interesting. Unit root tests characterise time series as short or long in memory depending on whether the root is closer to zero or unity, the extreme unit root case implying an infinitely-long memory. With hysteresis the memory is derivative of the pattern of shocks. If the major expansionary and contractionary shocks have occurred recently the memory will be short, the previous shocks having been dominated and therefore eliminated from the memory. Otherwise undominated major shocks from the more distant past can impart a long memory.

Figure 4 reproduces the results of simulating the model in (1) and (2), with the aggregate demand shock x and the distributions of a and b switching values being specified by random number generators (Piscitelli, Grinfield, Lamba and Cross 1996). The non-stochastic component of the inverse demand function was specified as $f(q_t)=\alpha(\beta q_t+1)^{-1}$, with the α intercept being set at 0.8 and the β slope parameter at 1.4, and 1,200 iterations were used. In response to the price fluctuations indicated by the dark line the proportion of active firms

FIGURE 4: PRICE FLUCTUATIONS AND THE PROPORTION OF ACTIVE FIRMS



Ranges for Equilibrium Unemployment

It is, of course, a major step to move from the simple model of hysteresis above to the equilibrium rate of unemployment. The model, however, does offer some guidelines. The "structural characteristics of labour and commodity markets" or "z" variables stressed in the natural rate literature can be seen as affecting the (a, b) values of firms. Thus a more favourable set of z characteristics would tend to reduce the a and b values of firms, making firms more likely to be active and employing labour for any given sequence of aggregate shocks, thus reducing the rate of inactivity or "unemployment" (M-N)/M. The innovation is that the sequence of shocks experienced, in the form of the non-dominated extremum values, also shapes the rate of inactivity or unemployment. The implication is that, for any given set of "z" characteristics, there will be a range of feasible equilibrium unemployment rates. Each new extremum value of the shocks will, by changing the partition between active and inactive firms, change the equilibrium unemployment rate.

The inflation-unemployment interaction suggested by this hysteretic equilibrium can be written as:

$$p_t - p_t^e = \Delta^2 p_t \quad = \quad F[u_t - uh_t] \tag{3}$$

$$u_t \quad = \quad g[z_t, x_t] \tag{4}$$

$$uh_t^* \quad = \quad f[z_t, h_t(x)] \tag{5}$$

Equation (3) follows the natural rate hypothesis except in specifying the equilibrium as hysteresis-haunted; equation (4) is standard in saying that actual unemployment depends on the contemporary value of the aggregate shock x as well as on the z variables; and equation (5) specifies the hysteresis equilibrium for unemployment, uh*, as depending on a hysteresis index of the past non-dominated extremum values of the shocks h(x), as well as on z.

To illustrate how this system works consider the sequence of aggregate shocks, to interest rates or exchange rates say, illustrated in Figure 5. Whether or not a particular interest rate or exchange rate is expansionary or contractionary depends on z. The horizontal line on the diagram is thus conditioned on a particular value of z, $\bar{z}$. A less favourable set of z characteristics, z^+, would shift the horizontal line upwards, making any given interest rate or exchange rate more contractionary or equivalently less expansionary; and vice-versa for a more favourable shift in z characteristics to z^-.

FIGURE 5: CONTRACTIONARY AND EXPANSIONARY SHOCKS

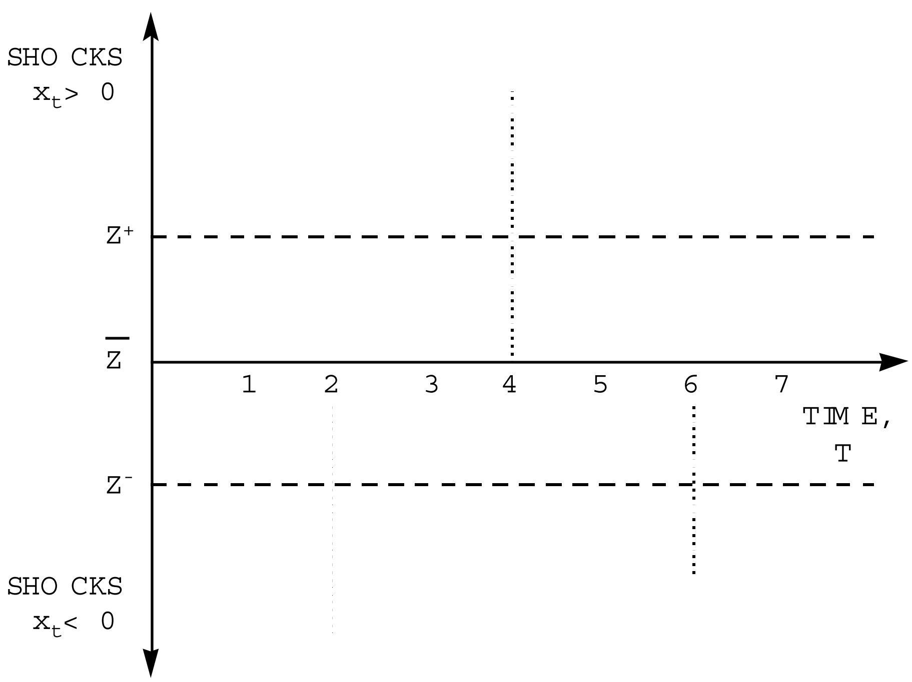
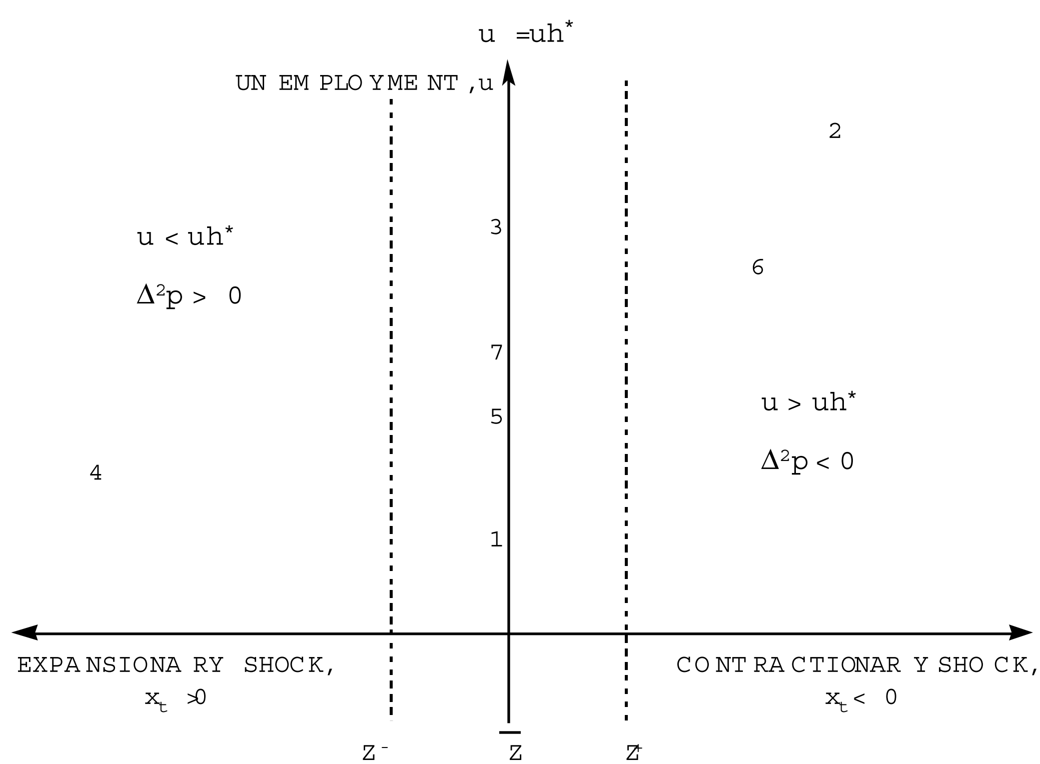


Figure 6 illustrates the implications for unemployment and inflation of the shocks given in Figure 5. The contractionary shock reaching a trough in period 2 sees unemployment rise from 1 to 2. The actual is above the equilibrium rate of unemployment, so the rate of inflation falls. As the contractionary shock fades in the face of real balance effects, the system does not retrace its steps back to the original equilibrium, as the natural rate hypothesis would imply, but instead reverts to a higher equilibrium unemployment rate in period 3: the extremum value of the shock experienced in period 2 remains in the memory bank. The expansionary shock in period 4 sees unemployment fall below the equilibrium rate, the rate of inflation rises, which stimulates adverse real balance effects. Unemployment rises to a new equilibrium rate in period 5, which is lower than the preceding equilibrium rate because the period 4 shock is retained in the memory bank. And so on. Take this economy through a further sequence of shocks and a range of equilibrium unemployment rates is traced, corresponding to the intersections with the vertical axis.

FIGURE 6: HYSTERESIS AND RANGES FOR EQUILIBRIUM UNEMPLOYMENT



The position of the vertical axis itself in Figure 6 is conditioned on a particular value of z, $\bar{z}$. If the z characteristics move favourably to z^- the equilibrium rates of unemployment would be lower for any given x shocks, and vice-versa if there are unfavourable developments to z^+.

Thus in an economy exposed to shocks there is a range of possible values for equilibrium unemployment. Rather than leaving the equilibrium indeterminate, the hysteresis hypothesis specifies the specific sub-set of the values of the shocks that determine the realised equilibrium. The "z" variables invoked by the natural rate hypothesis are also included, and would be the sole determinants of equilibrium unemployment in a shockless economy. The implications for policy in a world of shocks, however, are substantially different, macro policy measures being reinstated as having potentially lasting influences on unemployment as well as on inflation.

II. COMPUTING THE HYSTERESIS VARIABLE

The output of the system considered in the previous section can be expressed as

$$y(t) = \iint_{N(t)} g(a,b)\,da\,db = \sum_{k=1}^{n(t)} \iint_{Q_k(t)} g(a,b)\,da\,db \,, \tag{6}$$

where the set N is considered as the union of n(t) rectangular trapezoids Q_k. From visual inspection of Figure 3, it is clear that calculating the area N requires at least four steps. Step 1 involves specifying the (a_0, b_0) vertex of the right-angled triangle above a=b. Step 2 requires selecting the non-dominated extremum values from the time series for the input variable. Step 3 involves calculating the N area under the staircase partition between N and (M-N) as the union of the rectangular trapezoids specified as Q_k in (6) above. Step 4 requires the specification of a weight function g(a,b) that specifies how much each (a,b) switching combination contributes to aggregate output. This last step is perhaps the most troublesome. Ideally we would have cross-sectional information on the distribution of (a,b) switching points amongst agents. Then, by applying the means of the order statistics, the distribution for the ordered set N of (a,b) points would be determined. Without such information, a sensible procedure would start with a simple specification for the weight function g(a,b), which could then be varied to test for sensitivity to the assumption used. Accordingly in what follows it is assumed initially that the weight function is uniform, i.e. takes a constant value 1. In this case, the output is given by the unweighted sum of the areas of the trapezoids

$$y(t) = \sum_{k=1}^{n(t)} Q_k(t) \,, \tag{7}$$

Step 1: This step is made at the beginning of the programme, whereas Steps 2 and 3 are performed recursively, so that, at any point of time, the sequences of extrema and the value of the hysteresis output are updated. Mayergoyz (1991) defines $a_0=\max\{a, (a,b)\in T\}$ and $b_0=\min\{b, (a,b)\in T\}$, where T is the right-angle triangle. Given that we cannot compute a_0 and b_0 on the basis of experimental evidence, we scale the triangle T by assuming

$$b_0=\min\{x(t), t=1,2,...,T_{max}\} \tag{8}$$

and

$$a_0=\max\{x(t), t=1,2,...,T_{max}\}. \tag{9}$$

These assumptions would be problematic if they caused the hysteresis output measure to be highly sensitive to the initial value of the input variable. To see, however, that this is not a major problem, consider the alternative initial values for the input variable illustrated in Figures 7 and 8.

Figure 7

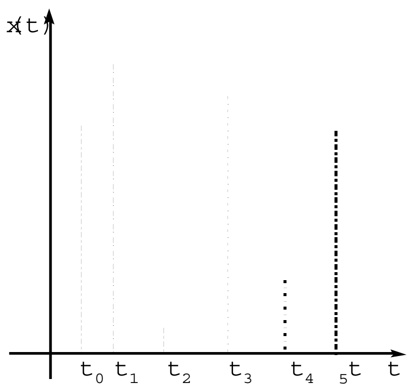
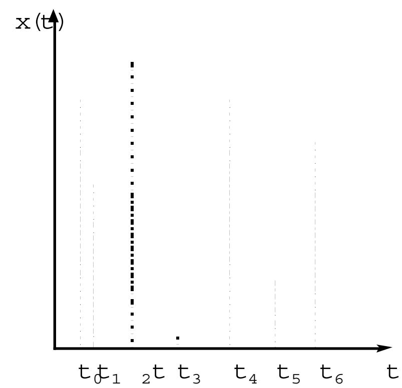


Figure 8



The time evolution of the set N for the input sequence in Figure 7 is given in Figure 9, while the equivalent evolution for the sequence in Figure 8 is provided in Figure 10. By comparing Figures 9 and 10 it can be seen that the different initial conditions only affect N as long as the global maximum and minimum have not been hit.

Figure 9 :

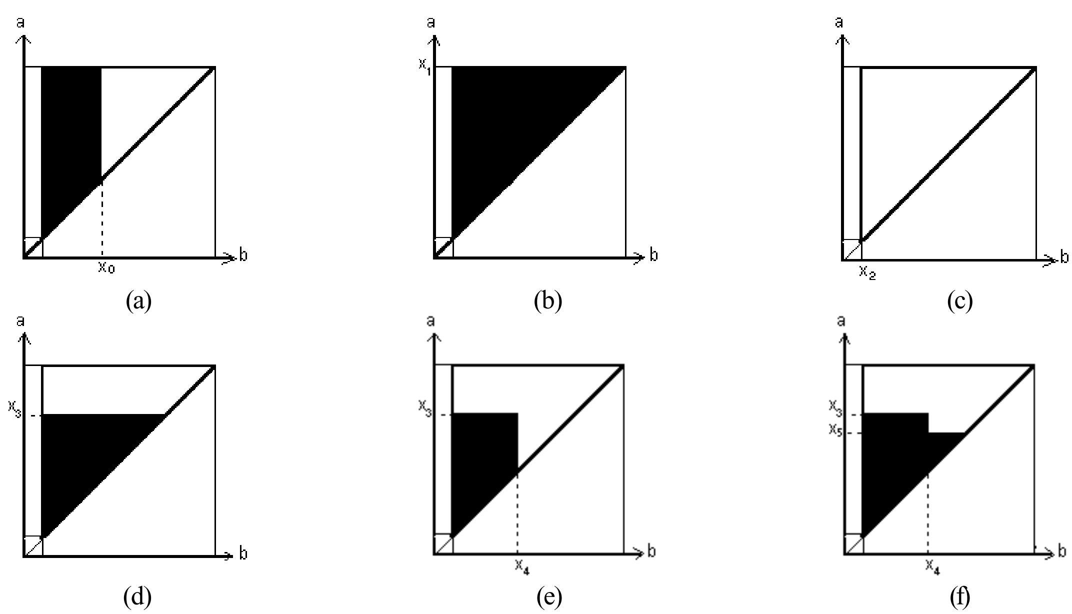
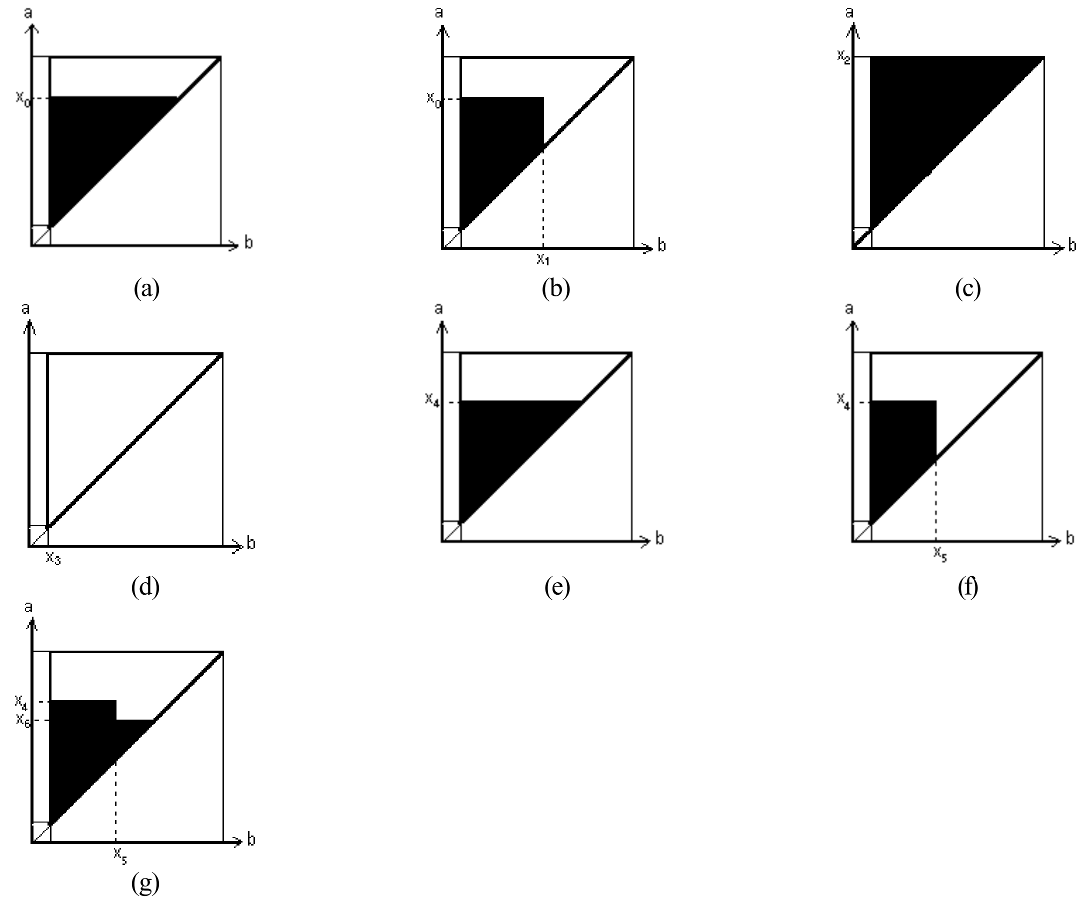
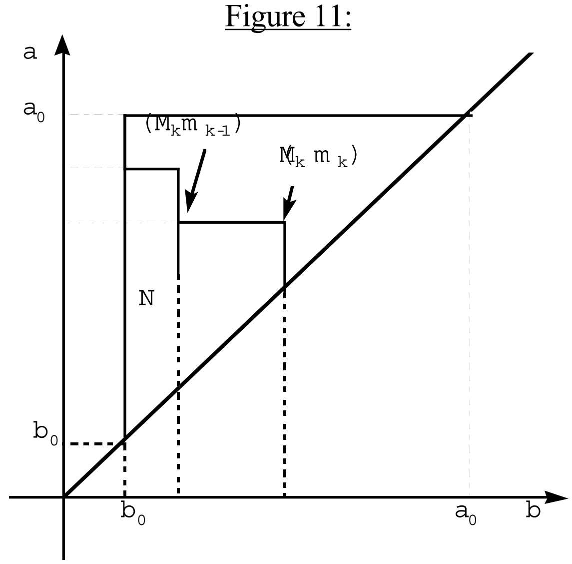


Figure 10:



Step 2. Let M be the matrix of elements M(i,t), i=1,2,..., t=1,2,...,t_{max}, where M(i,t) is the ith non-dominated maximum at time t, where we assume t_0=1 for computational reasons. And let m be the matrix of elements m(i,t), i=1,2,..., t=1,2,...,t_{max}, where m(i,t) is the ith non-dominated minimum at time t as illustrated in Figure 11.

Figure 11:



At any time t, the first non-dominated maximum is

$$m(2,t) = \min_{j \in [t_2^+, t]} x(j) \,. \tag{10}$$

Define t_1^+ by M(1,t)=x(t_1^+). Obviously, this is the first non-dominated maximum since it dominates all previous maxima and it is not dominated by any following maxima. By definition, it also removes the previous minima, so that the first non-dominated minimum is

$$m(1,t) = \min_{j \in [t_1^+, t]} x(j) \,, \tag{11}$$

where t_1^- is defined by m(1,t')=x(t_1^-) and m(1,t) removes all non-dominated extrema that are between M(1,t) and itself. The next non-dominated maximum is then

$$M(2,t) = \max_{j \in [t_1^-, t]} x(j) \,, \tag{12}$$

where t_2^+ is defined by M(2,t)=x(t_2^+). The second non-dominated minimum is

$$m(2,t) = \min_{j \in [t_2^+, t]} x(j) \,, \tag{13}$$

with t_2^- defined by m(2,t)=x(t_2^-). The complete sequence of non-dominated maxima and minima is then

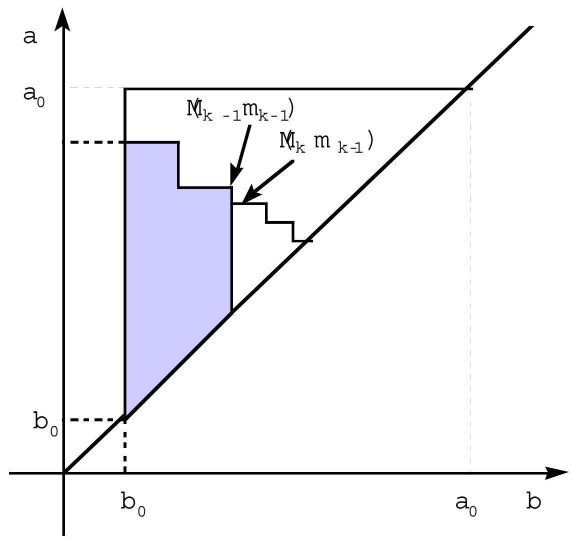
$$M(k,t) = \max_{j \in [t_{k-1}^-, t]} x(j) \,, \quad \text{such that} \quad M(k,t) = x(t_k^+) \,, \tag{14}$$

$$m(k,t) = \min_{j \in [t_k^+, t]} x(j) \,, \quad \text{such that} \quad m(k,t) = x(t_k^-) \,. \tag{15}$$

<u>Step 3</u>. Once the sequence of non-dominated extrema has been selected, the computation of the areas of trapezoids in Figure 11 is relatively straightforward. Referring to Figure 12, suppose at time t we have computed the sequence of non-dominated extrema up to i=k-1, and

let T(t) be the shaded area of the set N. Use M_k and m_k as shorthand for M(k,t) and m(k,t), respectively.

Figure 12:



The programme for calculating the hysteresis variable updates the sequence of extrema and the area T(t) at the same time: first, the maximum M_k is computed and then the area of the triangle having a vertex (M_k,m_{k-1}) is added to T(t), so that

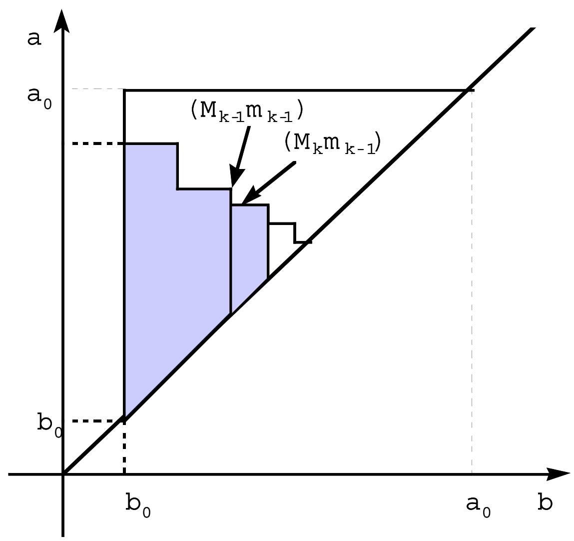
$$T(t) = T_p(t) + \frac{(M_k - m_{k-1})^2}{2}, \tag{16}$$

where $T_p(t)$ denotes the previous value of T(t). Note that the programme as constructed is more general since it allows for the cases where b_0 and/or the non-dominated extrema are negative. Next, the minimum m_k is computed and the area of the triangle with vertex in (M_k,m_k) is subtracted from the value computed in (16)

$$T(t) = T_p(t) - \frac{(M_k - m_k)^2}{2}. \tag{17}$$

Thus the shaded area in Figure 12 is augmented by the area of a trapezoid, as illustrated in Figure 13.

Figure 13:



Applying the same procedure recursively, the evolution of the set N over the rest of the time series can be computed.

The Programme

The following programme is written in RLAB and is available from the authors on request (the RLAB programming language is ©copyright 1993-94 Ian R. Searle). For programming reasons v denotes the input vector, tp(j) denotes t_j^+ and tm(j) denotes t_j^-.

```
Clear
format(10)
v=readm ( "v.dat" );
n=length(v);
u=v;

a0=abs(max(u));
b0=abs(min(u));

for (t in 1:length(u))
{
k=1;
M[k;t]=max(u[1:t]);
tp[k]=maxi(u[1:t]);
if (min(u)>0)
{
T[t]=(M[k;t]-b0)^2/2;
else
T[t]=(M[k;t]+b0)^2/2;
}
tt=tp[k];
while (tt<t)
{
z=u[tp[k]:t];
m[k;t]=min(z);
tm[k]=tp[k]-1+mini(z);
T[t]=T[t]-(M[k;t]-m[k;t])^2/2;
if (tm[k]<t)
{
w=u[tm[k]:t];
M[k+1;t]=max(w);
tp[k+1]=tm[k]-1+maxi(w);
T[t]=T[t]+(M[k+1;t]-m[k;t])^2/2;
tt=tp[k+1];
k=k+1;
else
tt=tm[k];
k=k+1;
}
}
}
```

Step 4. The sensitivity of the hysteresis index variable as calculated in the above programme to the assumption of the form of the weight function g(a,b) is best investigated using an actual time series example. Figure 14 graphs a time series for the UK real effective exchange rate as calculated by the IMF 1950-1996 (IMF, International Financial Statistics with the pre-1965 data kindly provided by the IMF Research Department).

Figure 14: UK Real Effective Exchange Rate

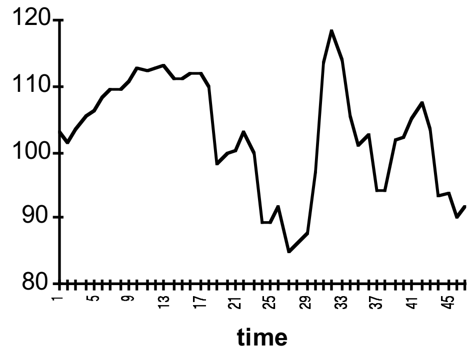


Figure 15: Uniform Distribution

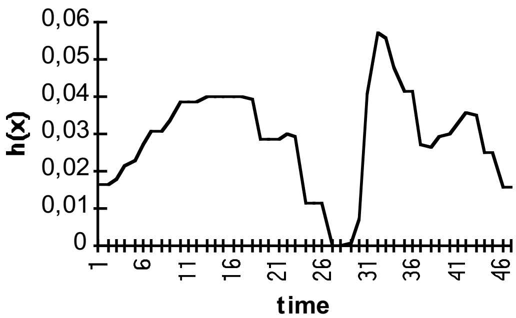


Figure 16: Normal Distribution

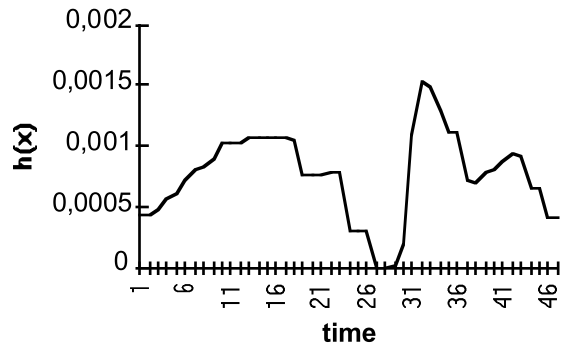


Figure 17: Exponential Distribution

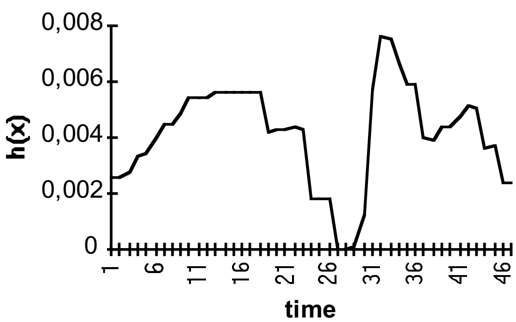


Figure 18: Poisson Uniform Distribution

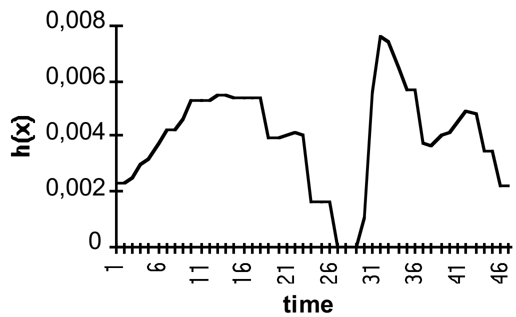


Figure 15 graphs the hysteresis index derived from this variable $h_t(x)$, under the assumption that g(a,b) is uniform. The h(x) variable generated under the assumption of the uniform distribution can then be compared with those generated under alternative assumptions on g(a,b). In Figure 16 a normal distribution is used, in Figure 17 an exponential, and in Figure 18 a Poisson. Visual inspection suggests that h(x) is not hyper-sensitive to the specification of the independence of the output of the Preisach model with respect to the distribution of micro-units is well known to experimental physicists and is often referred to as the "statistical stability" of the Preisach model (Bate (1962) and Wholfarth (1964)).

The hysteresis variable computed by the previous programme is the output of a non-linear transformation of the input variable that detects the non-dominated extrema and combines them in a non-linear way. The hysteresis variable appears smoother than the input variable, reflecting the fact that it records only the non-dominated extrema. This is also confirmed by comparing a plot of the spectral density function of the input variable with that of its hysteretic transformation. Plots of the autocorrelation function and conventional unit root tests also indicate that the hysteresis variable tends to reflect any stationarity properties of the input variable.

III. HYSTERESIS AND UK UNEMPLOMENT

Despite the conventional wisdom on the status of the natural rate hypothesis, evidence that the "z" variables invoked by the hypothesis, such as unemployment benefits, minimum wages and wage bargaining power cointegrate with unemployment is rather thin on the ground. On relation to UK unemployment, Darby and Wren-Lewis (1993) failed to find cointegration, for example, though Westaway (1996) was somewhat more successful in an equation involving the proportion of the population of working age not in work. In view of this it is interesting to see if hysteresis variables of the type suggested by the analysis of Section I of this paper, and calculated according to the programme in Section II, figure in the cointegrating, or equilibrium relationship for UK unemployment.

In the econometric results that follow, Johnasen's procedure was used to see if a unique cointegrating vector could be identified involving the log of the UK unemployment rate (LUPC) and to see of a valid error correction process could be estimated for LUPC. A vector error correction model underlies the estimation and testing framework employed. The data are annual and the estimation period is 1959-1996.

We were able to identify a single cointegrating vector. Data admissable restrictions on the loadings "alpha" matrix were imposed such that the cointegrating vector only feeds into the dynamic equation for LUPC, supporting the idea that there is a single valid ECM which determines movements in the log of the unemployment rate. The restricted cointegrating vector is reported below. Full details of these results, and in addition some single equation analysis which add further support to the Johansen results, are provided in an Appendix.

RESTRICTED COINTEGRATING VECTOR

Likelihood Ratio Test

H_0: there is a single cointegrating vector which only feeds into the LUPC equation.

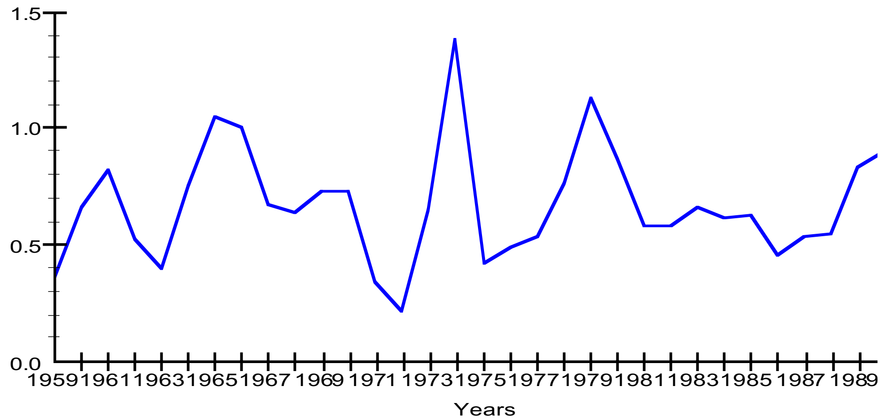
Rank=1: Chi-Sq(4) = 5.6224 [0.23] - the restrictions cannot be rejected.

$$
\underset{}{\mathrm{LUPC}} = \text{constant} + \underset{(0.42)}{2.2977}\,\mathrm{REPR} - \underset{(2.00)}{11.599}\,\mathrm{HLNER} + \underset{(.00063)}{.004704}\,\mathrm{HRR} + \underset{(0.40)}{3.8666}\,\mathrm{HLRPOIL}
$$

Restricted Alpha'

LUPC	REPR	HLNER	HRR	HLRPOIL
-0.86554 (0.13)	0.0	0.0	0.0	0.0

FIGURE 10: THE RESTRICTED COINTEGRATING RELATION



The "natural" variable in the cointegrating vector is the replacement ratio (REPR). The "unnatural" variables entering the cointegrating vector reflect the non-dominated extremum values of two types of shock. The first type of shock is that registered in the *nominal* exchange rate. The programme outlined in Section II of this paper was used to construct a hysteresis variable for the log of the nominal effective exchange rate (HLNER). The presence of this variable in the cointegrating vector suggests that money neutrality does not hold for equilibrium unemployment in the UK. The other type of shock registered in the cointegrating vector involves perturbations in *real* variables. Hysteresis variable were constructed for the log of the real price of oil, measured as the average of the Brent, Dubai and Alaskan prices in US \$ per barrel, and converted to £ using the spot exchange rate, then deflated by the GDP deflator; and for the real *ex post* interest rate, constructed from the three month Treasury Bill rate and the CPI inflation rate (HRR). The presence of these hysteresis variables in the cointegrating vector suggests that perturbations in, and not necessarily sustained shifts in, such real variables change the equilibrium rate of unemployment in the UK. These results are preliminary, but certainly of some interest.

IV CONCLUDING REMARKS

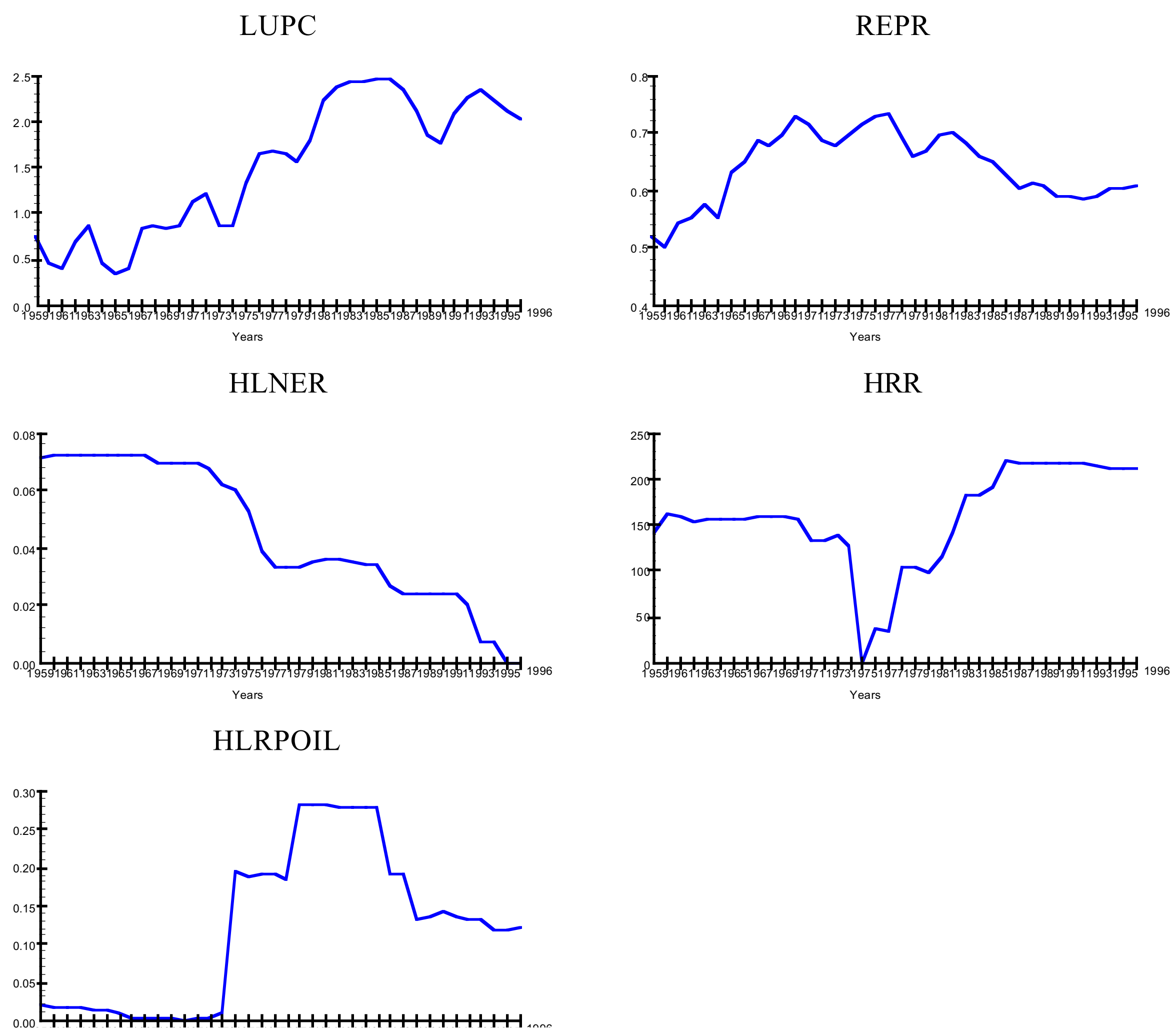
This paper has investigated the implications of hysteresis for the equilibrium rate of unemployment in the UK. The starting point was a simple model in which firms respond discontinuously and heterogeneously to some aggregate shock. The implication is hysteresis in aggregate output, employment, and by extension aggregate unemployment, in that the equilibrium rates of output, employment and unemployment are shaped, *inter alia*, by the non-dominated extremum values of the shocks experienced. A programme for calculating hysteresis time series variables that caputure the relevant profiles of the shocks experienced was then presented.

The "shocking" finding was that hysteresis variables reflecting both nominal shocks and perturbations in real variables figure in a cointegrating or equilibrium vector for UK unemployment 1959-1996. The empirical results are of interest for two reasons. The first is that they cast doubt on whether the monetary neutrality axiom, imposed by the natural rate hypothesis, applies to the equilibrium rate of unemployment. The second is that they suggest that the equilibrium rate of unemployment may display remanence in that when real shocks disappear their effects on equilibrium unemployment do not. It will be interesting to see if these results also apply to other countries and different time periods.

APPENDIX

The results that follow relate to the cointegrating properties of the log of the UK unemployment rate (LUPC), the replacement ratio (REPR) and hysteresis transforms of the log of the real price of oil, the log of the nominal effective exchange rate, and the ex-post real interest rate (these hysteresis variables are named HLRPOIL, HLNER and HRR respectively). The data are annual, and the estimation period runs from 1959-1996. The oil price, GDP deflator, exchange rate and interest data are from the IMF International Financial Statistics CD-ROM, December 1997 release, while the unemployment rate and CPI inflation are from ONS Economic Trends and the replacement rate is based on DSS data and constructed by HM Treasury. The data are graphed in Figure A1 below.

FIGURE A1:



Johansen Cointegration results are conditional on the specification of the initial unrestricted VAR, known as the UVAR. As a result we intially performed a number of diagnostic checks on a 5 variable VAR. The first set of tests are directed at determining the appropriate lag length:

TABLE A1: SPECIFICATION OF THE UVAR:

Test Statistics and Choice Criteria for Selecting the Order of the VAR Model

Based on 37 observations from 1960 to 1996. Order of VAR = 3
List of variables in the unrestricted VAR: LUPC HLNER HLRPOIL HRR REPR
List of deterministic and/or exogenous variables : C

Order	LL	AIC	SBC	Adjusted LR test
3	255.3574	175.357	110.921	n.a.
2	221.8598	166.860	122.560	CHI-SQ(25) = 38.02[.046]
1	182.4560	152.456	128.292	CHI-SQ(50) = 82.75[.002]
0	- 0.2713	- 5.2713	- 9.2986	CHI-SQ(75) = 290.2[.000]

Akaike's Information Criterion (AIC) and the adjusted Likelihood Ratio test favour three lags, while Schwarz's Bayesian Information Criterion (SBC) favours a one lag VAR. SBC always penalises additional lags more than AIC, but closer examination of the diagnostics for the individual VAR equations suggests that a single lag is insufficient, in that significant serial correlation remains (as revealed by Lagrange Multiplier statistics). A two lag UVAR is more satisfactory in this regard, but the diagnostics reveal some problems with non-normality of the residuals. These non-normality problems can be traced to outliers in the residuals of the UVAR equations which model the behaviour of the hysteresis transformed variables. Specifically, the UVAR residuals can be improved through the inclusion of three 0,1,-1,0 dummy variables.

The first of these dummy variables relates to the timing of the discrete jump in the real oil price following the first OPEC shock (required in the HLRPOIL equation). The second dummy relates to the rapid interest rate hike which occurred at the peak of the Lawson boom in the late 1980s (required in the HRR equation) and the final dummy is needed in the period equation). Importantly, none of these dummies are statistically significant in the LUPC equation of the UVAR, nor are they statistically significant in any other equations of the UVAR, though each is clearly statistically significant in the UVAR as a whole (see Table A2 below).

TABLE A2: SIGNIFICANCE OF DUMMIES IN UVAR EQUATIONS

DD73				
Equation	Coefficient	s.e.	t-value	t-probability
LUPC	-0.13474	0.10	-1.40	0.17
REPR	-0.00042	0.02	-0.03	0.98
HLNER	-0.00227	0.00	-1.12	0.27
HRR	2.96780	12.04	0.25	0.81
HLRPOIL	-0.09712	0.02	-4.66**	0.00

DX86				
Equation	Coefficient	s.e.	t-value	t-probability
LUPC	0.06171	0.11	0.56	0.58
REPR	0.00016	0.02	0.01	0.99
HLNER	-0.00447	0.00	-1.93	0.07
HRR	38.81200	13.83	2.81**	0.01
HLRPOIL	-0.05418	0.02	-2.26*	0.03

DD93				
Equation	Coefficient	s.e.	t-value	t-probability
LUPC	0.07719	0.09	0.82	0.42
REPR	0.00543	0.02	0.33	0.74
HLNER	-0.00852	0.00	-4.31	0.00
HRR	-6.41700	11.78	-0.55	0.59
HLRPOIL	0.00144	0.02	0.07	0.94

Joint Significance Tests:

All dummies in each equation		Dummy in all equations	
Equation	F-statistic F(3,24)	Dummy	F-statistic F(5,20)
LUPC	1.088 [.373]	DD73	5.67 [.002]**
REPR	0.038 [.990]	DX86	6.33 [.001]**
HLNER	7.266 [.001]**	DD93	4.56 [.006]**
HRR	2.794 [.062]		
HLRPOIL	8.701 [.000]**	Joint significance of all 3 dummies:	Chi-Sq (15) 80.08[.000]**

Normality and serial correlation tests on the residuals from the dummied UVAR are presented in Table A3.

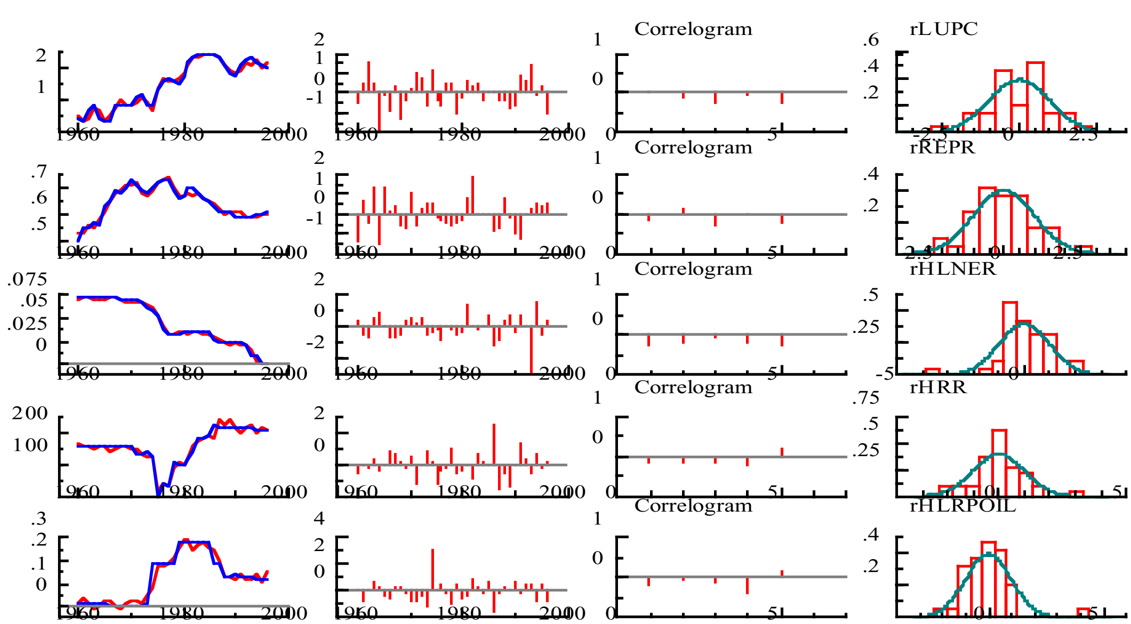
TABLE A3: UVAR DIAGNOSTICS

correlation of actual & fitted	0.990	0.959	0.996	0.973	0.978
Portmanteau 5 lags	7.534	5.765	7.516	5.793	10.65
AR1-2 F(2,22)	0.722[.50]	0.082[.92]	3.340[.06]	1.530[.24]	0.442[.65]
Normality Chi-Sq(2)	3.677[.16]	1.577[.45]	2.719[.26]	1.000[.61]	4.206[.12]

System Diagnostics

Vector portmanteau 5 lags	118.48
Vector AR1-2	F(50,48) = 1.221 [.24]
Vector normality	Chi-Sq(10) = 13.49 [.20]
F-tests on retained regressors	F(5, 20)
LUPC_1	6.69 [.001] **
LUPC_2	3.74 [.015] *
REPR_1	3.22 [.027] *
REPR_2	0.94 [.475]
HLNER_1	21.6 [.000] **
HLNER_2	2.92 [.039] *
HRR_1	3.61 [.017] *
HRR_2	6.36 [.001] **
HLRPOIL_1	20.7 [.000] **
HLRPOIL_2	6.53 [.001] **

FIGURE A2: PERFORMANCE OF THE UVAR



Having estimated a UVAR with Gaussian residuals, it is feasible to test for the number of linearly independent cointegrating vectors among the five variables.

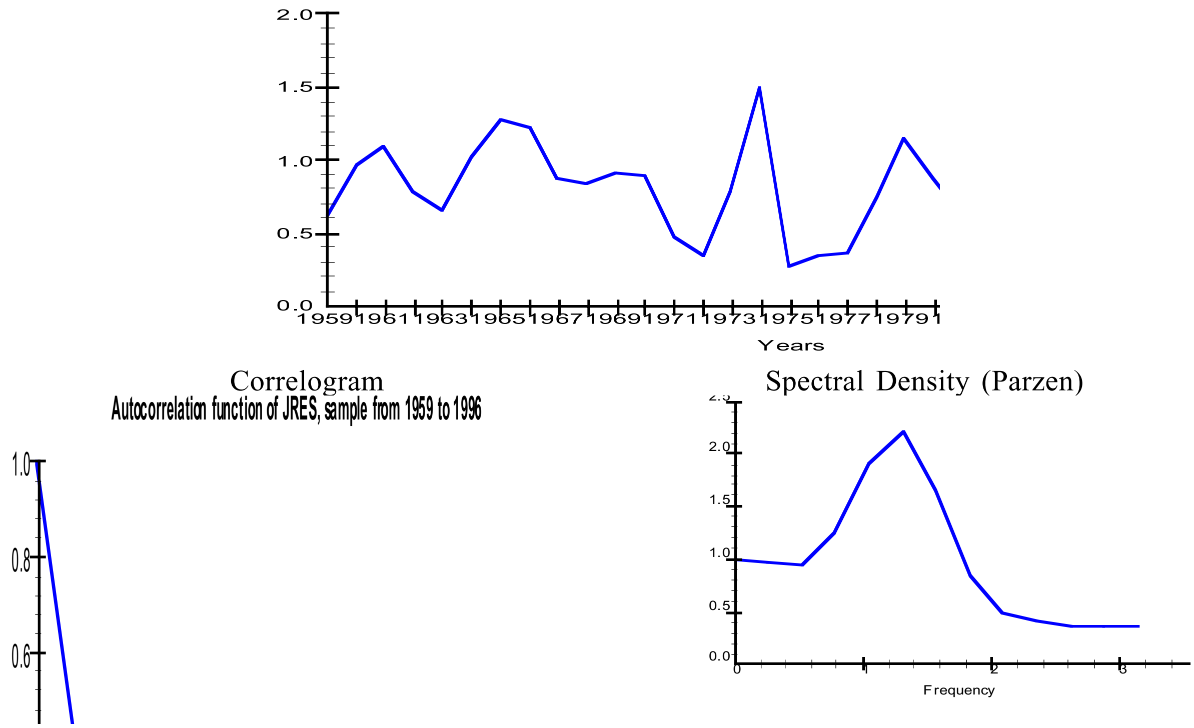
The UVAR is still conditioned on the three dummy variables, none of which have a long run effect, but all of which will affect the relevant critical values for the Johansen maximal know from work by Perron and Phillips that a positive adjustment to the standard critical values is probably appropriate. The cointegration tests on the dummied UVAR are reported in Table A4 and these are followed by cointegration tests based on a UVAR with no conditioning dummies. Whilst estimates based on the undummied UVAR are likely to be adversely effected by the non-normality of the system residuals, the cointegration tests are less subject to bias, so may have some value in clarifying inference.

TABLE A4: COINTEGRATION ANALYSIS 1959-1996

COINTEGRATION TESTS based on UVAR conditioned on 3 dummy variables						
	eigenvalue			loglik for rank		
				482.257 0		
	0.656235			502.546 1		
	0.580331			519.043 2		
	0.492231			531.920 3		
	0.243391			537.219 4		
	0.002610			537.269 5		
	Maximum Eigenvalue Test			Trace Test		
Ho:rank=p	-Tlog(1-\mu)	using T-nm	95%	-T\Sum log(.)	using T-nm	95%
p == 0	40.58 **	29.90	33.5	110.02 **	81.07 **	68.5
p <= 1	33.00 **	24.31	27.1	69.45 **	51.17 *	47.2
p <= 2	25.75 **	18.98	21.0	36.45 **	26.86	29.7
p <= 3	10.60	7.81	14.1	10.70	7.88	15.4
COINTEGRATION TESTS based on unconditioned UVAR						
	Maximum Eigenvalue Test			Trace Test		
Ho:rank=p	-Tlog(1-\mu)	using T-nm	95%	-T\Sum log(.)	using T-nm	95%
p == 0	41.65 **	30.69	33.5	92.55 **	68.19	68.5
p <= 1	26.58	19.58	27.1	50.90 *	37.5	47.2
p <= 2	16.51	12.17	21.0	24.32	17.92	29.7

Tests based on the unconditioned UVAR are certainly easiest to interpret, with the maximal eigenvalue and trace tests both supporting the conclusion of a single cointegrating vector using 99% critical values, the small sample corrected tests reject cointegration at the 95% but not the 90% level. In the case of the dummy adjusted UVAR, we clearly need to adjust the critical values up by some unknown amount. The large sample tests suggest up to 3 cointegrating vectors at standard 99% critical values, though plots of the residuals would not vector, as is the informal graphical evidence. The Johansen cointegrating vector is shown in Figure A3 along with its correlogram and spectral density, all of which are supportive of cointegration.

FIGURE A3: THE JOHANSEN BASED COINTEGRATING RELATION



ESTIMATED COINTEGRATING VECTOR (NORMALISED ON LUPC)

LUPC = constant + 1.7598 REPR -7.8407 HLNER + 0.006582 HRR + 4.0466 HLRPOIL

RESTRICTED COINTEGRATING VECTOR

Likelihood Ratio Test

H_0: there is a single cointegrating vector which only feeds into the LUPC equation.

Rank=1: Chi-Sq(4) = 5.6224 [0.23] - the restrictions cannot be rejected.

LUPC = constant + 2.2977 REPR - 11.599 HLNER +0.0047 HRR + 3.8666 HLRPOIL

(0.42)	(2.00)	(.00063)	(0.40)

Restricted Alpha'

LUPC	REPR	HLNER	HRR	HLRPOIL
-0.86554 (0.13)	0.0	0.0	0.0	0.0

The Likelihood Ratio test presented above supports the hypothesis that there is a single cointegrating vector which only feeds into the DLUPC equation of the VEC-ECM. Formally, imposing this restriction cannot be rejected, and imposition improves the efficiency of the estimated cointegrating vector and allows us to identify valid LR standard errors. These standard errors reveal that each of the five variables plays a significant role in the vector. The conclusion that the cointegrating vector feeds into solely the DLUPC equation is also supported through examination of each equation of the unrestricted VEC-ECM (still conditioned on the dummies). The t_{ECM} statistics from the individual equations of the VEC-ECM are reported below, and importantly, the only statistically significant ECM term appears in the LUPC equation.

TABLE A5: ERROR CORRECTION COEF. AND THE t_{ECM} FROM THE VEC-ECM

Equation	Coefficient	s.e.	t-value	t-probability
DLUPC	-0.8901	0.138	-6.45**	0.00
DREPR	-0.0338	0.029	-1.17	0.25
DHLNER	-0.0025	0.003	-0.84	0.40
DHRR	3.6765	24.48	0.15	0.88
DHLRPOIL	0.0324	0.039	0.84	0.41

These results suggest that non-system based estimation and testing procedures are applicable to the data. In Table A6 we present Granger-Engle estimates of the same cointegrating relation. Once again the results are supportive of the existence of cointegration and the parameter estimates are close to those obtained using Johansen's procedure.

TABLE A6: GRANGER-ENGLE ESTIMATES OF THE COINTEGRATING VECTOR

38 observations used for estimation from 1959 to 1996

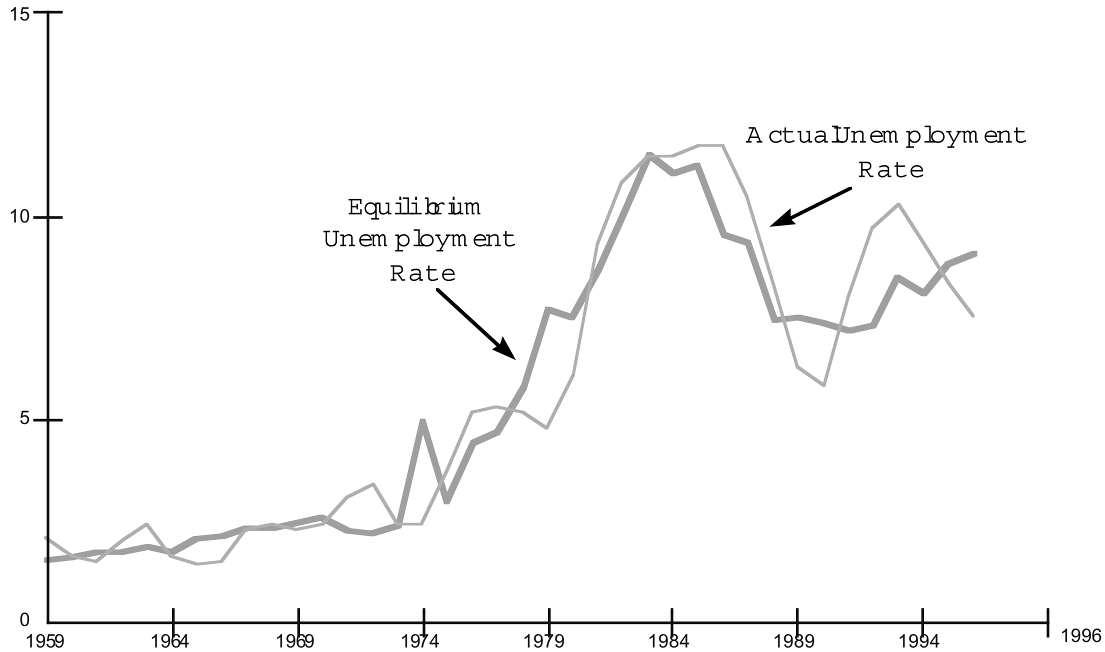
Regressor	Coefficient	Standard Error	T-Ratio[Prob]
C	-0.1583	0.6405	-.24720[.806]
REPR	2.0977	0.8005	2.6205[.013]
HLNER	-14.8072	2.6794	-5.5263[.000]
HRR	0.0039	0.0010	3.7592[.001]
HLRPOIL	3.1198	0.5875	5.3108[.000]
R-Squared	0.9001	DW-statistic	1.3435

Unit root tests for residuals: 5% critical value for the Dickey-Fuller statistic = -4.8380

	Test Statistic	Log Likelihood	AIC	SBC	HQC	LR Test
DF	-3.9779	3.4004	2.4004	1.6373	2.1402	F(1,36)=10.88[.00]
ADF(1)	-5.5319	8.1523	6.1523	4.6259	5.6318	F(1,35)=.001[.97]
ADF(2)	-3.9446	8.1613	5.1613	2.8717	4.3805	

So, to summarise, using Johansen's cointegration testing and estimation procedure we were able to identify a single cointegrating vector involving all five variables LUPC, REPR, HLRPOIL, HLNER and HRR. This vector feeds significantly into a dynamic equation for unemployment, but is statistically insignificant in the other equations of the VEC-ECM. These results are additionally supported by Granger Engle estimates of the cointegrating vector. Our empirical results are therefore supportive of an "equilibrium" log unemployment rate which is determined by the replacement ratio and to non-dominated extremum in the real price of oil, the nominal effective exchange rate and the real interest rate. The implied equilibrium rate of unemployment is graphed in Figure A4, along with the actual unemployment rate over the period 1959-1996.

FIGURE A4: ACTUAL AND "EQUILIBRIUM" UNEMPLOYMENT RATES



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