Geometry-Driven Parametric Sensitivity Analysis for Free-Form Marine Shapes

(Work In Progress)

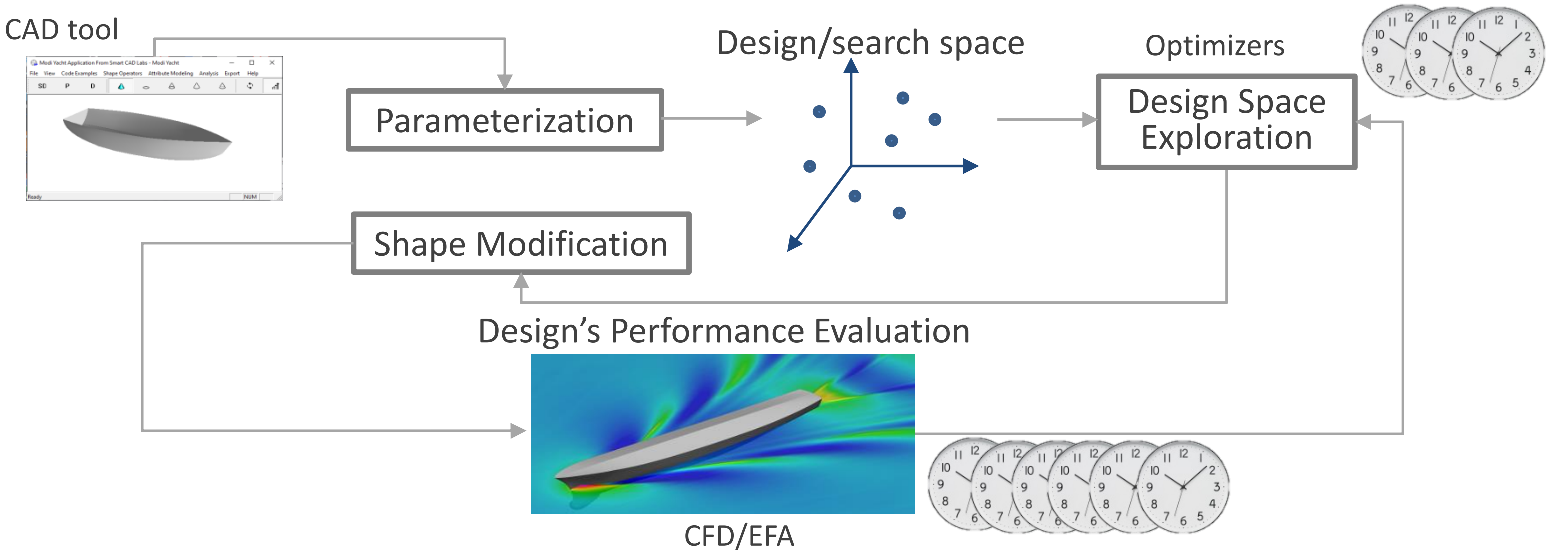
Shahroz Khan Panagiotis Kaklis Andrea Serani Matteo Diez

Department of Naval Architecture, Ocean and Marine Engineering, University of Strathclyde Glasgow, UK

CNR-INM, National Research Council - Institute of Marine Engineering, Rome, Italy

GRAPES
learninG, pRocessing And oPtimising shapES

Simulation-Driven Optimization (SDO)



High computational cost 

Rises exponentially with **design space dimensionality**

Existing Approaches

Design Space Dimensionality Reduction

- Unsupervised – PCA, Auto-encoders

  Latent **GEOMETRIC** features for lower dimensional representation of original design space.

- Supervised – Sensitivity Analysis (Sobol's method)

  Parameters with high variability impact on **performance**.

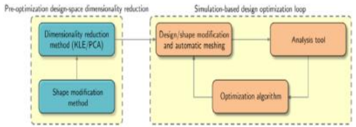
  Quantify uncertainty in performance.

Surrogate Modelling

- Supervised – Deep/Machine Learning (PINN, NN, CNN, GAN)

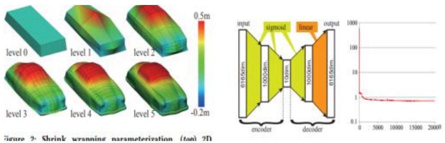
  Bypass the design's evaluation with CFD/FEA.

[D'Agostino et al., 2020]



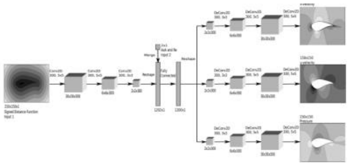
Principal Component Analysis (PCA)

[Umetani, 2017]



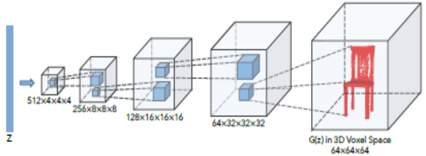
Autoencoders

[Bhatnagar et al., 2019]



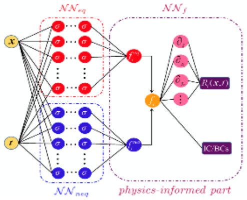
Convolutional Neural Network (CNN)

[Wu et al., 2016]



Generative Adversarial Network (GAN)

[Loua et al., 2020]



Physics Informed Neural Network (PINN)

Drawbacks

Supervised Techniques

- Design-Space Dimensionality Reduction – sensitivity analysis
- Surrogate Modelling

Require big datasets for reliable training

1 simulation → 1 hour (low fidelity)

n – dimensional design space

100 simulations → 100 hours

$n \times 10$ design instance

(least requirement for reliable training)

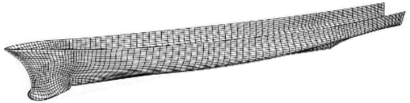
computational complexity still exists

Objective

- Compliment physics with computationally less expensive property?

quantity $\approx$ Physics and computationally less expensive

- Substituting design's physical properties by geometric properties (moments)?



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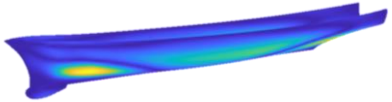


computational cost of geometrical properties <<< computational cost of physics

- Can we make a preliminary decision on **sensitivity of parameters** with geometrical properties?

Methodology – Geometric Integrals

Geometric moments of a shape

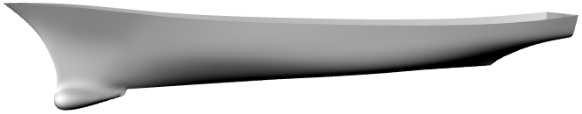
1. are intrinsic properties of its underlying geometry
2. provide a unifying medium between its **geometry** and **physics**.

$(l+m+n)th$ – order moment (**Riemann integrals**):

$$M_{lmn}(\mathcal{G}) = \iiint x^l y^m z^n \, \rho(x,y,z)\, dx\, dy\, dz$$

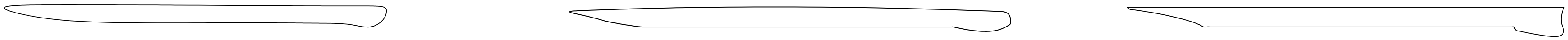
$$\rho(x,y,z) = \begin{cases} 1 & \text{if } x,y,z \in \mathcal{G} \\ 0 & \text{otherwise} \end{cases}$$

Geometric domain: $\mathcal{G}$



Methodology – Geometric Integrals

1st order　　　2nd order　　　….　　　$(l+m+n)$th order



Moments are invariant to transformation (Translation, Scaling, Rotation,)

$(l+m+n)-th$ order central moment:

$$\mu_{lmn}(\mathcal{G}) = \iiint (x-x_c)^l (y-y_c)^m (z-z_c)^n \, dx\, dy\, dz$$

(Invariant to translation)

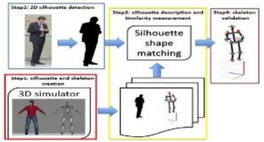
Methodology – Applications of Geometric Integrals

Computer-Aided Design and Computer Vision:

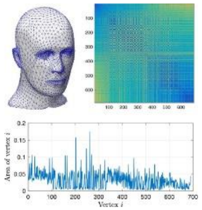
- Object Recognition [Atrevi et al., 2017]
- Shape Retrieval [Luciano & Hamza, 2019]
- Rigid Body Transformation [Bronstein & Bronstein, 2018]

Geometric foundation for many physical analyses:

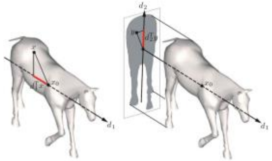
- Structural analysis [Kim et al., 2007]
- Meshless physical analysis [Taber et al., 2018]
- Governing equations of motion [Newman, 2008]
- Fluid simulations [Jin et al., 2019]
- Hydrodynamic and Hydrostatic stability [Biran & Pulido, 2013]



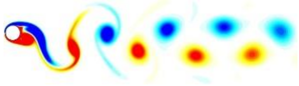
[Atrevi et al., 2017]



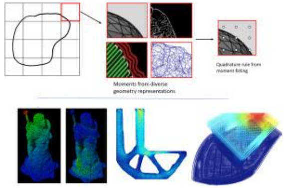
[Luciano & Hamza, 2019]



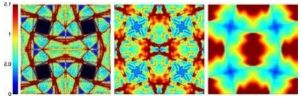
[Bronstein & Bronstein, 2018]



[Jin et al., 2019]



[Taber et al., 2018]



[Fox et al., 2018]

Methodology – Parametric Sensitivity Analysis (PSA)

Sobol' total sensitivity [Borgonovoa & Plischkeb, 2016]

- Variance-based method
- Quantifies parameter's direct contribution to QoI variance
- Sensitivity indices

Sensitive parameters: Sensitivity Indices ≥ 0.05

Dimension reduction

Perform optimisation with sensitive parameters (reduced dimensionality)

Uncertainty Quantification

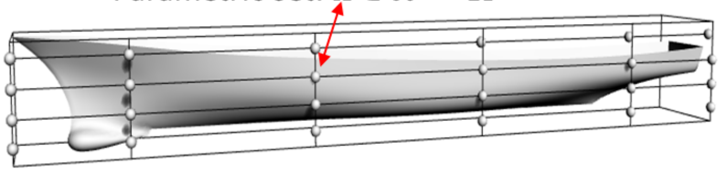
Refine the model to reduce variance caused by sensitive parameters

Methodology – Sensitivity Indices

Sensitivity indices of n design parameters with moments of up to $p = (l + m + n) - th$ order.

Design Parameterization

Parametric set: $\mathbf{x} \in \mathcal{X} \subset \mathbb{R}^n$



$$\boldsymbol{\mu} = [\mu_{200} \quad \mu_{020} \quad \mu_{002} \quad \cdots \quad \mu_p]$$

$$\begin{bmatrix} i_1^{200} & i_2^{200} & i_3^{200} & \cdots & i_n^{200} \\ i_1^{020} & i_2^{020} & i_3^{020} & \cdots & i_n^{020} \\ i_1^{002} & i_2^{002} & i_3^{002} & \cdots & i_n^{002} \\ & & \vdots & & \\ i_1^{p} & i_2^{p} & i_3^{p} & \cdots & i_n^{p} \end{bmatrix}$$

Sum

$$\begin{bmatrix} s^{200} = \sum(i_1^{200}, i_2^{200}, \ldots, i_{27}^{200}) \\ s^{020} \\ s^{002} \\ \vdots \\ s^n \end{bmatrix}$$

Sum

$$\mathcal{S} = \sum(s^{200}, s^{020}, \ldots, s^n)$$

Normalised Sensitivity indices

$$\begin{bmatrix} I_1 = \frac{\mathcal{S}}{s^{200}} \\ I_2 = \frac{\mathcal{S}}{s^{020}} \\ I_3 = \frac{\mathcal{S}}{s^{002}} \\ \vdots \\ I_n = \frac{\mathcal{S}}{s^{n}} \end{bmatrix}$$

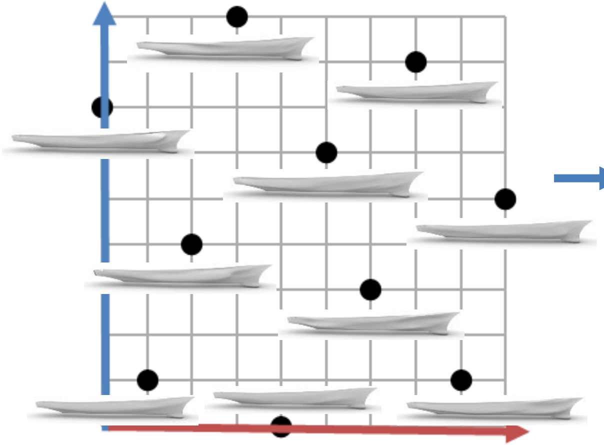
Methodology – Sensitive Parameters

Sensitive/Significant Parameters

- Sensitivity indices greater than significant threshold ($\varphi = 0.05$).
- m significant parameters with $I \geq \varphi$.

If $m < n$ (n: original number of design parameters)

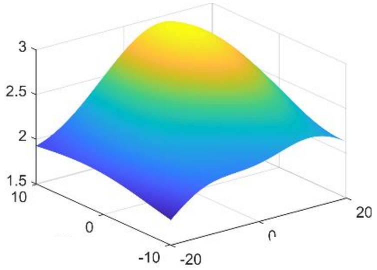
Construct m −dimensional design space



Supervised Learning

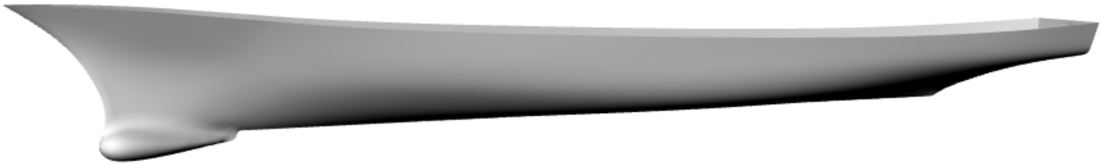
Optimisation

Surrogate model



Test Case

DTMB 5415 Naval Ship Model



- Parameterised with 27 design parameters
- Objective:

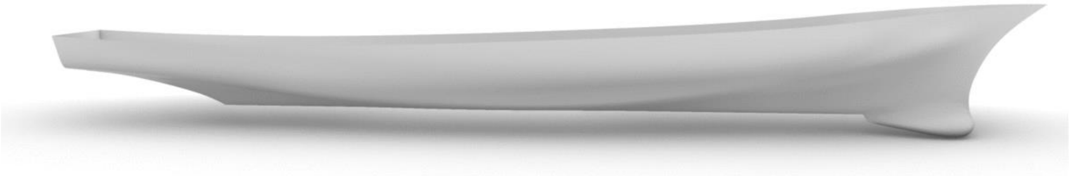
  Sensitivity of design parameters w.r.t.

  calm-water wave resistance coefficient (c_w)

Quantity	Value
Displacement	0.549 m^3
Length between perpendiculars	5.720 m
Beam	0.760 m
Draft	0.248 m
Longitudinal centre of gravity	2.881 m
Vertical centre of gravity	0.056 m
Water density	998.5 kg/m^3
Kinematic viscosity	1.09E-6 m^2/s
Gravity acceleration	9.803 m/s^2
Froude Number	0.250

Test Case

- 27-**Dimensional original** design space



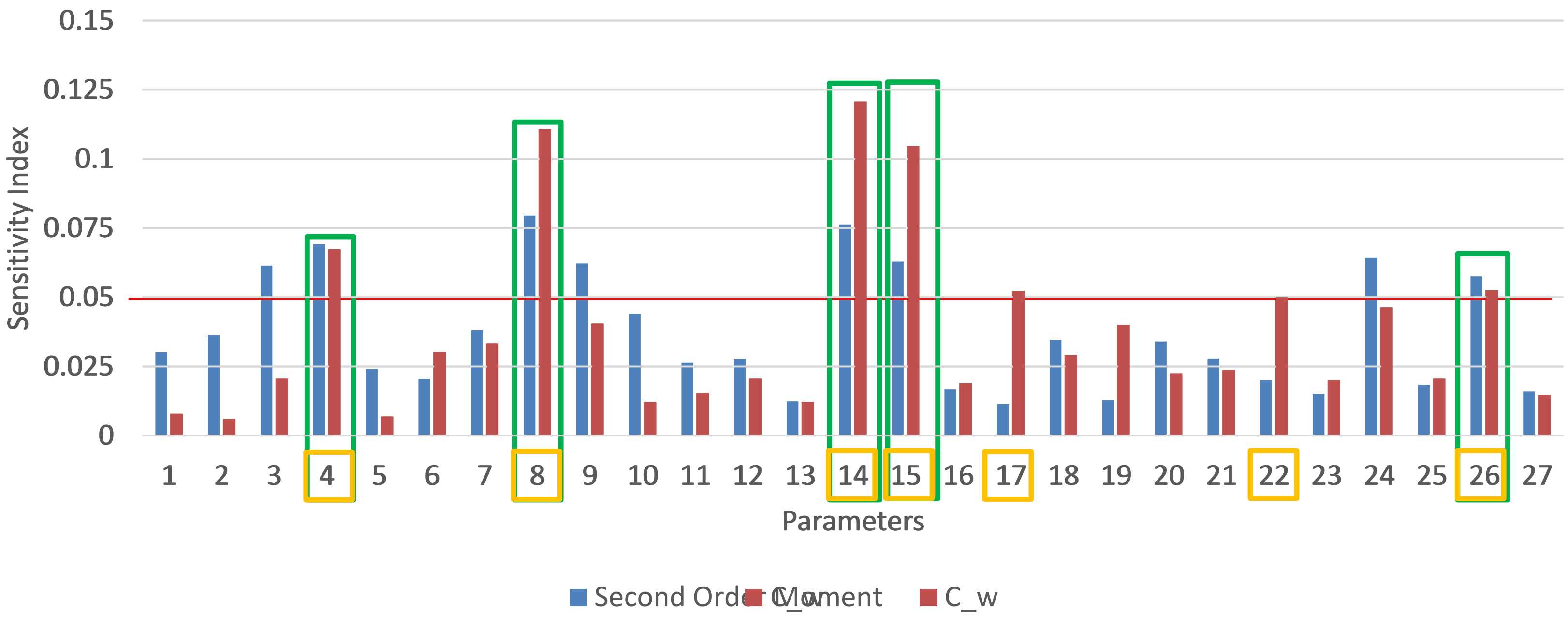
- Dataset Size:

  9000 uniformly distributed **designs** – sampled with **Monte Carlo method**

- **Hydrodynamic simulations:**
  - Performed with WARP (Wave Resistance Program), developed at CNR-INSEAN [Bassanini et al., 1994].

- **Moments of Second Order:**
  - Evaluated with Divergence Theorem [Krishnamurthy & McMains., 2011].

Results – Parametric Sensitivity



Top four sensitive parameters w.r.t. c_w are also sensitive w.r.t. 2nd order moments

Results - Surrogate Modelling

Gaussian process regression - [Williams & Rasmussen, 2006]

Hyper-parameter (θ) optimization using maximum likelihood method:

$$\theta_{optimum} = \arg\max \log p(\mathrm{y}|\theta) = -\mathcal{L}(\theta),$$

$$\mathcal{L}(\theta) = \frac{1}{2}\log|\mathrm{K}_D(\theta)| + \frac{1}{2}\mathrm{y}^T\mathrm{K}_D^{-1}(\theta)\mathrm{y} + \frac{n}{2}\log(2\pi)$$

K_D: Kernel function - Squared Exponential

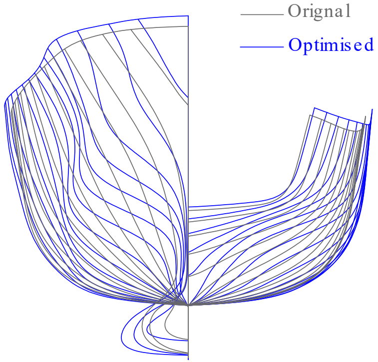
Optimisation - Projected gradient decent method

$R^2 = 0.9576$ **Cross-Validation MSE = 0.26836**

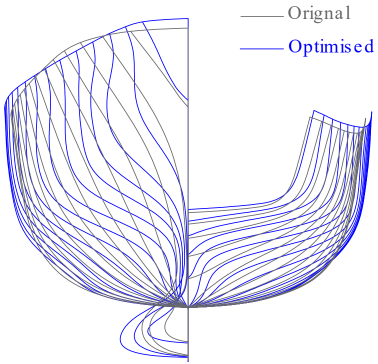
Results – Optimisation

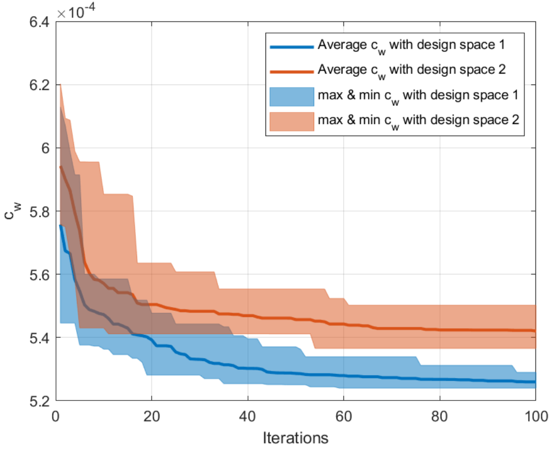
	PSA with c_w (Design Space 1)	PSA with 2nd order moment (Design Space 2)
Sensitive parameters (Index>0.05)	7	7
Design space dimensionality	7	7
Optimisation Iterations	500	500
Optimised design c_w	$5.2241e-04$	$5.3578e-04$
Difference (Absolute Percentage Error)	2.5589%	
Computational Cost	~375 Hours	~9.5 Hours

Design Space 1



Design Space 2





Conclusions & Future Work

Conclusion:

Computationally efficient geometry-based quantity to compliment design's physics during parametric sensitivity analysis.

Future Work:

- Implementation sensitivity analysis with higher order moments, i.e., forth, fifth, etc.
- Integration of high-order moments in Surrogate modelling, especially during Physics-Informed learning.

QUESTIONS?

Funding

University of Strathclyde:

- European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant GRAPES (agreement No. 860843).

CNR-INM:

- US Office of Naval Research through NICOP grant N62909-18-1-2033.

Study's Details

https://www.shahrozkhan.info/research