Supporting Expensive Physical Models With Geometric Moment Invariants to Accelerate Sensitivity Analysis for Shape Optimisation

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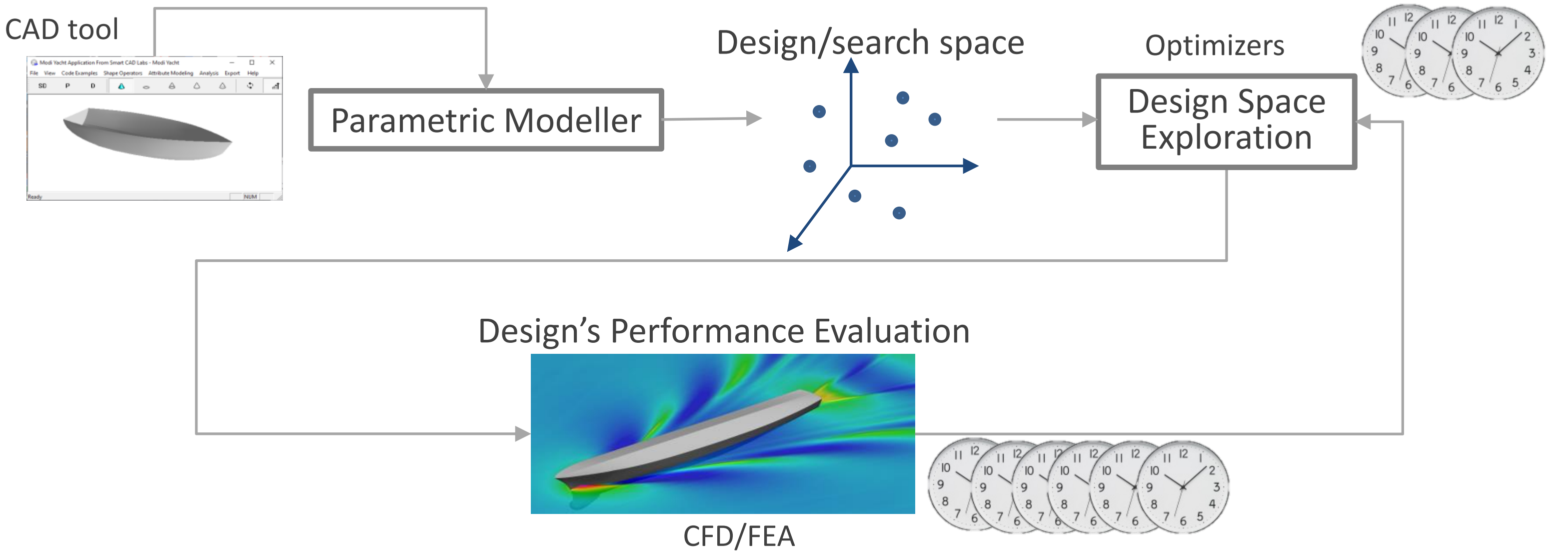
GRAPES
learninG, pRocessing And oPtimising shapES

Motivation

Simulation-Driven Optimization (SDO)



High computational cost 

Rises exponentially with the **dimension of the Design space**

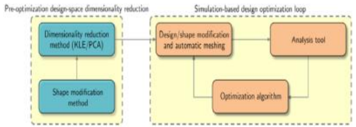
Existing Approaches

Design Space Dimensionality Reduction

- Unsupervised – PCA, Auto-encoders

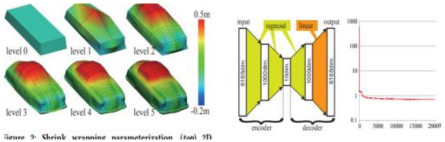
  Latent **GEOMETRIC** features

[D'Agostino et al., 2020]



Principal Component Analysis (PCA)

[Umetani, 2017]

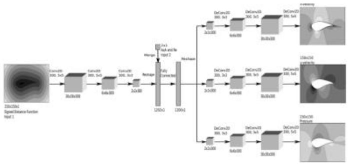


Autoencoders

- Supervised – Sensitivity Analysis

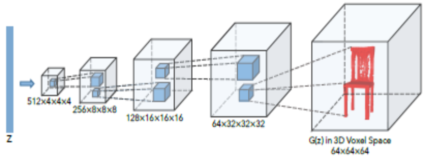
  Parameters with high variability impact on **performance**.

[Bhatnagar et al., 2019]



Convolutional Neural Network (CNN)

[Wu et al., 2016]



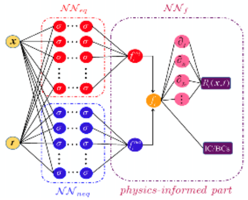
Generative Adversarial Network (GAN)

Surrogate Modelling

- Supervised – Deep/Machine Learning (PINN, NN, CNN, GAN)

  Bypass the design's evaluation with CFD/FEA.

[Loua et al., 2020]



Physics Informed Neural Network (PINN)

Drawbacks

Supervised Techniques

- Design-Space Dimensionality Reduction – sensitivity analysis
- Surrogate Modelling

Need big datasets for reliable training (High-dimensional problems)

- High fidelity simulation: single run is expensive

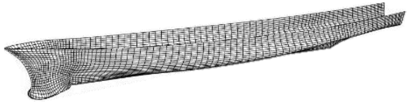
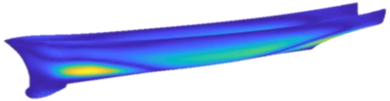
computational complexity still exists

Objective

- Support physics with computationally less expensive property?

quantity $\approx$ **Physics and computationally less expensive**

Geometric integrals (moments)

 ∞ 

- A preliminary decision on **sensitivity of parameters** with geometrical properties?

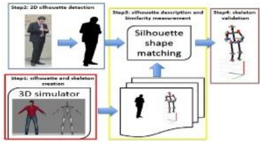
Applications of Geometric Integrals (moments)

Computer-Aided Design and Computer Vision:

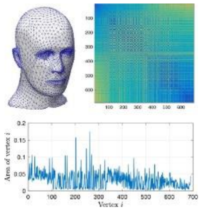
- Object Recognition [Atrevi et al., 2017]
- Shape Retrieval [Luciano & Hamza, 2019]
- Rigid Body Transformation [Bronstein & Bronstein, 2018]

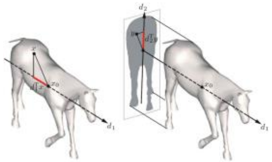
Geometric foundation for many physical analyses:

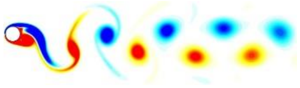
- Structural analysis [Kim et al., 2007]
- Meshless physical analysis [Taber et al., 2018]
- Governing equations of motion [Newman, 2008]
- Fluid simulations [Jin et al., 2019]
- Hydrodynamic and Hydrostatic stability [Biran & Pulido, 2013]

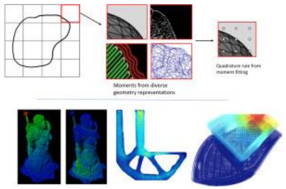


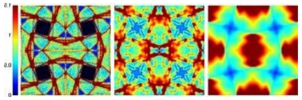
[Atrevi et al., 2017]


[Luciano & Hamza, 2019]


[Bronstein & Bronstein, 2018]


[Jin et al., 2019]


[Taber et al., 2018]


[Fox et al., 2018]

Methodology – Geometric Integrals

Geometric moments of a shape

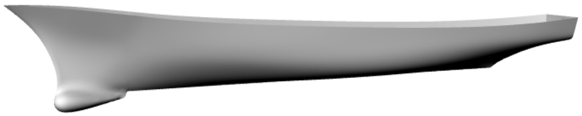
1. are intrinsic properties of its underlying geometry
2. provide a medium for interoperability between **geometry** and **physics**.

$(l + m + n)th -$ order moment (**Riemann integrals**):

$$M_{lmn}(\mathcal{G}) = \iiint x^l y^m z^n \rho(x, y, z)\, dx\, dy\, dz$$

$$\rho(x, y, z) = \begin{cases} 1 & \text{if } x, y, z \in \mathcal{G} \\ 0 & \text{otherwise} \end{cases}$$

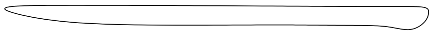
Geometric domain: $\mathcal{G}$



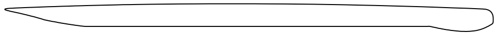
Zeroth order moment: $M_{000} =$ volume of $\mathcal{G}$

Methodology – Geometric Integrals

1st order

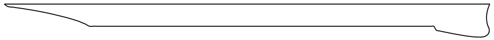


2nd order



....

$(l+m+n)$th order



Moments are variant to transformation (Translation, Scaling, Rotation,)

$(l+m+n)-th$ order central moment:

$$MI^{lmn}(\mathcal{G}) = \iiint (x-x_c)^l (y-y_c)^m (z-z_c)^n \, dx \, dy \, dz$$

(Invariant to translation)

where x_c, y_c and z_c are the centroidal coordinates of $\mathcal{G}$

($MI^{lmn}(\mathcal{G})$ can be also regularised in order to become invariant to uniform scaling)

$$\mathcal{MI}^s = [\mathbf{MI^2}, \mathbf{MI^3}, \dots, \mathbf{MI^s}]$$

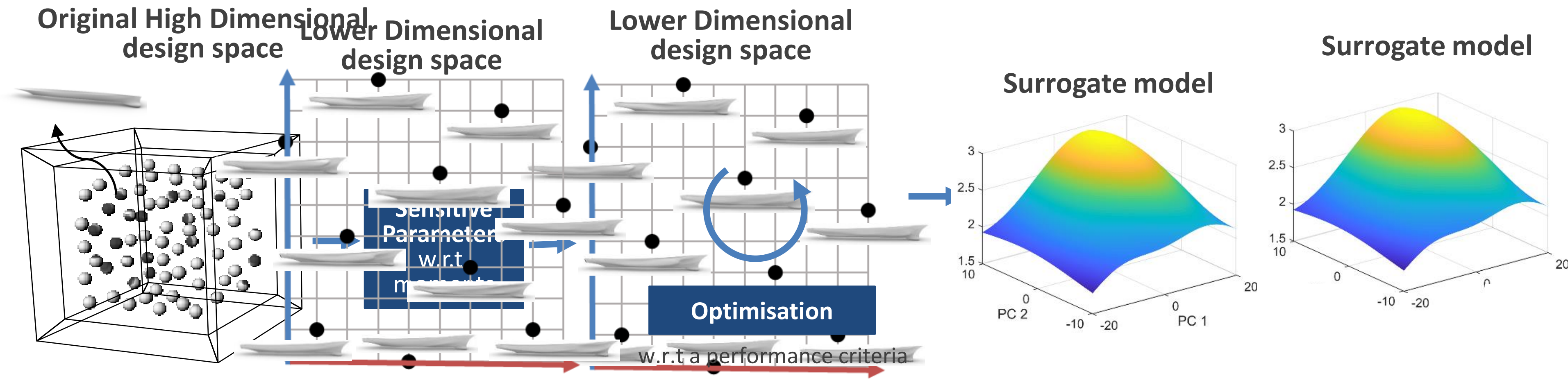
shape signature vector containing all component of moments up to s^{th} order

$\mathbf{MI^s}$ contain all moments $MI^{lmn}(\mathcal{G})$ such that $l+m+n=s$

Methodology – Sensitive Parameters

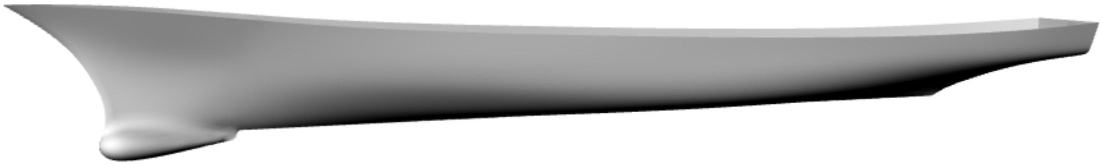
Sensitive/Significant Parameters

- Global Variance-Based Sensitivity Analysis (Sobol's Method)
- Sensitivity indices of each parameter w.r.t. QoI (Geometric Moments)
- Sensitive parameters: sensitivity indices $\geq$ threshold (φ)



Test Case

DTMB 5415 Naval Ship Model



- Parametric modeler of GMF type depending on 27 design parameters
- GMF (global modification function) is a grid modification approach

- Objective:

  Optimised design for

  calm-water wave resistance coefficient (c_w)

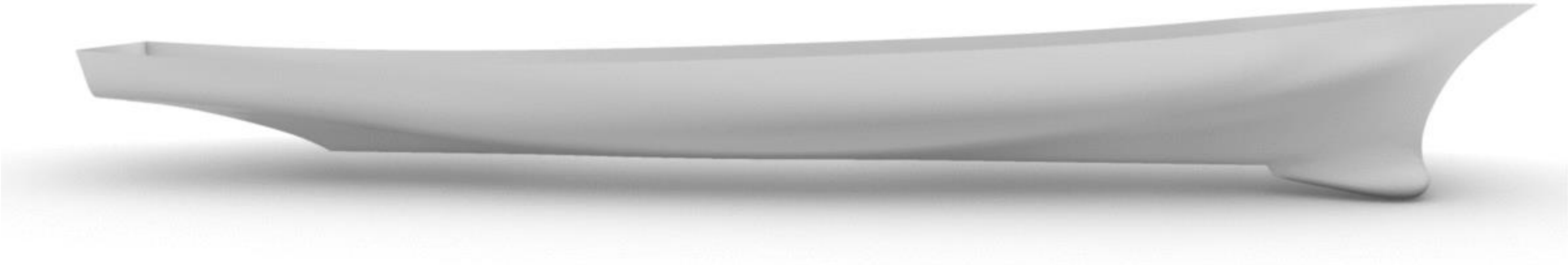
Quantity	Value
Displacement	0.549 m^3
Length between perpendiculars	5.720 m
Beam	0.760 m
Draft	0.248 m
Longitudinal centre of gravity	2.881 m
Vertical centre of gravity	0.056 m
Water density	998.5 kg/m^3
Kinematic viscosity	1.09E-6 m^2/s
Gravity acceleration	9.803 m/s^2
Froude Number	0.250

Test Case

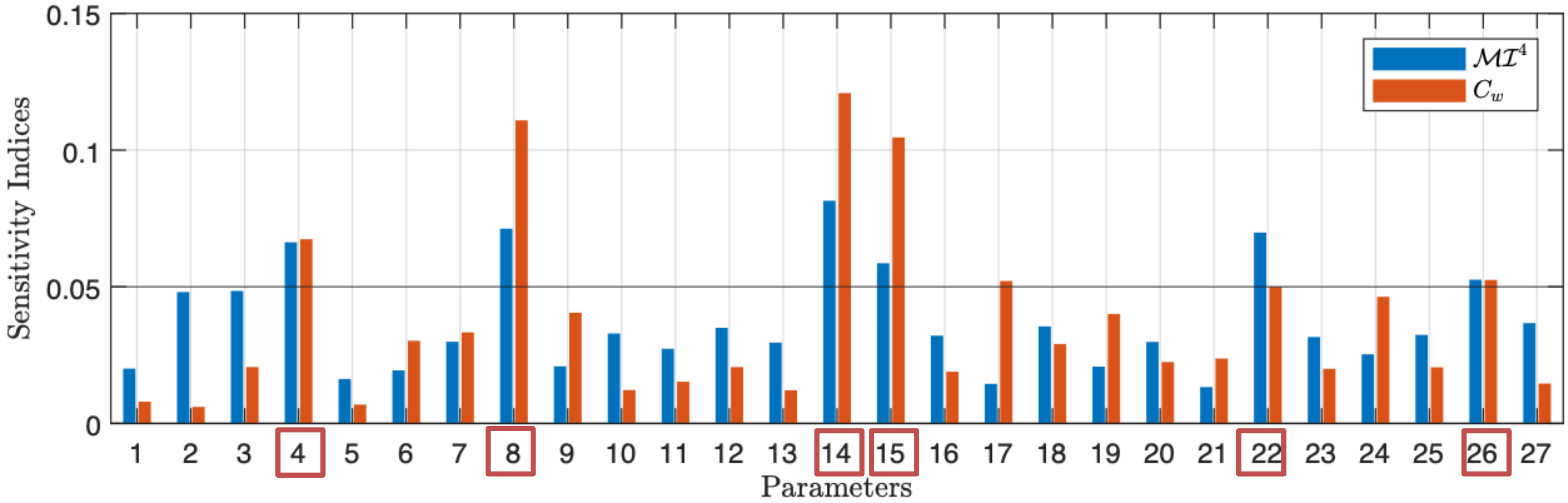
- **27-Dimensional original** design space

- Dataset Size:

  9000 uniformly distributed **designs** – sampled with **Monte Carlo method**



Results – Parametric Sensitivity



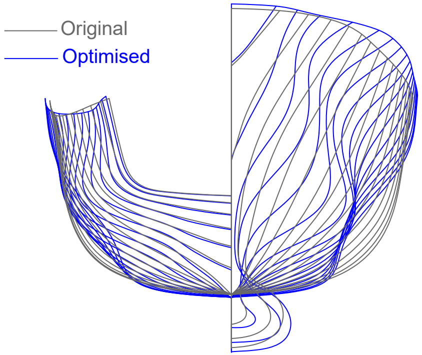
All parameters sensitive w.r.t. 4th order moments ($\mathcal{MJ}^4$) are also sensitive w.r.t. c_w

6 – lower dimensional design space with $\mathcal{MJ}^4$
7 – lower dimensional design space with c_w

Results – Optimisation

Design Space #1

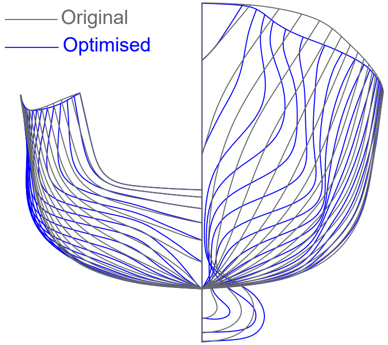
(a) (b)



Design Space #2

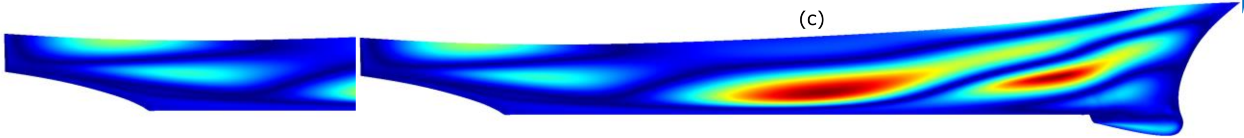
(a) (b)

Original
Optimised

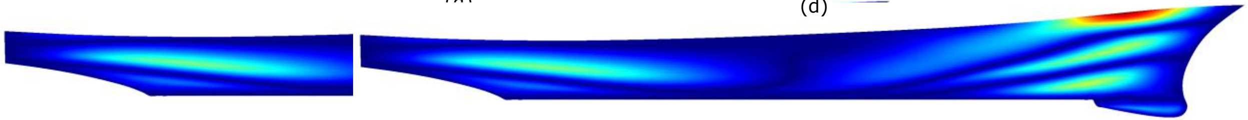


with

(c)



(d)



Computational cost of performing sensitivity analysis

~375 Hours

~9.5 Hours

Conclusions & Future Work

Conclusion:

Moments can be used to replace/complement design's physics for parametric sensitivity analysis.

Future Work:

- New test cases with different ships, parametric modelers and solvers
- Test the influence of specific ($MI^{k00}(\mathcal{G})$) or higher-order (>4) moments
- Investigate theoretically the correlation between moments and the physical model adopted by our solvers
- Integrate moments in surrogate modelling, especially during Physics-Informed learning.

QUESTIONS?

Funding

University of Strathclyde

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HINGE

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