1 2	EFFECTS OF AXIAL LOADS AND HIGHER ORDER MODES ON THE SEISMIC RESPONSE OF <mark>TALL</mark> BRIDGE PIERS
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15 ABSTRACT

Tall piers are essential components of the earthquake resisting system of bridges. The dynamic 16 17 behaviour of tall piers differs significantly from that of short piers due to a number of factors, 18 such as their high flexibility and inertia. This paper aims to quantify the influence of axial loads 19 and higher order modes on the seismic response of bridges tall piers and to provide results 20 useful for a more informed design and assessment. For this purpose, an analytical formulation 21 of the dynamic problem, developed and validated in a previous study, is employed to analyse a 22 wide range of piers and bridge configurations. In the first part of the paper, a thorough 23 parametric investigation is carried out to evaluate the influence of axial loads and higher order 24 modes on both the modal properties and the seismic response of tall piers with different 25 geometries and vertical loads. Subsequently, three realistic case studies representing bridges 26 with different geometrical, mechanical and dynamic conditions are analysed and seismic time-27 history analyses are performed to further investigate the problem. The obtained results provide 28 useful insights into the seismic behaviour of bridges with tall piers, identify the relevant 29 governing parameters and shed light on the accuracy of simplified approaches suggested by the 30 Eurocode 8 to account for the second order effects.

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34 1 INTRODUCTION

35 Many bridges in the world are characterized by the presence of tall piers as part of the 36 earthquake resisting system. The dynamic behaviour of tall piers may be very different from 37 that of short piers due to a number of factors, such as their higher flexibility, the higher ratio 38 between the pier mass and the deck mass they have to withstand, and the influence of the axial 39 loads on the vibrational properties.

The large flexibility of tall piers leads to large values of the fundamental period in the horizontal direction, which in turn results in low seismic response spectral accelerations for the bridge. In this situation, it is not cost-effective nor necessary to resort to seismic isolation or to design the piers for increased ductility [1]-[3]. Therefore, tall piers are usually designed to maintain an elastic (or limitedly ductile) behaviour under seismic actions.

45 In many design codes, a single-degree-of-freedom (SDOF) approximation of the bridge 46 behaviour is allowed under certain conditions. This is reported, for instance, in Eurocode 8 47 ("Fundamental mode method" reported in §4.2.2 of EC8-Part 2 [4]), in the AASHTO LRFD 48 bridge design specifications (e.g. in the "Single-Mode Spectral Method" and in the "Uniform 49 Load Method" in §4.7.4.3.2 of [5]), as well as in many national codes such as the recent Italian 50 Building Code NTC-2018 [6]. However, the SDOF model is allowed only if the pier mass is 51 relatively small compared to the deck tributary mass, so that higher order modes have negligible 52 influence on the bridge dynamic behaviour. As an example, according to the EC8 the pier mass 53 does not have to exceed 20% of the tributary mass of the deck. On the contrary, in bridges with 54 tall piers, the high ratio between the pier mass and the deck tributary mass often results in a 55 significant influence of the higher order modes on the seismic response, so that an accurate 56 description of the pier geometry and inertia distribution is strictly required [7]-[11]. Recent 57 results from shake table tests [12] as well as from model-updating hybrid tests [13], [14] on 58 tall-pier models confirmed that the contribution of higher modes may significantly affect the 59 seismic response of tall piers. One important effect is the increase of bending moment demand 60 at pier mid-height, which may trigger additional plastic regions for very high seismic intensities, such as peak ground acceleration (PGA) values higher than 0.8g [12]. 61 Moreover, bridges with tall piers are sensitive to axial loads, which may significantly influence 62

63 the dynamic properties of the system both in the elastic range [15] and at the collapse conditions

64 [16]-[19]. Thus, the effects of axial loads acting on the deformed bridge configuration need to

- 65 be included in the analysis by using a geometrical formulation coherent with the range of
- 66 displacement and rotation of interest (e.g., fully non-linear, moderate rotations, p-delta effects)

[7], [20]. In bridge design practice, the axial-load effects are usually taken into account in a simplified manner by introducing an amplification factor for the pier seismic moments (also called moment magnification) evaluated via first-order analysis, as reported, for instance, in §5.4 of EC8-Part 2 [4], in §4.5.3.2.2b of AASHTO LRFD bridge design specifications [5], and in §7.9.4 of NTC-2018 [6]. The formulations available for the amplification factors are, however, based on simple hysteretic SDOF models [21]-[26], and thus they do not adequately represent the behaviour of tall piers.

Also, the features of the seismic events might affect the response of tall piers, as recently analysed in [27], [28], where the authors observed that near-fault motions generally lead to higher seismic vulnerability for piers with height from 40 to 80 m; such study was performed by analysing the response of a single demand parameter, the curvature ductility demand, and by developing fragility curves for comparing the performances under near fault and far field records.

- 80 Recently, an analytical model and a related dimensionless formulation was proposed in [15] to 81 shed light on the main characteristics of the dynamic and seismic behaviour of tall piers 82 vibrating in their linear elastic range, by accounting for both the influence of axial loads and 83 higher order modes. The model also allowed the derivation of an analytical solution for the 84 eigenvalue and the seismic problem by extending previous results for similar problems [29]-[31], in particular by application of the Frobenius method [31]. The reliability of the results 85 86 achieved through the adoption of such analytical formulation was also assessed and the 87 proposed model and the related kinematic assumptions were validated in [15] by comparison 88 with a large displacement formulation approach.
- The present study builds on this work and aims at exploiting the formulation developed in [15] in an extensive parametric analysis aiming to achieve the following main objectives:
- evaluating the influence of higher order modes and axial load effects on the seismic
 response of tall piers for different values of the characteristic parameters identified in [15]
 varying in a range of practical interest;
- 94 2. evaluating whether and to which extent the use of the amplification factor suggested by
 95 EC8-Part 2 overestimates the seismic demand;
- 96 3. providing results useful for a more informed design and assessment of the seismic response
- 97 of tall piers by accounting for the influence of higher modes and second order effects.
- 98 In the second part of the paper, three realistic piers with different geometrical and mechanical 99 properties and corresponding to different values of the characteristic non-dimensional

100 parameters are selected and analysed, to reinforce the findings of the parametric analyses and 101 better explain the implications on the assessment and design of bridges with tall piers.

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ANALYTICAL PROBLEM FORMULATION AND SOLUTION 2

103 In this section, the analytical formulation developed in [15] and used for the purposes of the 104 present study is briefly recalled. This formulation is based on a continuous modelling approach 105 for the tall pier (Fig. 1), which consists of a linear-elastic Euler-Bernoulli cantilever beam with 106 bending stiffness b(x), mass per unit length m(x), and tip mass M_T at the top, with $x \in [0, H]$. The 107 formulation describes the perturbed motion starting from a reference configuration (Fig. 1-a) 108 where the beam axis lies over the x-axis; the beam is subjected to a concentrated compression 109 force, P, at the free end, and to a distributed compressive load, m(x)g, along its height. The term 110 P describes the vertical force related to the weight sustained by the pier supports, whereas the 111 term M_T represents the mass associated to the horizontal inertial forces and depends on the static 112 scheme of the deck in the horizontal plane. It is worth noting that the tip mass at the pier top 113 M_T and the deck vertical reaction P are two independent parameters, differently from mass and 114 weight of the pier which are, instead, related through the acceleration of gravity and can thus 115 be described by a single parameter. The proposed model is consistent with the "Individual pier" 116 modelling approach according to §4.2.2.6 of EC8-Part 2 [4], and it can be employed to describe 117 the seismic longitudinal and transversal behaviour of piers under some regularity conditions 118 that allow to consider a single pier with the tributary deck mass to represent the whole bridge; 119 for example, under the transverse seismic input the model is suitable for the case in which there 120 is no significant interaction between the adjacent piers (e.g., long and/or transversally flexible 121 decks).



Fig. 1. a) Pier model and undeformed configuration; b) deformed pier configuration. 122

123 The formulation developed in [15] describes the infinitesimal perturbed motion of the continuous system in the neighbourhood of the axially loaded reference configuration, under 124 the hypothesis of small strains and displacements, and linear elastic behaviour of the pier. It is 125 126 noteworthy that these assumptions are valid for most of tall piers, which are characterized by a 127 long vibration period such that the inelastic behaviour of the system is activated only for very 128 high seismic intensities (for example, in the shaking-table experiments of [12], the first plastic 129 hinge at the pier base formed for PGA>0.6g and the one at mid height was observed for PGA 130 levels higher than 0.8g). Under these assumptions, it is possible to define vibration modes of 131 the system, even though these are influenced by axial load effects, and thus the response to the 132 seismic input can be expressed by superimposing the contribution of the various modes.

In [15], the following analytical expression of the circular frequency for the *s*-th vibration wasderived:

135
$$\omega_s^2 = \frac{\mathsf{K} (\psi_s, \psi_s) - \mathsf{N} (\psi_s, \psi_s)}{\mathsf{M} (\psi_s, \psi_s)} = \frac{\overline{k}_s}{\overline{m}_s} \tag{1}$$

where \overline{k}_s and \overline{m}_s are the generalized stiffness and mass of the *s*-th mode, whose shape is described by Ψ_s , a spatial function representing the *s*-th mode shape and belonging to the space U of transverse displacement functions satisfying the essential boundary conditions; K, N, M are bilinear symmetric forms mapping pairs of functions belonging to U into the space R of real numbers. The expression of \overline{k}_s considers the reduction of stiffness due to the so-called P-delta effects.

142 The dynamic properties of the system are also described by the modal participation factor, ρ_s , 143 and by the modal mass participation factor, MPF_s , defined as follows, respectively:

144
$$\rho_{s} = \frac{\mathsf{M}^{*}(\psi_{s})}{\mathsf{M}^{'}(\psi_{s},\psi_{s})} = \frac{\int_{0}^{H} m(x)\psi_{s}(x)dx + M_{T}\psi_{s}(H)}{\int_{0}^{H} m(x)\psi_{s}^{2}(x)dx + M_{T}\psi_{s}^{2}(H)}$$
(2)

145
$$MPF_{s} = \frac{\left[\mathsf{M}^{*}(\psi_{s})\right]^{2}}{\mathsf{M}^{*}(\psi_{s},\psi_{s})}$$
(3)

According to the mode superposition method, the motion is expressed by the following summation series:

148
$$u(x,t) = \sum_{s=1}^{\infty} \psi_s(x) q_s(t)$$
(4)

149 where $q_s(t):[t_0,t_1] \rightarrow R$ is the generalized coordinate corresponding to Ψ_s .

150 The *s*-th decoupled equation for $q_s(t)$, obtained by employing the orthogonality conditions and 151 by adding a source of inherent damping, reads as follows:

152
$$\ddot{q}_{s}(t) + 2\xi_{s}\omega_{s}\dot{q}_{s}(t) + \omega_{s}^{2}q_{s}(t) = -\rho_{s}\ddot{u}_{g}(t)$$
(5)

153 where ω_s^2 is the circular frequency whose expression is given in Equation (1), ξ_s is the *s*-th 154 mode damping factor, and ρ_s is the *s*-th mode participation factor. The values of ξ_s for the 155 various vibration modes can be calibrated based on experimental observations [32].

Once the displacement response history is evaluated, the histories of the bending moment andof the shear along the beam can be obtained as follows:

158
$$M(x,t) = -b(x)u''(x,t) = -b(x)\sum_{s=1}^{\infty} \psi''_s(x) \cdot q_s(t)$$
(6)

159
$$V(x,t) = \left[-b(x)u''(x,t)\right]' - N(x)u'(x,t) = -\sum_{s=1}^{\infty} \left\{ \left[b(x)\psi_s''(x)\right]' + N(x)\psi_s'(x) \right\} q_s(t)$$
(7)

160 3 PARAMETRIC STUDY

In this section, the characteristic parameters controlling the problem are first introduced. Then, the influence of higher order modes on the pier dynamic behaviour is analysed by considering the relation between the characteristic parameters and the modal properties. Successively, a parametric study is carried out to evaluate the influence of higher order modes and axial load effects on the seismic response.

166 3.1 Characteristic parameters

167 Under the assumption of homogeneous pier mass and stiffness (i.e., m(x)=m and b(x)=EI), the 168 internal compressive action simplifies to N(x) = P + m(H-x) and the characteristic non-169 dimensional parameters controlling the dynamic behaviour of the system in Fig. 1, evaluated 170 by applying the Buckingham's Pi-theorem, are only three [15], namely:

171
$$\alpha^{2} = \frac{P}{P_{cr}} = \frac{P \cdot 4H^{2}}{\pi^{2} EI}, \quad \beta = \frac{\overline{m}H}{M_{T}}, \quad \gamma = \frac{\overline{m}gH}{P_{cr}} = \frac{\overline{m}gH \cdot 4H^{2}}{\pi^{2} EI}$$
(8)

172 Parameter α^2 represents the ratio between the load at the pier top (deck reaction), *P*, and the 173 Euler buckling load, P_{cr} , of a cantilever beam loaded by a concentrated force at its top. 174 Parameter γ denotes the ratio between the total pier weight $\overline{m}gH$ and P_{cr} . Thus, the sum α^2 +

- γ provides a non-dimensional measure of the total vertical loads acting on the pier in relation 175 to the buckling load and expresses the propensity to second order effects, so that the adjective 176 "slender" can be attributed to piers for which $\alpha^2 + \gamma$ is significantly larger than 0 (conversely, 177 stocky piers are characterized by low $\alpha^2 + \gamma$ values). Finally, β describes the ratio between 178 179 the distributed pier mass and the top concentrated mass M_T . It is noteworthy that in the particular case of $P = M_T g$ one has $\gamma = \beta \alpha^2$, hence the problem is governed by two parameters only, 180 whereas in the case of zero distributed mass $\beta = \gamma = 0$ and the problem is governed by a single 181 182 parameter.
- 183 While the modal properties of the system depend only on these non-dimensional parameters, 184 the response to a seismic input also depends on the system fundamental vibration period. This 185 can be described by $T_0 = \frac{2\pi}{\omega_0}$, where ω_0 is the circular frequency of the pier obtained by
- 186 neglecting the distributed mass and the axial load effects, i.e.:

187
$$\omega_0 = \sqrt{\frac{3EI}{M_T H^3}} \tag{9}$$

In this study, the values assumed for the pier characteristic parameters are chosen to cover awide range of behaviours typical of real bridge configurations.

190 The parameter α^2 is varied between 0.01 and 0.4. Higher values are not considered because 191 they would result in piers close to buckle under the non-seismic load combinations.

192 The values assumed for β are in the range between 0.01 and 3. The lower limit corresponds to 193 a pier with a negligible mass compared to the mass at its top, whereas the higher limit 194 corresponds to a very high pier mass and a low mass at the pier top. This situation is typical of 195 piers that are disconnected by the deck through a sliding bearing at their top.

In order to limit the parameters to be varied in the parametric study, it is assumed that $\gamma = \beta \alpha^2$. The period T_0 is assumed to vary in the range between 3s and 8s. Lower vibration periods are not considered because they would correspond to short piers, which are expected to exhibit an elastoplastic behaviour, not considered by the model. On the other hand, vibration periods higher than 8.0 s are not considered, because they would correspond to very slender piers, whose response may be more affected by loads different from the seismic loads (e.g., wind).

203 3.2 Modal properties

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The ratio between the *i*-th circular frequency of the pier, ω_i , and ω_0 , can be expressed in terms of a non-dimensional frequency ratio as:

$$\frac{\omega_i}{\omega_0} = f\left(\alpha^2, \beta, \gamma\right) \tag{10}$$

It can be shown that also the modal shapes and other modal properties (e.g., the modalparticipation factors) depend only on the three identified non-dimensional parameters.

209 In [15], the relation between the non-dimensional parameters and the modal properties of the system was investigated and the study outcomes can be synthesized as follows: the circular 210 frequencies of the system decrease with increasing α^2 and the axial load significantly affects 211 only the fundamental circular frequency, whereas the circular frequency of the higher modes 212 are weakly sensitive to variations of α^2 . The study is herein extended to assess the influence 213 of the axial loads on the modal participating mass, which is a useful parameter providing direct 214 215 information on the contribution of the vibration modes to the total base shear of the system; in 216 fact, in the case of unit base acceleration, the base shear is exactly equal to the sum of all modal 217 direction participating masses in а given [33]. То this aim,



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Fig. 2 plots the variation with α^2 and β of the first modal participating mass, MPF_1 , and of the sum of the first two participating masses $MPF_1 + MPF_2$, normalized with respect to the total mass of the system, M_{tot} .

In general, these modal properties are equal to 1 in the case of zero pier mass, i.e., for $\beta = 0$ (corresponding to one vibration mode only) and decrease for increasing β values (i.e.,

increasing pier mass) due to the increase of the higher modes' contribution to the pier motion. 224 225 Furthermore, the first mode contribution is very high and reaches values higher than 65% for 226 any β value, whereas the contribution of the first two modes is higher than 70% for any combination of β and α^2 value considered. Finally, it is noted that these two modal properties 227 228 are affected by axial load effects. In particular, the first modal participating mass MPF_1 is 229 almost unaffected by the axial load effects, whereas the second modal participating mass (and 230 hence the sum $MPF_1 + MPF_2$) is reduced from 85% to 70% by increasing the load at the pier top through the α^2 value. 231 Considering these results, it can be stated that high values of β are a necessary condition for 232 233 having a contribution of higher order modes to the response, and that for $\beta = 0$, the system

behaves as a single degree of freedom system. In most cases, high values of β are also a sufficient condition for higher order modes contribution, but in some particular situations (e.g., extremely rigid piers) the system moves with the ground and thus higher order modes become less important. These special cases can be accounted for by also analysing the parameters α^2

238 and γ , which attain values close to zeros when the pier's stiffness is very high.



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e normalized (by the total mass M_{tot} .) values of MPF_1 (a) and of the sum $MPF_1 + MPF_2$ (b) vs. β , for different α^2 values.

242 3.3 Seismic response

In this subsection, a parametric study is carried out to evaluate the influence of axial load effects on the seismic response of tall piers with various geometrical and mechanical properties. The seismic input considered for all the combinations of the above parameters is described by the 246 EC8 type I soil type B spectrum (soil factor S = 1.20), for an importance factor $\gamma_I = 1$ and a peak ground acceleration PGA = 0.30g (Fig. 3). The shape and amplitude of the spectrum at long 247 248 periods is controlled by the corner period $T_D = 2s$, identifying the beginning of the maximum 249 spectral displacement (MSD) plateau, by the period $T_E = 5$ s, corresponding to the end of the 250 plateau and the beginning of the linear decreasing branch of the spectrum, and by the period 251 $T_F=10$ s, beyond which the spectral displacement tends to a constant value (ground displacement 252 d_{g}). Appendix A of EC8-Part 1 provides the expressions for the displacement spectrum 253 ordinates for periods $T > T_E$.



Fig. 3. Displacement response spectrum as per EC8 Part 1 [4] (MSD: maximum spectral displacement).

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257 In the parametric study, three different values of T_0 are considered (consistent with the 258 fundamental periods of the piers analysed in Section 4), corresponding to the ratios $T_0/T_E = 0.6$, 259 1, and 1.4. The period elongation due to axial load effects and the higher order modes are 260 expected to have a different effect on the systems corresponding to these periods. Seven natural 261 ground motion records compatible with this displacement response spectrum have been selected 262 through the software tool Rexel-Disp [34], which is the result of many recent studies focused 263 on the characterization of ground motions for the seismic design and assessment of long-period 264 structures. The structural response quantities considered are the average values (among the 265 seven excitation scenarios) of the peak transverse displacement at the pier top, of the peak base 266 shear and of the peak bending moment.

In order to shed light on the influence of axial load effects, the ratio between the values assumed by these quantities by accounting for and by disregarding axial load effects are evaluated. These ratios are denoted as r_d , r_V , r_M , and refer to the pier top displacement, to the base shear, and to the base bending moment, respectively. The base bending moment is also evaluated in compliance with the EC8-Part 2 provisions [4], i.e. by increasing the bending moment obtained by a first-order analysis (neglecting axial force contribution) through the moment magnification factor $\Delta M = \frac{1+q}{2} d_{Ed} N_{Ed}$, where N_{Ed} is the axial force, d_{Ed} is the relative transverse displacement of the ends of the member, and *q* is the behaviour factor considered for the design. A similar moment magnification factor is prescribed also in the Italian building code NTC2018 [6], expressed as $\Delta M = d_{Ed} N_{Ed}$, which thus coincides with the previous one if *q*=1 (no ductile behaviour).

Therefore, a fourth response ratio r_{MC} is considered in this parametric study as the ratio between the code-based bending moment at the pier base calculated as per the EC8-Part 2 provisions (assuming q=1) and the base bending moment obtained by the proposed formulation, in order to evaluate the accuracy of the EC8 approach.

Fig. 4 reports the values assumed by the ratios r_d and r_V for different values of α^2 and of β 282 varying in the previously identified range and for $T_0/T_E = 0.6$, 1, and 1.4. Based on these results, 283 284 it is noted that for all the examined periods the pier top displacement response decreases when the axial load effects are taken into account, since values $r_d < 1$ are obtained for all the 285 combinations of α^2 and β different from zero. More specifically, the top displacement 286 response is reduced up to 30% for combination of shorter periods $T_0 / T_E = 0.6$ and large 287 distributed pier mass compared to the top concentrated mass ($\beta = 3$). However, while the 288 influence of the pier top load (α^2 parameter) is qualitatively similar for the examined periods 289 290 (both short and long), the displacement response is sensitive to the pier mass distribution only 291 in the relatively short period range $(T_0/T_E = 0.6 \text{ and } 1.0)$ and is negligible for periods lying in the linear decreasing branch of the spectrum (as in the case $T_0 / T_E = 1.4$). Also, the base shear 292 293 response, like the pier top displacement response, is reduced if the axial load effects are taken 294 into account. However, in contrast to the displacement response, the greatest reductions effects 295 of base shear are observed for lower values (rather than for higher values) of β . This is related 296 to the fact that the shear demand is significantly influenced by higher order modes, whose 297 relative contribution gets higher for larger distributed pier mass, which is in line with the results 298 presented in the previous paper [15]. In particular, it is observed that values of relatively large distributed pier mass (say $\beta > 0.5$) compensate for the reduction of the shear demand caused 299 by the axial load effects so that the shear response ratio r_V keeps in the neighbourhood of the 300 unity for all the range of α^2 values examined. Overall, the variation of the base shear demand 301 is dramatically influenced by the distributed pier mass when higher axial loads are considered 302 (say in the range $\alpha^2 > 0.2$). In the extreme case of $\alpha^2 = 0.4$ the base shear is reduced up to 303 more than 40% (compared to the case disregarding the axial load effects) for all the three 304

305 considered periods when the mass is entirely concentrated at the pier top and not distributed 306 along the pier ($\beta \approx 0$). Instead, in the same extreme situation the increase of β would 307 considerably mitigate base shear reductions. These cases are useful to illustrate the importance 308 of axial load effects (related to α^2) and higher order modes (related to β) on the base shear 309 demand of tall piers.

- Fig. 5 reports the values assumed by the ratios r_M and r_{MC} for different values of α^2 and of β 310 311 varying in the previously identified range and for $T_0/T_{\rm E} = 0.6, 1$, and 1.4. The qualitative trends 312 of base bending moment ratio r_M follow those of the base shear ratio r_V . This is reasonable since 313 base shear and bending moment response are similarly affected by axial load effects and by 314 higher mode effects. In particular, the increase of the pier top load generates a decrease of the bending moment response ratio (up to 20% in the extreme case $\alpha^2 = 0.4$ and $\beta \approx 0$), whereas 315 316 the increase of the pier distributed mass compensates for this reduction and may also produce an increment of the base bending moment response (e.g., case of $T_0/T_E = 1.4$ in Fig. 5c). 317
- 318 It is also interesting to observe that using the EC8-Part 2 formula to account for second order 319 effects (assuming q=1) yields to a significant overestimation of the bending moment demand, 320 compared to the values obtained with the proposed formulation. The highest overestimations (i.e., r_{MC} values) are observed for structural systems with high top loads (large α^2) and 321 negligible distributed mass (low β), which are characterized by r_{MC} values up to 70%. This 322 means that the base bending moment obtained by the simplified EC8 approach is overestimated 323 324 by more than 70% in comparison to that obtained with the proposed model. This is especially 325 true for larger periods (e.g., $T_0/T_E = 1$, and 1.4).



327 Fig. 4. Variation with α^2 and β of r_d for $T_0/T_E = 0.6$ (a), 1 (b), and 1.4 (c) and of r_V for T_0/T_E 328 = 0.6 (d), 1 (e), and 1.4 (f).



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330 Fig. 5. Variation with α^2 and β of r_M for $T_0/T_E = 0.6$ (a), 1 (b), and 1.4 (c) and of r_{MC} for T_0 331 $/T_E = 0.6$ (d), 1 (e), and 1.4 (f).

On the other hand, the discrepancy between the EC8 approach and the proposed formulation decreases when the pier distributed mass increases (higher β). As an example, for β approaching 3 and for the extreme case of $\alpha^2 = 0.4$, the overestimation reduces to around 30%, 20% and 5%, respectively for the three considered periods $T_0/T_E = 0.6$, 1, and 1.4. Thus, the higher the stiffness of the system, the higher the overestimation. In all the cases investigated, the amplification factors of EC8 leads to conservative estimates of the bending moments.

339 4 CASE STUDIES

- 340 In this section, the proposed analysis technique is employed to evaluate the influence of axial 341 load effects and higher order modes on the seismic response of three case studies of tall piers 342 belonging to realistic bridge models taken from the literature.
- 343 The main geometrical and mechanical characteristics of the three considered case studies are 344 summarized in the sketch of Fig. 6. In particular, a tall pier taken from Kolias [1], a stockier 345 pier taken from Wei et al. [26] and a very tall pier taken from Li et al. [35] are analysed. The three piers are characterized by different combinations of parameters α^2, β, γ , as reported in 346 Fig. 6, which are representative of different design scenarios discussed in the parametric study 347 348 above and are thus selected to cover a wide range of bridge configurations and seismic 349 behaviours. In this manner, it is possible to scrutinize the influence of the axial load effects and 350 of the higher order mode effects depending on the different mass and stiffness characteristics 351 of the piers of these three examples.
- The seismic input is the same as that employed for the parametric study, i.e., based on the EC8 type I soil type B spectrum soil and a PGA = 0.3 g. Fig. 7 shows the acceleration-displacement response spectrum (ADRS spectrum) of the spectrum-compatible records and of the mean spectrum. The plot in Fig. 7-b provides important information regarding the effect of the modal vibration periods and of the changes due to axial load effect on the seismic demand and it is useful to explain the results of the following seismic analyses. In particular, the axial loads effect in terms of period elongations can be also appreciated.
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Fig. 7 (a) Acceleration response spectra of natural records and (b) mean ADRS response
 spectrum with identification of the first two vibration periods of the three case studies
 analysed below.

366 **4.1** Case study 1 ($T_0=5s$, $T_0/T_E \approx 1$)

This case study consists of a tall pier belonging to a bridge, whose properties are taken from [1]. The bridge has a three-span steel-concrete composite deck (with span length of 60 m+80 m+60 m) and two identical RC piers. These two piers, of height H = 40 m, have a circular hollow transverse section with external diameter of 4.0 m and internal diameter of 3.2 m. The pier head has a rectangular transverse section with dimensions 4.0 m x 8.0 m, and is 1.5 m high. Class C35/45 concrete is adopted for concrete, and S500 steel grade is used for the longitudinal rebars, with a reinforcement ratio equal to 1.5% of the cross-section area [1].

- The formulation described in Section 2 is used to analyse the longitudinal bridge response.
- 375 Assuming a rigid deck behaviour in line with § 4.2.2.3 of EC8-Part 2 provisions [4], only the 376 response of a single pier is investigated, benefitting from the problem symmetry. The pier 377 effective stiffness accounting for concrete cracking is $EI = 7.3086 \cdot 10^7 \text{ kNm}^2$, whereas the pier distributed mass is $\overline{m} = 11.53$ ton/m. The mass concentrated at the pier top and describing the 378 379 deck and pier head inertia is $M_T = 2145$ ton. Since the cross section is uniform along the pier length, the axial force is a linear function $N(x) = P + \overline{mg}(H - x)$ with P = 15470 kN, the pier 380 381 height H = 40m and the pier mass $\overline{m}H = 461.2$ ton. It is worth noting that in this case study 382 M_T is different from P/g=1576 ton, because the longitudinal inertial force is resisted by the two
- 383 piers only, whereas the vertical weight is distributed among the piers and the abutments.
- 384 The buckling load of a cantilever beam with the same flexural stiffness of the pier is $P_{cr} = \pi^2 EI/4H^2 = 1.17 \cdot 10^8$ kN. This corresponds to a ratio $\alpha^2 = P/P_{cr} = 0.132$. The other two 386 characteristic parameters, related to the distributed mass, have the following values: $\beta = 0.215$ 387 and $\gamma = 0.039$.
- The Frobenius method is applied to solve the eigenvalue problem. The first three longitudinal vibration periods of the pier in the longitudinal direction disregarding axial load effects are 5.003s, 0.251s, and 0.078s. A comparison between the modal vibration periods T_i and mass participating factors MPF_i obtained by accounting for and by disregarding axial load effects (corresponding to assuming N(x) = 0 in Equation (2)) are reported in Table 1.
- By inspection of the results listed in Table 1, it can be noticed that only the first vibration mode is significantly affected by the axial load effects. This is expected from the results of the parametric study of the eigenvalue problem discussed in Section 3.2 and also reported in [15]. As an opposite trend, the discrepancies in terms of *MPF*s between the model with and without
- 397 axial load effects increase for increasing mode order.
- 398 The pier seismic behaviour is evaluated by computing, for each of the 7 response-spectrum-

- 399 compatible records representing the seismic input, the response envelopes (i.e., the peak400 absolute values) of the displacements, bending moments and shear observed along the pier.
- 401 Fig. 8 reports the average response obtained from all the considered records, i.e., the average
- 402 peak values of the displacement $u_{max}(x)$, bending moment, $M_{max}(x)$, and shear, $V_{max}(x)$ for each
- 403 location *x* along the pier length.

		-		-
	T_i [s]		MPF_i [%]	
Mode	w/o axial load	w/ axial load	w/o axial load	w/ axial load
1	5.003	5.402	91.491	91.462
2	0.251	0.253	4.966	4.548
3	0.078	0.078	1.249	1.412
4	0.038	0.038	0.767	0.677
5	0.022	0.022	0.345	0.398
6	0.014	0.014	0.297	0.259

Table 1. Second-order effects on periods and mass participating factors.





406 Fig. 8 Average seismic response of case study 1: a) modal contributions to the displacements,
407 bending moments and shear, b) axial load effects on displacements, bending moments and
408 shear profiles.

In particular, Fig. 8-a analyses the contribution of different modes to the seismic response: a comparison of the response envelopes obtained by truncated models that consider the contributions of mode 1, of modes 1 and 2, and of the first 8 modes in terms of displacements, bending moments and shear is illustrated. It is worth noting that the higher modes, related to the mass distributed along the pier, do not considerably influence the displacements, whereas they play a key role in the profiles of both the bending moment and shear force along the pier. It can be seen that the higher mode contribution to the shear demand at the base and at the top

416 of the pier is significant. More specifically, the second-mode contribution significantly affects 417 such shear demand at the two pier ends, as can be seen by comparing the response obtained by 418 including the first two modes with that obtained by including eight modes. Moreover, the shear 419 demand calculated by the first mode only would be largely underestimated.

420 Fig. 8-b compares the profiles of peak displacement, bending moment and shear force evaluated 421 by the proposed formulation against the corresponding results obtained by neglecting the axial 422 load effects. Notable differences can be observed for all the considered response quantities. 423 Unlike what occurs in static problems, the axial load effects give rise to lower seismic response 424 values along the whole pier height. This result can be explained in view of the fundamental 425 vibration period elongation caused by the additional axial loads, which shifts the structural 426 response on regions of the response spectrum with a lower spectral acceleration (cf. the ADRS 427 response spectrum in Fig. 7-b). As an example, the average peak bending moment value at the 428 base of the pier is 26904 kNm by disregarding axial loads and reduces to the value of 23923 429 kNm if axial loads are taken into account. Following the EC8-Part 2 approach [4], an 430 amplification of 2475 kNm would be required, corresponding to a ratio r_{MC} =1.22 between the 431 amplified first order base moment and the base moment resulting from the analysis accounting 432 for second order effects. It is observed how this r_{MC} value is very close to the value of $r_{MC}=1.19$ resulting from the parametric study of section 3.3 (see Fig. 5-e for $\alpha^2 = 0.132$ and $\beta = 0.215$). 433 434 The slight difference between these two ratios can be attributed to the different values of the characteristic parameter γ , i.e., $\gamma = 0.039$ in the present analysis and $\gamma = \beta \alpha^2 = 0.028$ in the 435 parametric study. This demonstrates the validity and usefulness of the above parametric study 436 437 for a more informative assessment of the bridge response including axial loads and higher mode 438 effects. Moreover, it is worth noting that in this case the bending moment reduction due to the 439 axial load effect is in net contrast to the amplification obtained by applying the EC8-Part 2 440 approach [4].

For what concerns the shear demand, the corresponding reduction is a bit less marked than those observed in other response parameters. This is due to the significant influence of higher order modes on the shear force of this pier, and the almost negligible influence of axial load effects on modes of order higher than one.

445 4.2 Case study 2 ($T_0=3s$, $T_0/T_E \approx 0.6$)

This section reports the results of the application of the proposed analysis technique to a bridgepier model taken from Wei et al. [26]. The reinforced concrete pier has geometrical properties

448 significantly different from those of the first case study. Its height is H = 16 m, and the cross 449 section is circular with diameter D = 1.5 m. The concrete has cylindrical mean strength of 35 450 MPa and the steel employed for the longitudinal rebars has strength of 470 MPa. The vertical 451 force at the pier top transmitted by the deck is equal to 5000 kN. The pier stiffness and distributed mass are respectively $b(x) = EI = 3.185 \cdot 10^6 \text{ kNm}^2$ and $m(x) = \overline{m} = 4.50 \text{ ton/m}$, 452 whereas the mass concentrated at the pier top is $M_T = 509.68$ ton. The value of the buckling 453 load of a cantilever beam with the same flexural stiffness of the pier is $P_{cr} = \pi^2 EI/4H^2 = 30692.9$ 454 kN, corresponding to a ratio $\alpha^2 = P/P_{cr} = 0.1629$, very similar to the value obtained for case 455 456 study 1 despite the different geometry. The values of the other characteristic parameters are $\beta = 0.141$, and $\gamma = 0.023$. The first three vibration periods of the pier disregarding axial load 457 458 effects are 2.986s, 0.123s, 0.0382s, whereas the corresponding values obtained by accounting 459 for axial load effects are 3.273s, 0.124s, 0.0383s. As observed for the previous case study, axial 460 load effects influence significantly only the first vibration period. The mass participation factors 461 of the first three modes obtained by accounting for and by disregarding axial loads effects 462 almost coincide and their values are 94.02%, 3.18%, and 0.1%.

Fig. 9-a shows the average of the peak absolute responses obtained by accounting for axial load effects for the set of ground motion records compatible with the spectrum of Fig. 7 a). Fig. 9-b compares the average response envelopes obtained by accounting for and by disregarding axial load effects.



467 Fig. 9. Average seismic response of case study 2: a) modal contributions to the displacements,
468 bending moments and shear, b) comparison between the response evaluated by accounting for
469 and disregarding axial load effects.

470 The displacement demand for the pier is inferior to the yield limit of 0.3 m given in Wei et al. 471 [26]; consequently, the elastic behaviour assumption is accurate for this system, despite the 472 quite severe seismic input considered (PGA = 0.3g). In this application example, the 473 contribution of higher vibration modes to the response is practically negligible for the 474 displacement and the bending moment demand, and modest also for the shear demand (Fig. 9 475 a). This is related to the lower value of β for the case study 2 in comparison to that of the 476 previous case study 1, which directly affects the importance of higher order modes due to the 477 distributed pier mass.

478 The axial load effects influence only the shear demand (Fig. 9 b), and the value of the base 479 shear reduces of about 17% when axial load effects are taken into account. This is explained 480 again by the period elongation effect due to axial loads, which results in a reduction of the 481 acceleration spectrum ordinates. The bending moments obtained by accounting for and by 482 disregarding axial load effects are very similar. This can be justified in view of two 483 counteracting effects related to axial loads: the bending moment reduction due to the decrease 484 of spectral ordinates, and the increment of bending moment demand due to the vertical force at 485 the pier top acting on the deformed configuration.

The base section bending moment demand, evaluated via first order analysis and amplified by the EC8-part 2 moment magnification factor [4] (i.e., 1401.5 kNm) is equal to 11898.3 kNm. The corresponding value evaluated with the proposed model is 10639.4 kNm. Thus, also in this case the EC8-Part 2 approach provides overconservative estimates of the effects of axial loads on the moment demand. The value of r_{MC} =1.11 is consistent with the value shown in Fig. 5-d of the parametric study of section 3.3 for $\alpha^2 = 0.163$ and $\beta = 0.141$.

492 **4.3 Case study 3 (T_0=7s, T_0/T_E \approx 1.4)**

493 This case study consists of a very tall bridge pier belonging to a regular multi-span bridge, 494 whose properties are taken from Li et al. [35]. The pier is 90 m high and has a hollow square 495 cross-section of dimensions 4.4 m x 4.4 m and thickness 0.5m. The longitudinal reinforcement 496 ratio is 1.48%. The concrete has cylindrical mean strength of 48 MPa. The vertical force at the pier top transmitted by the deck is equal to 6867 kN. The pier cracked stiffness and distributed 497 mass are respectively $b(x) = EI = 2.225 \cdot 10^6$ kNm² and $m(x) = \overline{m} = 19.87$ ton/m, whereas the 498 mass concentrated at the pier top is $M_T = 700$ ton. The values of the pier characteristic 499 parameters are $\alpha^2 = P/P_{cr} = 0.101$, $\beta = 2.556$, and $\gamma = 0.259$. The value of the sum $\alpha^2 + \gamma$ is 500

501 equal to 0.36, denoting an higher slenderness of this pier with respect to the case studies

previously analysed, characterized by lower $\alpha^2 + \gamma$ values, of the order of 0.17. Moreover, the 502 503 high value of β anticipates that, in this specific case, higher order modes are likely to contribute 504 significantly to the response. The first three vibration periods of the pier disregarding axial load effects are 6.968 s, 0.884 s, and 0.292 s, whereas the corresponding values obtained by 505 accounting for axial load effects are 7.692 s, 0.895 s, 0.293 s. In this case, also the higher modes 506 507 vibration period are slightly influenced by axial load effects. The mass participation factors of 508 the first three modes obtained by accounting for and by disregarding axial loads effects almost 509 coincide and their values are 67.43%, 16.76%, and 5.46%. This also confirms that the second 510 and third mode of vibration are expected to contribute significantly to the response.

511 Fig. 10a shows the average of the peak absolute responses obtained by accounting for axial load 512 effects and by considering the different modal contributions. As expected, the higher modes of 513 vibration dominate the pier seismic response. In particular, the second mode gives a significant 514 contribution to the bending moment and shear demand. The value of the moment demand at x 515 = 55m including the axial load effects is only slightly lower than the value at the pier base. 516 Thus, plastic hinges are expected to form also at this location, as already observed in [35]. 517 However, it should be pointed out that the pier responds elastically to the assumed seismic input and plastic hinges will form only for very severe excitations. In particular, plastic hinges were 518 519 reported to form only for PGA values higher than 0.4g [35].

520 Fig. 10b compares the average response envelopes obtained by accounting for and by 521 disregarding axial load effects. These effects have a negligible influence on the response. This 522 is a consequence of the relevant contribution of higher modes, which are not significantly 523 affected by axial load effects. Finally, it is worth noting that also in this case the EC8-Part 2 524 approach [4] gives conservative estimates of the effects of axial loads on the moment demand, 525 with the increment equal to 1133.7 kNm. The $r_{\rm MC}$ ratio in this case is equal to 1.08, and it is 526 consistent to the value $r_{\rm MC} = 1.05$ obtained from the parametric study results shown in Fig. 5-f, for $\alpha^2 = 0.101$ and $\beta = 2.556$. 527



Fig. 10. Average seismic response of case study 3: a) modal contributions to the
displacements, bending moments and shear, b) comparison between the responses evaluated
by accounting for and by disregarding axial load effects.

531 5 CONCLUSIONS

532 The study performed in this paper aims to quantify the influence of both axial loads and higher

order modes on the seismic response of tall piers, which are the most important components of
the earthquake resisting system of bridges.

535 The analytical formulation validated in a previous study is herein adopted to analyse a wide

range of piers and bridge configurations to provide a global overview of the problem. First, a

thorough parametric investigation is carried out to evaluate how the system modal and seismic

538 response is influenced by the main characteristic parameters. Afterwards, three realistic case

539 studies, representative of different geometrical and dynamic conditions, are selected and

540 seismic time-history analyses are performed to further investigate the influence of the aforesaid

541 parameters.

542 Based on the results from both parametric investigation and case studies, the following general

543 conclusions can be drawn.

544 Regarding the modal properties:

- the first period of vibration is the only one significantly affected by both axial loads and
 pier distributed mass;
- conversely, the first mode mass participation factor is not sensitive to the axial load, whereas
 the higher modes participation factors show significant variations.

549 Regarding the seismic response:

• the internal forces of the pier (i.e., shear and bending demand) are differently affected by

- the axial loads and higher order modes depending on the specific values assumed by the two main governing parameters, i.e., the pier top load intensity (governing the axial load effects) and the distributed mass (governing the contribution of higher order modes).
- The base shear reduces if the axial load effects are taken into account, and this effect can be very important in piers with low distributed mass values; however, this reduction may be compensated by the counteraction exerted by the higher order modes in case of piers with relatively large values of distributed mass.
- The bending moment response is similarly affected by axial loads and higher modes; in
 particular, the pier top load increment generates a decrease of the bending moment response,
 whereas the rise of the pier distributed mass compensates for this reduction and may also
 lead, in case of very long period systems, to an increment at the base of the pier.
- It is also worth mentioning that the bending moment reduction due to the decrease of
 spectral ordinates is often high enough to balance the increment of bending moment demand
 produced by the pier top vertical force acting on the deformed configuration.
- The shear and bending demand assessed neglecting higher modes contribution may be
 notably underestimated, in particular for piers with high fundamental period; more
 specifically, the second mode provides the highest contribution, while modes from 3 to 8
 do not significantly affect the response.
- Moreover, in case of high distributed mass values and long period systems, a first mode based estimation might be not adequate to correctly describe the internal actions distribution
 along the pier height, thus potential plastic region at locations different from the base of the
 pier might be not identified.
- From the case study analyses it can be concluded that at least the first two modes have to
 be considered to correctly estimate the shear and bending demand of tall piers.
- 575 Finally, with regard to the effectiveness of the simplified design approaches suggested by the 576 Eurocode 8 to account for the second order effects, the following points deserve to be 577 highlighted:
- the application of the EC8 design procedure to piers with low distributed mass brings to
 results extremely conservative and this trend increases with the pier top load intensity and
 system fundamental periods;
- conversely, the overestimation produced by the EC8 design procedure is lower in piers with
 high distributed mass and the conservativism of results reduces for smaller axial loads and
 with the system fundamental period;

• in all cases investigated, the amplification factors of EC8 are always from the safety side.

585 Due to the relevance of problem, it might be useful to extend the present study within a 586 probabilistic framework aimed at characterizing the seismic response of tall piers beyond the 587 design condition, by consequently removing the hypothesis of elastic response.

588 Moreover, given the observed sensitivity to the axial loads, future studies might be

589 recommended to consider near-fault pulse-like ground motions, in particular aimed at exploring

590 the influence of their relatively high vertical component of the excitation.

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