Comparative Study of Offshore Spar-Buoy Oscillating Water Column Dynamic Models for Captured Power Estimation

Seif Bayoumi[1*], Erkan Oterkus[2], Hassan El-gamal[3], Atilla Incecik[2]

[1] Arab Academy for Science, Technology and Maritime Transport, Alexandria 1029, Egypt. Seif.bayoumi@aast.edu

[2] Naval Architecture, Ocean and Marine Engineering, University of Strathclyde, Glasgow G4 0LZ, UK

[3] Department of Mechanical Engineering, Faculty of Engineering, Alexandria University, Alexandria 21544, Egypt.

* Corresponding author

Keywords: Wave energy; ocean power

Abstract

The prompt estimation of power and geometrical aspects enables faster and more accurate financial assessment of wave energy converters to be deployed. This may lead to better commercialization of wave energy technologies, as they require location-based customization, unlike the mature wind energy technologies with developed benchmark. The adopted approach provides simple and efficient modelling tool allowing the study of the system from different perspective. The aim of this study is to select the optimum dynamic model to predict the captured power of a spar-buoy Oscillating Water Column (OWC) wave energy converter. Four dynamic models were developed to predict the system dynamics and results were validated experimentally. In-depth investigations on the effect of the mass and damping ratios of the oscillating bodies on the accuracy of the adopted models were performed. Such investigations included the proposed one-way coupling model and three two-degree of freedom models and three reduced-scale models, in addition to analytical and numerical solutions. Pneumatic power was calculated for the reduced-scale model where orifices' covers simulated the power take-off mechanism damping experimentally. Analysis and comparisons between the adopted models are finally provided.

Keywords: Wave energy; Floating structures; Spar-buoy; Oscillating Water Column

1. Introduction

Several studies investigated the performance of Oscillating Water Columns (OWCs) whether they are offshore (Zabihi et al., 2021) or onshore OWCs (Doyle, 2020), and considering mooring system interactions (Wu et al., 2019 and Kisacik et al., 2020). Spar-buoy are attractive for being simple, axi-symmetrical, and equally efficient at capturing energy from all directions (Bayoumi et al., 2014).

The adoption of linear wave theory provides many benefits; Folley & Whittaker (2009) reported that it allows exact solutions to be produced with relative ease for both monochromatic and mixed seas. It also allows the system dynamics to be represented in the frequency domain, using linear superposition and Fourier analysis. The efficacy and simplicity of linear wave theory has meant that few other procedures have been used in the design of wave energy converters. Therefore, from performance perspective the adoption of linear wave theory is valid since linear waves may be assumed for most of the Wave Energy Converter (WEC) operating times. However, from the survivability perspective linear wave theory may not be suitable for such modelling application.

Unlike conventional offshore structures and ships mathematical model where the wave loading on the structure and the ship response to certain sea conditions that are of interest; researchers are more concerned about the performance of the power conversion process in case of WECs.

Jefferys (1984) and Brendmo et al. (1996) presented in their publications the power flows between the OWC components and the corresponding information flows, in addition to the basic models used to describe the OWC, the limitation for applying each model and the transformation from one model to another. The general model used to describe the OWC is the so-called applied-pressure (or pressure distribution) description, where the power input to the air chamber can be obtained in terms of air pressure fluctuation and air volume flux in the chamber above the interior water surface. Models based on applied-pressure description are valid for a wide range of frequencies and valid also if the shape of the interior water surface is changing. The alternative rigid-model (spring-mass-damper model) adopted in this study is also applicable if the wavelength is large compared to the horizontal extension of the interior water surface (low frequency). In this case, it is assumed that the surface of the water column remains plane and can be considered as a rigid piston (heaving body). The errors introduced by this approximation are negligible. The power input to the air chamber can then be obtained in terms of the net wave force acting on the interior water surface and the vertical velocity of this surface. Obviously, mechanical, and electrical analogy of OWC system provided simple and efficient modelling tool allowing the study of the system from different perspective.

The performance of OWC may be evaluated stochastically or through frequency or time domain approach. Stochastic and frequency domain analysis allow a prompt evaluation of the device dynamics. For a more detailed analysis, in which, the forces imposed by the PTO and the anchoring system are strongly non-linear, a time domain approach is required, as stated by Alves Costa et al. (2010). From readings, one may conclude that frequency domain modelling is usually adopted in studies concerned about design and especially for geometry optimization, while time domain approach is implemented for operation control models.

Nunes et al. (2011) modelled the behaviour of a floating OWC including the hydrodynamic and the aerodynamic parts using MATLAB. To improve the device performance for a wide number of sea states they applied a dimensional optimisation technique to the turbine and the pneumatic chamber then developed a control strategy to improve the quality of the energy absorbed by the device. However, their optimization procedure was not applied to the whole device geometry. It included parameters related to the aerodynamic problem only and ignored the hydrodynamic behaviour of the structure.

Bayoumi et al., 2014 provided experimentally validated numerical model of spar buoy OWC based on Szumko model. The study focused on the water oscillations and completely ignored the assessment of the captured power. This model was also used to identify the numerical tool's validity applied in the prediction of the motion response of a floating cylindrical platform designed to carry a wind turbine and a desalination plant (Islam et al., 2020).

Similar to the aim of the current research; Henriques et al., 2016 aimed to develop a systematic methodology for optimum design of wave energy converters. Their idea was to use the power matrix and a set of performance indicators to design two self-powered sensor-buoys for long term monitoring based on the oscillating-water-column principle. The optimisation focussed on buoy hydrodynamic shape, sizing and selection of the turbine and the generator suitable for the proposed sensor-buoys.

Sundar et al., 2022 aimed to convert the single resonant frequency (shoreline) OWC into a multi resonant device by providing two harbour walls in front of the device to create a second peak and capture substantial energy from a wider range of frequency. Their research presented numerical and experimental results highlighting the advantages and limitations of the proposed modifications. Similar to the work presented herein, their assessment also ignored the performance of the power take-off mechanism (turbine). They also calculated the captured power based on the pressure of the OWC pneumatic chamber within a range of frequency.

Compared to other studies, the mechanical modelling approach adopted in the research reported herein helps in the preliminary design phase of full-scale devices as it allows the investigation of oscillation amplitudes and phase angle for optimum design conditions. The prompt estimation of power and geometrical aspects enables faster and more accurate financial assessment of wave energy converters to be deployed. Therefore, the aim of this study is to investigate several models and validate their results experimentally for different geometries and operating frequencies bandwidth.

2. Dynamic models of floating OWCs

Following the rigid piston model, captive OWC is best described by considering a single translational mode in heave direction. For offshore spar-buoy OWC, the dynamic coupling of the water column and the floating structure is very important to achieve the desired efficiency and therefore the system is described by considering two translational modes in heave direction. In this study, four different dynamic models will be used to describe floating OWC and will be referred to in the text as: a) *Simplified 2 Degree of Freedom (DOF) model*. b) *One-way coupling model*. c) *Szumko model.* and d) *Modified Szumko model*. Models are illustrated in Fig.1 and the reasons for adopting these models specifically will be explained throughout this study.

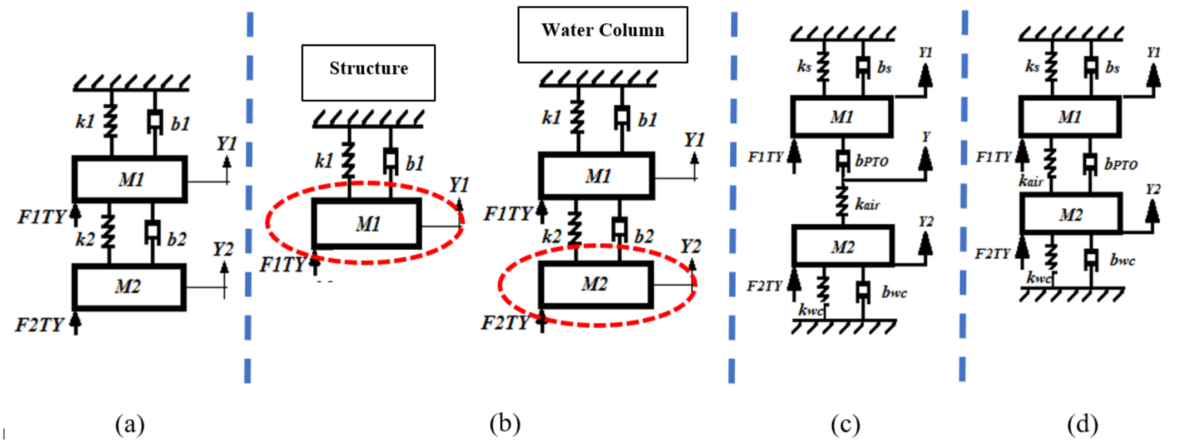


Fig.1. Dynamic models of floating OWCs: (a) Simplified 2DOF model (b) One-way coupling model (c) Szumko model (d) Modified Szumko model

In Fig.1 M_1 and M_2 are the total mass of the structure and water column, respectively, b_1 is the damping coefficient of the structure, b_2 is the damping coefficient of the water column b_{wc} and the Power Take-Off (PTO) mechanism b_{PTO}. k_1 is the structure's hydrostatic stiffness in heave mode and k_2 is the stiffness due to air compression k_{air}, and hydrostatic effects of the water column k_{wc}. $F_{1TY} \cos(\omega t)$ and $F_{2TY} \cos(\omega t)$ are the vertical wave forces acting on the structure and water column, respectively. $y_1, \dot{y}_1, \ddot{y}_1$ are the displacement, velocity, and acceleration of the structure and $y_2, \dot{y}_2, \ddot{y}_2$ are the displacement, velocity, and acceleration of the water column, respectively. The equations of motion of the four models are described as:

2.1. Simplified 2DOF Model

This model has been developed by Incecik (2003) in a report prepared for Wavegen Inverness ltd. The equations of motion of this model are:

$$M_1\ddot{y}_1 + (b_1 + b_2)\dot{y}_1 + (k_1 + k_2)y_1 - b_2\dot{y}_2 - k_2y_2 = F_{1TY}\cos(\omega t) \quad (1)$$

$$M_2\ddot{y}_2 + b_2\dot{y}_2 + k_2y_2 - b_2\dot{y}_1 - k_2y_1 = F_{2TY}\cos(\omega t) \quad (2)$$

Investigations of the effect of mass ratio and damping ratio on the behaviour of the system and the arrangement with the experimental results will be discussed in detail later including the development of a third experimental model. This investigation led to the following model.

2.2. One-way Coupling Model

This model has been introduced in this study after the failure of the first model (Simplified 2DOF model) to agree with the experimental results for certain mass ratios. Thus, one-way coupling between the two masses can be a reasonable assumption. This is achieved by treating the structure's heave motion as single DOF system while keeping the equation of motion of the water column as it is in the Simplified 2DOF Model. In other words, the structure's heave motion was considered in the water column oscillation modelling but not the other way around. Investigations led to this model will be discussed in detail in the results section of this study. The equations of motion of this model are:

$$M_1\ddot{y}_1 + (b_1 + 0)\dot{y}_1 + (k_1 + 0)y_1 - 0\dot{y}_2 - 0y_2 = F_{1TY}\, cos(\,\omega t) \tag{3}$$

$$M_2\ddot{y}_2 + b_2\dot{y}_2 + k_2 y_2 - b_2\dot{y}_1 - k_2 y_1 = F_{2TY}\, cos(\,\omega t) \tag{4}$$

2.3. Szumko Model

This model was originally developed by Szumko (1989) and recently adopted by Folley & Whittaker (2005), Stappenbelt & Cooper (2010) and Bayoumi et al., (2014). Unlike the simplified 2DOF model the turbine damping is modelled by the linear damping separately, not in conjunction with the water column damping. In addition, the air compressibility is also modelled by the linear stiffness separately not in conjunction with the hydrostatic stiffness of the water column. In other words, the PTO damping and air compressibility are related to the relative velocity and displacement between the two masses. The equations of motion of this model are:

$$M_1\ddot{y}_1 + b_s\dot{y}_1 + b_{PTO}(\dot{y}_1 - \dot{y}) + k_s y_1 = F_{1TY}\, cos(\,\omega t) \tag{5}$$

$$b_{PTO}(\dot{y} - \dot{y}_1) + k_{air}(y - y_2) = 0 \tag{6}$$

$$M_2\ddot{y}_2 + b_{wc}\dot{y}_2 + k_{wc}y_2 + k_{air}(y_2 - y) = F_{2TY}\, cos(\,\omega t) \tag{7}$$

2.4. Modified Szumko Model

A modelling problem appeared during the solution of the equations of motion of Szumko model, which is sensitivity of the modelling results to the pneumatic stiffness and damping values. Therefore, Szumko model has been modified so that the pneumatic spring and damper are parallel to each other. This means that the phase caused by the spring in the original Szumko model has been ignored. The equations of motion of this model are:

$$M_1\ddot{y}_1 + (b_s + b_{PTO})\dot{y}_1 + (k_S + k_{air})y_1 - b_{PTO}\dot{y}_2 - k_{air}y_2 = F_{1TY}\,cos(\,\omega t) \quad (8)$$

$$M_2\ddot{y}_2 - b_{PTO}\dot{y}_1 + (b_{PTO} + b_{wc})\dot{y}_2 - k_{air}y_1 + (k_{air} + k_{wc})y_2 = F_{2TY}\,cos(\,\omega t) \quad (9)$$

3. Data reduction

Masses of the structure and the water column were measured while added masses were calculated using the frequency independent form of cylindrical shapes in vertical direction provided as:

$$\boldsymbol{m_{avm}} = \frac{4}{3}\,\boldsymbol{\rho}\left(\frac{D}{2}\right)^3 \quad (10)$$

where $\boldsymbol{\rho}$ is the water density, and D is the diameter of the section. Three damping coefficients appeared in the mathematical models adopted herein representing the structure damping in heave, the water column oscillations damping and the PTO damping which extracts the energy from the system. The simplified 2DOF model only combined the water column and PTO damping for simplification. Decay tests were performed to identify the damping coefficients experimentally. PTO damping was modelled experimentally by orifices plate on the top of the OWC tube.

Three stiffness terms also appeared in the models representing the hydrostatic stiffness of the structure and the water column corresponding to their waterplane areas respectively in addition to the pneumatic stiffness due to air compressibility inside the OWC chamber. This approach was adopted in several studies by many researchers (Brendmo et al., 1996, Ikoma et al., 2012, Bayoumi et al., 2014). The pneumatic stiffness is calculated as:

$$\boldsymbol{k_{air}} = \frac{\gamma\, p\, A_{wc}{}^2}{V_0} \quad (11)$$

where V_0 is the average air volume in the pneumatic chamber, p is the pressure and γ is the specific heat ratio for air. However, for full-scale OWC the influence of the air compressibility will be minor compared to the hydrostatic (restoration) force previously discussed (Suleman & Bin Khaleeq 2010).

The total vertical wave excitation forces acting on the structure, F_{1TY} and the water column, F_{2TY} are assumed to consist of dynamic pressure forces (incident) and acceleration forces (Incecik, 1982, Aalbers, 1984 and Sphaier et al., 2007). The pressure and acceleration components are expressed as:

$$\boldsymbol{f_p = 0.5\, H_w\, A\, \rho g \frac{cosh(-dK+w_dK)}{cosh(w_dK)}} \quad (12)$$

$$\boldsymbol{f_a = -0.5\, H_w\, m_{avm}\;\, g\, K \frac{sinh(-dK+w_dK)}{sinh(w_dK)}} \quad (13)$$

Where H_W is the wave height, d is the depth of the section below water level, K is the wave number, W_d is the water depth, and g is the gravitational acceleration. For floating OWC, phase difference between forces applied on the structure and water column should be introduced in the modelling for proper dynamic analysis. Stappenbelt & Cooper (2009) reported that wave forces on the water column, and the floating structure, are to be related via a complex parameter allowing for both a magnitude and phase difference between the forces. It can be noticed that this parameter is real. In the limit of large wavelength, or small wave number, this parameter can also be shown to be equivalent to the area ratio, R_A, of the OWC opening to the total base area of the floating wave energy converter. In this case, wave forces applied on the water column are multiplied by R_A and wave forces applied on the structure are multiplied by $(1\text{-}R_A)$.

Similar to the analysis adopted by Falnes (2002), Stappenbelt & Cooper (2009), Gomes et al. (2012) and Falcão (2012), the mean captured power from a 2 DOF heaving system is given by:

$$P_{cap} = 0.5\, \omega^2\, b_{PTO}\, RM^2 \quad (14)$$

where RM is the relative heave motion between the structure and the water column.

For regular incident waves, the wave power per unit crest width is obtained as:

$$P_W = \frac{\rho\, g^2 T_e\, H_S^2}{64\, \pi} \quad (15)$$

where T_e is the wave energy period, and H_S is the significant wave height. Then, the capture width which represents the equivalent width of incident wave power that is completely captured by the device and converted to mechanical power is defined as:

$$C_W = \frac{P_{cap}}{P_W} \tag{16}$$

Finally, the Capture Factor C_F for cylindrical wave energy converter, also known as Capture Width Ratio C_{WR}, is expressed as:

$$C_F = \frac{P_{cap}}{P_W\, D} = \frac{C_W}{D} \tag{17}$$

4. Experiments

Experimental work has been carried out in the tank located at the Kelvin Hydrodynamics Laboratory of the University of Strathclyde where several reduced-scale models have been developed. Each reduced scale model consists of a transparent acrylic tube for visualisation purpose inserted in a cylindrical buoy made of solid foam (floater) and a brass cylindrical collar is attached to the bottom of the tube to enhance the vertical stability of the structure and to increase the spar damping. Plastic orifices' covers (with four and two 10 mm orifices) are placed on top of the tube to model different turbine damping. Experimental model 1 is 1:100 reduced scale model of the devices investigated by Incecik, 2003. For the second experimental model, the horizontal dimensions of model 1 were doubled while the vertical dimensions remained the same. The reason for increasing the horizontal dimensions only in the second model is to avoid the effect of the tank floor on the bottom of the device. A third experimental model was developed for special purpose and will be discussed later. The schematic diagram with dimensions and a photo of the first experimental model are presented in Fig.2.

Inclining tests were performed to assess the stability of the spars since the metacentric height (GM position) can be experimentally determined by moving weights transversely and/or longitudinally to produce a known overturning moment. Knowing the restoring properties (buoyancy) of the structure from its dimensions and floating position and by measuring the equilibrium angle of the structure, the GM is calculated in addition to other stability index and reference heights. Decay (free oscillations) tests have been performed by exciting the structure in heave direction and letting the motion dies out freely to identify the natural frequency of the spar. This is in addition to the decay test of the water column itself in case of unloaded OWC (open tube) and in case of orifices' covers being fixed on top of the OWC tube. Forced oscillation tests have been conducted by letting the device freely floats in the tank while subjected to regular waves of 10 mm amplitude and the frequencies varied from 0.3 to 1 Hz. Forced oscillation tests are necessary to validate the adopted mechanical models.

Kelvin Hydrodynamics Laboratory is equipped with infrared optical tracking. Using data from many cameras, the system calculates the 3D positions of the markers attached to the reduced-scale model. The 6 DoF motions are calculated and transferred over a TCP-IP connection to be presented in real-time. Wave probes used for the measurement of the amplitude of the passing waves and the water elevation inside the tube are “resistance" type probes. These probes produce voltage proportional to the submerged length. Primary results regarding mass, damping, stiffness, and stability results are presented in Table 1.

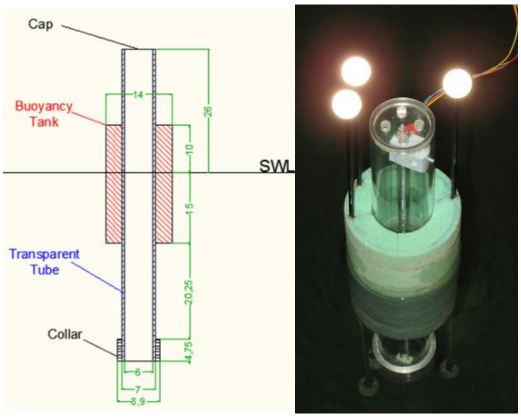


Fig.2 Schematic drawing and photo of the reduced-scale model

Table 1: Reduced-scale model experimental design parameters and results.

Results of reduced-scale models'	Model 1	Model 2	Notes
Mass and inertia			
Structure mass (kg)	2.065	9.045	Measured
Structure added mass in heave (kg)	0.502	4.595	Calculated
WC mass	1.131	4.599	Measured
WC added mass in heave (kg)	0.0360	0.295	Calculated
Damping ratio			
Structure in heave	0.056	0.070	Measured from decay test
WC (unloaded OWC)	0.041	0.043	Measured from decay test
WC + 4 orifices plate	0.043	0.059	Measured from decay test
WC + 2 orifices plate	0.046	0.082	Measured from decay test
Stiffness coefficients			
Structure hydrostatic in heave (N/m)	123.27	580.62	Calculated
WC hydrostatic in heave (N/m)	27.737	112.80	Calculated
Air compressibility	1.0875	4.4227	Calculated
Stability index and reference heights			
KG (m)	0.182	0.146	Measured from inclining test
KB (m)	0.287	0.294	Calculated
BM (m)	0.010	0.044	Calculated
GM (m)	0.115	0.193	Measured from inclining test

5. Results and Investigations

The simplified 2DOF model was solved analytically due to its simplicity. Predicted natural frequencies of this model for the two experimental models are presented in Table 2.

Table 2: Calculated natural frequencies of the simplified 2DOF model.

Natural frequencies	Model 1	Model 2
Structure's heave natural frequency (rad/s)	8.0	7.5
WC natural frequency (rad/s)	4.1	4.1

The results of the experimental model 1 will only be presented herein. Comparison between analytical and experimental results of structure's heave, water column oscillations and relative RAOs for experimental model 1 using simplified 2DOF model is presented in Fig.3. The water column RAO is defined as the response amplitude divided by the wave amplitude.

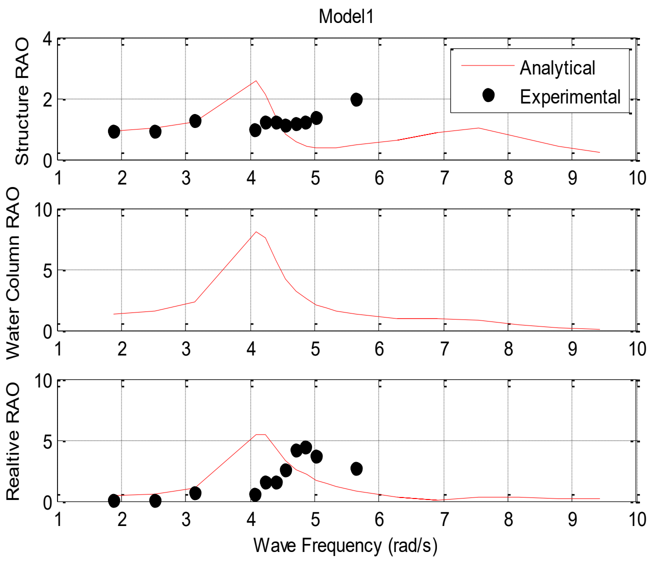


Fig.3. Analytical and experimental structure's heave, water column and relative motion RAOs vs. wave frequency for open tube; simplified 2DOF model

The predicted relative RAO peaks at 4.1 rad/s and 8 rad/s corresponding to the water column and structure's natural frequencies presented in Table 2. The predicted frequency responses of the spar and the relative motion are very close to each other and failed to match the measured peaks corresponding to the structure and water column natural frequencies. In order to validate the analytical procedure, Matlab scripts are developed to solve the same equations numerically according to the simulation presented in Bayoumi et al., (2014). Comparison between analytical, numerical and experimental results of structure's heave and relative motion RAOs for experimental model 1 is presented in Fig.4 within the range of frequencies tested experimentally.

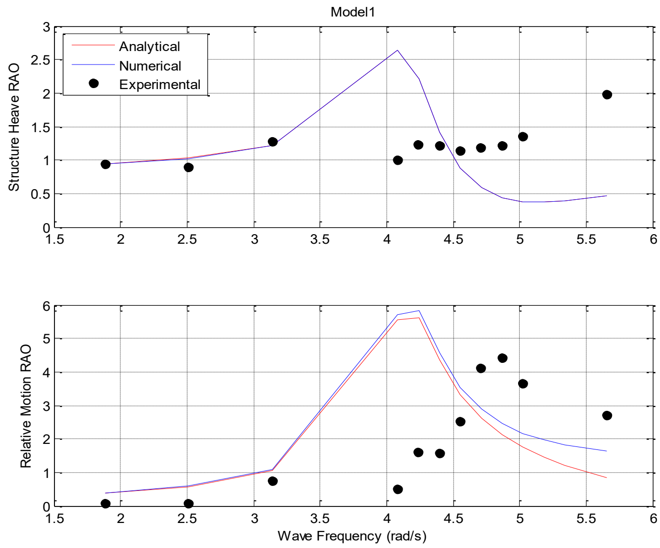


Fig.4. Analytical, numerical, and experimental structure's heave and relative motion RAOs vs. wave frequency for open tube; simplified 2DOF model

Fig.4 showed that both analytical and numerical procedures failed in predicting the spar heave and the relative motions responses. The results obtained from the simplified 2DOF model question the dynamic model originally and not the numerical procedures. However, two different approaches were adopted to investigate the awkward similarity between the responses of the two masses and improve the results obtained by the simplified 2DOF model.

First, the case that the damping applied is not high enough to model the coupling was considered. Therefore, simulations have been performed assuming higher damping ratios. Fig.5 presents a comparison between numerical results using constant structure damping ratio of 0.5 and higher water column damping ratio. On the other hand, Fig.6 presents a comparison between numerical results using constant water column damping ratio of 0.5 and higher structure damping ratio.

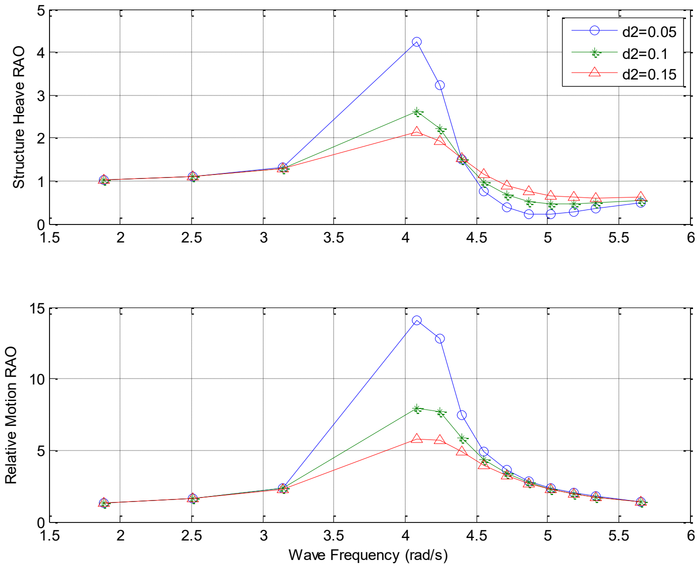


Fig.5. Numerical structure's heave and relative motion RAOs vs. wave frequency for simplified 2DOF model, constant structure damping ratio (d1=0.05) and different water column damping ratios (d2)

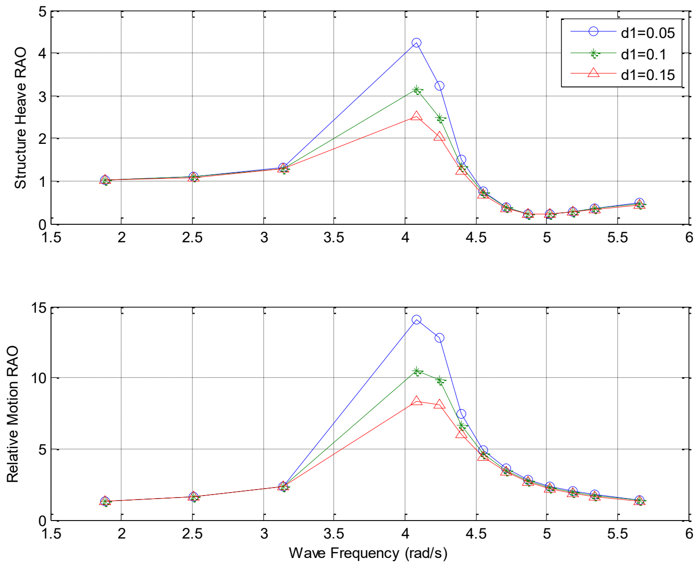


Fig.6. Numerical structure's heave and relative motion RAOs vs. wave frequency for simplified 2 DOF model, constant water column damping ratio (d2=0.05) and different structure damping ratio (d1)

Modelling of floating OWC by the simplified 2DOF model using higher damping values did not solve the problem since the structure's heave and relative motion peak frequencies occur at the same frequency as presented in Fig.5 and Fig.6 which does not agree with the experimental results.

In the second case, the effect of the mass ratio on the modelling was investigated. The special condition for harmonic excitation treated herein occurs when the mass ratio M_r (M_2/M_1) is large (>0.4 in this case) and the natural frequency ratio $(\omega_{n2}/\omega_{n1})$ is close to 1 with small damping ratio. In this case the predicted frequency responses of the masses become very close to each other as shown in Fig.4. Therefore, numerical modelling was performed using lower mass ratios by increasing the mass of the structure. Numerical results obtained are presented in Fig.7.

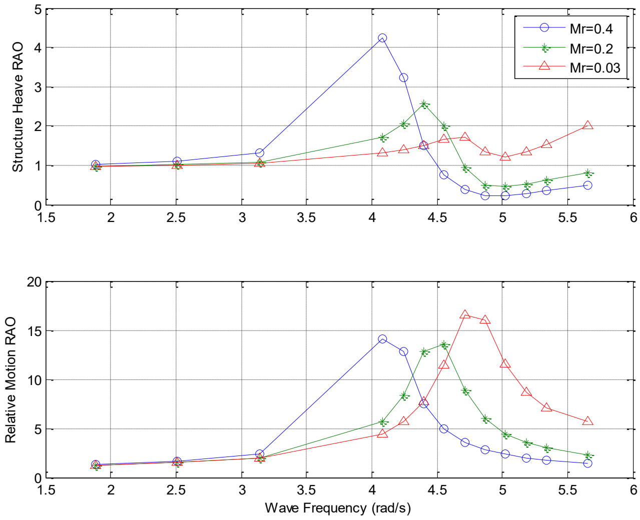
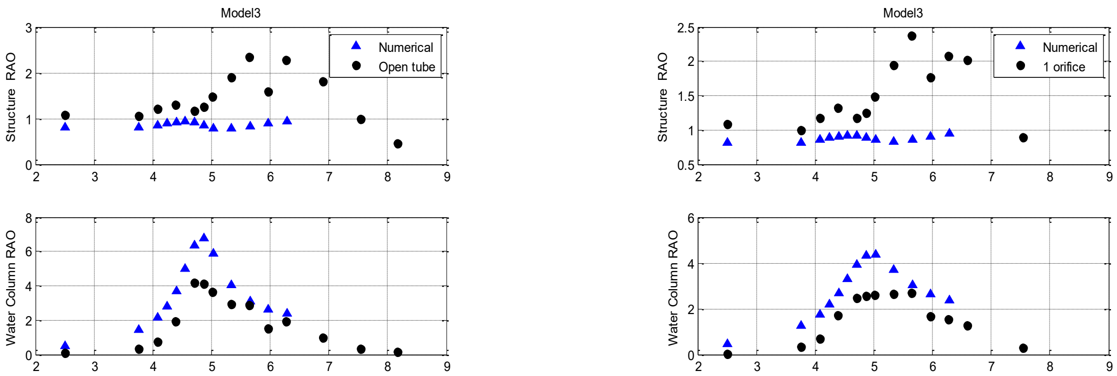


Fig.7. Numerical structure's heave and relative motion RAOs vs. wave frequency for simplified 2 DOF model, constant structure and water column damping ratios (d1=d2=0.05) and different mass ratio.

From Fig.7 it is noticed that as the mass ratio decreases, the relative motion peak frequency moves towards the expected correct value. To validate this approach, the previously mentioned experimental model 3 was used. Experimental model 1 was fitted inside a larger diameter floater (0.35m diameter instead of 0.14m) having the same draught and freeboard of the initial one. The model was tested undamped (open tube) and with cover plate containing one orifice only. Fig.8 presents comparison between the numerical and experimental spar heave and the relative motions RAOs. The damping in case of one orifice is assumed to be 15% of the critical damping and the spar damping is assumed to be 5%.



(a) (b)

Fig.8. Numerical and experimental structure’s heave and relative motion RAOs vs. wave frequency for experimental model 3; simplified 2 DOF model (a) Open tube, (b) one orifice.

Experimental model 3 was tested for a wider range of frequencies as we can see in Fig.8. Predicted relative motion response showed better agreement with experimental results for both cases: open tube and one orifice. In contrast, predicted spar heave motion response did not agree with the experimental results for frequencies over 5 rad/s. The first peak appeared correctly at 4.8 rad/s corresponding to the water column natural frequency. The second peak appeared at 6.5 rad/s which is the new structure’s natural frequency.

Due to the unsatisfactory agreements between experimental and numerical results a new model was developed which is the one-way coupling model. One-way coupling between the two masses is achieved by treating the structure’s heave motion as single DOF system while keeping the equation of motion of the water column as it is in case of the simplified 2DOF model. Natural frequencies calculated from this model are presented in Table 3 and comparison between numerical and experimental results of structure’s heave, water column oscillations and relative RAOs is presented in Fig.9.

Table 3: Calculated natural frequencies of the one-way coupling model.

Natural frequencies	**Model 1**	**Model 2**
Structure’s heave natural frequency (rad/s)	6.9	6.7
WC natural frequency (rad/s)	4.8	4.8

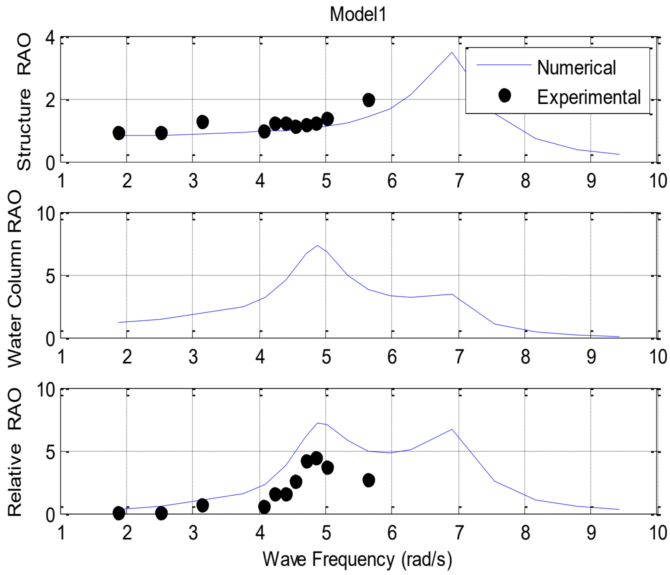
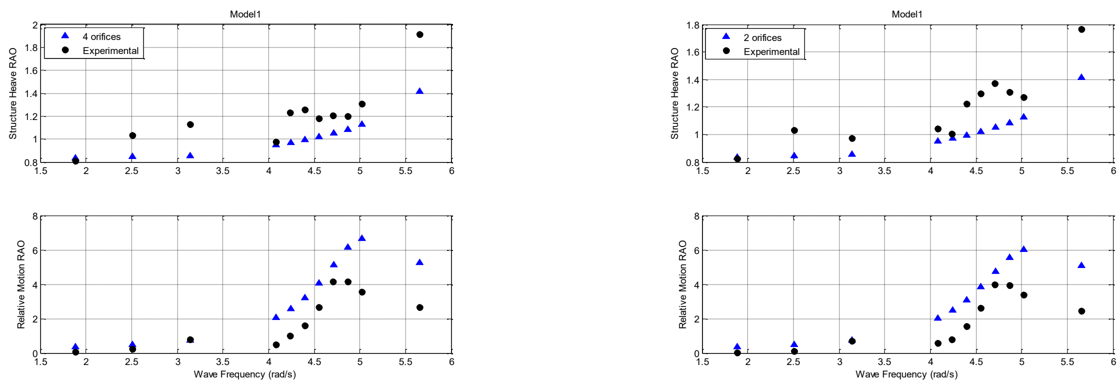


Fig.9. Numerical and experimental structure's heave, water oscillations and relative motion RAOs vs. wave frequency, open tube; one-way coupling model

Fig.9 showed that only one peak appeared in the structure's heave RAO at 7rad/s which corresponds to the uncoupled structure's heave natural frequency (as if the spar heave is modelled as a 1DOF system).

The water column and relative RAOs showed close behaviour with two peaks. The first peak at 4.8 rad/s, corresponds to the water column natural frequency and the second peak appeared at 7 rad/s corresponding to the structure's heave natural frequency.

Fig.10 presents comparison between numerical and experimental spar heave and relative RAOs within the experimentally validated range of frequencies using the two and four orifices' plates.



(a) (b)

Fig.10. Numerical and experimental structure heave and relative motion RAOs vs. wave frequency; one-way coupling model (a) four orifices, (c) two orifices

Predicted spar heave response obtained from the one-way coupling model showed better agreement with the experimental response than the response obtained from the simplified 2DOF model for the three experimental models. Predicted peak frequency agrees with the experimental peak unlike the relative motion peak frequency obtained from the simplified 2DOF model.

The third dynamic model used to describe the floating OWC motions is the Szumko model. Natural frequencies calculated from the Szumko model are presented in Table 4 and comparison between numerical and experimental results of structure's heave, water column oscillations and relative RAOs is presented in Fig.11.

Table 4: Calculated natural frequencies of the Szumko model.

Natural frequencies	Model 1	Model 2
Structure's heave natural frequency (rad/s)	7.0	6.9
WC natural frequency (rad/s)	4.8	4.8

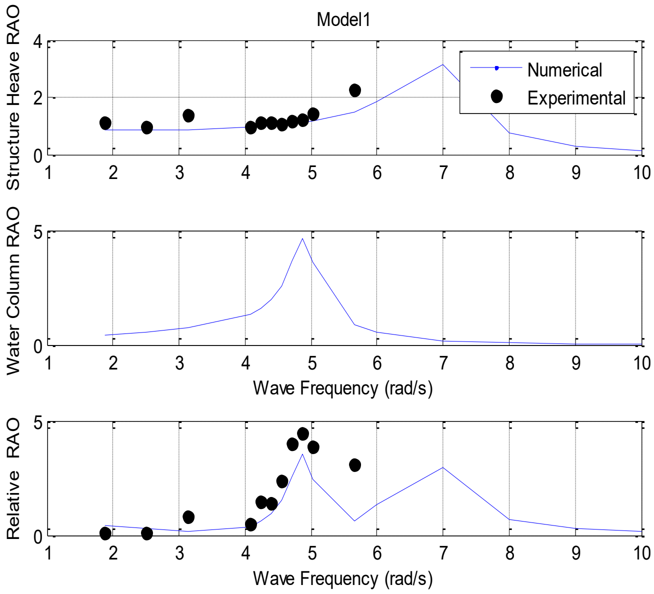
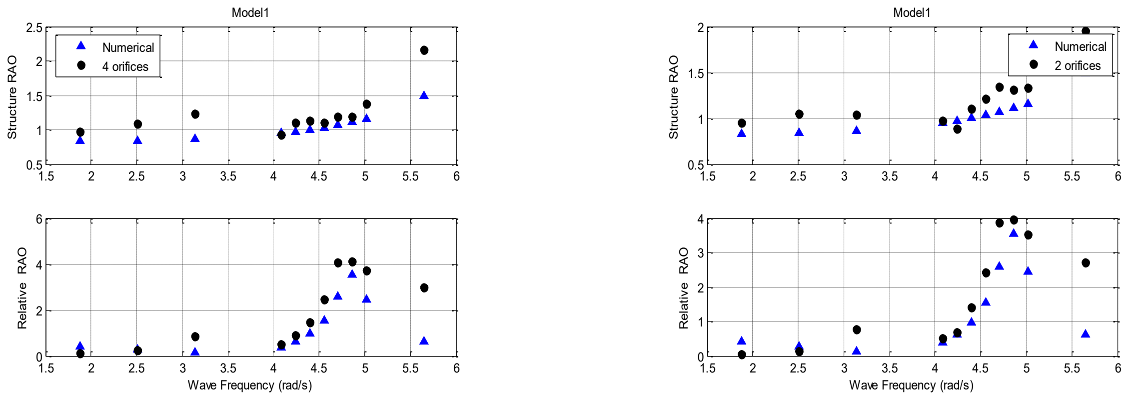


Fig.11. Numerical and experimental structure's heave, water oscillations and relative RAOs vs. wave frequency, open tube; Szumko model

Fig.12 presents comparison between the numerical and experimental spar heave and relative motions RAOs within the experimentally validated range of frequencies using the two and four orifices' plates.



(a) (b)

Fig.12. Comparison between numerical and experimental structure's heave and relative motion RAOs vs. wave frequency for experimental model 1; Szumko model (a) Open tube, (b) four orifices, (c) two orifices.

Modelling results of Szumko model showed better agreement with the experimental results. The disagreement between the predicted and measured relative RAO around the peak frequency is due to the adoption of viscous damping approach.

Seeking for a better agreement, Szumko model has been modified so that the pneumatic spring and damper are in parallel. The phase caused by the air compressibility has been ignored. However, the phase between the forces on the two masses is still introduced by the area ratio. Natural frequencies calculated from the modified Szumko model are presented in Table 5 and comparison between numerical and experimental results of structure's heave, water column oscillations and relative RAOs is presented in Fig.13.

Table 5: Calculated natural frequencies of the modified Szumko model.

Natural frequencies	**Model 1**	**Model 2**
Structure's heave natural frequency (rad/s)	7.6	7.4
WC natural frequency (rad/s)	4.8	4.8

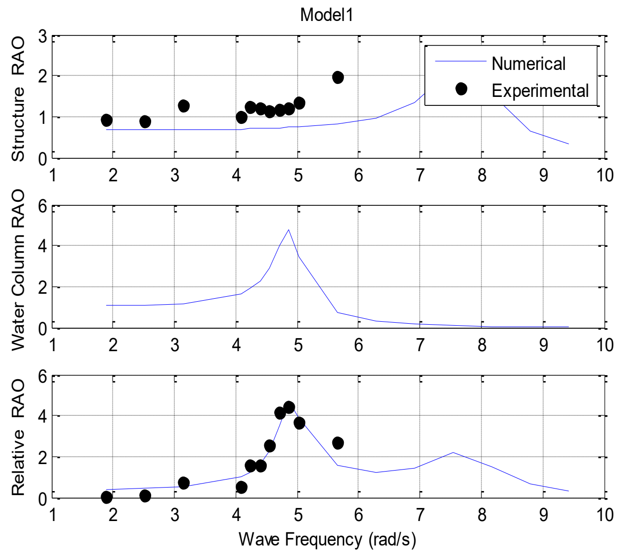
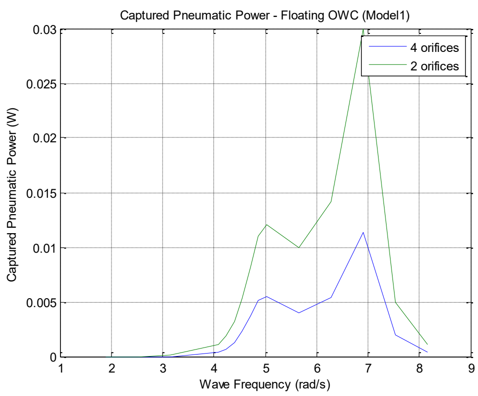


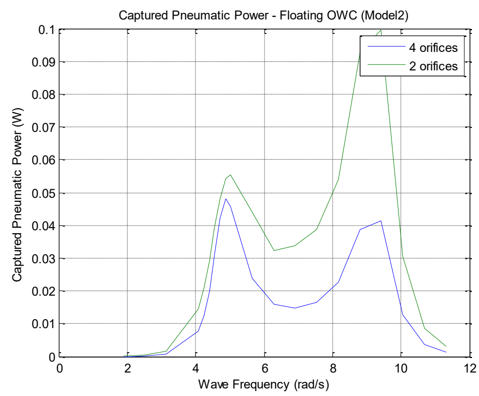
Fig.13. Comparison between numerical and experimental structure's heave, water oscillations and relative RAOs vs. wave frequency, open tube; Modified Szumko model

From Fig.13 it can be seen that the structure's natural frequency is shifted from 7 to 7.5 rad/s which caused the deviation between the predicted and measured structure's RAOs. Therefore, relative motion results of the Szumko model are used to assess the performance of the converter.

Finally, the pneumatic power captured by experimental models are calculated along with capture factor results are presented in Fig.14 and Fig.15. Wave height of 0.02m was used as input in captured power and capture factor calculations. The two damping values used are those obtained experimentally using the orifices covers simulating the turbine damping in different cases.

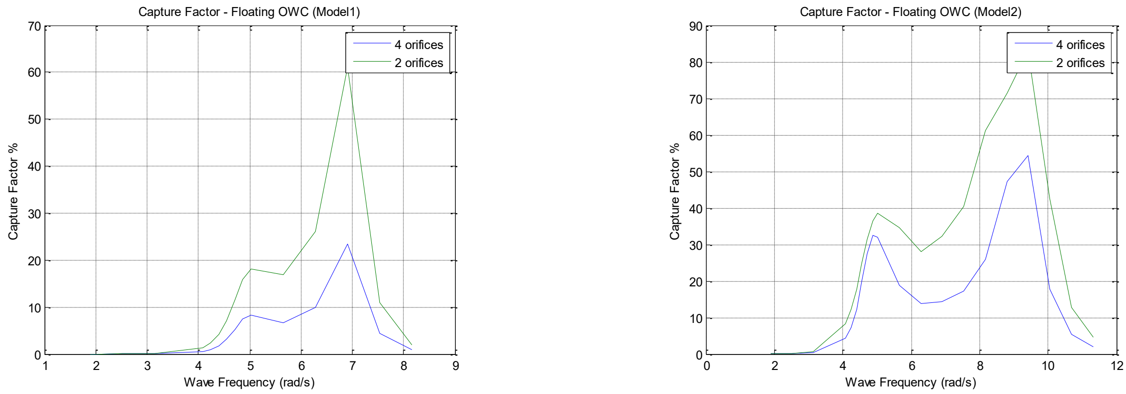


(a)



(b)

Fig.14. Captured pneumatic power a) Experimental model 1 b) Experimental model 2



(a) (b)

Fig.15. Capture factor a) Experimental model 1 b) Experimental model 2

The peaks corresponding to the OWC, and floating structure's natural frequencies are visible in the power capture plots and easy to be targeted during the preliminary design phase of the device. This is consistent with the floating OWC experimental results from the study by Sykes et al., (2009).

Conclusions and final remarks

Four different dynamic models have been adopted to predict the spar-buoy, water column and relative heave motions required for the estimation of the converter captured power.

A simplified 2DOF model was used and showed unsatisfactory agreement with experimental results. Intensive investigation of this model using different mass and damping ratios showed that this model is not suitable. Investigations of the mass ratio effect on the behaviour of the system highlighted the strong effect of the spar-buoy heave motion on the performance of the OWC but not the other way around which agrees with Ikoma et al., 2012 and Bayoumi et al., 2014; and led to the so-called one-way coupling model.

The one-way coupling model results showed fair agreement with experimental results. However, the model cannot be considered as a reliable model since the water column oscillations were not considered in the prediction of the structure's heave motions, which affects the accuracy of the relative motion prediction controlling the power calculations. The results obtained from this approach assured the inappropriateness of the simplified 2DOF model originally adopted and not the numerical procedures. That is why the model proposed by Szumko has been used.

Szumko model was adopted as it is the most trusted dynamic model to describe floating OWC. Unlike the simplified two DOF model coupling between the two masses in case of Szumko model did not include the water column stiffness and damping. The masses are assumed to be coupled by the pneumatic chamber only. Coupling between the structure and water column considered the air compressibility stiffness and PTO damping. Results obtained from Szumko model showed the best agreement with the experimentally measured responses.

More investigations motivated by the complexity of numerically modelling Szumko mathematical model and aiming to simplify it by ignoring the air compressibility led to the proposed modified Szumko model. The modified Szumko model did not achieve the desired aims as it failed to predict the spar heave response and consequently underestimated the relative motion near spar heave resonance.

Captured power and capture factor of the converter were predicted within the investigated frequency range using the damping ratios experimentally evaluated by the use of two different orifices covers modelling two different turbine damping values.

A significant increase in the relative motion and consequently the captured power is achieved when compared with the results in case of captive OWC reported by Bayoumi 2018 for the same experimental model where the relative motion RAO reached around 3. Another advantage of floating spar-buoy OWCs over captive ones is the ability of the device to harvest wave energy efficiently over a wider bandwidth of incident wave frequencies due to the separation of the two bodies' natural frequencies.

The proposed model can be important in the commercialization of WECs as it helps in the preliminary design phase of full-scale devices. Moreover, the incorporation of optimization procedure to the proposed model is important to allow quick customization of the device according to the deployment location and its environmental condition; and consequently, building faster commercial benchmarks.

References

Aalbers A.B. 1984, 'The water motions in a moonpool', Ocean Engineering Vol. 11 No. 6, pp 557-579

Islam Amin, Mohamed E. Aly, Seif Bayoumi, Ahmed Balah, Selda Oterkus, Hosam Shawky, Erkan Oterkus, Numerical Hydrodynamics-Based Design of an Offshore Platform to Support a Desalination Plant and a Wind Turbine in Egypt. Ocean Engineering

Bayoumi Seif, Atilla Incecik & Hassan El-Gamal (2014): Dynamic modelling of Spar-Buoy oscillating water column wave energy converter, Ships and Offshore Structures, DOI: 10.1080/17445302.2014.942086

Bayoumi Seif. (2018). Modelling of Cylindrical Oscillating Water Column Including Nonlinearities Due to Large Waves and Damping Forces in Time and Frequency Domain, Thirteenth International Conference of Fluid Dynamics, Cairo, Egypt.

Brendmo A., Falnes J., Lillebekken P.M., 1996, 'Linear modelling of OWCs including viscous loss'. Applied Ocean Research, Vol. 18, pp 65–75.

Doyle Simeon & Aggidis G.A. (2020). "Experimental investigation and performance comparison of a single OWC, array and M-OWC". Renewable Energy. 10.1016/j.renene.2020.12.032.

Falnes J., 2002, **Ocean waves and oscillating systems.** Cambridge: Cambridge University Press

Falcão António F.O., Henriques João C.C., Cândido José J., 2012, 'Dynamics and optimization of the OWC spar buoy wave energy converter'. Renewable Energy Vol. 48 pp 369 - 381

Folley, M. & Whittaker, T., 2005, 'The effect of plenum chamber volume and air turbine hysteresis on the optimal performance of oscillating water columns', In *Proceeding of 24th International Conference on Offshore Mechanics and Arctic Engineering* (OMAE2005), Halkidiki, Greece.

Gomes R.P.F., Henriques J.C.C., Gato, A.F.O. Falcão L.M.C., 2012, 'Hydrodynamic optimization of an axisymmetric floating oscillating water column for wave energy conversion' Renewable Energy Vol. 44 328-339

Henriques Joao & Portillo, Juan C. C. & Gato, L.M.C & Gomes, Rui & Neves Ferreira, Diogo & Falcao, Antonio. (2016). Design of oscillating-water-column wave energy converters with an application to self-powered sensor buoys. Energy. 112. 852–867. 10.1016/j.energy.2016.06.054.

Ikoma T, Masuda K, Watanabe Y, and Maeda H, 2012, 'Hydro-elastic responses and performance of owc type wecs of a large scale floating structure'. *In Proceeding of the 6th International Conference on HYDROELASTICITY in Marine Technology* (Hydroelasticity 2012), Tokyo, JAPAN

Incecik, A., 1982, 'Design aspects of hydrodynamic and structural loading on floating offshore platforms under wave excitation', Ph.D. thesis, University of Glasgow, U.K

Incecik A. 2003. Dynamic motion response analysis of a sparbuoy. Report prepared for WAVEGEN Limited, Inverness.

Jefferys E.R., 1984, 'Simulation of wave power devices'. Applied Ocean Research, Vol. 6, No. 1, pp 31-39.

Kisacik Dogan & Stratigaki Vasiliki & Wu Minghao & Cappietti Lorenzo & Simonetti Irene & Troch Peter & Crespo Alejandro & Altomare Corrado & Domínguez J.M. & Hall Matthew & Gómez-Gesteira Moncho & Canelas, Ricardo & Stansby P.K.. (2020). Efficiency and Survivability of a Floating Oscillating Water Column Wave Energy Converter Moored to the Seabed: An Overview of the EsflOWC MaRINET2 Database. Water. 12. 992. 10.3390/w12040992.

Nunes Guilherme, Duarte Valério, Pedro Beirão, and José Sá da Costa, 2011, Modelling and control of a wave energy converter, Renewable Energy, Vol 36, pp 1913-1921.

Sphaier S. H., Torres F.G.S, Masetti I.Q., Costa A.P. &Levi C, 2007, 'Monocolumn behaviour in waves: Experimental analysis', Ocean Engineering Vol. 34, pp 1724-1733

Stappenbelt Brad and Cooper Paul, 2009, 'Mechanical Model of a Floating Oscillating Water Column Wave Energy Conversion Device', Annual Bulletin of the Australian Institute of High Energetic Materials pp. 34-45. USBN: 978-0-9806811-3-0

Suleman zahid, and Bin khaleeq hammad, 2010, 'Design analysis of power extracting unit of an onshore OWC based wave energy power plant using numerical simulation'. Mehran university research journal of engineering & technology, volume 30, no. 3, [ISSN 0254-7821]

Sundar V., Sannasiraj S.A., David D.R. (2022) Performance Characteristics of an OWC in Regular and Random Waves. In: Samad A., Sannasiraj S., Sundar V., Halder P. (eds) Ocean Wave Energy Systems. Ocean Engineering & Oceanography, vol 14. Springer, Cham. https://doi.org/10.1007/978-3-030-78716-5_5

Szumko, S., 1989, 'Mechanical Model for Oscillating Water Column with Compressibility', *Journal of Engineering Mechanics,* Vol. 115, No.9, pp 1851-1865

Wu Minghao & Stratigaki Vasiliki & Troch Peter & Altomare Corrado & Verbrugghe Tim & Crespo Alejandro & Cappietti Lorenzo & Hall Matthew & Gómez-Gesteira Moncho. (2019). Experimental Study of a Moored Floating Oscillating Water Column Wave-Energy Converter and of a Moored Cubic Box. Energies. 12. 1834. 10.3390/en12101834..

Zabihi Milad & Mazaheri Said & Montazeri Namin Masoud & Mazyak Ahmad. (2021). Irregular wave interaction with an offshore OWC wave energy converter. Ocean Engineering. 222. 108619. 10.1016/j.oceaneng.2021.108619