SME investment and financing under asymmetric information

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Abstract

We investigate the investment timing and nancing decisions of nancially constrained Small and Medium-sized Enterprises (SMEs) in a a real-option setting with asymmetric information. Bad rms can sell over-priced securities by mimicking in a pooling equilibrium. However, good rms can separate from bad rms by imposing an adverse selection cost for mimicry only when the bene t of being recognized as the good type outweighs the investment distortion costs. Further, asymmetric information induces good rms to accelerate investment, leading to investment distortion and higher guarantee costs. Equity-for-guarantee swap not only mitigates SMEs nancing constraints but also reduces the investment and nance distortions.

Keywords: Asymmetric information, equity-for-guarantee swap, least-cost equilibrium, real option, SME nancing
JEL: G11, G14, G32

1. Introduction

The development of Small and Medium-sized Enterprises(SMEs) plays an essential role in promoting a country s economic development, innovation, and employment. However, significant nancing constraints due to nancial frictions have disproportionately a ected SMEs in the wake of economic shocks(Christodoulou, Ho and Prokhorov, 2021; Ferrucci, Guida and Meliciani, 2021). Innovative nancial contracts, such as equity-for-guarantee swaps (EGS), are important in mitigating such nancing constraints.[2] More precisely, an SME nances a risky project using equity-for-guarantee swaps (EGS), which secures guaranteed debt at the

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[2] For example, Song, Zhang and Zhao (2021) demonstrate that innovative credit guarantee scheme with EGS can effectively increase SMEs' borrowing capacity.

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expense of equity dilution in favor of an external insurer.[3] However, virtually all the existing models consider EGS financial instrument only in a perfect information setting. As information asymmetry between SMEs and external investors exacerbates financing constraints (Andrikopoulos, 2009), it is important to investigate the impact of asymmetric information on SME investment and financing decisions. The following research questions are of special concern: How does EGS affect SMEs' investment strategies? How does the information advantage party, SMEs owners, create a credible signal to external investors? What role does asymmetric information play in affecting SMEs' external financing cost?

To answer these questions, we develop a real option model of SME investment and financing decisions with EGS under asymmetric information. Our model generates a rich set of testable predictions and offers insights in how asymmetric information affects SMEs' equilibrium investment strategies and financing constraints. To be specific, EGS is a financial contract signed among three parties—a bank/lender, an insurer, and an SME/borrower—and it specifies that an SME gets bank loan guaranteed by the insurer and it pays the insurer equity shares instead of the normal guarantee fees. Most existing researches of EGS, see Yang and Zhang (2013), Wang, Yang and Zhang (2015) and Song, Zhang and Zhao (2021) among others, show that EGS effectively alleviates SMEs' financial constraints and dominates traditional credit-guarantee-schemes in markets with perfect information. However, those models provides almost no insight regarding whether such mitigating effects of EGS still hold under asymmetric information and how information asymmetry affects SMEs investment and financing decisions.

Following Morellec and Schrhoff (2011), we adopt a signaling approach to analyze the internal mechanism of the market games of SME investment. We assume that the market has two types of SMEs: high-type (high cash flow) firms and low-type (low cash flow) firms. The quality of cash flow is private information known only to SMEs owners, creating asymmetric information between the SMEs and external investors. After investment, the project generates a cash flow that follows an arithmetic Brownian motion (ABM). ABM was chosen because of the following reasons. First, cash flow approximated by geometric Brownian motion (GBM) is always positive while SME cash flow might be negative. Second, for GBM cash flow, investors' response to price changes follows Weber's Law, driving variation to zero if price approaches zero (Blaug, 1997). However, with the usual constant-volatility in ABM, variation in price remains constant regardless of the price level. Third, ABM has also been extensively used in option pricing research, for instance, in Smith (1976), Alexander, Moffatt and Stent (2012), Hugonnier, Malamud and Morellec (2014), Brooks and Brooks (2017), and Choi, Liu and Seo (2019), among others.

Unlike large mature firms, we assume financially constrained SMEs can only finance the investment by using guaranteed debt via EGS. The firm type information is known only to SMEs' owners, the rational external investors use Bayes' rule to update their beliefs about the firm's profitability. Therefore, the low-type firm has an incentive to mimic as it benefits from information asymmetry by selling over-priced securities. Given the expectation of

[3] See details in Yang and Zhang (2013), Wang, Yang and Zhang (2015), and Yang (2020) among others.

mimic by low-type firms, high-type firms might separate from low-type firms by accelerating investment, which deters mimicry and serves as a credible signal to external investors. Such a separating equilibrium only exists when the following two incentive compatibility conditions hold: First, the benefit of mimicking for low-type firms must be less than the benefit of being themselves; Second, the high-type firm's benefit of separation equilibrium is higher than the benefits of being recognized as a low-type firm. Similarly, we derive the conditions to ensure the existence of a pooling equilibrium where the outside insurers cannot identify firm type information as both high-type and low-type firms invest at the same time. Interestingly, EGS significantly alleviates the degree of asymmetric information and reduces the motivation of low-type companies to mimic, which leads to that the least-cost equilibrium overlaps the first-best strategy in most cases. We also explore the empirical implications of our model by and conclude that firms with a higher market-to-book ratio and higher growth potential are more likely to invest. Further, firm external financing costs tend to be higher for firms with a higher market-to-book ratio or a higher default cost.

Moreover, we also extend Morellec and Schrhoff (2011)'s model by introducing the borrowing constraint in our setting. Unlike Morellec and Schrhoff (2011)'s model, we assume a financing constrained SME cannot fully finance the investment cost without the help of EGS, therefore, it might forgo a profitable investment project if the borrowing constraint is severe. Even if the project were partially launched with limited capital, its cash flow will be scaled down (discounted) as the firm does not operate with its maximum production capacity. Our model shows that SMEs without EGS suffer a significantly larger loss of value for higher growth opportunities as SMEs are unable to launch the investment opportunity at its full potential. Moreover, an SME with EGS tends to have smaller abnormal returns compared with its peers without guarantee, which implies that EGS effectively mitigates the negative impact of information asymmetry.

Finally, we conduct a comprehensive simulation to fully explore SMEs' financing and investment behaviors with EGS under asymmetric information. Among the 60,000 artificial firms generated in our simulation, we find 47% (28,165) of SMEs can be identified as their "ture" type by outside investors using the timings of their investment, which leads to least-cost separating equilibria. Unlike extant empirical literature, our simulation provides novel empirical predictions that market optimism (a market with more high-type SMEs) diminishes SMEs' investment probability in incomplete markets as a higher belief leads to pool equilibria which delay firm investment.

Our paper is closely related to the literature on firm investment under asymmetric information. Myers and Majluf (1984) apply the signaling model in corporate finance and found that asymmetric information leads to a distortion in firms' investment and financing strategy. Grenadier and Wang (2005) find that asymmetric information induces underinvestment. Cui and Shibata (2017) show that the presence of agency conflicts delays investment and increases the quantum of investment. Morellec and Schrhoff (2011) develop a real option model of corporate investment and financing where the firm owner's information is superior to that of external investors. Morellec and Schrhoff (2011) claim that firms can credibly signal their type to lenders by the timing of investment and capital structure. Further, Clausen and Flor (2015) extend the work of Morellec and Schrho (2011) by incorporating an abandonment option and assets-in-place and found that these extensions make debt more attractive. Subsequently, Lemmon and Zender (2019) point out that debt structure choice can balance the ex-ante adverse selection against the ex-post moral hazard. Almost all of these studies consider only rms with exible nancing tools (equity, debt), however, we relax the SME borrowing constraints by introducing innovative nancing contracts (EGS) and investigate the impact of EGS on SME investment and nancing choices under information asymmetry.

Our paper also extends the recent research on SME nancing using EGS. Yang and Zhang (2013) is the rst paper to investigate EGS pricing and its role in alleviating SME nancing constraints and enhancing rm value. Liu, Song and Tang (2021) extend it into a dynamic growth model with asymmetric information and found that high-pro t rms sacri ce pro ts to send a signal of separation from low-pro t rms by increasing the latter s mimicking cost. Unlike Liu, Song and Tang (2021), our model builds a real option pricing model, which mainly focuses on the dynamics of joint investment timing and SME nancing decisions. Similar to this paper, Wang and Kwok (2020) extend Morellec and Schrho (2011) s model by inducing EGS in SME nancing, an innovative idea which was rstly introduced by Yang and Zhang (2013). Wang and Kwok s real option model assumes a nite time window of the investment opportunity and explores further how the information cost and nature of separating and pooling equilibria evolve over the nite time span of the investment opportunity. Unlike Wang and Kwok (2020), our model mainly focuses on how does EGS mitigate SMEs borrowing constraints and add value to incumbent shareholders. Moreover, we generate a large arti cial data set of SMEs characteristics to further test our model implications on SME nancing under asymmetric information.

The rest of this paper is organized as follows. Section 2 describes the benchmark model under perfect information, followed by a general model featuring SME investment and nancing using EGS under asymmetric information in Section 3. Section 4 contains the discussions and Section 5 develops the empirical predictions of the model. Finally, Section 6 concludes. All technical developments and proofs are given in the Appendices.

2. Benchmark model: A rst-best case with perfect information

Here, we develop a benchmark model of SME investment with EGS under a perfect information setting. Unlike mature companies that can issue risky debt, we assume nancially constrained SMEs can only nance an irreversible risky project with both equity and the guaranteed debt using EGS. The project, once completed, produces a continuous stream of cash ows, the level of which depends on the speci c rm type k. Under perfect information, rm type is observable for both the SME owner and external investors.

We consider a set of rms, each of which has an option to invest in a risky project that requires a constant irreversible investment cost, I. Time is continuous and indexed by $t \in [0 \ \infty)$. After investment, a rm of type k generates a pro t ow given by $\ _k x_t$, where

$_k > 0$ and x_t are publicly observable. The cash ow follows an ABM given by

$$dx_t = \ dt + \ dB_t \qquad x_0 > 0$$

where the volatility > 0 are constant over time, B_t is a Standard Brownian Motion under the risk-neutral measure, x_0 is the initial value of cash ow. We further assume that there are two types of rms in nancial markets: high-growth (high-type $k = h$) and low-growth (low-type $k = l$) rms with $_h > \ _l > 0$ and the probability of high-type h is $\Pr(\ _k = \ _h) = p$.

Following Goldstein, Ju and Leland (2001), we assume a simple tax structure that includes personal and corporate taxes, where interest payments are taxed at personal tax rate $_i$, e ective dividends are taxed at $_d$, and corporate pro ts are taxed at $_c$, with full loss o set provisions.

Let (x) denote the present value of a perpetual stream of cash ows at any time $t \geq 0$, then we immediately get

$$(x) = \mathbb{E}\left[\int_t^{\infty} e^{-r(s-t)} x_s ds \big| x_t = x\right] = \frac{\ + rx}{r^2}$$

The sunk cost of the investment I is nanced by risk-free perpetual debt from a bank using EGS. The coupon of the guaranteed debt is c_k. After investment, the cash ow accruing to the SME over each time interval is $(\ _k x - c_k - f)dt$, where $f > 0$ represents constant operating expenses. Therefore, the value of equity after investment, $E_k(x)$, satis es the following ordinary di erential equation (ODE) subject to the following two boundary conditions:

$$rE_k = \ \frac{\ E_k}{\ x} + \frac{^2}{2}\frac{\ ^2 E_k}{\ x^2} + (1 - \ )(\ _k x - c_k - f) \quad x > x_k^d(c_k); \ k = h \ l$$

$$s\ t \begin{cases} (value - matching) : E_k(x_k^d(c_k)) = 0 \\ (no - bubble\ condition) : \lim\limits_{x \to \infty} E_k(x) < \infty \end{cases}$$

where $x_k^d(c_k)$ is the default threshold of the type k rm given by solving the rst-order condition $\ E_k / \ x|_{x = x_k^d(c_k)} = 0$. $F = f/r$ represents the present value of constant operating expenses and is the e ective tax rate de ned by $1 - \ = (1 - \ _c)(1 - \ _d)$. The condition of value-matching implies that as the cash ow approaches the endogenous default threshold, the option of equity becomes worthless.

Solving the above ODE yields the expression of equity value

$$E_k(x) = (1 - \ )\left[\ _k \ (x) - \frac{f + c_k}{r} - \left(\ _k \ (x_k^d(c_k)) - \frac{f + c_k}{r}\right) e^{- \ _2 \left(x - x_k^d(c_k)\right)}\right] \tag{1}$$

and the optimal default threshold is given by

$$x_k^d(c_k) = \frac{f + c_k}{\ _k} - \frac{}{r} - \frac{1}{\ _2} \tag{2}$$

where $_1 = \frac{-\sqrt{^2+2r\ ^2}}{^2} < 0$ and $_2 = \frac{+\sqrt{^2+2r\ ^2}}{^2} > 0$. It is obvious that the low-type rm defaults earlier than the high-type rm under symmetric information (i.e., $x_l^d > x_h^d$) as $_h > \ _l$.

Similarly, the value of risky debt (without guarantee), $D_k(x)$, satis es the following ODE subject two boundary condition

$$rD_k = \quad \frac{D_k}{x} + \frac{^2}{2}\frac{^2D_k}{x^2} + (1- \ _i)c_k \quad x > x_k^d(c_k); \ k = h \ \ l$$

$$s\ t \begin{cases} (value - matching) : D_k(x_k^d) = (1- \quad)[(1- \quad) \ _k \quad (x_k^d) - F] \\ (no - bubble\ condition) : \lim_{x\to\infty} D_k(x) = (1- \ _i)c_k/r \end{cases}$$

Thus, the debt value under perfect information is given by

$$D_k(x) = (1- \ _i)\frac{c_k}{r} - \left[(1- \ _i)\frac{c_k}{r} - (1- \quad)[(1- \quad) \ _k \quad (x_k^d) - F]\right] e^{- \ _2\left(x - x_k^d\right)} \qquad (3)$$

The last term in the above equation represents the bankruptcy cost for the debt holder.

2.1. SME nancing with guaranteed debt

With the help of EGS, SMEs borrowing constraints have been fully lifted up due to guarantee. Therefore, an SME with EGS can raise all the requested investment cost, I, with guaranteed debt. Similar to Yang and Zhang (2013); Wang, Yang and Zhang (2015) among others, the insurer requests $_k$ fraction of the SME s equity as guarantee fees to compensate for its payment to the lender should the SME default. To make such debt risk-free, the insurer s compensatory payment $D_{guar\ k}$ to the lender satis es

$$D_k(x) + (1- \ _i)D_{guar\ k}(x) = D_0(c_k) \equiv \frac{c_k}{r}(1- \ _i) \qquad (4)$$

Arranging the terms we have

$$D_{guar\ k}(x) = \left[\frac{c_k}{r} - A[(1- \quad) \ _k \quad (x_k^d) - F]\right] e^{- \ _2(x - x_k^d)}$$

where $A = (1- \quad)/(1- \ _i)$.

Under the assumption of a highly competitive market, an insurer will sign a large number of swap contracts with many di erent SMEs. In order to spread risk and ensure the smooth execution of the swap contracts, the guarantee cost, denoted by a fraction of equity $_kE_k(x)$, should be equal to the insurer s compensation $(1- \ _i)D_{guar\ k}(x)$ at the investment threshold x_k^i. Thus, we have

$$_kE_k(x_k^i) = (1- \ _i)D_{guar\ k}(x_k^i) \qquad (5)$$

where $_k$ is explicitly given by

$$_k(x_k^i) = \frac{(1- \ _i)\left[\frac{c_k}{r} - A[(1- \quad) \ _k \quad (x_k^d) - F]\right] e^{- \ _2(x_k^i - x_k^d)}}{(1- \quad)\left[\ _k \quad (x_k^i) - \frac{f+c_k}{r} - \left(\ _k \quad (x_k^d) - \frac{f+c_k}{r}\right) e^{- \ _2\left(x_k^i - x_k^d\right)}\right]} \qquad (6)$$

Obviously, the guarantee cost is signi cantly driven by the rm type as the low-type rm gives up more equity in exchange for guarantee.

With Eq.(1) and Eq.(3), the value of k type rm after investment is de ned by the sum of the equity and debt values, i.e.,

$$V_k^a(x) = E_k(x) + D_k(x)$$

Using Eq.(4) and Eq.(5), $V_k^a(x)$ can be rewritten as

$$V_k^a(x) = (1 - \ _k(x_k^i))E_k(x) + D_0(c_k)$$

The above equation shows that the rm assets can be divided into diluted equity and risk-free debt values. Finally, the market leverage ratio of the rm with type k is given by

$$L_k = \frac{D_0(c_k)}{(1 - \ _k)E_k(x) + D_0(c_k)}$$

Furthermore, the value of type k rm at any time before investment is given by

$$V_k^b(x) = \left[(1 - \ _k(x_k^i))E_k(x_k^i) - (I - D_0(c_k))\right] e^{- \ _1(x - x_k^i)} \tag{7}$$

Thanks to EGS, any type of SMEs could nance the full amount of the irreversible investment cost $D_0(c_k) = I$. Therefore, the coupons c_k selected by k-type s owners at the investment threshold are identical, given by

$$c = c_l = c_h = Ir/(1 - \ _i) \tag{8}$$

However, the guarantee cost varies signi cantly with di erent rm types.

In addition, the smooth-pasting condition requires $V_k^a / \ x|_{x=x_k^i(c)} = \ V_k^b / \ x|_{x=x_k^i(c)}$. Applying Eq.(7) to the smooth-pasting condition yields

$$V_k^a / \ x|_{x=x_k^i(c)} = - \ _1\left[(1 - \ _k(x_k^i))E_k(x_k^i) - (I - D_0(c))\right]$$

It implies that the optimal investment threshold x_k^i can be obtained by solving the following equation:

$$(1 - \ )\frac{k}{r} + (1 - \ ) \ _2\left[\ _k \ (x_k^d) - \frac{f + c}{r}\right] e^{- \ _2(x_k^i - x_k^d)} = \left[(\ _1 - \ _2) \ _k(x_k^i) - \ _1\right] E_k(x_k^i) \tag{9}$$

2.2. SME nancing without guaranteed debt

As discussed earlier, SMEs often face severe nancial constraints with limited debt capacities, leading to underinvestment. In the previous section 2.1, we assume the SME with EGS can raise su cient capital to fully launch the investment. What if an SME can only partially launch its investment due to its borrowing constraints (i,e, no access to EGS)? To fully address this question we extend Morellec and Schrho (2011) s model by incorporating nancial constraints in this section.

More generally, we formulate the investment decision problem for a constrained levered-firm without guaranteed debt. We assume that the SME s borrowing constraint (debt capacity) is qI, where $q \in [0\ 1]$ is the maximum percentage rate of the total cost that the SME can cover. As the SME with borrowing constraints can only raise up to qI capital, the investment project will not be operated at its full capacity. Therefore, we assume the cash flow scale of the SME will be discounted. That is to say, at any time t after investment, the total pre-tax profit flow generated by normal operation of the firm is shrunk down to $_k x_t$. With reference to Wong (2010), we establish the relationship between the borrowing constraint, q and the discounted cash flow scale as follows,

$$q = \frac{(1 + \ ^2)}{2} \tag{10}$$

among which, $0 \leq \ \leq 1$. As shown in Eq.(10), we assume an extreme case that the SME cannot proceed its investment when it can only raise up to 50% of the investment cost, i.e. $= 0$ for $q = 50\%$. This means that once the firm s debt capacity is below 50%, the firm is restricted to be all-equity financed.

Let E_N denote the equity option value for the constrained levered firm without guarantee, where subscript N refers to the firm without guarantee. Following the same procedure as with EGS (see Eq.(2)), we obtain the default threshold

$$x^d_{N\ k} = \frac{f + c_{N\ k}}{\ _k} - \frac{}{r} - \frac{1}{\ _2} \tag{11}$$

By comparing the default level of the EGS funded investment in Eq.(11) with Eq.(2), we find that bankruptcy costs induce an earlier default of production. However, the existence of guarantee makes default less attractive for equity holders.

Beside that, the debt financing capacity constraint is assumed to be fully binding as

$$D_k\,(x\ \ c_{N\ k}) = qI$$

where D_k, the value of debt without guarantee, is given by Eq.(3). Further, we have the value of the option to invest yields

$$\begin{aligned} V^b_{N\ k}(x\ \ c_{N\ k}) = \{(1 - \ )[\ \ _k\ \ (x^i_{N\ k}) - F] + (\ - \ _i)c_{N\ k}/r - qI \\ -[(1 - \ )\ \ \ _k\ \ (x^d_{N\ k}) + (\ - \ _i)c_{N\ k}/r]e^{-\ _2(x^i_{N,k} - x^d_{N,k})}\}e^{-\ _1(x - x^i_{N,k})} \end{aligned}$$

Consistent with EGS, the investment threshold $x^i_{N\ k}$ satisfies the smooth-pasting condition and the coupon rate is determined by the budget constraint. Unlike the setting with EGS, this extension considers a more complex problem in which we numerically solve for the coupon rate and the investment threshold simultaneously.

3. General model with asymmetric information

Now, we relax the perfect information assumption and assume that the firm type information is known only to SMEs owners. The external investors interpret the firm s actions rationally and have to use Bayes rule to update their beliefs about the rm s pro tability. Here, we develop an equilibrium model of SME investment and nancing choices with EGS under asymmetric information.

3.1. The timing of investment as a signal

In the benchmark model with perfect information, a rm makes investment decisions based on its own project quality fully observed by the market. However, in a dynamic setting with asymmetric information, the low-type rm has an incentive to mimic, which reduces its guarantee cost for debt nancing, while the high-type rm could impose an adverse selection cost on low-type rms. Therefore, there exists a least-cost separating (lcs) equilibrium where the high-type rms invest earlier and the low-type rms invest like the rst-best case.

We rst check the existence of such an equilibrium. Assuming the rm s type perceived by the insurer is $\ $, $\ _l < \ < \ _h$, the guarantee cost for a rm investing at x^i is given by

$$(x^i) = \frac{(1-\ _i)D_{guar}(x^i)}{E(x^i)} = \frac{(1-\ _i)\left[\frac{c}{r} - A[(1-\ )\ (x^d)-F]\right]e^{-\ _2(x^i-x^d)}}{(1-\ )\left[\ (x^i)-\frac{f+c}{r}-\left(\ (x^d)-\frac{f+c}{r}\right)e^{-\ _2\left(x^i-x^d\right)}\right]} \tag{12}$$

According to Appendix A, Eq.(12) shows that the higher the type $\ $, the lower the guarantee cost and the lower the ownership dilution, the larger the equity stake for old shareholders. The valuation of type k rm when signaling by investing at x^i and when the perceived type is $\ $ equals

$$V_k(x; x^i\ ) = (1-\ (x^i))E_k(x^i)e^{-\ _1(x-x^i)} \tag{13}$$

where $x^d = \frac{f+c}{\ } - \frac{\ }{r} - \frac{1}{\ _2}$.

The following lemma shows the conditions under which an (high-type) SME would prefer to send a credible signal to avoid ownership dilution (nancing distortion) at the cost of investment distortion (all proofs are given in Appendix A):

Lemma 3.1. *EGS enables the high-type rm to separate itself from the low-type rm by distorting investment (speeding up investment) such that the single-crossing property holds globally:*

$$\frac{\ }{\ _k}\left(\frac{V_k/\ }{V_k/\ x^i}\right) > 0 \ for\ all\ (\ \ x)$$

According to Lemma 3.1, it is feasible for high-type rms to separate from low-type ones by changing their investment threshold, leading to investment distortion. Consequently, the timing of investment can be considered a credible signal of the rm s type. Therefore, each rm balances the tradeo between ownership dilution and investment distortions. The least-cost separation equilibrium exists if a high-type rm invests with an appropriate investment threshold such that the undervaluation from being wrongly recognized as a low-type rm outweighs the cost of investment distortion.

3.2. The separating equilibrium

The existence of separation equilibrium depends further on two sufficient and necessary conditions. First, the benefits of mimicking for low-type firms must be less than the benefits of being themselves. Second, the high-type firm s benefit of separation equilibrium is higher than the benefits of being recognized as a low-type firm: the following incentive compatibility constraints should be checked (ICC):

$$(1-{}_h(x))E_l(x)-(I-D_0(c))\leq\left\{(1-{}_l(x_l^i))E_l(x_l^i)-(I-D_0(c))\right\}e^{-{}_1(x-x_l^i)} \tag{14}$$

$$(1-{}_h(x))E_h(x)-(I-D_0(c))\geq\left\{(1-{}_l(x_l^i))E_h(x_l^i)-(I-D_0(c))\right\}e^{-{}_1(x-x_l^i)} \tag{15}$$

First, according to the optimal investment timing of the low-type firm, x_l^i, solved using Eq.(9), it is better off mimicking. Next, when the threshold x' for which Eq.(14) is binding, the low-type firm is indifferent between mimicking or waiting to invest at its first-best timing. For $x<x'$, the low-type firm prefers to wait until its first-best threshold than mimicking the high-type firm, while when the cash flow is above the lower bound x'' of an interval solved by (15), the high-type firm prefers to separate from the low-type firm. Therefore, there is a separating equilibrium, if the value of cash flow shock x satisfies $x'\leq x\leq x''$. To minimize the high-type firm s cost of separation, the separating threshold chosen by the high-type firm should be as close as possible to its optimal investment timing x_h^i, that is,

$$x_{sep}^i=\min(x_h^i\;\; x') \tag{16}$$

We now summarize our key results in the following proposition:

Proposition 3.2. *Under the budget constraint $D_0(c)=I$, there exists a unique least-cost separating equilibrium for SMEs in which the contract offered by the high-type firm is $(x_{sep}^i\;\; {}_h(x_{sep}^i))$ with the contract $(x_l^i\;\; {}_l(x_l^i))$ offered by the low-type firm, where x_{sep}^i, x_l^i, c, is given by (16), (9), (8), (6), respectively.*

Before investment, the intrinsic value of the high-type firm $V_{sep\,h}^b(x)$ and low-type firm $V_l^b(x)$ are given by

$$V_{sep\,h}^b(x)=\left(E_h(x_{sep}^i)+D_h(x_{sep}^i)-I\right)e^{-{}_1(x-x_{sep}^i)}$$

$$V_l^b(x)=\left(E_l(x_l^i)+D_l(x_l^i)-I\right)e^{-{}_1(x-x_l^i)}$$

The market value of the firm that is independent of project quality follows

$$V_{sep}^b(x)=pV_{sep\,h}^b(x)+(1-p)V_l^b(x)$$

Similar to Morellec and Schrhoff (2011), the cost of adverse selection depicting the reduction in value of high-type firms distorting investment is defined by

$$AC_{sep}=(V_h^b(x)-V_{sep\,h}^b(x))/V_h^b(x) \tag{17}$$

The abnormal return, change in the value of type k at the time of investment, is formulated as follows

$$AR_k=(V_k^b(x)-V_{sep}^b(x))/V_{sep}^b(x)$$

3.3. The pooling equilibrium

Although high-type rms might signal their private information to external investors through the separating equilibria, this does not apply to all market settings. Here, we examine the pooling equilibria in which both rm types o er an identical contract $(x^i_p \quad {}_p(x^i_p))$ to the insurer. Thus, the insurers cannot distinguish the high-type rms from the low-type ones. Importantly, the high-type rm chooses whether to separate or pool with a low-type rm by trading o the cost of signaling (investment distortion) against the cost of being imitated (nancing distortion). Now we turn to investigate the pool equibibria where the investment distortion costs outweigh the nancing distortion costs.

We assume that the prior belief of insurers on the rm type is given by ${}_p = p \ {}_h + (1-p) \ {}_l$. Then, the pooled value of the SMEs after investment is given by

$$V^a_p(x) = (1 - \ {}_p(x^i_p))E_p(x) + D_0(c)$$

where ${}_p$ and E_p are given by (6) and (1), respectively.

To show that a pooling equilibrium exists, we need to check the incentive compatibility constraint of the low-type rm as follows

$$(1 - \ {}_p(x))E_l(x) - (I - D_0(c)) \geq \left\{(1 - \ {}_l(x^i_l))E_l(x^i_l) - (I - D_0(c))\right\} e^{- \ {}_1(x - x^i_l)}$$

Similarly, for a given contract, the high-type rm prefers to pool with the low-type if the value of the high-type rm executing the contract is higher than in the least-cost separating equilibrium:

$$\left\{(1 - \ {}_p(x))E_h(x) - (I - D_0(c))\right\} e^{- \ {}_1(x^i_{sep} - x)} \geq (1 - \ {}_h(x^i_{sep}))E_h(x^i_{sep}) - (I - D_0(c))$$

To verify that the contract $(x^i_p \ c \quad {}_p(x^i_p))$ is the optimal strategy for both rm types, this contract must maximize the intrinsic values of the high-type rms in the pooling equilibrium, that is,

$$V^b_{p\,h}(x) = \sup_{x^i_p\ c\quad p} \left\{(1 - \ {}_p(x^i_p))E_h(x^i_p) - (I - D_0(c))\right\} e^{- \ {}_1(x - x^i_p)}$$

We then immediately get the following results:

Proposition 3.3. *There exists a least-cost pooling equilibrium if the pair $(x^i_p \ c \quad {}_p(x^i_p))$ satisfy*

$$\sup_{x^i_p} (1 - \ {}_p(x^i_p))E_h(x^i_p)e^{- \ {}_1(x - x^i_p)}$$

$$s\ t \begin{cases} (ICC_{pl}): \ (1 - \ {}_p(x^i_p))E_l(x^i_p) \geq (1 - \ {}_l(x^i_l))E_l(x^i_l)e^{- \ {}_1(x^i_p - x^i_l)} \\ (ICC_{ph}): \ (1 - \ {}_p(x^i_p))E_h(x^i_p)e^{- \ {}_1(x^i_{sep} - x^i_p)} \geq (1 - \ {}_h(x^i_{sep}))E_h(x^i_{sep}) \\ (\textit{the budget constraint}): \ D_0(c) = I \end{cases}$$

Furthermore, the market and intrinsic values in the pooling equilibrium are given by

$$V^b_p(x) = (1 - \ {}_p(x^i_p))E_p(x)e^{- \ {}_1(x - x^i_p)}$$

$$V^b_{p\,k}(x) = (1 - \ {}_p(x^i_p))E_k(x^i_p)e^{- \ {}_1(x - x^i_p)}$$

Like Proposition 3.2, the cost of adverse selection for the high-type firm is defined by

$$AC_p = (V_h^b(x) - V_{p\,h}^b(x))/V_h^b(x) \tag{18}$$

Following Morellec and Schrhoff (2011), we calculate investment probability, the probability of a firm developing an investment over the next t years, to further test the impact of asymmetric information on SME financing decisions.

$$H(t) = P\left[\sup_{s\in[0\ t]} x_s \geq K\right] = \Phi\left[\frac{x_0 - K + \mu t}{\sqrt{t}}\right] + e^{\frac{2\mu(K-x_0)}{\sigma^2}}\Phi\left[\frac{x_0 - K - \mu t}{\sqrt{t}}\right] \tag{19}$$

where Φ is the standard normal cumulative density function and K is the timing of investment under the corresponding equilibrium.

We also examine the hazard rate of an investment project, denoted by $H'(t)/(1 - H(t))$, which sheds deeper insights on the role of adverse selection or moral hazard on SME investment and financing choices under information asymmetry.

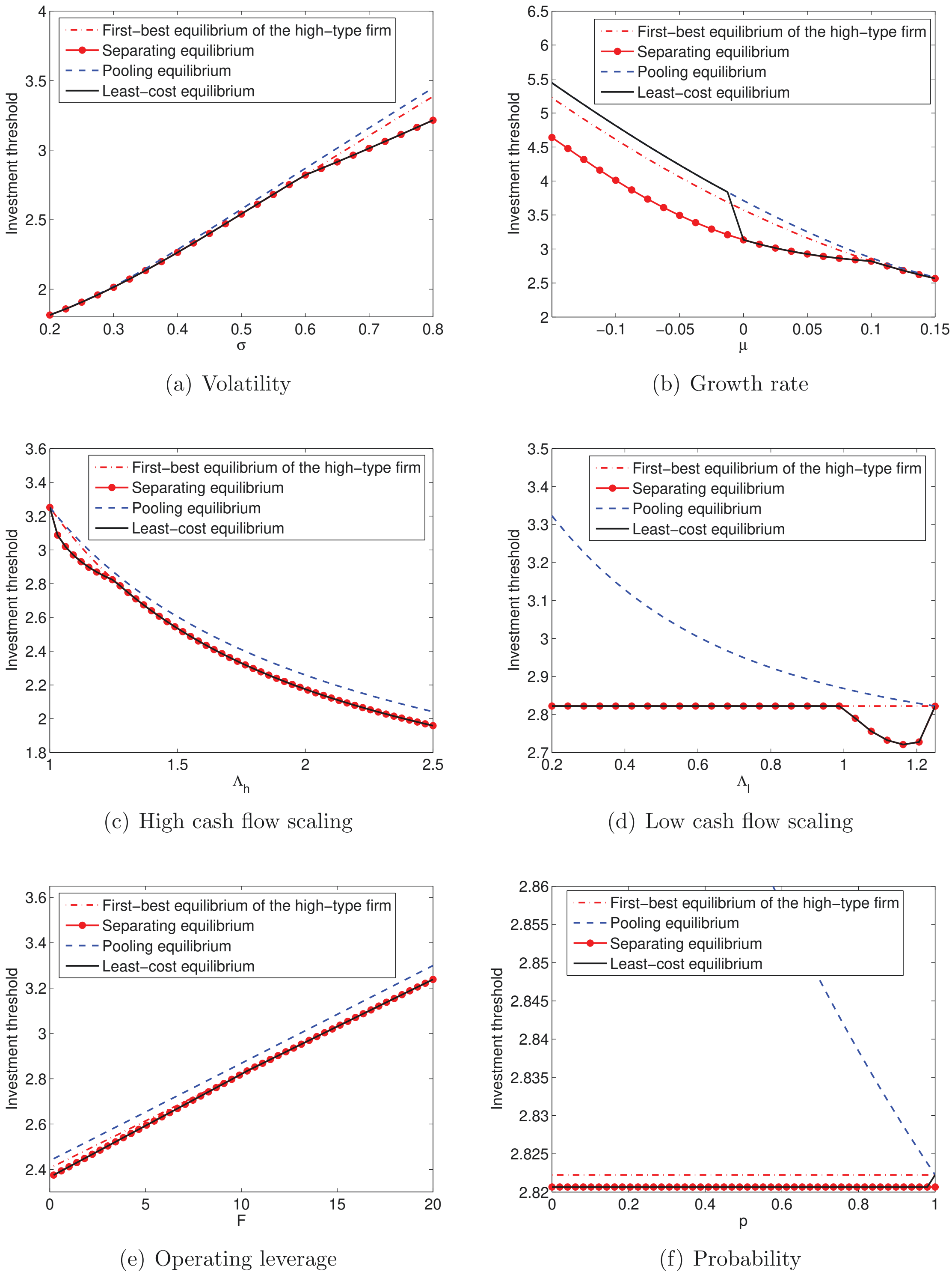
4. Numerical results

Here, we conduct numerical analyses to provide more insight on the impact of asymmetric information on SME investment and financing decisions with EGS. Most of our baseline parameter values are borrowed from Yang and Zhang (2013) and Morellec and Schrhoff (2011) for a better comparison. To highlight the characteristics of SMEs, we assume a higher volatility, lower operating expenses, and lower sunk cost compared to the case of Morellec and Schrhoff (2011). Table 1 summarizes all the baseline parameter values.

4.1. Investment timing under different equilibria with information asymmetry

Figure 1 presents the investment threshold in the first-best case (solid red line), pooling equilibrium (blue dotted line), and least-cost equilibrium (bold black line) for the high-type firm. The least-cost equilibrium investment threshold is a combination of the other three lines in the figure. If the investment threshold in the least-cost equilibrium is lesser than in its first-best case, the high-type firm accelerates investment, implying that it has an incentive to separate. Unlike the separation equilibrium, the pooling equilibrium reduces the high-type firm value while the low-type firm is better off thanks to the over-priced securities, which is consistent with Morellec and Schrhoff (2011).

Interestingly, even under information asymmetry, there are cases that the least-cost and first-best investment thresholds are identical such that the signal is cost-free. According to Figure 1, the impact of asymmetric information on SME investment is slim, i.e., the investment threshold gap between the first best case (solid red line) and the least-cost separating equilibrium case (bold black line), when the high-type firm has lower systemic risk, higher operating leverage, or higher cash flow scaling. However, as volatility increases or operating leverage decreases, asymmetric information plays an increasingly important role in SME investment decisions. As a result, the high-type firms suffer investment distortion, reflecting as the cost of credible signaling in incomplete markets.



(a) Volatility

(b) Growth rate

(c) High cash flow scaling

(d) Low cash flow scaling

(e) Operating leverage

(f) Probability

Figure 1: The impact of project volatility (a), growth rate (b), cash flow scaling (c-d), operating leverage (e), and belief(f) on investment threshold under different equilibria.

Table 1: Baseline parameter values
The table reports the parameter values for the benchmark model and all the parameters take the baseline parameter values unless otherwise stated.

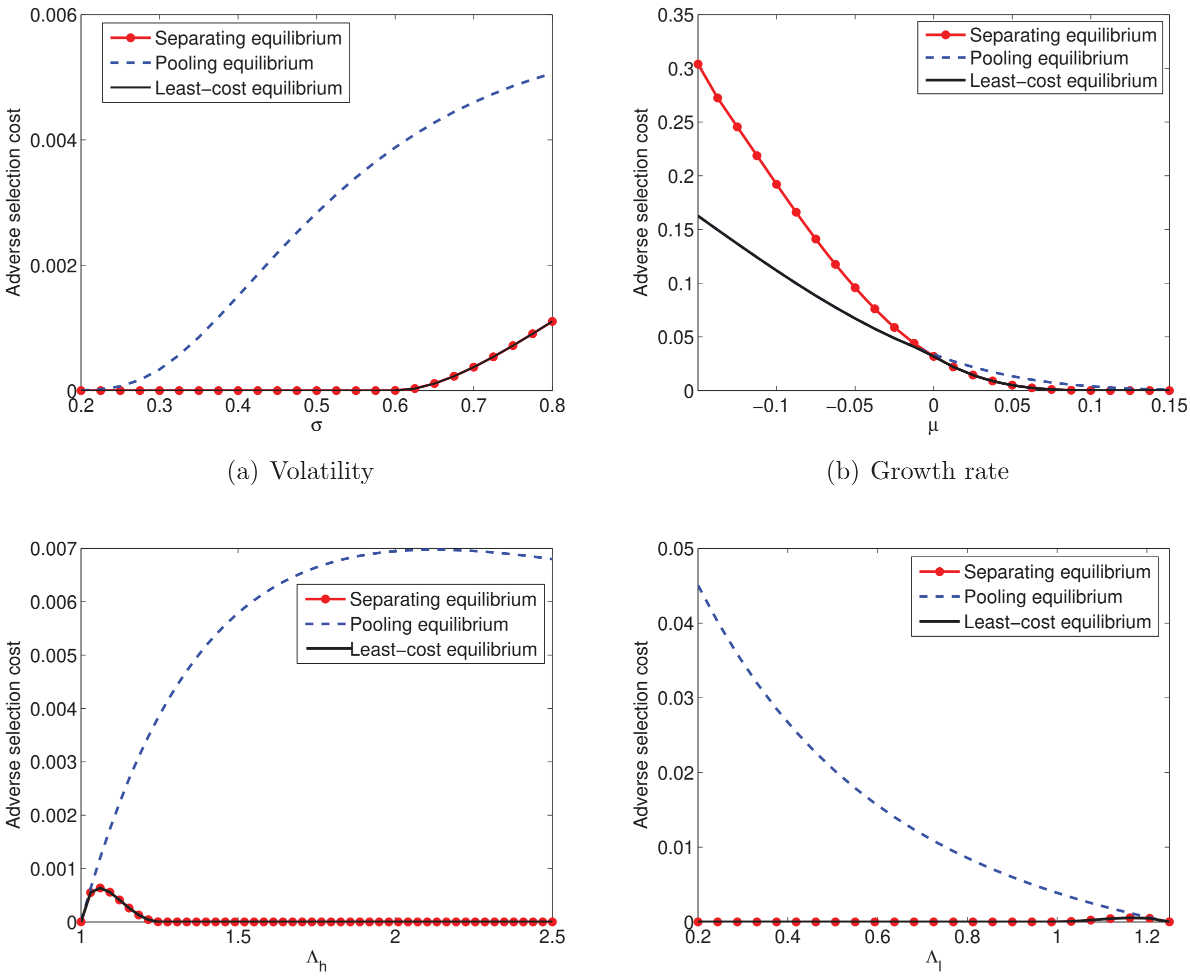
Variable	Symbol	Value	References
Risk-adjusted growth rate		0.10	Miao and Wang (2007)
Volatility		0.60	Yang and Zhang (2015)
Risk-free interest rate	r	0.05	Yang and Zhang (2013)
Personal tax rate	$_i$	0.35	
Corporate pro t tax rate	$_c$	0.35	
Dividend tax rate	$_d$	0.20	
Default loss rate		0.50	
High cash ow scaling	$_h$	1.25	Morellec and Schrho (2011)
Low cash ow scaling	$_l$	1.00	
Proportion of high-type rm	p	0.50	
Initial value of cash ow	x_0	1.00	Wang, Yang and Zhang (2015)
Sunk cost	I	20.00	
Operating expenses	f	0.50	Nishihara and Shibata (2018)

From Figure 1(c), when rms are identical in type ($_h = 1 =$ $_l$), all rms will invest at their rst-best threshold. When the high-type rm s scaling increases, the incentive to mimic for the low-type rms rst increases and then, eventually disappears because the cost of mimicking for the low-type rm outweighs the bene t from being recognized as the high-type rm. Further, the gure shows that the investment threshold rises with volatility and the value of operating leverage F and declines with the growth di erential $_h/$ $_l$ and growth rate .

4.2. Adverse selection cost

In incomplete nancial markets with asymmetric information, the price that the high-type rm pays to separate itself from low-type rms is de ned as the cost of adverse selection. The cost of adverse selection in the least-cost equilibrium is de ned as $\min(AC_{sep}\ AC_p)$. In general, the high-type rm s best strategy is the separation equilibrium, which ensures an e ective signal under asymmetric information, as indicated in Figure 2.

Figures 2(c) 2(d) indicate that the cost of a high-type rm in least-cost equilibrium rst increases and then declines as the gap between the two rms increases. Indeed, the cost of adverse selection disappears when the growth di erential $_h$ $_l$ is high enough because the larger growth gap tends to either increase the mimicking costs or decrease the bene ts of mimicking. Either way, the low-type rms are increasingly reluctant to mimic, allowing the separating equilibrium to approach the high-type rm s rst-best investment strategy. Figure 2 also reveals that the cost of adverse selection decreases as the growth rate of cash ow increases.



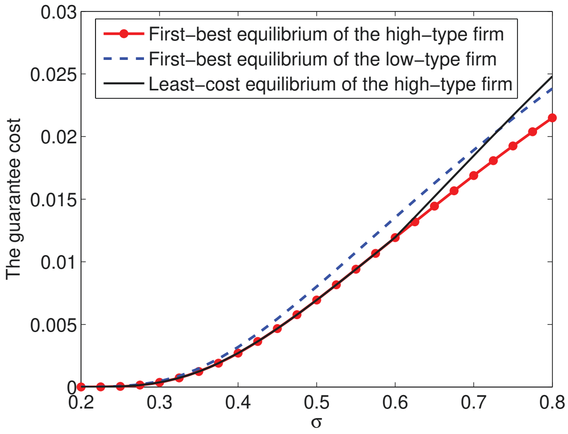
(a) Volatility
(b) Growth rate
(c) High cash flow scaling
(d) Low cash flow scaling

Figure 2: The impact of project volatility (a), growth rate (b), and cash flow scaling (c-d) on the cost of adverse selection under different equilibria.

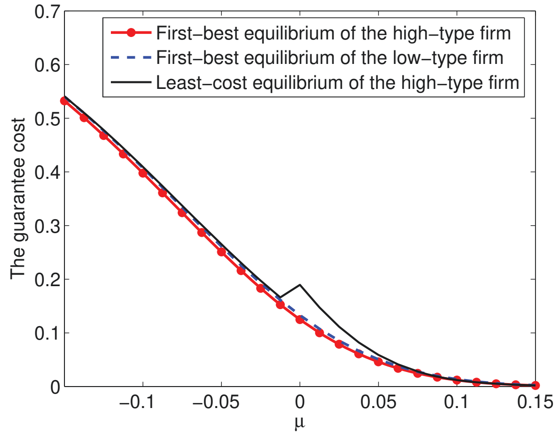
4.3. The guarantee cost as a measure of equity dilution

Figure 3 graphs the guarantee cost of the high-type firms in the least-cost equilibrium for different parameter values. First, the guarantee cost for high type firms under asymmetric information is higher than in the benchmark case, reflecting the investment distortion caused by information asymmetry. To be more specific, if the high-type firm cannot freely communicate its positive information to the market, it will face a higher equity dilution (guarantee cost) for risk-free debt issuance. This is mainly because the high type firm under asymmetric information accelerates investment to signal the positive information and results in a devaluation of equity, which in turn increases the guarantee cost.

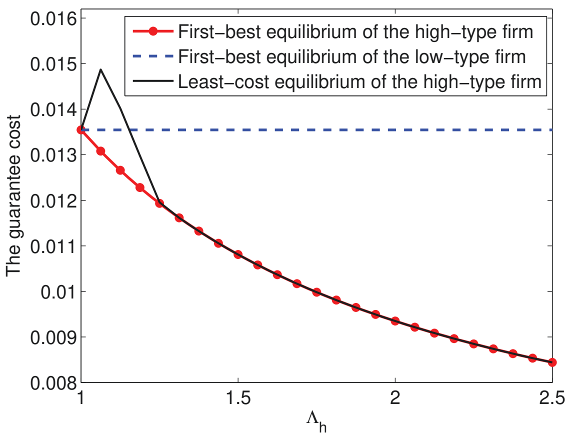
Next, the bold black line in Figure 3(b) overlaps the dashed blue line, suggesting that the fraction of equity offered by the market to the insurer is unique and that private firm information cannot be effectively communicated to the market. Figure 3 also reveals that the guarantee cost first increases with cash flow scaling, verifying condition (A.2). Moreover, the guarantee cost declines with growth rate and rises with volatility σ due to the negative correlation between the credibility of SMEs and market risk.



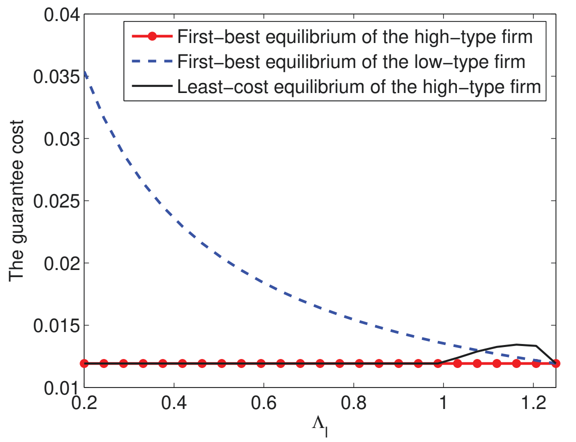
(a) Volatility



(b) Growth rate



(c) High cash flow scaling



(d) Low cash flow scaling

Figure 3: The impact of project volatility (a), growth rate (b), and cash flow scaling (c-d) on an SME's guarantee cost under different equilibria.

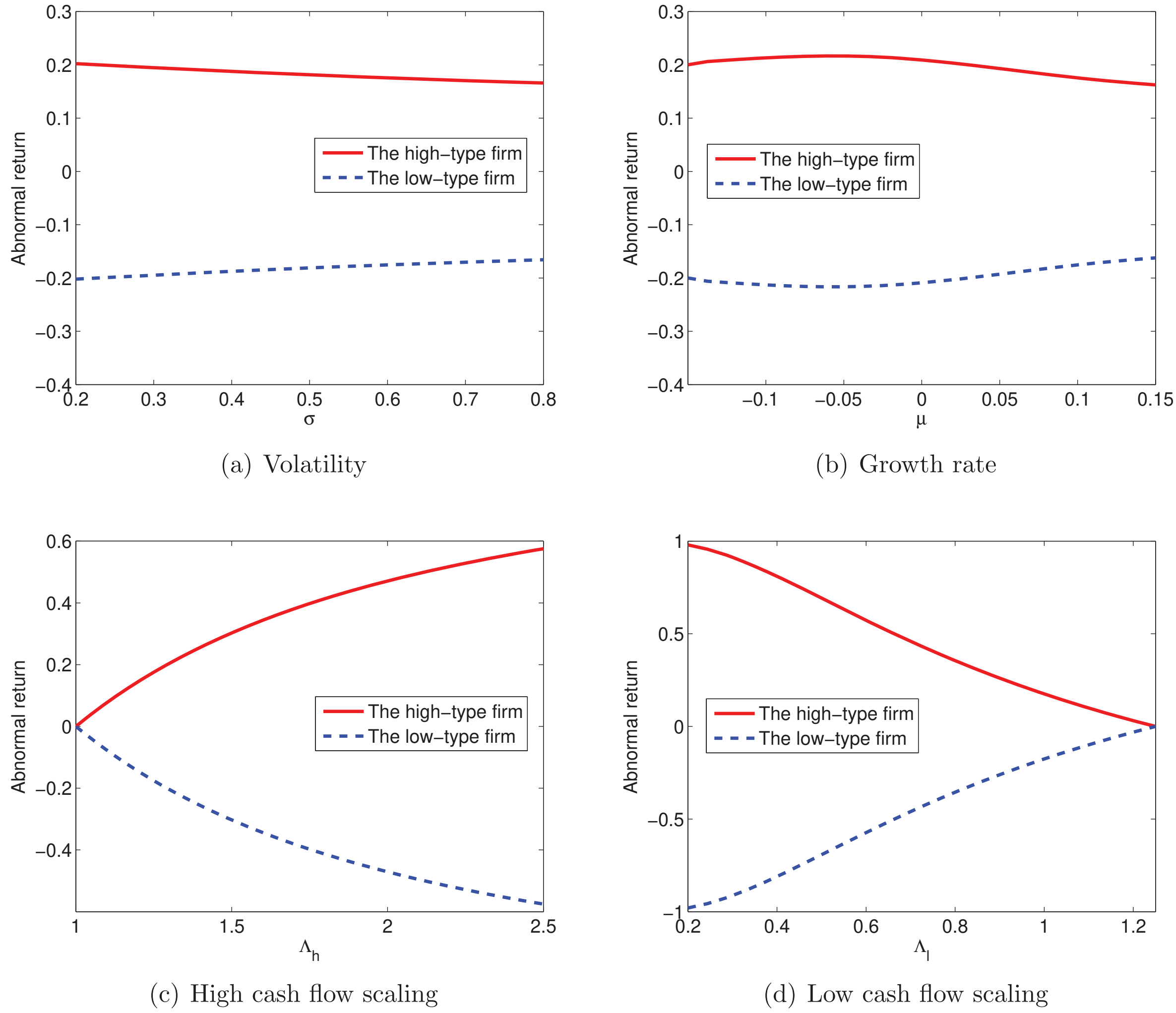
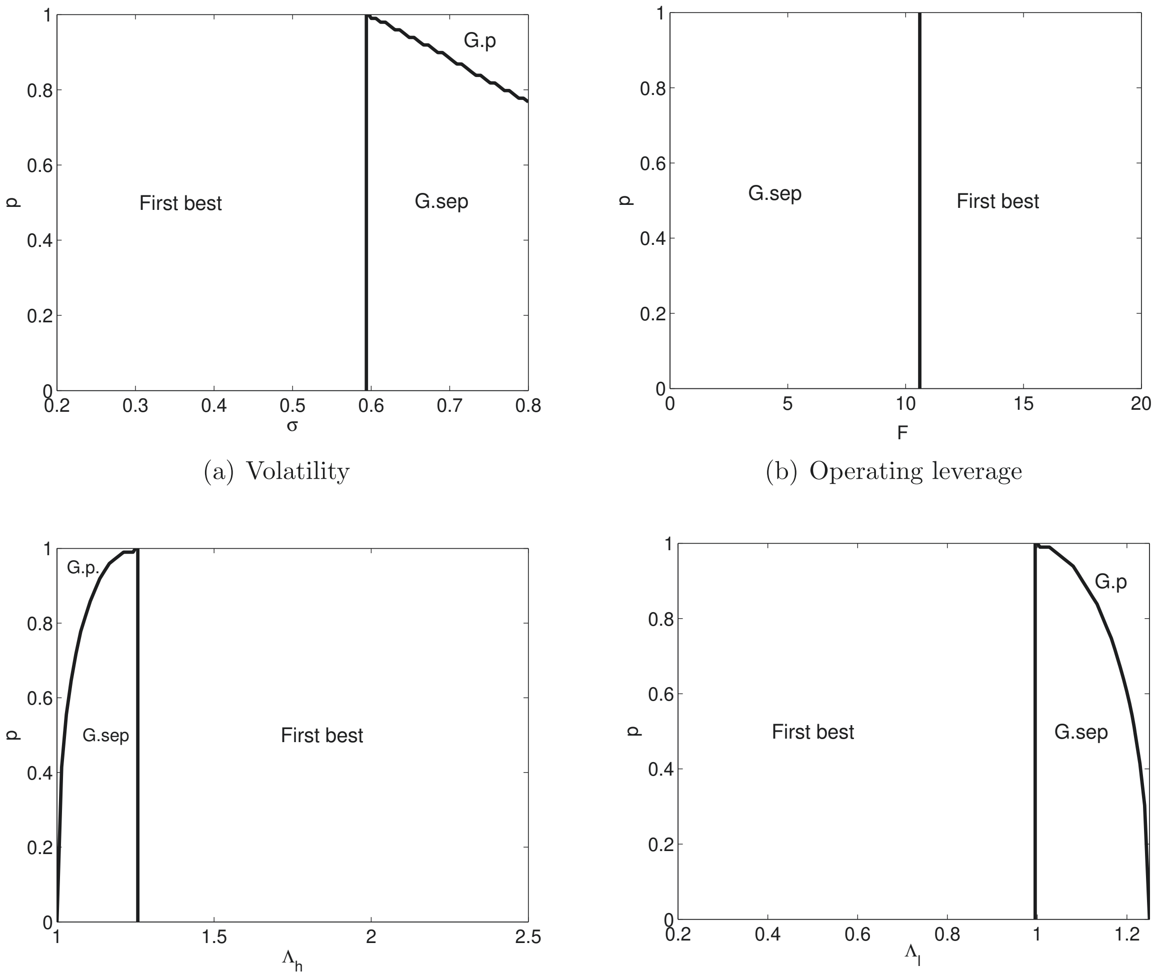


Figure 4: The impact of project volatility (a), growth rate (b), and cash flow scaling (c-d) on the abnormal return for different firms.

4.4. Abnormal return and the least-cost equilibrium

We adopt abnormal return to measure the costs of asymmetric information in the separation equilibrium. Abnormal return reflects a jump in the value of firms at the time of investment versus the different parameter values; it is positive (negative) for the high-type (low-type) firm. When the high-type firm invests at the threshold x^i_{sep} in the separation equilibrium, it signals to the external insurer who accordingly modifies the belief regarding the firm's type, which in turn places a higher (lower) equity value for the high-type (low-type) firm. Figure 4 shows that the abnormal return of the high-type firm decreases with cash flow volatility and cash flow scaling of the low-type, while it increases with the cash flow scaling of the high-type.



(a) Volatility (b) Operating leverage

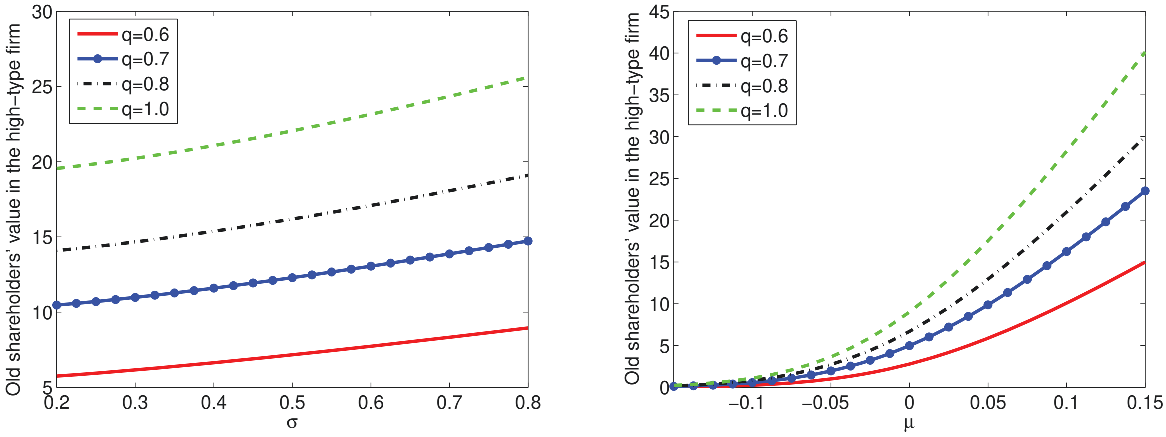
(c) High cash flow scaling (d) Low cash flow scaling

Figure 5: The least-cost equilibrium.
The least-cost equilibrium as a function of the parameters σ, F, Λ_h and Λ_l on the $x-axis$ and of the insurers' beliefs about the fraction of high-type firms p on the $y-axis$. "First best" in the figure means that asymmetric information has no impact on SMEs' corporate decisions. In this case, the investment decision of a high-type firm is identical to that made under the symmetric information. "G.p" in the figure means that the high-type firm achieves the pooling equilibrium under asymmetric information. Relatively, "G.sep" means that the high-type firm reaches a separating equilibrium.

According to Figure 5, the optimal strategy for the high-type firm is to pool with low-type firms when the insurers' belief is higher (i.e., the market consists of mostly high-type firms), as it is cost ineffective to signal to the external investors. In particular, the first-best equilibrium becomes the optimal strategy for high-type firms in most cases, which suggests that EGS helps the high-type SMEs to alleviate their financing constraints and reduces the moral hazard problem in the market as well.

4.5. The impact of EGS on SME investment

When introducing borrowing constraints in Morellec and Schrhoff (2011)'s model, an SME with limited borrowing capacity has lower firm value as it cannot raise sufficient capital to run its production with full capacity. Both Figure 6(a) and 6(b) show that this negative effect is stronger when the borrowing constraints is much severer (say $q = 0.6$). To be more specific, the investment distortion (the gap between different lines in the Figure) is larger when the volatility is higher, which is consistent with the real option effect for volatility. Moreover, as indicated in Figure 6(b), SMEs suffer significantly larger loss of value for higher growth opportunities as SMEs are unable to launch the investment opportunity at its full potential.

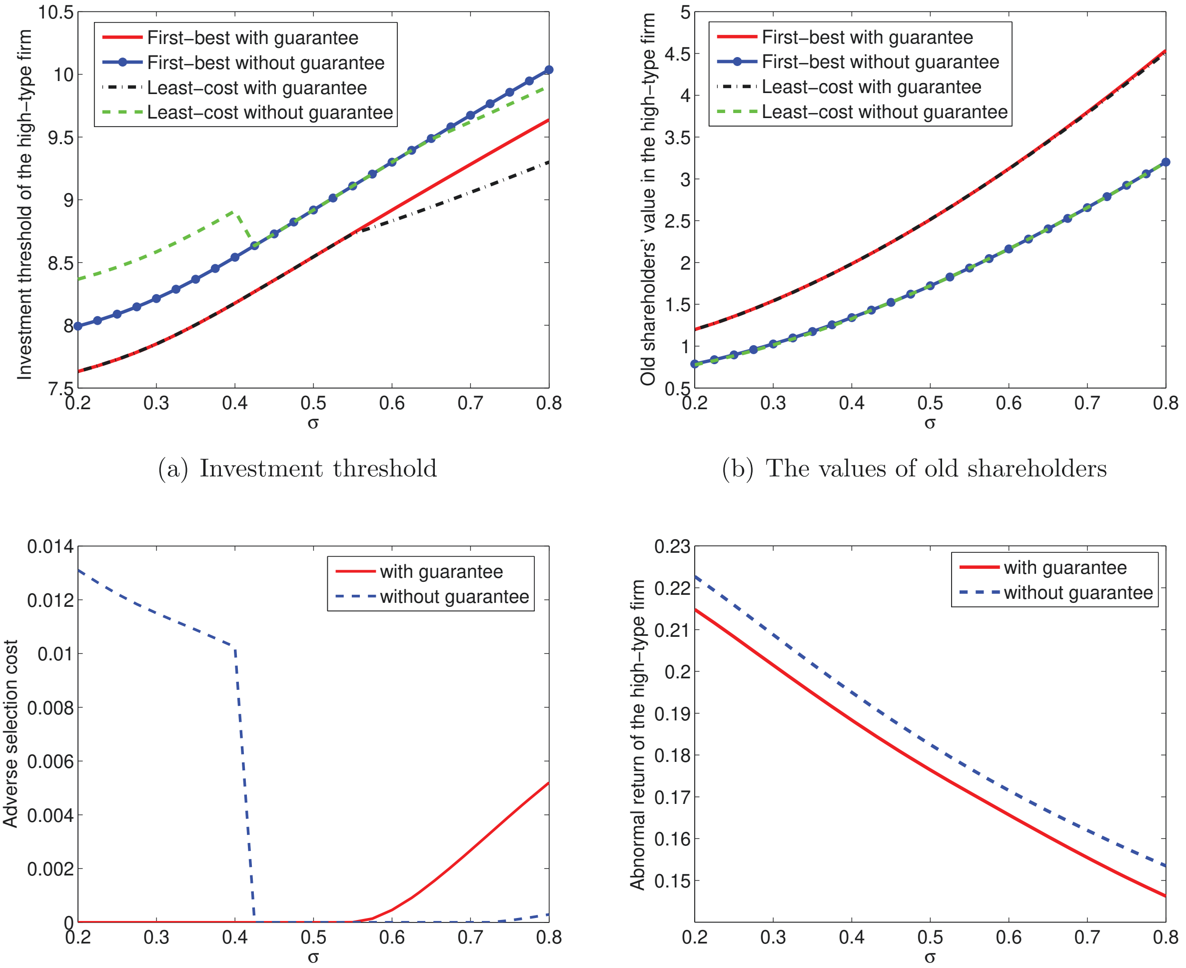


(a) Volatility

(b) Growth rate

Figure 6: The old shareholders' values as a function of volatility σ and growth rate μ for different levels of debt capacity.

We further explore the benefit of EGS in Figure 7. Overall, EGS has a significant effect on SMEs' investment policies and firm value. As shown in Figure 7(a) and 7(b), regardless of the type of equilibrium (separate or pool), firms with EGS accelerate investment and have higher firm values. Also, we find less risky firms (with lower volatility) without EGS tend to have a higher cost of adverse selection while the opposite is true for riskier firms (with higher volatility) with guarantee, see Figure 7(c). This is mainly because it is more likely to have polled equilibrium for riskier firms with EGS (less risky firms without EGS). As expected, Figure 7(d) shows firms with EGS have a smaller abnormal return compared with the case of no guarantee, which provides further evidence that EGS mitigates the negative impact of asymmetric information.



(a) Investment threshold

(b) The values of old shareholders

(c) Adverse selection cost

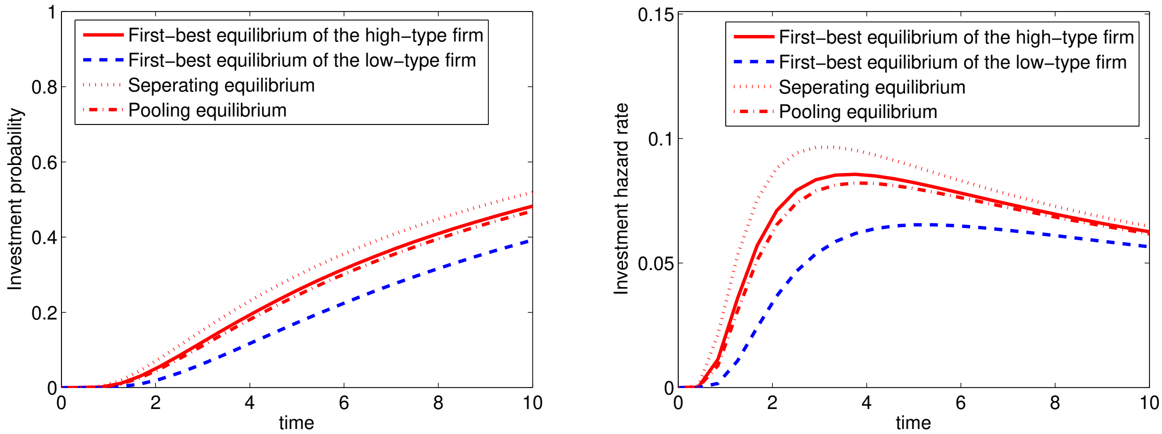
(d) Abnormal return

Figure 7: The difference between an SME with guarantee (EGS) and without guarantee. To highlight the difference between separating and pooling equilibria, the base parameters here are chosen as follows: $p = 0.5$, $\Lambda_h = 1.1$, $\Lambda_l = 1$, $r = 0.05$, $\mu = 0.1$, $I = 100$, $f = 0.5$, $\alpha = 0.5$, $\tau_i = 0.35$, $\tau_c = 0.35$, $\tau_d = 0.2$, $q = 0.8$.

4.6. The impact of asymmetric information on SME investment

Figure 8 plots the probability of investment and hazard rate as a function of time. This figure contains the first-best investment policy for two different firm types (high-type: solid line, low-type: dashed line), separating equilibrium (dotted line), and pooling equilibrium (dash-dotted line). Overall, asymmetric information speeds up firm investment, as indicated by Figure 8(a), because at any given point in time following $t = 1$, high-type firms are more likely to invest.

Similarly, Figure 8(b) implies that EGS makes the rm invest more ready in the separating equilibrium. By contrast, Figure 8 shows that in the pooling equilibrium when rms cannot commute information e ciently, the insurer will accept the unique contract such that the high-type rms underinvest compared to the rst-best case. Figure 8 also demonstrates that under the rst-best equilibrium, low-type rms invest later than high-type ones as expected.



(a) Investment probability (b) Hazard rate

Figure 8: Investment probability (a) and hazard rate (b) versus time.
To highlight the di erence between separating and pooling equilibria, the base parameters here are chosen as follows: p=0.5, r=0.05, =0.5, $_i$=0.35, $_c$ = 0.35, $_d$=0.2, =0.1, =0.8, $_h$=1.25, $_l$=1, I=20, f=0.5 and x_0=1.

5. Empirical implications and simulation results

Due to the nonlinear feature of the important state variables in our model, some of the moments of interest, such as the correlation between investment probability and market-to-book value, are di cult to obtain analytically. In this section, we test the implications of our model on SME nance under asymmetric information via simulation.[4]

We generate 60,000 pieces of data based on our model; each piece of simulation data contains information on rm characteristics and market beliefs as well. Thus, one could consider a piece of data as an arti cial rm. The summary statistics of the arti cial rms are reported in Table 2. Overall, 47% of SMEs (Panel A and B) can be identi ed by outside investors by the timing of their investments and 39% of SMEs (Panel B) do not su er from asymmetric information.

[4]Our simulation builds on Berk, Green and Naik (1999), Clausen and Flor (2015), Strebulaev (2007) and Morellec and Schrhoff (2011), the details of the simulation experiment are given in Appendix B.

Table 2: Descriptive Statistics.
This table presents descriptive statistics of 60,000 arti cial SMEs generated via a simulation approach. Panel A presents the summary statistics for the entire sample of the valid rms. Panels B, C, and D present the summary statistics for the subsamples of valid rms in the rst-best equilibrium, the separating equilibrium, and the pooling equilibrium, respectively.

Variables	Count	Mean	Std.Dev	Min	Max
Panel A: The entire sample					
Market-to-book $[M/B]$	60000	2.10	0.68	1.00	5.18
Volatility $[\sigma]$	60000	0.53	0.17	0.20	0.80
Firm size $[x^i]$	60000	2.41	0.76	1.00	6.38
Operating leverage $[F]$	60000	9.62	5.69	0.20	20.00
Cash flow growth $[\mu]$	60000	0.08	0.06	-0.15	0.15
Default loss rate $[\alpha]$	60000	0.51	0.29	0.00	1.00
Growth potential $[\Lambda_h/\Lambda_l]$	60000	3.33	2.07	1.00	12.46
Belief $[p]$	60000	0.59	0.27	0.01	1.00
Panel B: The first-best equilibrium subsample					
Market-to-book $[M/B]$	23374	2.48	0.66	1.00	5.18
Volatility $[\sigma]$	23374	0.49	0.17	0.20	0.80
Firm size $[x^i]$	23374	2.01	0.55	1.00	5.74
Operating leverage $[F]$	23374	9.16	5.63	0.20	19.99
Cash flow growth $[\mu]$	23374	0.11	0.04	-0.15	0.15
Default loss rate $[\alpha]$	23374	0.50	0.29	0.00	1.00
Growth potential $[\Lambda_h/\Lambda_l]$	23374	2.77	2.06	1.00	12.20
Belief $[p]$	23374	0.54	0.29	0.01	1.00
Panel C: The separating equilibrium subsample					
Market-to-book $[M/B]$	4791	1.94	0.44	1.02	4.50
Volatility $[\sigma]$	4791	0.64	0.13	0.20	0.80
Firm size $[x^i]$	4791	2.58	0.50	1.01	4.96
Operating leverage $[F]$	4791	7.99	5.45	0.20	19.98
Cash flow growth $[\mu]$	4791	0.06	0.05	-0.14	0.15
Default loss rate $[\alpha]$	4791	0.53	0.29	0.00	1.00
Growth potential $[\Lambda_h/\Lambda_l]$	4791	2.09	1.49	1.01	11.25
Belief $[p]$	4791	0.45	0.26	0.01	0.99
Panel D: The pooling equilibrium subsample					
Market-to-book $[M/B]$	31835	1.84	0.59	1.00	5.08
Volatility $[\sigma]$	31835	0.53	0.16	0.20	0.80
Firm size $[x^i]$	31835	2.66	0.80	1.35	6.38
Operating leverage $[F]$	31835	10.20	5.69	0.20	20.00
Cash flow growth $[\mu]$	31835	0.05	0.07	-0.15	0.15
Default loss rate $[\alpha]$	31835	0.51	0.29	0.00	1.00
Growth potential $[\Lambda_h/\Lambda_l]$	31835	3.92	1.95	1.00	12.46
Belief $[p]$	31835	0.65	0.24	0.01	0.99

Table 3: Determinants of SME investment probability.
The table presents coe cients from a linear regression of investment probability at di erent time (T=1, 2 or 5) using the simulated data. The investment probability is given by Eq.(19) and de ned as the probability of project implementation as a function of time. Each of the three columns under the speci ed time (T=1, 2 or 5) takes into account di erent speci cations. Statistically insigni cant if the p-value is greater than 0.001 and is denoted by ×.

	T=1			T=2			T=5		
Market-to-book[M B]	0.1697	0.0888	0.2028	0.2196	0.1165	0.2539	0.1803	0.0949	0.1882
Volatility[ ]	-0.3752	-0.1318	-0.1228	-0.3148	−0 0103$^{\times}$	0 0255$^{\times}$	-0.1202	0.1250	0.1573
Firm size[x^i]	-0.0063	-0.0654	-0.1260	-0.0701	-0.1439	-0.2320	-0.1524	-0.2118	-0.3010
Operating leverage[F]	-0.0045	-0.0039	-0.0026	-0.0051	-0.0044	-0.0024	-0.0038	-0.0033	-0.0010
Cash ow growth[ ]	-0.6641	-0.2921	-0.5866	-0.7324	-0.2325	-0.5547	0.1891	0.6356	0.3424
Default loss rate[ ]	0 0033$^{\times}$	−0 0005$^{\times}$	0 0054$^{\times}$	0 0014$^{\times}$	−0 0031$^{\times}$	0 0023$^{\times}$	−0 0017$^{\times}$	-0.0050	−0 0029$^{\times}$
Growth potential[$_h$/ $_l$]	0.0130	-	0.0449	0.0149	-	0.0537	0.0102	-	0.0378
Belief[p]	-0.2103	-	-0.3567	-0.2677	-	-0.4941	-0.2215	-	-0.5321
interaction terms :									
$p \times M$ B	-	-	0 0094$^{\times}$	-	-	0.0233	-	-	0.0437
$p \times$	-	-	-0.5857	-	-	-0.7762	-	-	-0.6218
$p \times x^i$	-	-	0.2474	-	-	0.3312	-	-	0.2956
$p \times F$	-	-	-0.0040	-	-	-0.0057	-	-	-0.0056
$p \times$	-	-	-0.1369	-	-	-0.2680	-	-	-0.1795
$p \times$	-	-	−0 0014$^{\times}$	-	-	0 0014$^{\times}$	-	-	0 0052$^{\times}$
$p \times$ $_h$/ $_l$	-	-	-0.0475	-	-	-0.0576	-	-	-0.0408
Constant	0.1117	0.1828	0.0930	0.2718	0.3547	0.2707	0.5791	0.6382	0.6610
N	60000	60000	60000	60000	60000	60000	60000	60000	60000
R^2	0.5226	0.4346	0.5878	0.7233	0.6461	0.7855	0.8762	0.8419	0.9052

Table 4: Determinants of SME external nancing costs.
This table reports regression results for the external nancing costs presented in Section 3, concerning the impact of asymmetric information on SME for investment and nancing. The dependent variable, external nancing costs, is a variable equal to Eq.(17) or Eq.(18) if high-type rm achieve separation equilibrium or achieve pooling equilibrium, respectively. So,the cost measure is evaluated in present value terms, measured in the ratio of rst-best value, and given by $Cost = (V_h^b - \max(V_{p\,h}^b\ \ V_{sep\,h}^b))\ \ V_h^b$. Statistically insigni cant if the p-value is greater than 0.001 and is denoted by $\times$.

	Speci cation 1	*Speci cation 2*	*Speci cation 3*
Market-to-book[$M\ \ B$]	0.0200	-0.0032	0.0306
Volatility[ ]	-0.1198	-0.0445	-0.1905
Firm size[x^i]	0.0433	0.0249	0.0746
Operating leverage[F]	-0.0008	-0.0006	-0.0012
Cash ow growth[ ]	-0.2583	-0.1773	-0.5472
Default loss rate[ ]	0.0095	0.0081	0.0115
Growth potential[$_h$/ $_l$]	0.0054	-	0.0179
Belief[p]	-0.0605	-	0.0631
interaction terms :			
$p \times M\ \ B$	-	-	-0.0146
$p \times$	-	-	0.1170
$p \times x^i$	-	-	-0.0561
$p \times F$	-	-	0.0008
$p \times$	-	-	0.2907
$p \times$	-	-	-0.0043
$p \times$ $_h$/ $_l$	-	-	-0.0198
Constant	-0.0238	0.0041	-0.0856
N	60000	60000	60000
R^2	0.5596	0.4187	0.6942

Table 3 reports the key determinants of SME investment probability for di erent time periods (T=1, 2, or 5) given by Eq.(19). Our model indicates that a higher investment threshold induces a lower investment probability. Any explanatory variable that accelerates (postpones) investment is expected to increase (reduce) the probability of investment. Consistent with our theoretical analysis, rms with a higher market-to-book ratio or higher growth-potential have a higher probability of investment. In particular, the in uence of these two variables on the explained variable rst increases and then decreases with an increase in time T.

The simulation results also con rm that project volatility, operating leverage, and default cost reduce SMEs investment probability. Interestingly, cash ow growth diminishes investment probability for a short time period while it boosts the investment incentive for a longer horizon. Another novel prediction of our empirical simulation is that market optimism reduces firm investment probability in incomplete markets as a higher belief that the market has more high-type firms induces a pooling equilibrium that delays investment for high-type firms.

We now investigate the determinants of firms external financing costs, which is defined as $Cost = \left(V_h^b - \max(V_{p\ h}^b\ V_{sep\ h}^b)\right)/V_h^b$. Table 4 reports the ordinary least squares regression results of the coefficients of independent variables on the external financing costs. It indicates that SMEs with higher market-to-book ratios or larger default costs face higher external financing costs and suffer more from asymmetric information. However, the operating leverage and cash flow growth reduce firms financing costs. Additionally, it reports that project volatility has a greater impact on financing cost while growth potential has less impact on external financing cost when the market belief is higher.

6. Conclusion

The impact of EGS on SME investment and financing decisions under perfect information has been extensively explored in previous literature. However, the effectiveness of ESG on mitigating SMEs financial constraints in incomplete markets, particularly with information asymmetry, receives less attention. In this paper, we develop a real option model of SME investment with EGS in a dynamic setting where the market is incomplete with information asymmetry. We further derive the conditions for the separating and pooling equilibria and identify firms optimal investment strategies under different economic conditions.

Moreover, we find information asymmetry is indeed good for low-type (low cash flow) firms as they can sell over-priced securities by mimicking high-growth firms in a pooling equilibrium. To deter the mimicry by low-type firms, high-type firms can separate from low-type firms by accelerating investment and imposing an adverse selection cost for the mimicry only when the benefit of being recognized as good type outweighs the investment distortion costs. In such cases, asymmetric information induces high-type firms to accelerate investment, leading to investment distortion and higher guarantee costs.

To fully illustrate the benefit of EGS, we extend Morellec and Schrhoff (2011) s model by introducing borrowing constraints for SMEs. Overall, we find EGS not only mitigates SMEs financing constraints but also reduces the investment and finance distortions under information asymmetry. We further explore the testable implications of our model in a simulation study. Unlike the extant empirical literature, our simulation provides novel empirical predictions of SMEs investment and financing behaviors. In particular, our model predicts market optimism (a market with more high-type SMEs) diminishes SMEs investment probability in incomplete markets as a higher belief leads to pool equilibria which delay firm investment.

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Appendices

Appendix A Single-crossing property

When the rm type perceived by insurers is , the guarantee cost of signaling by investing at x^i equals Eq.(12) and the valuation of type k before investment is Eq.(13).

To simplify the exposition, de ne the coe cients A_1 and B_1 as follows:

$$A_1 = (\quad (x^d) - (f+c)/r/ \quad)e^{- \ _2(x^i - x^d)} \text{ and } B_1 = (\quad (x_k^d) - (f+c)/r/ \ _k)e^{- \ _2(x^i - x^d)}$$

Then, we have

$$\frac{}{x^i} \quad (x^i \quad) = \frac{(1 - \ _i)}{E^2}\left[E\frac{D_{guar}}{x^i} - D_{guar}\frac{E}{x^i}\right]$$

$$= -\frac{(1 - \ _i)}{E^2}\left[\ _2 E D_{guar} + D_{guar}\,(1 - \quad)\left(\frac{}{r} + \quad _2 A_1\right)\right] < 0 \tag{A.1}$$

$$\frac{}{} \quad (x^i \quad) = \frac{(1 - \ _i)}{E^2}[E\frac{D_{guar}}{} - D_{guar}\frac{E}{}]$$

$$= -\frac{(1 - \ _i)}{E^2}\{\left[AA_1(1 - \quad) + \ _2 D_{guar}\frac{f+c}{_2}\right]E$$

$$+ \frac{(1 - \ _i)}{E^2}\left[\frac{E}{} + (1 - \quad)\left(\frac{f+c}{r} + \ _2 A_1\frac{f+c}{}\right)\right]D_{guar}\}$$

$$< 0 \tag{A.2}$$

and

$$\frac{}{x^i}V_k(x; x^i \quad) = V_k\left\{\frac{}{1 - }\left[\ _2 + \frac{E(x^i \quad)/ \ x^i}{E(x^i \quad)}\right] + \frac{E_k(x^i \quad)/ \ x^i}{E_k(x^i \quad)} + \ _1\right\} \tag{A.3}$$

$$\frac{}{}V_k(x; x^i \quad) = V_k\frac{}{1 - }\left[\frac{E(x^i \quad)/}{E(x^i \quad)} - \frac{D_{guar}(x^i \quad)/}{D_{guar}(x^i \quad)}\right] \tag{A.4}$$

The single-crossing property is given by

$$\frac{}{_k}\left(\frac{V_k/}{V_k/ \ x^i}\right)$$

$$= \left(\frac{V_k}{V_k/}\right)^2\frac{}{1 - }\left[\frac{D_{guar}/}{D_{guar}} - \frac{E/}{E}\right]\left[\frac{(E_k)^2/ \quad _k \ x^i}{E_k} - \frac{E_k/ \ x^i}{E_k}\frac{E_k/ \quad _k}{E_k}\right]$$

From (A.2), we have

$$\frac{D_{guar}/}{D_{guar}} - \frac{E/}{E} < 0$$

Then,

$$
\begin{aligned}
&\frac{(E_k)^2/\quad_k\ x^i}{E_k} - \frac{E_k/\ x^i}{E_k}\frac{E_k/\quad_k}{E_k} = (E_k)^2\left[\frac{(E_k)^2}{_k\ x^i}E_k - \frac{E_k}{x^i}\frac{E_k}{_k}\right] \\
&= (1-\quad)(E_k)^2\left\{\left[C_1 - {}_2{}^2B_1\frac{f+c}{_k{}^2}\right]E_k - C_1\left[E_k + (1-\quad)\ {}_2B_1\frac{f+c}{_k} + (1-\quad)\frac{f+c}{r}\right]\right\} \\
&< 0
\end{aligned}
$$

where $C_1 = 1/r + \ {}_2B_1$. Obviously, $0 < \quad < 1$ and $[V_k/(\ V_k/\quad)]^2 > 0$, so the single-crossing property

$$\frac{\quad}{_k}\left(\frac{V_k/}{V_k/\ x^i}\right) > 0$$

Condition (A.1) and (A.2) show that the greater the investment threshold and the higher the rm type scaling result in a lower ownership dilution (i.e., a lower guarantee cost). The possibility for the high-type rm to separate from the low-type rm depends on each type s willingness to exchange equity stakes for changes in the investment threshold. Further, the single-crossing condition indicates that the marginal rate of substitution between being perceived as type and investment threshold x^i depends positively on the actual type k.

Appendix B Details of the Simulation Analysis

Ordinary least squares (OLS) regression is used for empirically predicting the determinants of investment probability and external nancing costs. The fundamental independent variables are volatility of cash ows ( ), operating leverage (F), cash ow growth ( ), default cost ( ), growth potential ($_h/\ _l$), belief (p), rm size (x^i), and market-to-book ratio ($M\ B$).

We divide the independent variables into parameter set ($_h\ \ _l$ $F\ p$), and ($x^i\ M\ B$). It is obvious that , F, , . and $_h$ represent rm characteristics and the industry speci c is characterized by $_l$ and p. To obtain the data for empirical prediction, we draw each of the parameters of ($_h\ \ _l$ F) from the uniform distributions with the same bounds as in Table 1 while keeping the other ve parameters xed. The belief p varies from zero to one in steps of 1%. Next, we measure the market-to-book ratio ($M\ B$) at the time of investment x^i. Through this, we generate 60,000 valid pieces of data, which represent 60000 valid rms. At the beginning of the simulation, all rms have no assets, investment expenditure is I=20, and the risk-free rate is r=0.05.