Non-linear revenue evaluation*

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Abstract

In this article we investigate different market structures where decision makers are incentivized by both profit and revenue. Our innovation is that we consider managers that evaluate revenue in a non-linear way, exhibiting diminishing marginal utility. This implies that incremental changes in revenue—for example due to demand shocks—generate production choices that depend on the existing revenue base of the firm. We show that this intuitively appealing extension reverses some conventional results: decision makers may increase output in the presence of negative demand shocks, which depends on the concavity of their utility function with respect to revenue.

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1 Introduction

The traditional approach in industrial organization is to view firms as profit-maximizing entities. Even though this seems a reasonable assumption from the shareholders' or owner's perspective, critical strategic decisions that determine a firm's performance are more often than not in the hands of CEOs or managers who may pursue different objectives. This feature has been recognized early on and integrated in the development of the theory of the firm and organization economics (e.g., Baumol, 1958; Williamson, 1963). Under this alternative paradigm, the firm is still seen as maximizing profits, but the agents contracted for fulfilling this goal can be endowed with complementary objectives such as sales (i.e., total revenue) or market share maximization, or production cost minimization. This, admittedly more realistic, modeling strategy alters the predictions relating to the degree of competition between firms, as well as the effect of policy measures, or changes in market conditions.

A manager maximizing a combination of profits and some other performance can be justified by the managers' need to increase their power, status, or prestige, which can be symptomatic of a desire for empire-building considerations (Baumol, 1959; Williamson, 1963; Jensen, 1986; Baker et al., 1988; Hope and Thomas, 2008), to reduce their unemployment risk (Amihud and Lev, 1981), or to enhance promotion objectives (Cao et al., 2018) because of internal scrutiny on these measures of firm 'success'. While the original contributions conceived of firm decision makers having general preferences over multiple objectives, existing approaches that account for strategic behavior among firms have assumed linear evaluation of these alternative objectives (e.g., Sklivas, 1987; Fershtman and Judd, 1987), essentially assuming decision makers care about a weighted average of profit and alternative objectives. Such an assumption, however, is neither innocuous nor necessarily realistic.

There is no *a priori* justification, other than perhaps convenience of modeling, for restricting analysis to linear evaluations and, indeed, it is very reasonable in such circumstances to consider the manager's utility to be more general in alternative objective(s), as originally proposed within Williamson (1963) (but where his focus was not on the alternative objective of sales). The reason for considering that managers are motivated by alternative objectives as well as profit is that there is scrutiny on these objectives, either to determine remuneration, or in terms of the manager's internal or external reputation. In a model where managers care about a weighted combination of profit and revenue, we see it as highly plausible that managers' evaluation of revenue is concave, exhibiting diminishing marginal payoffs, so that changes in revenue have a greater impact when revenues are small than when they are large. From an external scrutiny, empire-building, perspective, a key assessment metric will be the manager's ability to make the firm grow (often judged by sales) in *relative* terms; so a given increase in revenue will have more gravitas when revenue is small than when it is already large.[1] Likewise for internal kudos of the manager among the organization's employees. An additional justification comes from recognizing that managers are often remunerated by stock options: empirical studies have uncovered a robust "firm-size effect", that is at play above and beyond firms' profitability measures, whereby the stocks of small firms tend to produce higher (but more volatile) expected returns (e.g., Reinganum, 1981; Asness et al., 2018). This, in turn, implies that the marginal return to managers is decreasing in the firm size, which is determined by the magnitude of revenues. The findings of Nourayi (2006) and Canarella and Nourayi (2008) further support these observations since they empirically find that, while managers' compensation is convex in the firm's profits, it is concave in the returns of the firms' stocks. This establishes a concave mapping from performance into remuneration itself, consistent with the manager's evaluation of revenue being concave.

Baumol (1958) considered that managers are motivated by sales maximization subject to a minimum profit constraint, meaning the manager's payoff is linear in sales. When managers value sales non-linearly, negative demand shocks may generate highly counter-intuitive results since we demonstrate that production may increase in some circumstances both under monopoly and oligopolistic market structures. The intuition for this result is clear. If the sole objective of a decision maker is either to maximize profits or sales, a negative demand shock will always lead to lower production. Indeed, a negative demand shock reduces marginal sales, and thus incentivizes both the profit-maximizing and the revenue-maximizing decision makers to reduce their production. If, however, a manager values sales non-linearly, a negative demand shock may reduce marginal profits while *increasing* the marginal utility of sales (since sales have reduced). When the latter effect is stronger than the former, the manager will opt for *higher* production. Interestingly, this result survives in oligopolistic settings where the decision variables (quantities) are strategic substitutes. The same result, however, cannot be obtained in existing linear valuation models of alternative objectives because the marginal utility of sales would always track the change in demand.

[1] Asplund (2002) considers risk-averse firms in uncertain environments, thus also diverging from the traditional modeling assumptions. In contrast to the literature considering alternative objectives to profits, risk-aversion in profits tends to make firms less aggressive.

2 Related literature

Our main focus in this article is investigating how managers—with multiple objectives, including profit maximization—adjust their output decisions in the event of demand shocks. While we show that output choices depend on managers' preferences over profit and revenue, the idea that positive demand shocks may result in output contractions has been previously considered (Quirmbach, 1988; Hamilton, 1999; Baldenius and Reichelstein, 2000; Cowan, 2004). By imposing restrictions on the demand function—such as the type of shock occurring or the characteristics of the demand function itself—it has been shown that output can decrease in the presence of 'positive' demand shocks. In our framework, however, we investigate these output choices not by placing restrictions on the demand function or subsequent shocks, but by allowing a more general setting for a manager to make decisions. In particular, we allow managers to have more general preferences over how they maximize revenue and profit. By doing this, we complement this literature by providing a new perspective on how output choices are determined, linking a more general structure of managers' utility functions with their output choices in the presence of demand shocks.[2]

A large literature has explored the consequences of endowing firms (or their managers) with alternative incentives and objectives other than simply profit. These objectives include maximizing market share (Jansen et al., 2007; Ritz, 2008), or maximizing the relative performance of the firm (Salas Fumas, 1992; Miller and Pazgal, 2002; Vroom, 2006; Matsumura and Matushima, 2012; Nakamura, 2014). Alternatively, scholars have developed principal-agent theories in strategic settings with effort-minimizing managers (e.g., Hart, 1983). Yet, the majority of the literature has focused on sales (i.e., total revenue) maximization as an alternative or complementary objective to profit maximization, following the initial contribution of Baumol (1958).

The empirical evidence testifying that the evaluation of managers and CEOs performance is indeed a function of both the firm's profit—through its effect on shareholders—and sales is compelling (Murphy, 1985; Baker et al., 1988; Huang et al., 2015; Bloomfield, 2016). Conyon (2014), for instance, estimates the elasticity of executive pay to firm sales to be approximately 35%. Given this background, it is not surprising that an important theoretical literature, known as the *strategic delegation* literature, studied the strategic implications of endowing managers with such incentives. This literature explores firm owners' incentives to endoge-

[2] Other scholars have concluded that changing market conditions can produce results going against the common wisdom: in the context of product differentiation (Chen and Riordan, 2008); in the presence of search costs (Janssen and Moraga-Gonzalez, 2004); or when consumers, as well as firms, have market power (Dickson, 2013).

nously assign managers strategic decisions and corresponding contracts to alter their firm's strategy by committing via delegation, and to thereby expand profits. Hence, by rewarding managers according to a combination of profit and sales, a firm can be rendered more aggressive in a Cournot competition setting to potentially gain a competitive edge. All the major initial contributions to this literature consider contracts featuring linear combinations of profits and sales, as well as a capacity on the firm's behalf to commit to the announced contracts concluded with the agents/managers (Sklivas, 1987; Vickers, 1985; Fershtman, 1985; Fershtman and Judd, 1987).

More recently, the works of Kaneda and Matsui (2003) and Cornes and Itaya (2016) both consider an oligopoly model where the firms' managers maximize the weighted average of profit and some other alternative objective. Kaneda and Matsui (2003) consider how firms' behavior changes when they put different weights on alternative objectives. Cornes and Itaya (2016) characterize the equilibria of such games with weaker conditions on the functional forms and also allow for asymmetric firms. Again, their focus is on understanding the relative behavior of firms when they care more or less about alternative objectives.

In this article we focus on the manager's payoff function and work with a model that incorporates non-linear evaluation of revenue. By relaxing this linearity assumption—in line with the initial intuition of Williamson (1963)—we explore the consequences of exogenous demand shocks on the equilibrium under both monopoly and Cournot oligopoly. Allowing for this more general framework shows that firm output can either increase or decrease in the presence of an exogenous demand shock. Our framework, then, has the ability to nest the existing literature within our general model as a special linear case. Therefore our major contribution is the development of a framework that provides a broader understanding of production choices under the presence of demand shocks.

The remainder of the article is organized as follows. In Section 3 we analyze the monopoly case, in Section 4 we extend the analysis to a Cournot oligopoly. The analysis of each of these market structures is accompanied by a worked example that illustrates the validity of the result. Section 5 provides some concluding remarks. The appendix generalizes the Cournot setting to the case of heterogeneous firms.

3 Non-linear revenue evaluation in a monopoly

3.1 The setup

Consider a market in which a monopolist chooses its level of output $X \geq 0$. The price is given by $P(X, \alpha)$, where α is a demand parameter, and we define the firm's revenue as $r(X; \alpha) \equiv$

$XP(X;\alpha)$. We assume that $P_X < 0$, $r_{XX} \equiv 2P_X + XP_{XX} < 0$, and marginal revenue is such that either $r_X \equiv P + XP_X = 0$ for some $X < \infty$ or $r_X \to 0$ as $X \to \infty$. The demand parameter α is an exogenous demand shock described by $P_\alpha > 0$ and $r_{X\alpha} \equiv P_\alpha + XP_{X\alpha} \geq 0$, so an increase in α both increases the price and marginal revenue.[3] The monopolist's cost function $C(X)$ is such that $C' > 0$ and $C'' \geq 0$. The firm's profit is given by $\pi(X;\alpha) \equiv r(X;\alpha) - C(X)$.

Consider a decision-maker's payoff given by their utility $U(\pi, r)$, which is influenced by both profit and revenue. Following the existing literature, we suppose that the decision-maker's utility is a weighted average of the two components but, in contrast to the existing literature, we allow for the evaluation of the revenue component to be non-linear and given by $v(r)$, where we assume $v' > 0$ and $v'' \leq 0$. This allows us to capture more general preferences, which includes the intuitively compelling case where incremental gains in revenue are more valuable when revenue is scarce compared to when it is abundant. As such, for a parameter $\beta \in [0, 1]$, we suppose that the decision maker has the payoff function[4]

$$\begin{aligned} U(\pi, r) &= [1-\beta]\pi(X;\alpha) + \beta v(r(X;\alpha)) \\ &= [1-\beta][r(X;\alpha) - C(X)] + \beta v(r(X;\alpha)). \end{aligned}$$

If $\beta = 0$ the decision maker is only concerned about profit, whereas if $\beta > 0$ there is also a revenue motive. If $\beta = 1$ the decision maker only cares about revenue. We also suppose that the decision maker is subject to a minimum profit constraint, that we assume to be zero for simplicity.

The decision-maker chooses output $X \geq 0$ to maximize their payoff $U(\pi, r)$ subject to the constraint that profit is non-negative:

$$X^m \in \arg\max_{X\geq 0}\{[1-\beta][r(X;\alpha) - C(X)] + \beta v(r(X;\alpha)) \text{ s.t. } r(X;\alpha) - C(X) \geq 0\},$$

where the superscript m denotes the case of a monopolist.

[3]If we allow for $P_\alpha + XP_{X\alpha} < 0$ as in Cowan (2004), a 'positive' demand-shock reduces the firm's marginal revenue, thereby pushing a pure profit-maximizing monopolist to reduce equilibrium output. We instead focus on the more realistic case where marginal revenue increases with demand.

[4]We are assuming β is given in our setting, as opposed to the strategic delegation literature whose focus is on the endogenous determination of these weights. Vickers (1985) and Sklivas (1987), for instance, establish that it is optimal for firm owners to set $\beta \in (0, 1)$ in oligopolistic settings, with the exact equilibrium value depending on demand and technology parameters. Conceptualizing β as a parameter in the context of a monopoly is reasonable because β is often a subjective weight allocated to revenue maximization by the decision maker valuing both the remuneration that is aligned with profit maximization, and alternative goals such as enhancing power, status or prestige.

3.2 The equilibrium

A decision maker's optimal output, ignoring for the moment the non-negative profit condition, is given by $X^m = \max\{0, X\}$, where X is the solution to the first-order condition[5]

$$\phi(X;\alpha) \equiv \frac{\partial U}{\partial X} = [1-\beta][r_X(X;\alpha) - C'(X)] + \beta v'(r(X;\alpha))r_X(X;\alpha) = 0. \tag{1}$$

Decision makers are subject to the constraint that profit is positive. As such, optimal output will actually be given by the minimum of that defined above, and $\bar{X}^m$, which is the output at which profit is zero. We suppose throughout that at equilibrium the positive profit constraint is not binding so that when we consider a shock to the environment the decision maker can respond following the incentives they face in our setup.

Note that if $\beta = 0$, the problem becomes one of equating marginal revenue, r_X, to marginal cost, C', as in a standard profit-maximization context. If $\beta = 1$ the problem collapses to $v'r_X = 0$, the solution to which (since $v' > 0$) necessitates equating marginal revenue to zero, as would be expected because in this case the payoff function is $v(r)$, which is a monotonic transformation of revenue, maximized where revenue is maximized. In intermediate cases where $\beta \in (0, 1)$ the decision-maker cares about both profit and revenue. Observe that in these cases marginal revenue must be positive at the chosen level of output, for if $r_X \leq 0$ equality with zero in the first-order condition can never be achieved, and by the same reasoning it must be the case that marginal revenue is less than marginal cost.

Figure 1 illustrates both profit (π) and revenue functions (r), as well as the decision maker's value of revenue $\tilde{v}(X;\alpha) \equiv v(r(X;\alpha))$. Under our assumptions both the revenue and profit functions are concave. When $\beta = 0$, we obtain the standard monopoly solution in which the firm maximizes profit by producing output where its marginal revenue is equal to its marginal cost, as observed in Figure 1 at point k. When $\beta = 1$, the equilibrium is such that the firm maximizes revenue, which, for graphical purposes, we have assumed to be downward sloping for large enough levels of output. In that instance, the monopoly will produce at point l, where the firm's marginal revenue is zero, which by extension also implies that the marginal valuation of revenue is zero. Observe that since both revenue and profit functions are concave, and at the profit-maximizing level of output marginal revenue is positive, this necessitates a higher level of output under revenue maximization than under profit maximization. For intermediate cases where $\beta \in (0, 1)$, the decision maker will produce output until its weighted marginal valuation of profit and revenue is zero, as shown as output $\hat{X}$ in Figure 1, which will

[5] $\phi_X = [1 - \beta + \beta v']r_{XX} + \beta v''[r_X]^2 - [1-\beta]C'' < 0$ under our assumptions, so the second-order condition is satisfied.

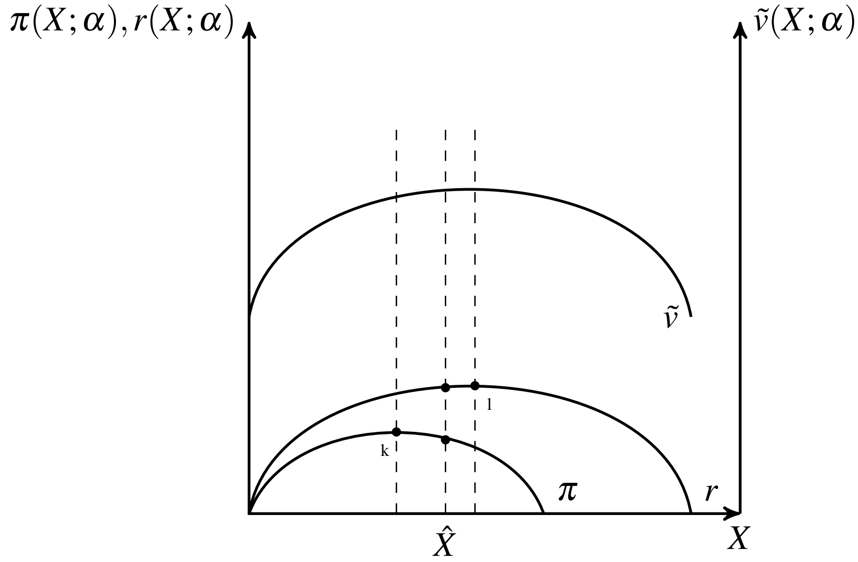

Figure 1: The firm's functions for profit π, revenue r, and evaluation of revenue $\tilde{v}$. Profit and revenue are measured on the left-hand axis, while the evaluation of revenue is measured on the right-hand axis.

lie somewhere between the profit-maximizing level of output and the revenue-maximizing level of output.

As more importance is placed on revenue (i.e., β increases), ceteris paribus, equilibrium output increases from the profit-maximizing solution to the revenue-maximizing solution. This can be deduced from applying the implicit function theorem to (1):

$$\frac{\mathrm{d}X^m}{\mathrm{d}\beta} \equiv -\frac{\phi_\beta(X^m;\alpha)}{\phi_X(X^m;\alpha)} = -\frac{v'(r(X^m;\alpha))r_X(X^m;\alpha) - [r_X(X^m;\alpha) - C'(X^m)]}{\phi_X(X^m;\alpha)}. \tag{2}$$

Since $\phi_X < 0$ (see footnote 5) and in equilibrium marginal revenue must be positive—but at the same time less than marginal cost—this term is positive.

3.3 Exogenous demand shocks

We now turn to understand how the equilibrium supply of the decision maker responds to a change in market conditions. More specifically, we are interested in the effect of an exogenous demand shock on the equilibrium. Applying the implicit function theorem to (1) gives

$$\frac{\mathrm{d}X^m}{\mathrm{d}\alpha} = -\frac{\phi_\alpha(X^m;\alpha)}{\phi_X(X^m;\alpha)}.$$

Since we have deduced that $\phi_X < 0$, the sign of $\frac{dX^m}{d\alpha}$ will be given by the sign of ϕ_α. Now (suppressing notation),

$$\begin{aligned}\phi_\alpha &= [1-\beta]r_{X\alpha} + \beta[v' r_{X\alpha} + r_X v'' r_\alpha] \\ &= r_{X\alpha}\left[1-\beta+\beta\left[v' + r_X v'' \frac{r_\alpha}{r_{X\alpha}}\right]\right] \\ &= r_{X\alpha}\left[1-\beta+\beta v'\left[1-\gamma^m \frac{r_X}{r}\frac{r_\alpha}{r_{X\alpha}}\right]\right]\end{aligned} \tag{3}$$

where $\gamma^m(X;\alpha) \equiv -\frac{r(X;\alpha)v''(r(X;\alpha))}{v'(r(X;\alpha))} \geq 0$ is the coefficient of relative risk aversion.

We begin by analyzing the two polar cases; namely, the pure profit-maximizing case ($\beta = 0$) and the pure revenue-maximizing case ($\beta = 1$). In the pure profit-maximizing case where $\beta = 0$, we can see from (3) that the effect of the shock comes solely through the effect on marginal revenue since $\phi_\alpha(X^m;\alpha) = r_{X\alpha}(X^m;\alpha)$, which we reasonably assume to be positive (see Footnote 3). As such, and without surprise, the equilibrium supply of pure profit-maximizing firms will increase following a positive demand shock.

The same is true if firms are purely revenue maximizing. To see this note that if $\beta = 1$ in (3) then $\phi_\alpha(X^m;\alpha) = r_{X\alpha}v'\left[1-\gamma^m\frac{r_X}{r}\frac{r_\alpha}{r_{X\alpha}}\right] = r_{X\alpha}v' > 0$, since $r_X = 0$ in equilibrium. The effect of a positive demand shock on a firm's output decision is therefore dictated by the exact same forces that exist in the profit-maximizing case; namely, the change in the marginal revenue $r_{X\alpha}$. We can thus state the following result.

Proposition 1. *A pure profit-maximizing monopoly and a pure revenue-maximizing monopoly both increase their output with a positive demand shock.*

This finding is quite intuitive. A revenue-maximizing decision maker will increase firm production until the marginal revenue becomes zero, since this will maximize its utility irrespective of the shape of the utility function. Any increase in the marginal revenue will thus incentivize the monopolist to increase output. A profit-maximizing decision maker sets marginal revenue equal to the marginal cost. Since the demand shock leaves the marginal cost unaffected, but increases the firm's marginal revenue, this incentivizes the manager to increase output.

We now turn to the more interesting intermediate cases where $\beta \in (0,1)$ where, from (1) we can write $\beta v' = -[1-\beta]\frac{r_X - C'}{r_X}$ since $r_X > 0$ in equilibrium when $\beta < 1$, as we previously deduced. Substitution of this into (3) gives

$$\begin{aligned}\phi_\alpha &= r_{X\alpha}[1-\beta]\left[1-\frac{r_X - C'}{r_X}\left[1-\gamma^m\frac{r_X}{r}\frac{r_\alpha}{r_{X\alpha}}\right]\right] \\ &= \frac{r_{X\alpha}}{r_X}[1-\beta]\left[r_X\gamma^m\frac{r_X}{r}\frac{r_\alpha}{r_{X\alpha}} + C'\left[1-\gamma^m\frac{r_X}{r}\frac{r_\alpha}{r_{X\alpha}}\right]\right] \\ &= \frac{r_{X\alpha}}{r_X}[1-\beta]\left[C' - \gamma^m[C'-r_X]\frac{r_X}{r}\frac{r_\alpha}{r_{X\alpha}}\right].\end{aligned} \tag{4}$$

We thus have $\text{sgn}\{\frac{\mathrm{d}X^m(\alpha)}{\mathrm{d}\alpha}\} = \text{sgn}\{C' - \gamma^m[C' - r_X]\frac{r_X}{r}\frac{r_\alpha}{r_{X\alpha}}\}$, and therefore (recalling that in equilibrium $C' > r_X$),

$$\frac{\mathrm{d}X^m(\alpha)}{\mathrm{d}\alpha} < (>) 0 \Leftrightarrow \gamma^m \equiv -\frac{rv''}{v'} > (<) \frac{C'}{C' - r_X}\frac{r}{r_X}\frac{r_{X\alpha}}{r_\alpha}. \tag{5}$$

If the valuation of revenue is linear, $\gamma^m = 0$, so we will always have $\frac{\mathrm{d}X^m(\alpha)}{\mathrm{d}\alpha} > 0$ since in equilibrium $r_X > 0$. However, if the decision-maker's valuation of revenue is sufficiently concave the reverse will be true, implying that it is feasible to witness the counter-intuitive situation summarized in the following proposition.

Proposition 2. *In a monopoly setting, a decision maker that has non-linear evaluation of revenue and maximizes a combination of profit and revenue reduces firm output following a positive demand shock provided their valuation of revenue is sufficiently concave.*

In the special case where demand is multiplicative and $P(X;\alpha) = \alpha f(X)$, it is readily deduced that $\frac{r}{r_X}\frac{r_{X\alpha}}{r_\alpha} = 1$. In the special case where demand is additive and $P(X;\alpha) = \alpha + g(X)$ we can deduce that $\frac{r}{r_X}\frac{r_{X\alpha}}{r_\alpha} = \frac{\alpha+g}{\alpha+g+Xg'} > 1$. As such, since $\frac{C'}{C'-r_X} > 1$, in both of these cases a necessary condition for output to contract following a positive demand shock is for γ^m to exceed 1. Of course, since we have a model with general functions the necessary and sufficient condition in (5) depends on endogenous objects for which we cannot explicitly deduce equilibrium values. However, after some discussion of the result we provide a worked example in the following subsection that demonstrates the proposition is not vacuous.

The intuition behind this result is most easily understood by considering a *contraction* in demand (i.e., a reduction in α), where the conventional wisdom would suggest a firm decreases its output. Consider, however, a manager that is in a situation where they have strongly diminishing marginal utility of revenue (e.g., because they have a concave evaluation of revenue and currently generate a relatively low amount of revenue) and is facing such a shock. Without any reaction in terms of output, the manager is facing a reduction in their revenue which, because of diminishing marginal utility will have a large impact on their payoff. To counter this, they seek to reduce the impact on the revenue generated, and consequently their utility from such, by increasing the firm's supply to the market. As such, with a sufficiently concave valuation of revenue, the response of decision makers to a change in market conditions will break the conventional wisdom.

Figure 2 illustrates a decision maker's choice; dashed curves indicate post-demand expansion functions. In our most general case, the decision maker has a weighted preference for profit and revenue maximization. Denote the optimum output as $\hat{X}$. At this point the (weighted) slope of the profit function at point a must be equal to the (weighted and product

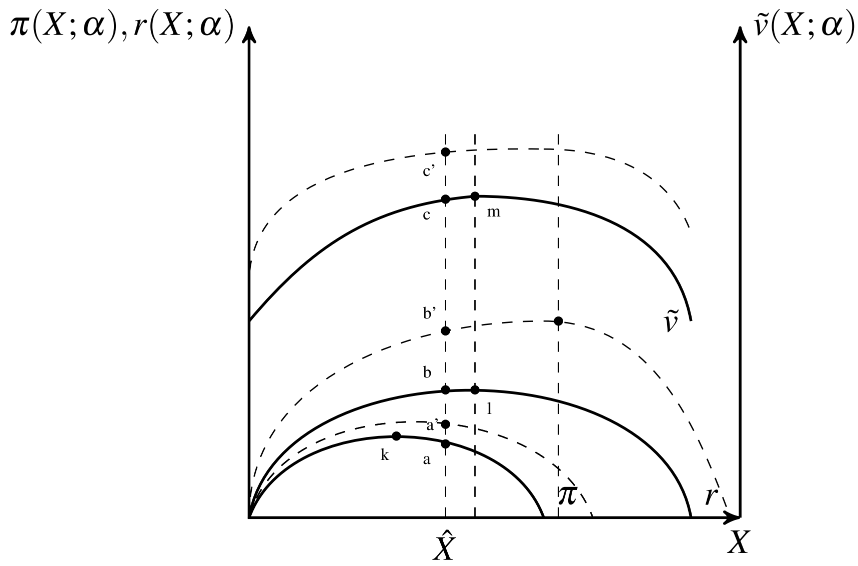

Figure 2: A positive demand shock results in the dashed expansion of profit, revenue, and the evaluation of revenue. Output will decrease if the decrease in the slope at point a is offset by the decrease in the product of the slopes at points b and c. This occurs if the evaluation of revenue $v(r)$ is sufficiently concave.

of) slopes of the revenue function at point b and the value function at point c. When demand expands both revenue and marginal revenue increase. Thus the profit function and the slope of the profit function increase, so at point a' the slope is less negative; the slope of the revenue function increases at point b'; and the slope of the value function decreases at point c'. Thus the slope at point a will decrease and the product of b and c can increase or decrease. If it decreases a lot, then at the previous equilibrium output the marginal payoff will be negative, so the optimum needs to move in the direction that increases the slopes of b and c and makes the slope at point a less negative, that is, have a lower $\hat{X}$. The rate at which the product of the slopes b and c decrease depends on the concavity of the value function $v(r)$.

To see the importance of the utility function's concavity, consider instead a linear utility function as has been assumed in the related literature (e.g. Sklivas, 1987; Vickers, 1985; Fershtman and Judd, 1987). Positive demand shocks will then leave the slope of the marginal revenue-related utility generated by an extra unit of output unchanged. In other words, the valuation of the total revenue will only be impacted through the effect the demand shock has on the firm's marginal revenue. Therefore, since both marginal profits and the marginal valuation of revenues are impacted in the same way—namely, through an increase in the firm's marginal revenue—with linear utility of revenue the firm's output will always increase with α.

3.4 Worked example

To illustrate the above results for the monopoly case, consider the following example in which demand is given by

$$P(X;\alpha) = \frac{\alpha}{\mu + X},$$

where $\mu > 0$. The valuation of revenue is assumed to be given by

$$v(r) = \delta r - \frac{\theta}{2} r^2,$$

with $\delta, \theta > 0$ and δ large enough so that $v' = \delta - \theta r > 0$ in equilibrium. Production costs are assumed to be given by $C(X) = cX$ with $c > 0$.

It follows that $r(X;\alpha) = \frac{\alpha X}{\mu + X}$ and $r_X(X;\alpha) = \frac{\mu\alpha}{[\mu+X]^2}$. If the manager exclusively maximizes profits ($\beta = 0$), the profit-maximizing condition $r_X = c$ gives $X^m = \left[\frac{\mu\alpha}{c}\right]^{1/2} - \mu$. For a pure revenue-maximizing manager ($\beta = 1$), $r_X = 0$ implies $X^m \to \infty$. We now turn to the intermediate cases where $\beta \in (0,1)$. The first-order condition is:

$$\phi(X;\alpha) \equiv \left[1 - \beta + \beta\left[\delta - \frac{\alpha\theta X}{\mu + X}\right]\right] \frac{\alpha\theta}{[\mu + X]^2} - [1-\beta]c = 0. \tag{6}$$

The above expression implicitly defines X^m. By applying the implicit function theorem to expression (6), we deduce that the sign of $\frac{\mathrm{d}X^m}{\mathrm{d}\alpha}$ is given by the sign of

$$\frac{\mathrm{d}\phi}{\mathrm{d}\alpha} = 1 - \beta + \beta\delta - \frac{2\alpha\beta\theta X^m}{\mu + X^m}.$$

As such,

$$\frac{\mathrm{d}X^m}{\mathrm{d}\alpha} < (>) 0 \Leftrightarrow \theta > (<) \frac{[1 - \beta + \beta\delta][\mu + X^m]}{2\alpha\beta X^m}. \tag{7}$$

While this condition depends on the equilibrium output (for which we do not have an explicit solution), if we take the limit case where $\mu \to 0$ the above expression then reads as:

$$\frac{\mathrm{d}X^m}{\mathrm{d}\alpha} < (>) 0 \Leftrightarrow \theta > (<) \frac{1 - \beta + \beta\delta}{2\alpha\beta},$$

which only depends on the parameters of the model. Observe next that the right-hand side of (7) is continuous in μ, so that if the parameter combination is such that $\frac{\mathrm{d}X^m}{\mathrm{d}\alpha} < 0$ (> 0) for $\mu \to 0$, then there will exist a $\bar{\mu} > 0$ such that for all $\mu \in (0, \bar{\mu}]$ the sign of the inequality is still respected.

To verify that there is a non-empty set of parameters such that the equilibrium exhibits output decreasing in a positive demand shock, we need to verify that there is a parameter combination where output is positive, the valuation of revenue is positive, profit is positive, and output is decreasing in α. We demonstrate that this can hold for the limit case where $\mu \to 0$ since, by continuity, this is true for some strictly positive values of μ. Since $r = \frac{\alpha X}{\mu + X} < \alpha$,

$v' = \delta - \theta r > 0 \Leftarrow \alpha < \delta/\theta$. So let $\alpha = \delta/\theta - \eta$ with $\eta \to 0$ and we consider the dual limit with $\mu, \eta \to 0$. In this case, $X^m \to [\alpha/c]^{1/2} > 0$ since $\alpha > 0$, and $\pi^m \to \alpha - [\alpha c]^{1/2} > 0$ when $\alpha > c$. Finally, $\frac{\mathrm{d}X^m}{\mathrm{d}\alpha} < 0$ requires $\alpha > \frac{1}{2}[\frac{\delta}{\theta} + \frac{1-\beta}{\beta}]$. Thus, so long as $\min\{c, \frac{1}{2}[\frac{\delta}{\theta} + \frac{1-\beta}{\beta}]\} < \alpha < \frac{\delta}{\theta}$, which by inspection is a non-empty set of parameters, all these conditions are satisfied. By continuity, we can relax the assumption on η and the results remain valid.

This confirms that Proposition 2 is not vacuous: demand expansion can result in a firm reducing output when the decision maker has a sufficiently concave valuation function over the firm's revenue, and likewise increasing output when demand contracts.

3.5 The implications of delegation

The problem at stake being decision-theoretic (as opposed to game-theoretic, as in the next section), it is necessarily the case that delegating output decisions to a decision maker with non-aligned preferences (i.e., $\beta > 0$) will always result in lower profits compared to when the firm's owner makes production choices. As such, with non-aligned preferences between the principal-owner and the agent-manager, delegation will never occur absent additional advantages such as more advanced expertise or capabilities on behalf of the manager.

An important question for investors, however, pertains to the variability of a firm's performance when market conditions change. While it is expected that a monopolist's profit will fluctuate with the demand level, in what follows we demonstrate that this variability is affected by whether decision making has been delegated to an agent or not. In general, the effect of a change in α can be decomposed into the direct effect on profit, and the indirect effect that comes through the change in activity as a result. Thus:

$$\frac{\mathrm{d}\pi}{\mathrm{d}\alpha} = \frac{\partial \pi}{\partial \alpha} + \frac{\partial \pi}{\partial X}\frac{\partial X^m}{\partial \alpha}. \tag{8}$$

Note that since $\pi = Xp(X,\alpha) - C(X)$, $\partial\pi/\partial\alpha = XP_\alpha$. When $\beta = 0$, so the decision-maker is motivated by profit, the envelope theorem implies $\partial\pi/\partial X = 0$ and therefore:

$$\left.\frac{\mathrm{d}\pi}{\mathrm{d}\alpha}\right|_{\beta=0} = XP_\alpha > 0. \tag{9}$$

When $\beta > 0$, the decision maker's payoff function is $U = [1-\beta]\pi + \beta v(r)$, and so the first-order condition requires $[1-\beta]\frac{\partial\pi}{\partial X} + \beta\frac{\partial v(r)}{\partial X} = 0$. This implies $\frac{\partial \pi}{\partial X} = -\frac{\beta}{1-\beta}\frac{\partial v(r)}{\partial X} < 0$ in equilibrium, since $\partial v(r)/\partial X > 0$. Moreover $X^m|_{\beta>0} > X^m|_{\beta=0}$. As such, so long as $P_{X\alpha} > 0$, $X^m P_\alpha|_{\beta>0} > X^m P_\alpha|_{\beta=0}$. Inspection of (8) therefore allows us to conclude that when a decision maker is motivated by revenue, and as a result of a positive shock reduces supply ($\frac{\partial X^m}{\partial\alpha} < 0$), not only does profit increase, but it increases by more than would be the case if the firm were operated by a profit-maximizing decision maker. In a similar way, a negative demand shock will equally map into higher profit-contraction for firms delegating decision-making compared to when decisions are taken by the firm's owners. Consequently, the variance of the firm's performance will be higher under delegation.

4 Non-linear revenue evaluation in an oligopoly

In a monopoly setting, we deduced that as a result of a positive demand shock the manager may reduce output. It is well-known that Cournot oligopoly with profit maximizing firms is a game of strategic substitutes under reasonable assumptions on demand. As such, it is not obvious that the results from monopoly carry over to an oligopoly setting: with a sufficiently concave revenue evaluation functions managers have a motive to reduce output, but if they anticipate that others will do so strategic substitutability implies there is an offsetting motive to increase output. Thus, we need to investigate the balance between these two effects. As we show in this section, however, the results from the monopoly setting carry over *mutatis mutandis* to the oligopoly setting.

We consider a Cournot oligopoly setting in which there are $i = 1, \dots, n$ firms who choose their level of output $x^i \geq 0$. We let $X = \sum_{i=1}^{n} x^i$ be total supply and $X^{-i} = \sum_{j \neq i} x^j$. We continue to denote inverse demand $P(X; \alpha)$ and retain the assumptions $P_X < 0$ and $P_\alpha > 0$. Firm i's revenue is given by $r^i(x^i, X^{-i}; \alpha) \equiv x^i P(X; \alpha)$, and we further assume demand is such that $P_X + x^i P_{XX} \leq 0$ and $r^i_{x^i \alpha} = P_\alpha + x^i P_{X\alpha} \geq 0$. We adapt the notation to the current setting so that the cost of production is now denoted by $C^i(x^i)$, and a firm i's profit is given by $\pi^i(x^i, X^{-i}; \alpha) \equiv r^i(x^i, X^{-i}; \alpha) - C^i(x^i)$.

The decision-maker's payoff is given by the utility function $U^i(\pi^i, r^i)$, and the evaluation of the revenue component is denoted by $v^i(r^i)$, where we assume $v^{i\prime} > 0$ and $v^{i\prime\prime} \leq 0$. Decision-maker i's specific weight assigned to profits in the utility function is denoted by β^i. We assume that each decision maker's β^i is common knowledge and that output decisions are taken simultaneously, and study the n-player simultaneous-move game of complete information where decision makers choose their output $x^i \geq 0$ subject to the constraint that profit is non-negative and payoffs are given by $U^i(\pi^i, r^i)$. We seek a Nash equilibrium in pure strategies $\{x^{i*}\}_{i=1}^{n}$ in which, for each firm $i = 1, \dots, n$,

$$x^{i*} \in \arg\max_{x^i \geq 0} \{[1 - \beta^i][r^i(x^i, X^{-i*}; \alpha) - C^i(x^i)] + \beta^i v^i(r^i(x^i, X^{-i*}; \alpha)) \text{ s.t. } r^i(x^i, X^{-i*}; \alpha) - C^i(x^i) \geq 0\}.$$

In the analysis that follows, for simplicity, we treat decision makers as symmetric where $v^i(\cdot) = v(\cdot)$, $\beta^i = \beta$ and $C^i(\cdot) = C(\cdot)$ for all $i = 1, \dots, n$. In the appendix, however, we extend the analysis to account for heterogeneity among firms and derive similar results.

4.1 The equilibrium

Assuming firms are symmetric, a typical decision-maker's payoff is given by

$$U = [1-\beta][r^i(x^i, X^{-i}; \alpha) - C(x^i)] + \beta v(r^i(x^i, X^{-i}; \alpha)).$$

A decision maker's best response, ignoring for the moment the non-negative profit condition, is given by the reaction function $\hat{x}^i(X^{-i}) = \max\{0, x^i\}$, where x^i is the solution to the first-order condition[6]

$$\phi(x^i, X^{-i}; \alpha) \equiv \frac{\partial U}{\partial x^i} = [1-\beta][r^i_{x^i}(x^i, X^{-i}; \alpha) - C'(x^i)] + \beta v'(r^i(x^i, X^{-i}; \alpha)) r^i_{x^i}(x^i, X^{-i}; \alpha) = 0. \tag{10}$$

Observe that, for the same reasons as in the monopoly setting, here too it must be the case that for $\beta \in (0,1)$ the marginal revenue is less than marginal cost. Moreover, similarly to the monopoly case the positive profit constraint imposes that reaction functions are actually given by the minimum of that defined above, and $\bar{x}^i(X^{-i})$, which is the output at which profit is zero, but we suppose throughout that this constraint is not binding.

The slope of the reaction function with respect to the action of any other player $j \neq i$ is given by

$$\frac{\partial \hat{x}^i(X^{-i})}{\partial x^j} = \frac{\mathrm{d}\hat{x}^i(X^{-i})}{\mathrm{d}X^{-i}} = -\frac{\phi_{X^{-i}}}{\phi_{x^i}},$$

from the implicit function theorem applied to (10), where $\phi_{x^i} < 0$ (see Footnote 6) and

$$\phi_{X^{-i}} = [1-\beta+\beta v'] r^i_{x^i X^{-i}} + \beta v'' r^i_{x^i} r^i_{X^{-i}}.$$

The sign of this is not determined under the current assumptions as while the first term is negative (since $r^i_{x^i X^{-i}} = P_X + x^i P_{XX} \leq 0$ by assumption), the second is positive (since $v'' \leq 0$, $r^i_{x^i} = P + x^i P_X \geq 0$ and $r^i_{X^{-i}} = x^i P_X < 0$).

To establish uniqueness of the equilibrium we require that the joint reaction function map is a contraction. A sufficient condition is that $\sum_{j\neq i} \frac{\partial \hat{x}^i(X^{-i})}{\partial x^j} < 1$ (see, for example, Vives (1999)), which requires $[n-1]\frac{\mathrm{d}\hat{x}^i(X^{-i})}{\mathrm{d}X^{-i}} < 1$, and therefore that $\phi_{x^i} + [n-1]\phi_{X^{-i}} < 0$. As such, uniqueness of equilibrium requires

$$[1-\beta+\beta v'] r^i_{x^i x^i} + \beta v'' [r^i_{x^i}]^2 - [1-\beta]C'' + [n-1][[1-\beta+\beta v'] r^i_{x^i X^{-i}} + \beta v'' r^i_{x^i} r^i_{X^{-i}}] < 0.$$

Expanding terms, this can be reduced to

$$[1-\beta][P_X + n[P_X + x^i P_{XX}] - C''] + \beta[v'[P_X + n[P_X + x^i P_{XX}]] + v''[P + x^i P_X][P + n x^i P_X]] < 0.$$

[6]Note that $\phi_{x^i} = [1-\beta+\beta v'] r^i_{x^i x^i} + \beta v'' [r^i_{x^i}]^2 - [1-\beta]C'' < 0$ as our assumption that $P_X + x^i P_{XX} \leq 0$ implies $r^i_{x^i x^i} \equiv 2P_X + x^i P_{XX} \leq P_X < 0$.

The first and second terms in this expression are negative by assumption. The third term requires more scrutiny. We know that on the reaction function $P + x^i P_X > 0$, so if $P + nx^i P_X > 0$ as well then the third term is negative. However, if $P + nx^i P_X < 0$ then a sufficient condition to ensure the whole expression is negative is that $P + x^i P_X < -\frac{v'}{v''} \frac{\frac{\partial}{\partial x^i}[P+nx^i P_X]}{P+nx^i P_X}$, which we assume to hold at least in equilibrium, for then uniqueness of equilibrium is guaranteed.

In a symmetric Nash equilibrium in which $x^{i*} = x^*$ for all $i = 1, ..., n$ equilibrium supply is implicitly defined by[7]

$$\phi(x^*, [n-1]x^*; \alpha) \equiv [1-\beta][r^i(x^*, [n-1]x^*; \alpha) - C'(x^*)] + \beta v'(r^i(x^*, [n-1]x^*; \alpha)) r^i_{x^i}(x^*, [n-1]x^*; \alpha) = 0. \tag{11}$$

4.2 Exogenous demand shocks

We next study the effect of a change in market conditions on our Cournot equilibrium. Applying the implicit function theorem to (11) gives

$$\frac{\mathrm{d}x^*}{\mathrm{d}\alpha} = -\frac{\phi_\alpha(x^*, [n-1]x^*; \alpha)}{\frac{\mathrm{d}\phi(x^*, [n-1]x^*; \alpha)}{\mathrm{d}x^*}}.$$

Our preceding consideration of uniqueness of equilibrium can be used to deduce that $\frac{\mathrm{d}\phi(x^*,[n-1]x^*;\alpha)}{\mathrm{d}x^*} < 0$ (see footnote 7), so the sign of $\frac{\mathrm{d}x^*}{\mathrm{d}\alpha}$ will be the same as the sign of $\phi_\alpha(x^*, [n-1]x^*; \alpha)$. Now

$$\phi_\alpha(x^*, [n-1]x^*; \alpha) = [1 - \beta + \beta v'] r^i_{x^i \alpha} + \beta v'' r^i_{x^i} r^i_\alpha. \tag{12}$$

Since this is analogous to the expression studied in the monopoly case (4), we can deduce that

$$\frac{\mathrm{d}x^*}{\mathrm{d}\alpha} < (>) 0 \Leftrightarrow \gamma \equiv -\frac{r^i v''}{v'} > (<) \frac{C'}{C' - r^i_{x^i}} \frac{r^i}{r^i_{x^i}} \frac{r^i_{x_i \alpha}}{r^i_\alpha}, \tag{13}$$

which allows us to draw the same conclusions for oligopoly as we presented in Propositions 1 and 2 for the case of monopoly.

Proposition 3. *Under Cournot competition, pure profit-maximizing firms and pure revenue-maximizing firms increase their output with a positive demand shock, while a decision maker that has non-linear evaluation of revenue and maximizes a combination of profit and revenue reduces firm output following a positive demand shock provided their valuation of revenue is sufficiently concave.*

Following a positive demand shock, because of the decreasing marginal utility derived from the revenue component, the marginal utility of output may decrease with sufficiently

[7] Note that $\frac{\mathrm{d}\phi(x^*,[n-1]x^*;\alpha)}{\mathrm{d}x^*} = \phi_{x^i} + [n-1]\phi_{X^{-i}}$, which is negative as this is exactly the condition that imposes the joint reaction function is a contraction.

concave utility functions, thereby incentivizing a manager to reduce its output for given opponents' aggregate output. The competitors' reaction functions will also shift inwards, thus producing a counter-effect whereby—since quantities are strategic substitutes—the focal firm increases its own output. Yet, with symmetric firms and given that the (unique) equilibrium has been shown to be stable under our modeling assumptions, it is necessarily the case that the former effect dominates the latter, hence implying a reduction in the equilibrium output for all firms.

4.3 Worked example

We now explore our example in which there are n competitors each of which evaluates revenue according to $v(r^i) = \delta - \frac{\theta}{2}[r^i]^2$, and demand is given by $P(X) = \frac{\alpha}{\mu + X}$ where $X = \sum_{i \in N} x^i$. In this Cournot competition setup we can assume $\mu = 0$ which allows us to derive explicit solutions from the outset.

For the symmetric profit-maximizing condition ($\beta = 0$) we have $x^* = \frac{\alpha[n-1]}{n^2 c}$ and for the symmetric revenue maximization output ($\beta = 1$) we have $x^* \to \infty$.

We now turn to the intermediate cases where $\beta \in (0, 1)$. The first-order condition is:

$$\phi(x^i, X^{-i}; \alpha) = \left[1 - \beta + \beta \left[\delta - \frac{\theta \alpha x^i}{X}\right]\right] \left[\frac{\alpha}{X} - \frac{x^i \alpha}{X^2}\right] - [1 - \beta]c = 0$$

and in a symmetric equilibrium this gives:

$$x^* = \frac{n-1}{n^2[1-\beta]c} \alpha \left[1 - \beta + \beta \left[\delta - \frac{\theta \alpha}{n}\right]\right].$$

We can now use this explicit solution to identify the conditions where the firm either expands or contracts output as product demand changes:

$$\frac{\mathrm{d}x^*}{\mathrm{d}\alpha} = \frac{n-1}{n^2[1-\beta]c} \left[1 - \beta + \beta \left[\delta - \frac{2\theta \alpha}{n}\right]\right], \tag{14}$$

and therefore

$$\frac{\mathrm{d}x^*}{\mathrm{d}\alpha} \lesseqgtr 0 \Leftrightarrow \theta \gtreqless \frac{n[1 - \beta + \beta\delta]}{2\alpha\beta}.$$

We now verify that for parameter combinations such that $0 < \delta - \frac{1-\beta}{\beta} < \frac{\theta\alpha}{n} < \delta$, which by inspection is non-empty for finite n, there is an equilibrium in which output is decreasing in α. First, note that $r^* = P^* x^* = \alpha / n$ since $P^* = \alpha / n x^*$. As such, in equilibrium $v' = \delta - \frac{\theta\alpha}{n} > 0$ by virtue of the fact we assume $\frac{\theta\alpha}{n} < \delta$. Note that this assumption also implies $x^* > 0$. Equilibrium profit is given by $\pi^* = r^* - cx^* = \frac{\alpha}{n^2[1-\beta]}[1 - \beta - \beta[\delta - \frac{\theta\alpha}{n}]]$ after some manipulation. This is positive by virtue of our assumption that $\frac{\theta\alpha}{n} > \delta - \frac{1-\beta}{\beta}$. Inspection of (14) allows us to deduce that $\frac{\mathrm{d}x^*}{\mathrm{d}\alpha} < 0$ requires $\frac{\theta\alpha}{n} > \frac{1}{2}[\delta + \frac{1-\beta}{\beta}]$. Our assumption that

$\delta - \frac{1-\beta}{\beta} > 0$ implies $\frac{1}{2}[\delta + \frac{1-\beta}{\beta}] < \delta$, so there is a non-empty range of parameter values for which $\max\{\delta - \frac{1-\beta}{\beta}, \frac{1}{2}[\delta + \frac{1-\beta}{\beta}]\} < \frac{\theta\alpha}{n} < \delta$ where there is an equilibrium in which output is positive, marginal evaluation of revenue is positive, profit is positive, and output reduces in response to a positive demand shock.

Similar to the monopoly case, here too we can deduce that the comparative statics results shown to hold for the extreme case $\mu = 0$ are necessarily verified—by continuity—for strictly positive (but small enough) values of μ. This confirms our findings within the general model, namely: demand expansion can result in a firm reducing output when the decision maker has a sufficiently concave benefit function over the firm's revenue, i.e., θ is sufficiently large.

4.4 The implications of delegation

Unlike the monopoly setting, in an oligopolistic market structure firm owners may have incentives to delegate strategic decisions to an agent-manager as shown in the strategic delegation literature (e.g., Sklivas, 1987) . In a Cournot setting, in particular, scholars have shown that contracting a manager and misaligning his incentives with the ones of the principal-owner by offering the latter a remuneration that is a linear combination of profits and revenues enables a firm to commit to a more aggressive strategy. From an individual standpoint, such a strategy has been shown to be profitable, since by committing to high output the principal-owner can force competitors to reduce their own production. Yet, as all firms will delegate output decisions and given that these decisions are strategic substitutes, each firm will end up with a lower profit than if none had delegated. Hence, while delegation is individually optimal, it is collectively suboptimal.

In our setting the valuation of the revenue-component is non-linear, but it is immediate to show that when agents assign a positive weight to the revenue component of the their utility function, their reaction function shifts outwards with sufficiently concave valuations of revenue. Indeed, implicitly differentiating the problem's first-order condition allows us to obtain:

$$\frac{\mathrm{d}\hat{x}^i(X^{-i})}{\mathrm{d}\beta} = -\frac{v' r^i_{x^i} - [r^i_{x^i} - C']}{\phi_{x^i}} > 0,$$

We thus deduce that delegating decisions in our setting enables firms to behave more aggressively, hence potentially conferring them a competitive edge, and making it individually rational to delegate output decisions to managers with non-aligned preferences. In other words, the incentives that justify strategic delegation via the use of contracts also command strategic delegation to decision-makers with intrinsically non-aligned preferences, as is the case in our setting. Moreover, such collective strategic delegation will produce lower profits to all firms.

A second important point that we also explored in the monopoly case pertains to the variability of profits when market conditions change. The effect of a change in α on equilibrium profits under Cournot competition is given by:

$$\frac{d\pi}{d\alpha} = \frac{\partial \pi}{\partial \alpha} + \frac{\partial \pi}{\partial x}\frac{\partial x^*}{\partial \alpha} \tag{15}$$

Since this takes the same form as our analysis of the monopoly case, we can therefore conclude that when a decision maker is motivated by revenue in a Cournot setting and as a result of a positive (negative) shock reduces (increases) supply, profits will exhibit a higher variance than if the firm were operated by a profit-maximizing decision maker.

5 Concluding remarks

A conventional assumption within the industrial organization literature is that firms maximize profits. Although this is a reasonable assumption from the point of view of the firm owners, many important strategic decisions are taken by agents such as managers, who hold complementary objectives, such as maximizing sales, market share, or performance relative to competitors. To investigate the consequences of these alternative objectives, current approaches often assume a linear evaluation of these goals. Yet there is no *a priori* rationale to assume such a restrictive setting and, indeed, a general approach may capture more of the realistic settings with which decision makers are faced, such as incentives to increase their power, status, prestige or empire building.

This article investigates both a monopoly and oligopoly framework in which decision makers' objectives are a weighted sum of profit and utility over revenue. Our major innovation has been to assume non-linear preferences over revenue; namely, we allow the decision maker to have diminishing marginal utility over revenue. This means that our model includes cases where incremental gains in revenue are more valuable when revenue is scarce compared to when it is abundant. The non-linearity in preferences has important consequences under both market structures considered in this article. Under monopoly, if the decision maker is *solely* either profit maximizing or revenue maximizing, a negative demand shock reduces marginal revenue and output will fall. Yet we find that if the decision maker's objective is a weighted average of profit and the utility derived from revenue, then an exogenous negative demand shock may result in *increases* in output. Intuitively, a contraction in market demand will (i) reduce marginal profits but (ii) increase the marginal utility of revenue. If the latter effect is stronger than the former then demand shocks can result in an increase in output. This occurs when the decision maker's utility over revenue is sufficiently concave. Similarly, positive demand shocks may result in reductions in output. The same logic is demonstrated to carry over to Cournot oligopolies, despite the strategic substitutability characterizing such markets and giving rise to a strategic effect incentivizing decision makers to expand their output when competitors contract their own.

Unlike important contributions to the literature on managerial delegation, in this paper we have not engaged with the question of the optimal degree of delegation in the monopoly or oligopoly model. Instead, we have taken the degree of delegation as given, and studied some implications of non-linear evaluation of revenues, namely understanding output choices in the presence of demand shocks. We believe that our contribution will open the way for further research on firms' optimal delegation schemes, that need not be simple convex combinations of profit and revenue.

Appendix: Heterogeneous firms

In Section 4 we assume homogeneous firms for ease of exposition, but in this appendix we extend the analysis to consider heterogeneous firms, where there is common knowledge of firms' characteristics.

The first-order condition for each firm is

$$\phi(x^i, X^{-i}; \alpha) \equiv \frac{\partial U}{\partial x^i} = [1-\beta][r^i_{x^i}(x^i, X^{-i}; \alpha) - C^{i\prime}(x^i)] + \beta v^{i\prime}(r^i(x^i, X^{-i}; \alpha)) r^i_{x^i}(x^i, X^{-i}; \alpha) = 0.$$

Following the approach of the aggregative games literature (see Corchon (1994), and the introduction of Dickson (2017) for a straightforward explanation), we now define, for each firm, a *share function* which is derived from the first-order condition in which X^{-i} is replaced with $X - x^i$ and x^i is replaced with $\sigma^i X$, with σ^i denoting firm i's share of total supply. Let us write marginal revenue in these terms as $\varphi^i(\sigma^i, X; \alpha) \equiv r^i_{x^i}(\sigma^i X, [1-\sigma^i]X; \alpha) = P(X; \alpha) + \sigma^i X P_X(X; \alpha)$. Firm i's share function, when multiplied by X, will give the supply of firm i consistent with a Nash equilibrium in which the aggregate supply of all firms is X. Share functions are defined only when their value is no larger than 1, and take the form $s^i(X; \alpha) = \max\{0, \sigma^i\}$ where σ^i is such that

$$\begin{aligned} &\phi^i(\sigma^i X, X[1-\sigma^i]; \alpha) = 0 \Rightarrow \\ &\qquad l^i(\sigma^i, X; \alpha) \equiv [1-\beta^i][\varphi^i(\sigma^i, X; \alpha) - C^{i\prime}] + \beta^i v^{i\prime} \varphi^i(\sigma^i, X; \alpha) = 0. \end{aligned}$$

Share functions identify individually consistent behavior. Nash equilibrium requires consistency at the aggregate level. It is a matter of definition-chasing to recognise that there is a Nash equilibrium with aggregate supply $X > 0$ if and only if

$$\sum_{i=1}^{n} X s^i(X; \alpha) = X \Leftrightarrow S(X; \alpha) \equiv \sum_{i=1}^{n} s^i(X; \alpha) = 1.$$

We now want to determine the properties of individual, and consequently aggregate, share functions. By the implicit function theorem,

$$s^i_X = -\frac{l^i_X}{l^i_{\sigma^i}}.$$

Now,

$$l^i_{\sigma^i} = \beta^i v^{i\prime\prime} \varphi^i r^i_{\sigma^i} + [1-\beta^i+\beta^i v^{i\prime}]\varphi^i_{\sigma^i} - [1-\beta^i] X C^{i\prime\prime} < 0$$

as $\varphi^i_{\sigma^i} = XP_X < 0$ and $r^i_{\sigma^i} = XP > 0$ so each term is negative by assumption, and

$$\begin{aligned} l^i_X &= \beta^i v^{i\prime\prime} \varphi^i r^i_X + [1-\beta^i+\beta^i v^{i\prime}]\varphi^i_X - [1-\beta^i]\sigma^i C^{i\prime\prime} \\ &= \beta^i v^{i\prime\prime} \sigma^i [P + XP_X][P + \sigma^i X P_X] + [1-\beta^i+\beta^i v^{i\prime}][P_X + \sigma^i[P_X + XP_{XX}]] - [1-\beta^i]\sigma^i C^{i\prime\prime} \\ &< \beta^i v^{i\prime\prime} \sigma^i [P + XP_X][P + \sigma^i X P_X] + [1-\beta^i+\beta^i v^{i\prime}]\sigma^i[2P_X + XP_{XX}] - [1-\beta^i]\sigma^i C^{i\prime\prime} < 0 \end{aligned}$$

as the second and third terms are negative by assumption, and the discussion on the additional assumption we require for uniqueness on p15 implies the first term is negative. This allows us to conclude that individual share functions are strictly decreasing in $X > 0$.

To derive the other properties of share functions, it is useful to envisage $l^i(\sigma^i, X; \alpha)$ plotted as a (strictly decreasing) function of σ^i. First, note that

$$\begin{aligned} l^i(0, X; \alpha) &= [1 - \beta^i][P - C^{i\prime}(0)] + \beta^i v^{i\prime} P \text{ and} \\ l^i(1, X; \alpha) &= [1 - \beta^i + v^{i\prime}\beta^i][P + XP_X] - [1 - \beta^i]C^{i\prime}(X). \end{aligned}$$

Let us define $\underline{X}^i$ by $l^i(1, \underline{X}^i; \alpha) = 1$, i.e., the value of X where the share function is just equal to 1. Such an $\underline{X}^i$ exists by virtue of the intermediate value theorem, noting that

$$\begin{aligned} l^i(1, 0; \alpha) &= [1 - \beta^i + v^{i\prime}(0)\beta^i]P(0) - [1 - \beta^i]C^{i\prime}(0) > 0 \text{ and} \\ \lim_{X\to\infty} l^i(1, X; \alpha) &= [1 - \beta^i + \lim_{X\to\infty} v^{i\prime}(XP)\beta^i] \lim_{X\to\infty}[P + XP_X] - [1 - \beta^i] \lim_{X\to\infty} C^{i\prime}(X) < 0, \end{aligned}$$

where the second line follows from marginal revenue in the large-X limit being either zero or negative. For $X < \underline{X}^i$ the share function would take a value that exceeds 1 and is therefore undefined. For $X \geq \underline{X}^i$ the share function, as previously noted, is strictly decreasing in X. As X increases without bound, note that $\lim_{X\to\infty} l^i(0, X; \alpha) \leq 0$ since $\lim_{X\to\infty} P(X) = 0$. If this is strictly negative then there is an $\bar{X}^i < \infty$, defined such that $l^i(0, \bar{X}^i; \alpha) = 0$ in which case the share function takes the value zero for all $X \geq \bar{X}^i$. If it is equal to zero then $s^i(X; \alpha) \to 0$ as $X \to \infty$.

To summarize, individual share functions are undefined for $X < \underline{X}^i$, take the value of 1 at $X = \underline{X}^i$, and then strictly decrease in X to take the value of zero at $\bar{X}^i < \infty$ if such a value exists, or converge to zero as X increases without bound.

The aggregate share function $S(X; \alpha)$ is defined only for $X \geq \max_i\{\underline{X}^i\}$ where it takes a value that exceeds 1. It then inherits the property of individual share functions that it is strictly decreasing in X and is either equal to zero for some large-enough X, or converges to zero as $X \to \infty$. Since it is continuous, the intermediate value theorem therefore allows us to conclude that there is a single value of X where $S(X; \alpha) = 1$, and therefore a unique Nash equilibrium.

When we consider a shock to demand which changes α, because share functions are strictly decreasing in X we can understand the effect of this from the effect of α on share functions. Formally, if we denote by X^* the equilibrium aggregate supply which satisfies $S(X^*; \alpha) = 1$, the implicit function theorem implies

$$\frac{\mathrm{d}X^*}{\mathrm{d}\alpha} = -\frac{\sum_{i=1}^n s^i_\alpha}{\sum_{i=1}^n s^i_X}$$

the sign of which, since we know $s^i_X < 0$, depends on the sign of the s^i_α. Using the implicit function theorem again,

$$s^i_\alpha = -\frac{l^i_\alpha}{l^i_{\sigma^i}},$$

the sign of which (since $l^i_{\sigma^i} < 0$) depends on the sign of l^i_α. Now,

$$\begin{aligned} l^i_\alpha &= [1 - \beta^i + \beta^i v^{i\prime}]\varphi^i_\alpha + \beta^i v^{i\prime\prime} r^i_\alpha \varphi^i \\ &= [1 - \beta^i + \beta^i v^{i\prime}] r^i_{x^i\alpha} + \beta^i v^{i\prime\prime} r^i_\alpha r^i_{x^i} \end{aligned}$$

(as $\varphi^i(\sigma^i, X; \alpha) \equiv r^i_{x^i}(\sigma^i X, [1 - \sigma^i]X; \alpha)$) which is analogous to our expression (12) in the symmetric analysis. As such, exactly the same conditions that govern whether a positive shock to demand results in a contraction in supply in the model with homogeneous firms apply to the case of heterogeneous firms if the sign of this effect is the same for all heterogeneous firms.

Notice that if the sign of s^i_α is negative for all firms then the aggregate supply decreases, but it could also be the case that $s^i_\alpha < 0$ only for a subset of firms and yet aggregate supply still declines so long as the aggregation of the effects makes $\frac{\mathrm{d}X^*}{\mathrm{d}\alpha} < 0$. Notice also that, even if $s^i_\alpha < 0$ for all firms, the individual supply of every firm does not necessarily decrease, but any increases are offset by sufficiently large reductions in supply by other firms so that the aggregate supply declines.

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