This is a peer-reviewed, author's accepted manuscript of the following conference paper:
Mauro, F., Paterson, D., Michalec, R., Boulougouris, E., & Vassalos, D. (2021).
A damage sampling method to reduce A-index standard deviation in the probabilistic assessment of ship survivability using a non-zonal approach.
Paper presented at 1st International Conference on the Stability and Safety of Ships and Ocean Vehicles.

A damage sampling method to reduce A-index standard deviation in the probabilistic assessment of ship survivability using a non-zonal approach

Francesco, Mauro, *University of Strathclyde* francesco.mauro@strath.ac.uk

Donald, Paterson, *University of Strathclyde* donald.paterson@strath.ac.uk

Romain, Michalec, *University of Strathclyde* romain.michalec@strath.ac.uk

Evangelos, Boulougouris, *University of Strathclyde* eboulougouris@strath.ac.uk

Dracos, Vassalos, *University of Strathclyde* dvassalos@strath.ac.uk

ABSTRACT

The present SOLAS damage stability regulations for passenger and dry cargo ships address vessel survivability after flooding due to collisions with a probabilistic framework. This concept has been extended to other possible hazards responsible for flooding of a ship, such as groundings (bottom or side). Therefore, probabilistic distributions have been provided for damage locations and dimensions, enabling ship survivability assessment to be based on Monte Carlo (MC) sampling of pertinent distributions for generation of damage breaches using a flexible non-zonal approach. Such a method introduces randomness into the process, leading to a dispersion of obtained A-indices within different batches of generated damages. In the present work, a Quasi Monte Carlo sampling method is applied to generate multiple sets of bottom grounding damages on a reference test barge available in literature. The obtained A-index has a significant data dispersion reduction compared to standard MC samples of equivalent size, reducing the number of cases necessary to obtain an engineering significant value for A-index.

Keywords: *Damage stability, non-zonal approach, damage breach distribution sampling, Quasi Monte Carlo method*

1. INTRODUCTION

Damage stability regulations for passenger and dry cargo ships address vessel survivability after flooding due to collisions with a probabilistic framework described by SOLAS (IMO, 2019). This framework has been extended by several EU-funded projects (HARDER, SAFEDOR, GOALDS, EMSA, eSAFE) to other possible source of flooding for a ship, such as bottom and side groundings. The analysis of dedicated database of accidents (IMO, 2004, 2012, Zaraphonitis et al., 2017) led to the definition of probabilistic distributions for damage locations and dimensions (Papanikolaou et al., 2011, Bulian et al., 2020). Moreover, these studies propose to abandon the classical SOLAS framework, based on a *zonal* approach (Lützen, 2001, Pawlowski, 2004), in favour of a flexible *non-zonal approach*, based on Monte Carlo (MC) generation of breaches.

The use of a MC approach for ship survivability assessment in damage condition has been widely applied not only for static analysis (Krüger et al., 2008, Krüger & Dankowski, 2019), but also for more advanced time-domain ship motion and flooding

simulation analyses, (Vassalos, D., 2008, Spanos & Papanikolaou, 2014, Ruponen et al., 2019, Atzampos et al., 2019). However, the application of MC sampling introduces randomness into the process, leading, to a dispersion of obtained survivability indices within different batches of generated damages. It is common practice to reduce this dispersion to acceptable levels by increasing the number of samples per each batch (about 10,000), considering at least five batches to obtain the resulting index (Bulian et al., 2016). In the present work, a Quasi Monte Carlo (QMC) sampling method is presented as an alternative to the conventional MC method, aimed to reduce the variance of the calculated survivability whilst reducing the number of samples.

The method has been applied on a reference test barge available in literature (Bulian et al., 2016), limiting the investigation to bottom groundings. Three different sample sizes have been used, comparing the results obtained according to standard MC sampling and the proposed QMC one on the final A- index.

2. THE ATTAINED SURVIVABILITY INDEX FOR A DAMAGED SHIP

According to the SOLAS probabilistic damage stability regulation, the ship survivability should be addressed in case of collision accidents. In such a case, the representative metrics of the damaged ship survivability is the attained index of subdivision *A*. The SOLAS framework prescribes the analysis of three draughts, namely:

- T_1 : deepest subdivision draught;
- T_2 : partial subdivision draught;
- T_3 : light subdivision draught.

For the three draughts, the index is given by the summation of the contribution of each damage case:

$$A_{T_j} = \sum_{i=1}^{N_c} p_i s_i \quad with\ j = 1,2,3 \tag{1}$$

Where i denotes each of the N_c unique groups of compartments describing a damage case, p_i is the occurrence of each of the damage case, and s_i is linked to the probability to survive the flooding of the i-case damaged compartments. SOLAS 2009 prescribes to determine p_i in analytical form, applying a preliminary zoning of the ship. The s_i are determined according to a *GZ*-based approach. The partial *A*-index, obtained according to eq. (1), should be weighted between the three draughts:

$$A = 0.4 A_{T_1} + 0.4 A_{T_2} + 0.2 A_{T_3} \tag{2}$$

The resulting index should then be compared with the required subdivision index *R*, to ensure satisfaction of $A \geq R$. The SOLAS framework can be extended to other damage type, also applying a direct approach (Bulian et. al., 2020). Hereafter, the case of bottom groundings will be described.

2.1 Direct approach for generation of bottom grounding breaches

Bottom grounding damages, usually referred to as *B00 damages*, can be analysed in a ship damage stability framework by applying a direct approach. To do this, it is necessary to follow the next steps:

- Determination of sample breaches;
- Determination of damage cases;
- Survivability assessment.

The determination of the sample breaches requires definition of a geometrical model of the damage breach. In accordance with GOALDS project findings, damages are supposed to be box-shaped, as they can be considered conservative for damage stability of ships compared to other possible damage geometries, such as triangular or parabolic penetrations (Papanikolaou et al., 2011). This is representative of an equivalent potential damage, considering the vessel region actually damaged also by multiple breaches. This assumption has been initially considered valid

for static calculation purposes (Bulian et al., 2016), but, in the absence of more detailed damaged data, it is used also in dynamic calculations (Atzampos et al., 2019). With this modelling assumptions, the geometric parameters that should be defined to describe a *B00* potential damage are:

- Longitudinal position of damage forward end X_F (m);
- Lateral position of measured damage forward end centre Y_F (m);
- Longitudinal extent of the potential damage *Lx* (m);
- Lateral potential damage extents *Ly* (m);
- Vertical penetration of the potential damage *Lz* (m).

The probabilistic framework for groundings according to non-zonal approach, prescribe to use specific damage distributions for the above-listed geometric characteristics. It is usual to refer to distributions given in non-dimensional form, as shown in Figure 1.

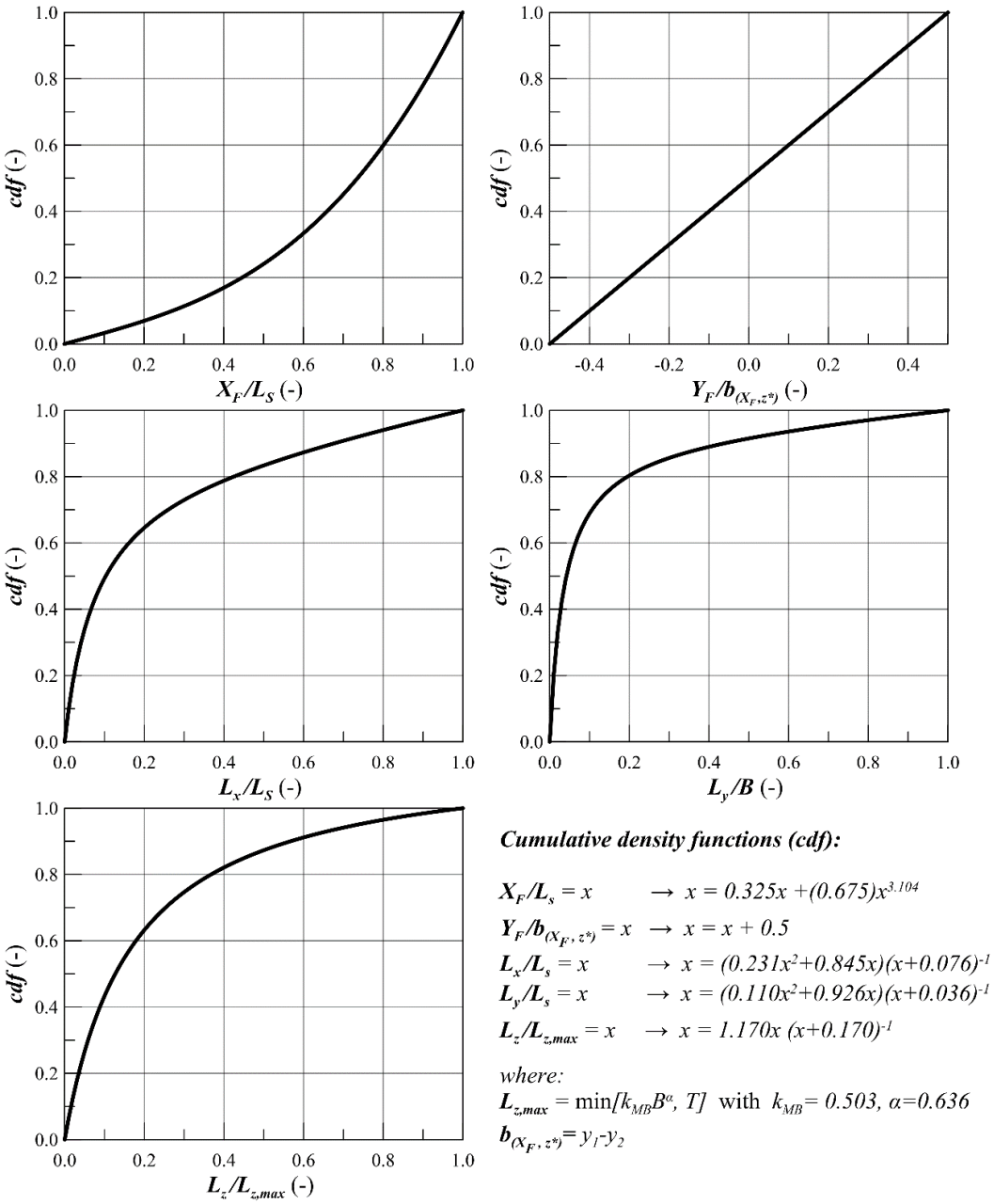

Figure 1 Cumulative density functions of dimensionless geometric parameters for bottom groundings B00 damages.

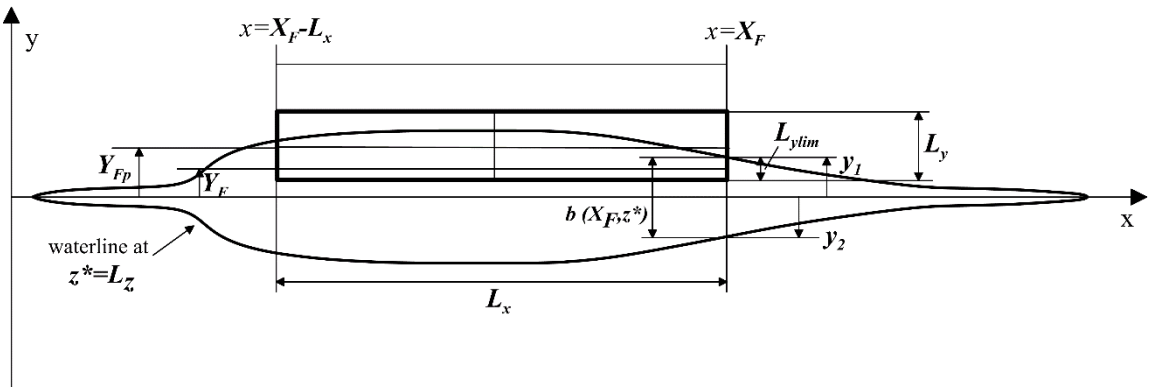

Figure 2 Geometrical parameters characterisation for B00 groundings damages.

However, a further step is needed to define and position the equivalent 'box shaped' damage from the given geometric characteristics. In fact, Y_F is the lateral position of the effective measured damage and not the lateral position of the centre of the potential damage. Therefore, defining Y_{Fp}, the lateral position of the potential damage centre, the following system needs to be solved for its evaluation:

$$\begin{cases} Y_{Fp} = Y_F + \dfrac{sign(\delta)}{2}\max[(Ly - Ly_{\lim});0] \\ \delta = Y_F - \dfrac{y_1 + y_2}{2} \\ Ly_{\lim} = \min[2(y_1 - Y_F);2(Y_F - y_2)] \end{cases} \quad (3)$$

Where, y_1 and y_2 are the portside and starboard side coordinates of the reference waterline at $z^*=z_{bottom}+Lz$ and $x=X_F$. As shown in Figure 1, Y_F is given in a non-dimensional form Y_F'; therefore, the actual framework requires further adjustment to find the final value as follows:

$$Y_F = \frac{y_1 + y_2}{2} + Y_F' b(X_F, z^*) \quad (4)$$

An explanatory representation of the resulting *B00* damage definition and positioning is given in Figure 2. It should be noted that the described geometrical characterization may still lead to a small amount of non-contact damages, meaning potential box damages outwith the ship length (Bulian et al., 2016). It is then possible sampling N_b breaches from the distributions, as all the above-mentioned quantities are supposed to be independent random variables. Regardless the method used to sample the data, damages should be regrouped in the N_c unique groups of compartments defining damage cases. From this regrouping, p_i values are determined and static stability calculations can be carried out on the N_c damage cases to evaluate the associated s_i. Calculation of A-index per each draft is then straight forward applying equation (1).

As the process includes five random variables, the final A-index is subjected to uncertainties. Therefore, it is worth considering possible methods to reduce uncertainties from the final result. This is possible considering more in detail the sampling of the independent random variables.

3. DAMAGE SAMPLING METHOD

The adoption of a probabilistic framework for generation of damage breaches requires the use of sampling techniques to derive damage characteristics from the marginal distributions described above. There are several methods that could be adopted to sample independent random variables according to a defined distribution (Devroye, 1986). The most commonly used in the framework of damage stability is the inversion of the cumulative density function. The sampling is based upon the following general property; if F is a continuous cumulative density function in $(-\infty,+\infty)$ with inverse F^{-1} defined by:

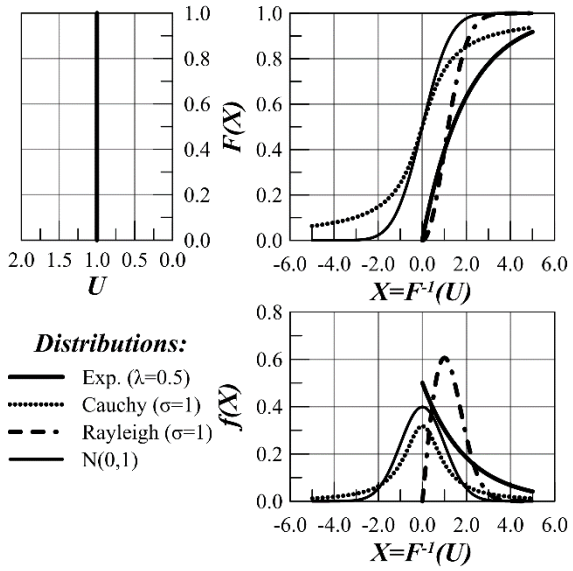

Figure 3 Inversion method example for general theoretical continuous statistic functions.

$$F^{-1}(u) = \inf\{x : F(x) = u, 0 < u < 1\} \quad (5)$$

If U is a uniform random variable in [0,1], then $F^{-1}(U)$ is distributed according to F, and also if a variable X has cumulative F (and associated density f), then $F(X)$ is uniformly distributed in [0,1]. The property is also extended to distributions with finite support. This useful inversion property is graphically described in Figure 3 for a set of theoretical continuous statistic distributions.

3.1 Pseudo-random approach

The adoption of an inversion method can be extended also to discrete distributions; thus, it is applicable to a sampling process, where U is generated with a discrete sequence of random numbers. When a MC simulation needs to be performed, the generation of U requires the use of pure pseudo-random sequences. For such cases, the reproduction of a uniform distribution requires the use of a high number of samples. In Figure 4 an example for the normal distribution $N(0,1)$ is given, showing the differences between the sampled U and obtained X using 10^2 and 10^5 samples.

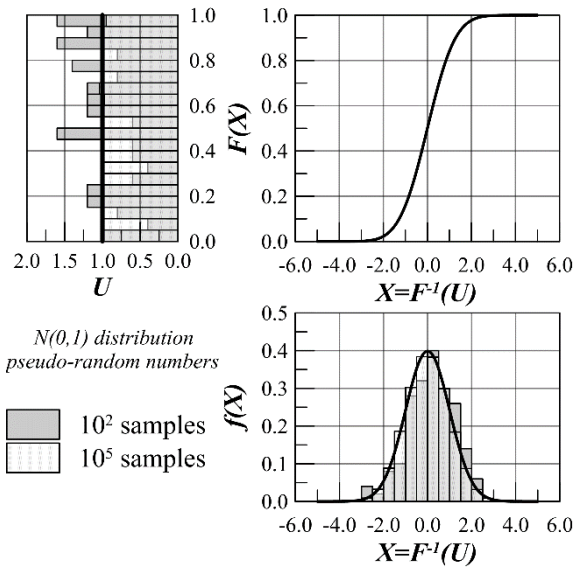

Figure 4 Inversion method example, using pseudo-random numbers on a $N(0,1)$ with different sample size.

It can be observed that, with a high number of samples, U is well reproduced with pseudo-random number generation. However, decreasing the samples number, the accuracy of fitting U decreases, reflecting this weakness also in the resulting random variable X. Moreover, the adoption of pseudo-random generation introduces additional uncertainties to the total process. As the final aim of a MC simulation is the evaluation of a quantity (in this case the A-index), a pure pseudo-random process produces results for multiple repetitions within a confidence interval related to the associated variance. With this method, the more samples are produced, the more the variance decreases. However, other sample approaches can be used to reduce the variance of the process.

3.2 Quasi-random approach

A convenient method to reduce variance in a sampling process is changing the methodology to generate random numbers. Different ways can be pursued to achieve a sequence of numbers representing a U distribution in [0,1]. Between them, a possible solution is given by adoption of quasi-random samples (Niederreiter, 1987).

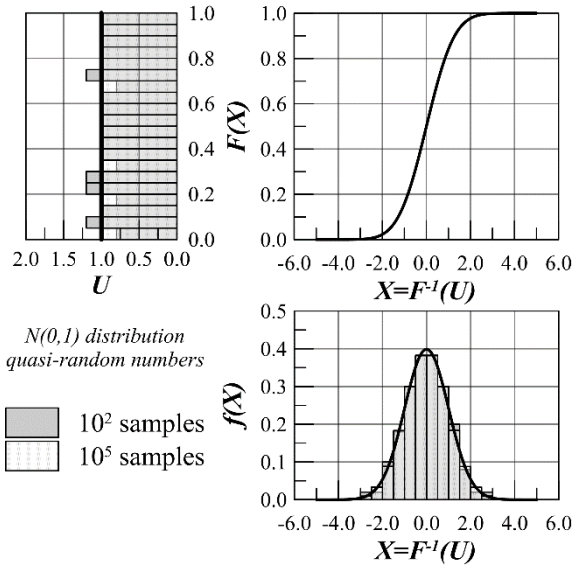

Figure 5 Inversion method example using quasi-random numbers on a $N(0,1)$ with different sample size.

Quasi-random samples differ from pseudo-random ones because of the fully deterministic nature of the sequence of generated numbers. Different methods can be used to generate deterministic low discrepancy number sequences (LDS), as the Halton, Faure, Sobol or Niederreiter ones (Niederreiter, 1988). In this work use is made of Sobol sequences, as several studies have proven its advantages compared to other LDSs (L'Ecuyer & Lemieux, 2002, Jaeckel, 2002). This is true because the sequence has been constructed such as to have a better uniformity of distribution with increasing sample but a good distribution even with fairly small initial samples with a very fast computational time (Sobol et al., 2011).

As a main consequence for the sampling of a random variable, this quasi-random approach allows to well reproduce a given theoretical distribution with a rather low number of samples. As an example, in Figure 5 the case of the $N(0,1)$ is shown, adopting the same sample sizes as for the pseudo-random case previously reported. It is evident that the quasi-random approach is capable to reproduce the distribution also with 10^2 samples. A comparison with the results in Figure 4 highlights that pseudo-random and quasi-random sampling are comparable only with a high number of samples.

The inversion process can be used also to produce multivariate random variables distributions. In case of statistically independent variables, the final outcome is a superposition of multiple inversion methods on the marginal distributions. In Figure 6, an example is given for the case of two independent random variables following a $N(0,0.25)$ and a $B(2,5)$ distributions, respectively. The reported case shows a more efficient coverage of the domain given by the adoption of the quasi-random approach. Also with a high number of samples, e.g. 10^4, where the single distributions are well reproduced by both methods, the quasi-random approach covers more uniformly the multivariate variables domain. In fact, this method is avoiding excessive sample agglomerations typical for pseudo-random methods. The quasi-random methods, avoiding agglomeration of samples, reduce also the possibility to have sample less holes in the domain, thus, theoretically granting a higher convergence of a Monte Carlo integration method that, in this case, is named Quasi-Monte Carlo method.

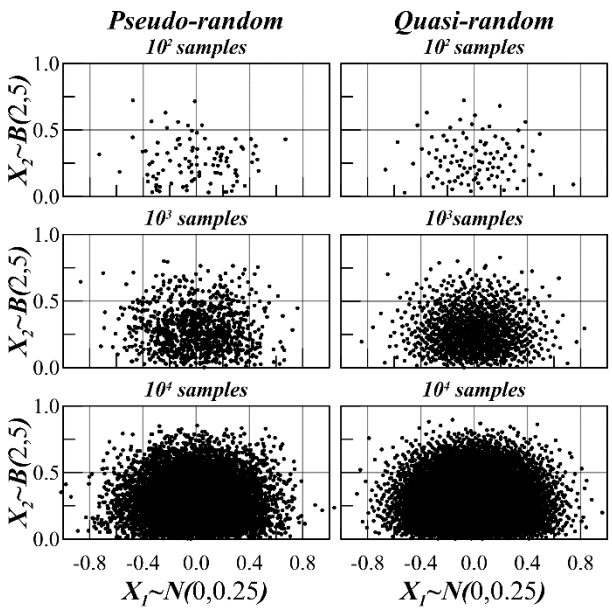

Figure 6 Pseudo-random and quasi-random sampling of a $N(0,0.25)$ and a $B(2,5)$ marginal distributions with different sample size.

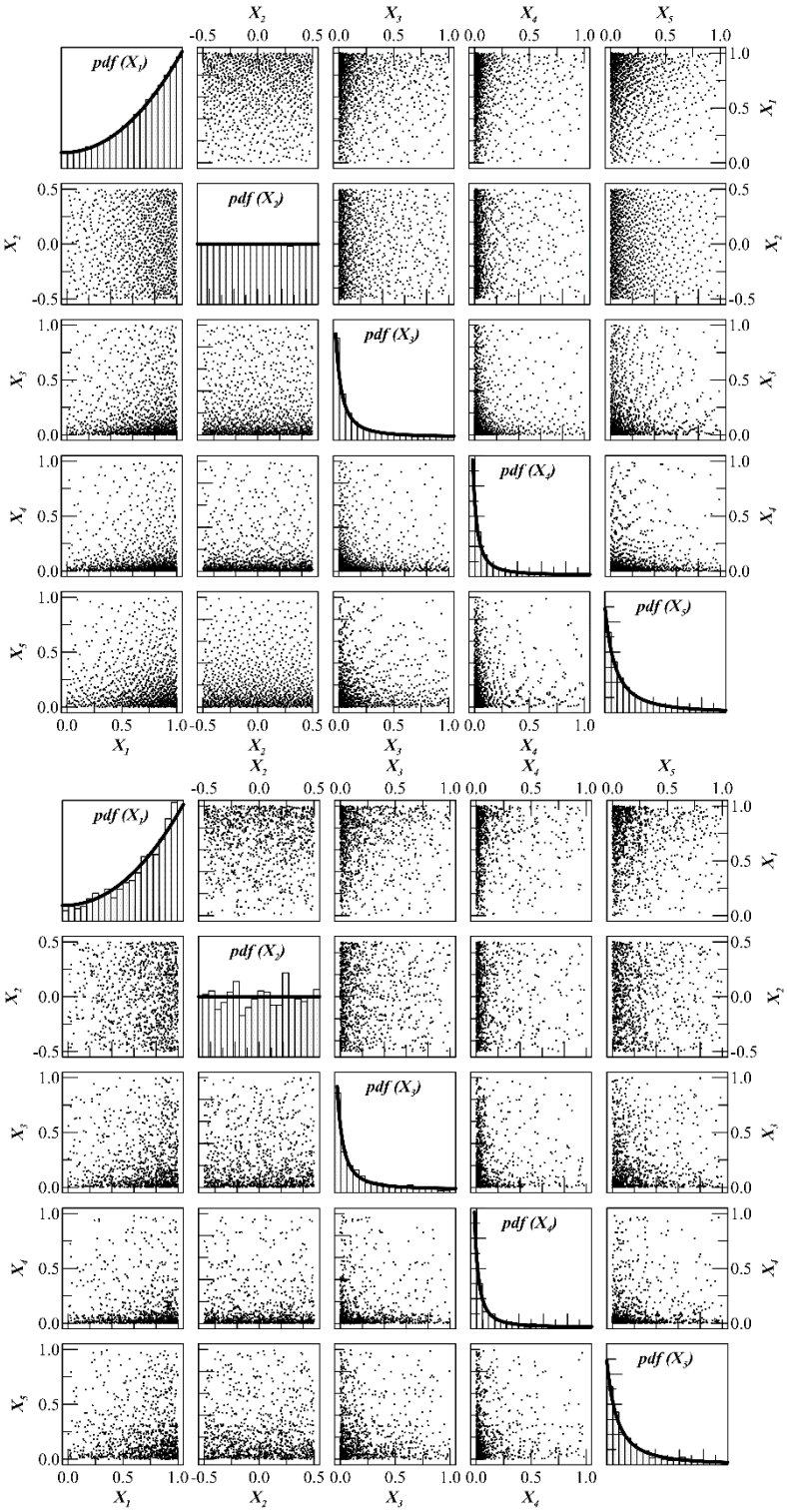

Figure 7 Quasi-random (*top*) and pseudo-random (*bottom*) sampling of 10^3 *B00* damages.

3.3 Sampling *B00* damages

The application of the proposed sampling procedure to the damage distributions for bottom groundings, requires a multivariate sample on a five-dimension hypercube. Considering the distributions presented in Figure 1, the quasi-random and the pseudo-random sampling methods have been used to generate damage cases. Figure 7 reports a comparison of the population obtained with the two sampling strategies for N_b=10^3 sample size.

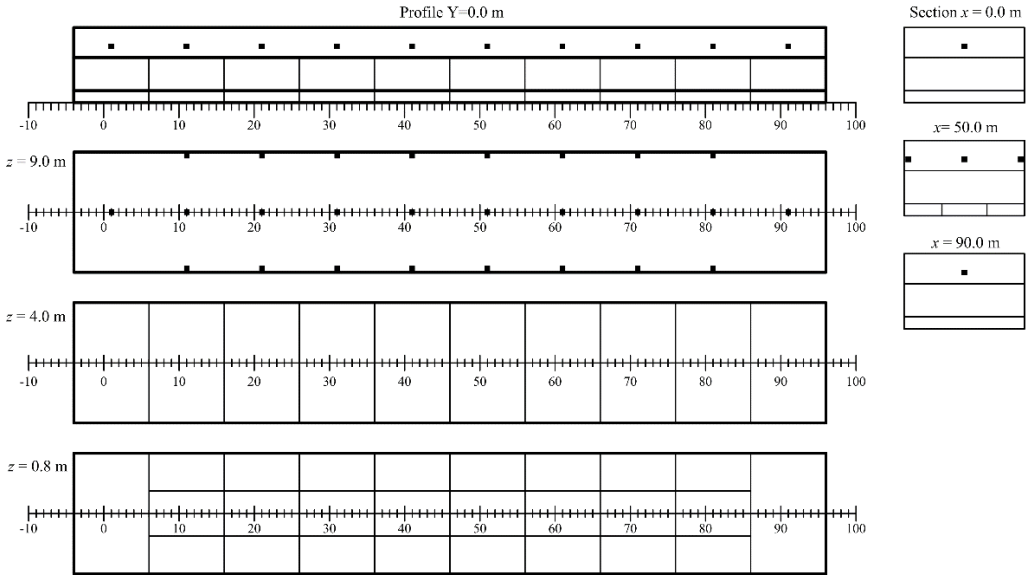

Figure 8 Test barge general layout and openings.

In Figure 7, non-dimensional values of the damage geometric dimension and location are reported with the following nomenclature: $X_1=X_F/L_s$, $X_2=Y_F/b$, $X_3=L_x/L_s$, $X_4=L_y/B$ and $X_5=L_z/L_{z,max}$. It can be observed that the quasi-random samples based on the Sobol sequences cover more evenly the domain compared to the pseudo-random ones. This confirms the trend previously observed for the mono and bi-variate cases earlier presented. For the sake of brevity, only the $N_b=10^3$ case is here reported. Increasing the number of samples to 10^5, then the quasi-random sampling covers all the domain, while pseudo-random sampling has difficulties to fill the entire design space. Therefore, the usage of different sampling strategies may affect the evaluation of the final survivability index for a vessel.

4. APPLIED EXAMPLE

The proposed sampling procedure is here applied to a reproducible barge available in literature for bottom groundings damage survivability (Bulian et al., 2016). This barge has been used because it represents a good benchmarking example for damage stability assessment. Therefore, it is the most indicative case for testing new procedures/methods for damage stability calculations.

Table 1. Test barge main characteristics

Quantity		***value***	***unit***
Length over all	L	100.0	m
Breadth	B	16.0	m
Construction height	D	10.0	m
Deepest subdivision draught	T_1	4.0	m
Partial subdivision draught	T_2	3.6	m
Lower subdivision draught	T_3	3.0	m
Metacentric height	GM_T	2.0	m

The main characteristics of the barge are summarised in Table 1, and a view of the general arrangement is given in Figure 8. The internal subdivision is quite simple, being composed only by box-shaped compartments. A double bottom is present with a height of 1.6 m, divided in 10 longitudinal zones, and, except for the fore and aft end, in 3 transversal zones. The double bottom compartments are associated with an unprotected opening (represented as black squares in Figure 8), vertically positioned at 7.5 m above the ship bottom and longitudinally positioned at the compartment centre. For the centre compartments, the opening is transversally centred, while the side one is located at 7.5 m from centreline to starboard or portside. The presence of unprotected openings influences the *GZ* curve for *s* factor calculation.

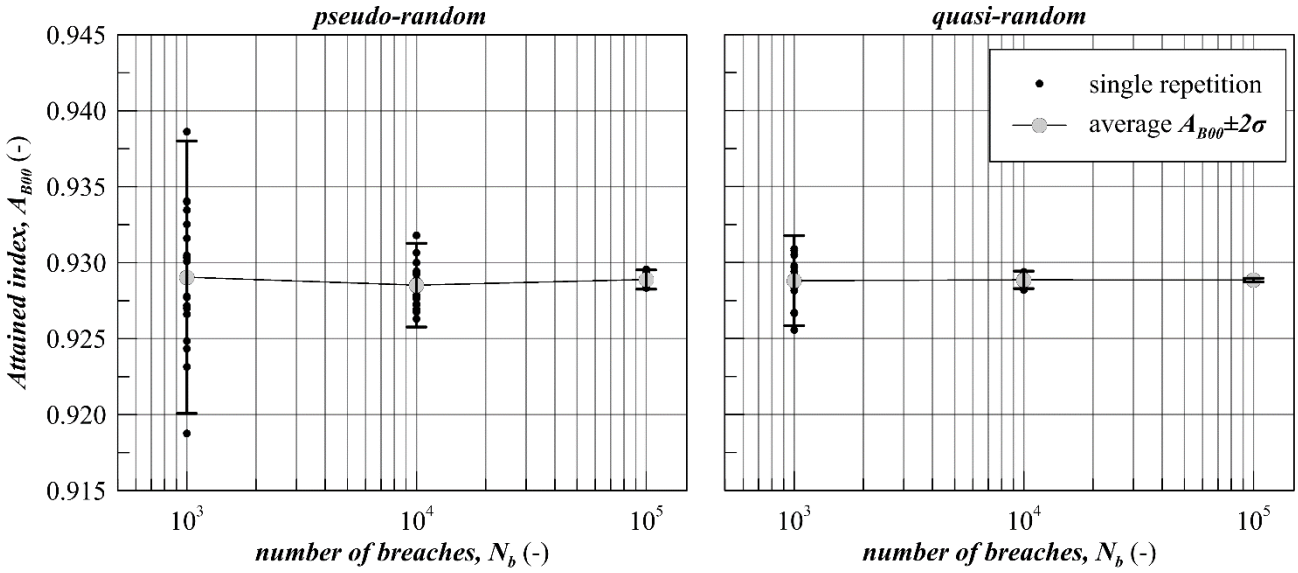

Figure 9 Attained index calculations for *B00* damages on the test barge.

The attained subdivision index for bottom groundings has been calculated according to the metrics described by equation (2), evaluating the partial indices with equation (1). Adopting this calculation, the partial A-indices are integrals obtained from a Monte Carlo (in case of pseudo-random samples) or Quasi-Monte Carlo (for quasi-random samples) process. To perform the calculations, the non-dimensional damages sampled according to the two proposed methods needs to be dimensioned as described in section 2, in this case using the total barge length as L_s. For all the tested cases, repetitions of 20 damage batches have been generated considering 10^3, 10^4 and 10^5 samples. The generated damages have been regrouped to determine the N_c unique damage cases for each calculation batch.

The calculation of the s factor is considering the final stage of flooding, as the simplified layout of the barge does not include cross-flooding or possible transitory phases. The calculation of the heeling moments includes the presence of passengers on side (750 persons, 75 kg each at 7.2 m from centreline) and wind effect. For compliance with the reference case, a *GM* of 2 metres has been considered for all the three draughts, together with an internal permeability of 95% for all the compartments.

An in-house tool has been used to generate the damage cases and the associated p values, using both pseudo-random and quasi-random procedures. The tool has been here used to generate *B00* damage cases; however, it is capable to produce damages also for collisions, side groundings or custom damages. The static calculations for the damage cases have been performed with the software PROTEUS3 (Jasionowski, 2001) available at MSRC.

The results obtained applying the different sample methods on multiple damage batches with different sizes are reported in Figure 9. For all the tested sampling size, the resulting A_{B00} of each single run is reported together with the associated mean and 2σ confidence interval, with σ being the corrected standard deviation for A_{B00} across the N_r repetitions:

$$\sigma = \sqrt{\frac{1}{N_r - 1}\sum_{i=1}^{N_r}\left(A_{B00_i} - \overline{A_{B00}}\right)^2} \tag{6}$$

This quantity is a simplified Gaussian confidence band, useful for the graphical understanding of the confidence and variability of the obtained results among different sample size and sampling methods. This gives a practical overview regarding the number of

breaches to be used to reach a given accuracy for A-index calculations. The reported results, clearly show the superiority of the quasi-random sample in comparison with the traditional pseudo-random sampling, as with only 10^3 samples, the standard deviation became the same as the 10^5 case with standard pseudo-random sampling. However, a more detailed discussion is needed to analyse possible implications for the design prospective.

5. DESIGN IMPLICATIONS

A simple evaluation of the results variance using standard deviation as expressed in equation (6) may result in a too simplistic analysis, leading to a misinterpretation of the advantages given by the quasi-random sampling process. First of all, a more detailed definition of the confidence level for the mean A-index can be adopted. In fact, according to the Central Limit Theorem, a normal approximation is valid only for a large amount of repetitions (i.e. more than 30). Here we are analysing a low number of repetitions; therefore, the confidence interval *CI* should be found using a Student distribution:

$$CI(c) = \overline{A_{B00}} \pm t \frac{s_A}{\sqrt{N_r}} \tag{7}$$

Where c is the desired confidence level, t is the inverse cumulative density function of the Student t-distribution with confidence level c and N_r-1 degrees of freedom, while s_A is the sample variance according to equation (6).

According to this definition, a confidence interval around the mean value of each sample can be determined, considering the number of repetitions performed. In Table 2, the values obtained for the adopted methods are reported and compared for a 95% confidence interval considering also data from the literature. The results highlight that the simulations performed with the conventional sampling method are in line with those obtained in the original study for this test barge (Bulian et al., 2020), not only for the final mean value but also for the partial draughts. However, these results were available only for the 10^4 sample size. Results in Table 2 refer to 20 repetitions of each case; therefore, the associated confidence interval is valid for these number of repetitions.

Table 1. Obtained A-indices with different sampling methods and sample size.

Method	***A-index case***	***Sample size***					
		10^3		***10^4***		***10^5***	
		mean	***CI (95%)***	***Mean***	***CI (95%)***	***mean***	***CI (95%)***
Bulian et al. 2020	***T_1***	***	***	0.9344	±1.10E-03	***	***
	T_2	***	***	0.9321	±1.00E-03	***	***
	T_3	***	***	0.9128	±9.50E-04	***	***
	Total	***	***	0.9292	±6.50E-04	***	***
Pseudo random	***T_1***	0.9344	±3.19E-03	0.9344	±1.07E-03	0.9340	±2.58E-04
	T_2	0.9314	±3.45E-03	0.9319	±1.12E-03	0.9321	±2.86E-04
	T_3	0.9136	±4.23E-03	0.9122	±1.01E-03	0.9122	±3.07E-04
	Total	0.9295	±2.10E-03	0.9285	±6.45E-04	0.9289	±1.48E-04
Quasi random	***T_1***	0.9335	±1.04E-03	0.9339	±2.07E-04	0.9338	±6.17E-05
	T_2	0.9324	±1.36E-03	0.9321	±2.04E-04	0.9321	±3.77E-05
	T_3	0.9122	±1.07E-03	0.9123	±2.08E-04	0.9123	±5.88E-05
	Total	0.9288	±6.93E-04	0.9289	±1.34E-04	0.9288	±2.75E-05

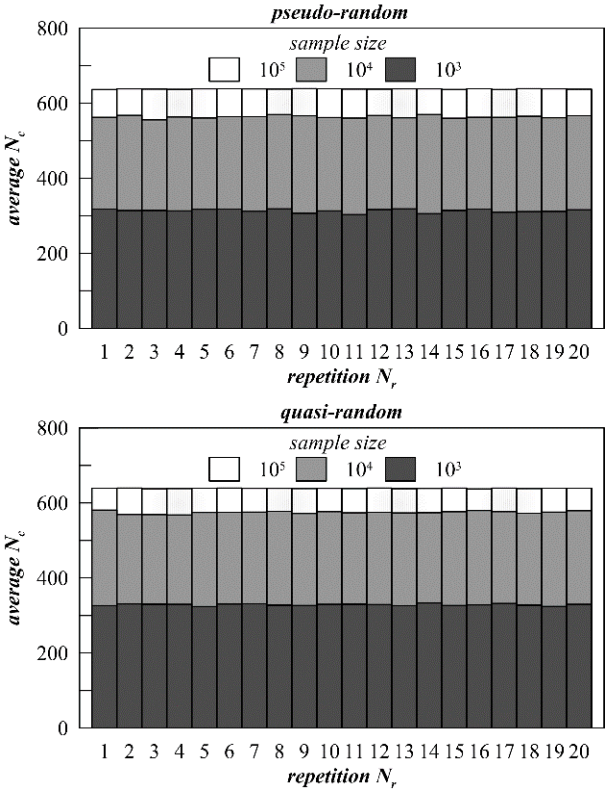

Figure 10 Average number of unique damage cases according to different sampling processes.

The collected data show that the quasi-random sampling generates really small *CI* around the mean value of the A-index, increasing the confidence in the obtained result. This property can be used to reduce the number of calculations needed to reach a certain confidence level on the obtained calculations, and there are two ways to achieve such target.

Taking as reference the *CI* values reported in Table 2, and assuming that the target *CI* is around the one resulting from the 20 repetitions with 10^4 samples, then it could be assumed that adopting the quasi-random samples, 20 repetitions with 10^3 samples are sufficient. However, this assumption is too simplistic, as it is not evident whether the smallest sample size is capable to identify a sufficient number of unique damage cases. Figure 10 shows the average number of unique cases identified by the different methods. It is evident that 10^3 samples are identifying only half of the possible damage cases. All the cases (640 for the tested barge) can be identified only with 10^5 samples, while 10^4 samples cover 87.9% of cases and 89.6% of cases with pseudo random and quasi-random samples respectively (which is a reasonable coverage).

Therefore, a wiser interpretation of the results may suggest a reduction of the repetitions needed to achieve a given confidence. Assuming as practical acceptable level of accuracy the one obtained with pseudo-random sampling of 5 repetition of 10^5 cases (Bulian et al., 2016), then, applying equation (7), 3 repetitions of 10^5 cases are giving a *CI*(95%) of ±5.61E-04 as average value, which is lower than ±1.74E-03 achievable with 5 repetitions of equivalent size with a conventional pseudo-random approach.

6. CONCLUSIONS

In the present work, the sampling process of damages within the SOLAS probabilistic framework has been analysed, proposing an alternative sampling process useful to reduce uncertainties while adopting a non-zonal approach. The present paper reports the case for bottom grounding damages on a simple reference barge, highlighting how the proposed method could significantly reduce the number of samples to be generated to achieve a target confidence level on the results. The same procedure, can be extended also to side groundings and collisions damages and adopted also for dynamic analysis, where the benefits in terms of calculation reduction could be even higher than for static calculations. The reduction of the number of breaches to be generated to reach a reasonable convergence level for the *A*-index is a significant improvement for the practical engineering application of damage stability.

7. ACKNOWLEDGEMENTS

The financial support of the EU project FLARE is acknowledged. The information and views set out in this paper are those of the

authors and do not necessarily reflect the official opinion of all FLARE partners.

8. REFERENCES

Atzampos, G., Vassalos, D., Cichowicz, J., Paterson, D. and Boulougouris, E., 2019, "eSAFE – cruise ship survivability in waves", Proceedings of the 17th International Ship Stability Workshop, pp. 265-274.

Bulian, G., Linderoth, D., Ruponen, P. and Zaraphonitis, G., 2016, "Probabilistic assessment of damaged ship survivability in case of groundings: development and testing of a non-zonal approach", Ocean Engineering, Vol. 120, pp. 331-338.

Bulian, G., Cardinale, M., Dafermos, G., Linderoth, D., Ruponen, P. and Zaraphonitis, G., 2020, "Probabilistic assessment of damaged survivability of passenger ships in case of grounding or contact", Ocean Engineering, Vol. 218, N. 107396.

Devroye, L., 1986, "Non-Uniform Random Variate Generation", Springler-Verlag, New York, NY.

IMO, 2004, "Bottom damage statistics for draft regulation 9", Technical report SLF 47/INF.4.

IMO, 2012, "Derivation of updated probability distributions of collisions and grounding damage characteristics for passenger ship", Technical report SLF 55/INF.7.

IMO, 2019, "International Convention for the Safety of Life at Sea (SOLAS)", Consolidated Edition as of 2019.

Jaeckel, P., 2002, "Monte Carlo methods in Finance", John Wiley & Sons, New York.

Jasionowski, A., 2001, "An Integrated Approach to Damage Ship Survivability Assessment", PhD Thesis, University of Strathclyde, Glasgow, Scotland, UK.

Krüger, S., Kehren, F.I. and Dankowski, H., 2008, "Leckstabilitätsberechnungen durch Monte Carlo Simulationen" (in German), Festschrift, Schriftenreiche Shiffbau, No. 641, pp. 60-85.

Krüger, S and Dankowski, H., 2019, "A Monte Carlo based simulation method for damage stability problems", Proceedings of the ASME 2019 38th International Conference on Ocean, Offshore and Artic Engineering (OMAE 2019), 9-14 June.

L'Ecuyer, P. and Lemieux, C., 2002, "Recent advances in randomized Quasi-Monte Carlo methods", in Modelling Uncertainties: An Examination of Stochastic Theory, Methods and Applications, Kluwer Academic Publisher, Boston, pp. 419-478.

Lützen, M., 2001, "Ship Collision Damage", PhD Thesis, Technical University of Denmark, Lyngby.

Niederreiter, H., 1987, "Point sets and sequences with small discrepancy", Monatshefte für Mathematik, Vol. 104, pp. 273-337.

Niederreiter, H., 1988, "Low-Discrepancies and Low-Dispersion Sequences", Journal of Number Theory, Vol. 30, pp.51-70.

Papanikolaou, A., Bulian, G. and Mains, C., 2011, "GOALDS – Goal based Damage Stability: collisions and grounding damages", Proceedings of the 12th International Ship Stability Workshop, pp. 37-44.

Pawlowski, M., 2004, "Subdivision and Damaged stability of Ships", Euro-MTEC book series, Gdansk, Poland.

Ruponen, P., Linderoth, D., Routi, A.L. and Aartovaara, M., 2019, "Simulation-based analysis method for ship survivability of

passenger ships", Ship Technology Research, Vol. 63 (3), pp. 182-194.

Sobol, I., Asotsky, D., Krenin, A. and Kucherenko, S., 2011, "Construction and Comparison of High-Dimensional Sobol' Generators", Wilmott Journal, pp.64-79.

Vassalos, D., 2008, Chapter 2: Risk-Based Ship Design - Methods, Tools and Applications, in A. Papanikolaou, Editor, Risk-Based Ship Design (pp. 17-98), Springer.

Zaraphonitis, G., Bulian, G., Hamann, R., Eliopoulou, E., Cardinale, M. and Luhmann, H., 2017, "eSAFE D2.2.1 – Description of methodology", Technical Report, Joint Industry Project eSAFE, Rev 2.