Control of Streamwise Vortices Developing in Compressible Boundary Layers

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ABSTRACT

We derive and test an optimal control algorithm in the context of compressible boundary layers, in an attempt to suppress or at least limit the growth of streamwise vortices caused by high-amplitude freestream disturbances. We aim to reduce the vortex energy and ultimately delay the transition to turbulent flow. We introduce flow instabilities to the flow either through roughness elements equally separated in the spanwise direction or via freestream disturbances. We analytically reduce the compressible Navier-Stokes equations to the compressible boundary region equations (CBRE) in a high Reynolds number asymptotic framework, based on the assumption that the streamwise wavenumber of the streaks is much smaller than the cross-flow wavenumbers. We employ Lagrange multipliers to derive the adjoint compressible boundary region equations, and the associated optimality conditions. The wall transpiration velocity represents the control variable, whereas the wall shear stress or the vortex energy designates the cost functional. We report and discuss results for different Mach numbers, wall conditions, and spanwise separations.

1. Introduction

Stream-wise oriented vortices and their accompanying streaks develop in boundary layers under the effect of freestream disturbances of wall nonuniformities. Examples of such applications include the flow around wings or turbo-machinery blades, flows in along the walls of wind tunnels or turbofan engine intakes. Not only that these instabilities result in the transition from laminar to turbulent flow, but this transition is also a significant source of noise in supersonic and hypersonic wind tunnels, subsequently causing high levels of interference with the measurements of the test section (Schneider [1]). One would want to lessen the energy associated with such stream-wise vortices, which is expected to delay early nonlinear breakdown and the transition to turbulence. Moreover, since the instability's transient part is the one influencing the growth of three-dimensional instabilities, responsible for the breakdown, the control strategy must focus on the restriction of the development of such transient modes.

Although significant achievements have been made in the last decades toward understanding the phenomenology behind the formation and the influence of the stream-wise vortices and streaks in incompressible boundary layers, the progress in the compressible regime has been slow and the bypass transition remains largely unexplored. A number of experiments aiming to control disturbances in laminar or turbulent boundary layers by blowing and suction have been conducted over the years. Several of them are briefly mentioned here. Gad-el-Hak & Blackwelder [2] used continuous or intermittent suction to eliminate artificially generated disturbances in a flat-plate boundary layer. The same idea was used in the experiments conducted by Myose & Blackwelder [3] to delay the breakdown of Görtler vortices. Jacobson & Reynolds [4] developed a new type of actuation based on a vortex generator to control disturbances generated by a cylinder with the axis normal to the wall, and unsteady boundary layer streaks generated by pulsed suction. Regarding the latter, the actuation was able to significantly reduce the span-wise gradients of the stream-wise velocity, which are known to be an important driving force of secondary instabilities (see Swearingen & Blackwelder [5]). In the experiments of Lundell & Alfredsson [6], stream-wise velocity streaks in a channel flow were controlled by localized regions of suctions in the downstream, which are found to be effective in delaying secondary instabilities and consequently the transition onset.

Optimal control in the framework of laminar or turbulent boundary layers has been utilized in a number of studies. There are numerous studies pertaining the application of optimal control of shear flows (see the review of Gunzburger [7] or a more recent review of Luchini & Bottaro [8], although the latter is in a slightly different context). Many studies have targeted the control of disturbances evolving in laminar or turbulent boundary layers using optimal control algorithms in the incompressible regime (*e.g.* [9, 10, 11, 12, 13, 14, 15]).

2. Scalings and Governing Equations

All dimensional spatial coordinates (x^*, y^*, z^*) are normalized by the spanwise separation of the disturbances λ^*, while the dependent variables by their respective freestream values, except the pressure, which is normalized by the dynamic pressure. Reynolds number based on the spanwise separation is defined as $R_\lambda = \frac{\rho^*_\infty V^*_\infty \lambda^*}{\mu^*_\infty}$ where μ^*_∞ is freestream dynamic viscosity. For boundary layer flows over curved surfaces, we define the global Görtler number as $G_\lambda = \frac{R^2_\lambda \lambda^*}{r^*}$ where r^* is the radius of the curvature.

We consider a compressible flow of uniform velocity V^*_∞ and temperature T^*_∞ past a flat or curved surface. The air is treated as a perfect gas so that the sound speed in the free-stream $c^*_\infty = \sqrt{\gamma R T^*_\infty}$, where $\gamma = 1.4$ is the ratio of the specific heats, and R is the gas constant; Mach number is assumed to be of order one. The compressible boundary region equations (CBRE) can be derived from the full steady-state compressible N-S equations, where the streamwise coordinate is scaled as $x = \bar{x}/R_\lambda$ while the other two coordinates are the same $y = \bar{y}$, $z = \bar{z}$, and the time is scaled

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as as $t = \bar{t}/R_\lambda$. Also, the crossflow components of velocity are expected to be small compared to the streamwise component, and variations of pressure are expected to be very small. This suggest the introduction of the scaling of dependent variables as: $u = \bar{u}; v = \bar{v}/R_\lambda; w = \bar{w}/R_\lambda; \rho = \bar{\rho}; p = \bar{p}/R_\lambda^2; T = \bar{T}; \mu = \bar{\mu}; k = \bar{k}$. CBRE are not included here for brevity.

For the optimal control approach, we first write CBRE in the generic and more compact form $\mathcal{G}(\mathbf{q}) = 0$, with appropriate initial and boundary conditions, where $\mathcal{G}()$ is the non-linear CBRE differential operator in abstract notation, $\mathbf{q} = (\rho, u, v, w, T)$ is the vector of state variables.

We define an objective (or cost) functional as $\mathcal{J}(\mathbf{q}, \phi) = \mathcal{E}(\mathbf{q}) + \sigma\left(\|\phi_x\|^{\beta_2} + \|\phi\|^{\beta_2}\right)$, where $\mathcal{E}(\mathbf{q})$ is a specified target function to be minimized. A common approach to transform a (nonlinear) constrained optimization problem into an unconstrained problem is by using the method of Lagrange multipliers (see, for example, [10], [7], [12]). To this end, we consider the Lagrangian $\mathcal{L}(\mathbf{q}, \phi, \mathbf{q}^a) = \mathcal{J}(\mathbf{q}, \phi) - \langle \mathcal{G}(\mathbf{q}), \mathbf{q}^a \rangle$, where $\mathbf{q}^a$ is the vector of Lagrange multipliers $(\rho^a, u^a, v^a, w^a, T^a)$, also known as the adjoint vector. In other words, the Lagrange multipliers are introduced in order to transform the minimization of $\mathcal{J}(\mathbf{q}, \phi)$ under the constraint $\mathcal{G}(\mathbf{q}) = 0$ into the unconstrained minimization of $\mathcal{L}(\mathbf{q}, \phi, \mathbf{q}^a)$ via $\delta\mathcal{L} = \frac{\partial \mathcal{L}}{\partial \mathbf{q}}\delta\mathbf{q} + \frac{\partial \mathcal{L}}{\partial \phi}\delta\phi + \frac{\partial \mathcal{L}}{\partial \mathbf{q}^a}\delta\mathbf{q}^a = 0$ All directional derivatives must vanish, providing different sets of equations: (i) adjoint CBRE equations are obtained by taking the derivative with respect to $\mathbf{q}$, $\frac{\partial \mathcal{L}}{\partial \mathbf{q}} = 0 \Rightarrow \mathcal{G}^a(\mathbf{q}^a) = 0$; (ii) optimality conditions are obtained by taking the derivatives with respect to ϕ, $\frac{\partial \mathcal{L}}{\partial \phi} = 0 \Rightarrow \mathcal{O}(\mathbf{q}^a, \mathbf{q}, \phi) = 0$; (iii) the original CBRE equations are obtained by taking the derivative with respect to $\mathbf{q}^a$, $\frac{\partial \mathcal{L}}{\partial \mathbf{q}_a} = 0 \Rightarrow \mathcal{G}(\mathbf{q}, \psi) = 0$. These equations form the optimal control system that can be utilized to determine the optimal states and the control variables.

The relationship between the state variables and the adjoint variables can be expressed by the adjoint identity, $\langle \mathcal{G}(\mathbf{q}), \mathbf{q}^a \rangle = \langle \mathbf{q}, \mathcal{G}^a(\mathbf{q}^a) \rangle + \mathcal{B}(\phi)$, where the last term, $\mathcal{B}$, represents a residual from the boundary conditions.

The adjoint CBRE are lengthy, so they are not included here for brevity (more details will be given on the presentation). Here, we present few results from the numerical solution of CBRE.

In figure 1, we show contours of temperature for Mach numbers of 0.8, 2, 4 and 6, for a boundary layer flow along a concave surface; the columns represent different spanwise separations. In figure 2, we show wall heat transfer distributions for Mach numbers 2 and 6. The wall heat flux shows a decrease as the spanwise separation is increased; there is also a decay of the wall heat flux in the streamwise coordinate range where the energy saturation occurs.

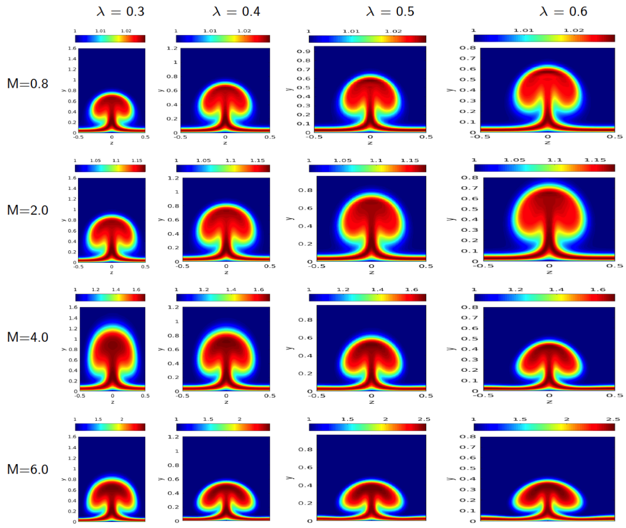

Fig. 1 Contours of streamwise velocity for different spanwise separations.

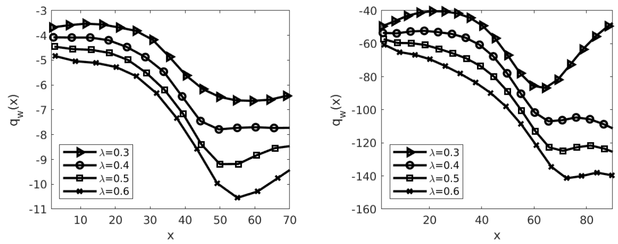

Fig. 2 Vortex energy: $M = 2.0$ (left); $M = 6.0$ (right).

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