Active Control of High-speed Boundary Layer Flows

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ABSTRACT
High-amplitude freestream disturbances, as well as surface roughness elements, trigger streamwise oriented vortices and streaks of varying amplitudes in laminar boundary layers, which can lead to secondary instabilities and ultimately to transition to turbulence. In this project we aim at deriving and testing a control algorithm based on the adjoint compressible boundary region equations, which are obtained in the assumption that the streamwise wavenumber of the disturbances is much smaller the the crossflow wavenumbers. In our control algorithm, the wall transpiration velocity represents the control variable, whereas the wall shear stress or the vortex energy designates the cost functional. We anticipate the optimal control of the streamwise vortices approach to lessen vortex energy and subsequently cause a delay of the occurrence of transition from laminar to turbulent flow. Here, we report the progress on the project.

1. Introduction

Flow control techniques have been the focus of many studies due to their wide range of applications as well as the impact that they could have on the various parameters of the flow, and thus on the efficiency, performance and noise levels. For instance, wings, engine inlets and nozzles, combustors, automobiles, aircraft, and marine vehicles all offer suitable configurations for the implementation of such control methods. Depending on the application and the desired result, one might want to reduce drag, increase lift, delay or accelerate the transition, enhance mixing, postpone flow separation or even manipulate a turbulence field.

Controlling transitional or fully developed turbulent boundary layers mainly intend to achieve a reduction in the energy carried by structures in the streamwise direction, which take the form of high- and low-velocity streaks originating in the near-wall region. For boundary layers over flat plates or wings, these streaks appear when the upstream roughness' height surpasses a specific critical value, or when the amplitude of the free-stream disturbances exceeds a certain threshold. Like many other types of instabilities, stream-wise oriented vortices and their accompanying streaks are present and very important in many engineering applications. Not only that these instabilities result in the transition from laminar to turbulent flow, but this transition is also a significant source of noise in supersonic and hypersonic wind tunnels, subsequently causing high levels of interference with the measurements of the test section, which in turn makes it a challenging task to compare between wind tunnel measurements and real flight conditions. With this in mind, one would want to lessen the energy associated with such stream-wise vortices, which is expected to delay early nonlinear breakdown and the transition to turbulence.

Although significant achievements have been made in the last decades toward understanding the phenomenology behind the formation and the influence of the stream-wise vortices and streaks in incompressible boundary layers, the progress in the compressible regime has been slow and the bypass transition remains largely unexplored.

Optimal control in the framework of laminar or turbulent boundary layers has been utilized in a number of studies. There are numerous studies pertaining the application of optimal control of shear flows (see the review of Gunzburger [3] or a more recent review of Luchini & Bottaro [5]).

Very few studies of active control techniques that can be applied in the framework of high-speed boundary layers were conducted, perhaps because of the technical challenges associated with real applications of such techniques. However, finding an optimal control variable that minimizes a given global quantity (e.g., the disturbance energy, the frictional drag, or the heat transfer) must be possible in any flow regime, and this is the main objective of this research. In general, for a given system, an optimal control scheme seeks to find a control law that determines a certain optimality criterion to be achieved, through a set of differential equations with solutions targeted to minimize a defined cost functional. Most of the studies in optimal control of shear flows were performed in the incompressible regime (e.g., Bewley & Moin [2], Gunzburger [3], or Sescu and Afsar [8]). There are several extensions to the compressible regime for various flows (e.g., Pralitis et al. [7], Barone & Lele [1]), but very few were applied in the context of compressible boundary layers.

2. Research Progress

We made progress on deriving the adjoint compressible boundary region equations that can be employed in an optimal control algorithm. We wrote a Fortran computer code, soling both the boundary region equations forward in the streamwise direction, and the corresponding adjoint equations, which are solved backwards. At this time, there are no conclusive results since more testing is needed; debugging of the code is also underway.

To fix ideas, we consider a compressible boundary layer flow over a flat or concave surface excited by upstream freestream disturbances or surface non-uniformities. The span-wise length scale, Λ^*, is in the same order of magnitude as the local boundary-layer thickness. The air is treated as a perfect gas so that the sound speed in the free-stream $c^*_\infty = \sqrt{\gamma R T^*_\infty}$,

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where $\gamma = 1.4$ is the ratio of the specific heats, and $R = 287.05 Nm/(kgK)$ is the universal gas constant; Mach number is assumed to be of order one.

All dimensional spatial coordinates (x^*, y^*, z^*) are normalized by the spanwise separation λ^*, while the dependent variables are nondimensionalized by their respective freestream values, except the pressure, which is normalized by the dynamic pressure. Reynolds number based on the spanwise separation, Mach number and Prandtl number are defined as $R_\lambda = \rho^*_\infty V^*_\infty \lambda^* / \mu^*_\infty, Ma = V^*_\infty / a^*_\infty, Pr = \mu^*_\infty C_p / k^*_\infty$, where μ^*_∞, a^*_∞ and k^*_∞ are freestream dynamic viscosity, speed of sound and thermal conductivity, respectively, and C_p is the specific heat at constant pressure.

We derived the compressible boundary region equations (BRE) from the full compressible Navier-Stokes equations based on the assumption, the streamwise, wall-normal and spanwise coordinates can be scaled as $x = \bar{x}/R_\lambda$ $y = \bar{y}$, $z = \bar{z}$, respectively, and the time is scaled as $t = \bar{t}/R_\lambda$. Also, the crossflow components of velocity are expected to be small compared to the streamwise component, and variations of pressure are expected to be very small. The BRE is parabolic in the streamwise direction and elliptic in the spanwise direction. Appropriate initial/upstream and boundary conditions are necessary to close the problem.

For the adjoint equations, we utilized the method of Lagrange multipliers, and transformed a (nonlinear) constrained optimization problem into an unconstrained problem ([4], [3]). To this end, we consider the Lagrangian $\mathcal{L}(\mathbf{q}, \phi, \mathbf{q}^a) = \mathcal{J}(\mathbf{q}, \phi) - \langle \mathcal{G}(\mathbf{q}), \mathbf{q}^a \rangle$,, where$\mathcal{J}(\mathbf{q}, \phi) = \mathcal{J}(\mathbf{q}, \phi) = \mathcal{E}(\mathbf{q}) + \sigma \left(\|\phi_x\|^{\beta_2} + \|\phi\|^{\beta_2} \right)$ is the cost functional based on the target function $\mathcal{E}(\mathbf{q})$, $\mathbf{q}$ is the state vector, and $\mathbf{q}^a$ is the vector of Lagrange multipliers $(\rho^a, u^a, v^a, w^a, T^a)$, also known as the adjoint vector. In other words, the Lagrange multipliers are introduced in order to transform the minimization of $\mathcal{J}(\mathbf{q}, \phi)$ under the constraint $\mathcal{G}(\mathbf{q}) = 0$ into the unconstrained minimization of $\mathcal{L}(\mathbf{q}, \phi, \mathbf{q}^a)$. The adjoint compressible boundary region equations are not included here for brevity (they are very lengthy).

Th control variables can be updated using optimality conditions or by a steepest descent method. The control algorithm starts with the solution to the compressible BRE for the uncontrolled boundary layer, followed by the solution to the adjoint compressible BRE (note that the adjoint BRE depend on the solution to the BRE). The difference between the wall shear stress and the original laminar wall shear stress is then compared to a desired value.

As a preliminary result, in figure 1, we show contours of temperature for a $M = 6$ boundary layer flow over a concave surface for different spanwise separations, by applying an adiabatic wall condition at the wall. These solutions were obtained by numerically solving the compressible boundary region equations. Figure 2 shows the kinetic energy distribution along the streamwise direction for the two wall temperatures. A reduction of the energy is noticed as the wall is heated at $T_w = 450$ K.

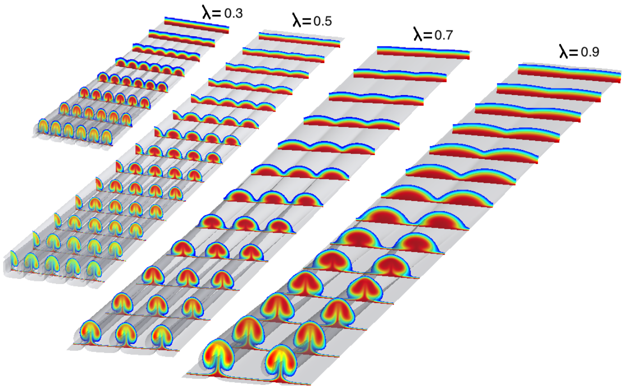

Fig. 1 Contours of streamwise velocity for different spanwise separations.

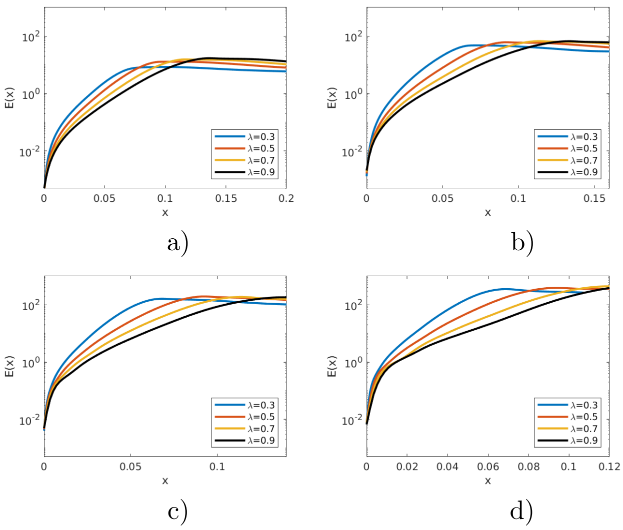

Fig. 2 Vortex energy: a) $M = 0.8$; b) $M = 2.0$; c) $M = 4.0$; d) $M = 6.0$.

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References

[1] Barone, M.F. & Lele, S. (2005) *J. Fluid Mech*, 540, pp. 301-335.

[2] Bewley, T. & Moin, P. (1994) *ASME DE*, 75, 221-227.

[3] Gunzburger, M. (2000) *Flow Turb. Comb.*, 65, 249.

[4] Joslin, R. et al. (1197), *AIAA J.*. 35, 816-824.

[5] Luchini, P., and Bottaro, A. (2014) *Annu. Rev. Fluid Mech.*. 46, 493-517.

[6] Paredes, P. et al. (2018) *AIAA Paper 2018-0057*.

[7] Pralitis, J.O. et al. (2000) *Flow Turb Comb.*, 65, 321-346.

[8] Sescu, A. & Afsar, M. (2018) *J. Fluid Mech*, 848, 5-41.

[9] Sescu, A. et al. (2018) *AIAAJ*, in press.