Control of Görtler vortices in high-speed boundary layer flows using nonlinear boundary region equations

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ABSTRACT

We formulate a mathematical framework for the optimal control of compressible boundary layers to suppress the growth rate of the streamwise vortex system before breakdown occurs. We introduce flow instabilities to the flow either through roughness elements equally separated in the spanwise direction or via freestream disturbances. We reduce the compressible Navier-Stokes equations to the boundary region equations (BRE) in a high Reynolds number asymptotic framework wherein the streamwise wavelengths of the disturbances are assumed to be much larger than the spanwise and wall-normal counterparts. We apply the method of Lagrange multipliers to derive the adjoint compressible boundary region equations and the associated optimality conditions. The wall transpiration velocity represents the control variable while the wall shear stress or the vortex energy designates the cost functional. The control approach induces a significant reduction in the kinetic energy and wall shear stress of the boundary layer flow. Contour plots visually demonstrate how the primary instabilities gradually flatten out as more control iterations are applied.

1. Introduction

Görtler vortices and the associated streaks develop in boundary layers due to freestream disturbances of wall non-uniformity. Examples of applications involving these flow structures include the flow around wings or turbo-machinery blades, flows along wind tunnel walls or turbofan engine intakes. Not only do these instabilities result in transition, but there is also a significant increase in noise for supersonic and hypersonic wind tunnels and this can cause interference with the measurements at the test section (Schneider [1]). This renders the comparison between wind tunnel measurements and real flight conditions a challenging task. Therefore, the objective function for a control algorithm in this context would be to lessen the streamwise vortex energy, which is expected to delay nonlinear breakdown and the transition. Since the transient part of the primary instability is responsible for the growth of the three-dimensional disturbances (and subsequent breakdown), the control strategy must focus on restricting the growth of the transient modes.

Although significant achievements have been made in the last decades toward understanding the phenomenology behind the formation and the influence of Görtler vortices and streaks in incompressible boundary layers, the progress in the compressible regime has been slow and the bypass transition at high upstream flow speeds remains largely unexplored. Gad-el-Hak & Blackwelder [2] used continuous or intermittent suction to eliminate artificially generated disturbances in a flat-plate boundary layer. The same idea was used in the experiments conducted by Myose & Blackwelder [3] to delay the breakdown of Görtler vortices. Jacobson & Reynolds [4] developed a new type of actuation based on a vortex generator to control disturbances generated by a cylinder with the axis normal to the wall, and unsteady boundary layer streaks generated by pulsed suction. Regarding the latter, the actuation was able to significantly reduce the span-wise gradients of the stream-wise velocity, which are known to be an important driving force of secondary instabilities (see Swearingen & Blackwelder [5]). In the experiments of Lundell & Alfredsson [6], stream-wise velocity streaks in a channel flow were controlled by localized regions of suctions in the downstream, which are found to be effective in delaying secondary instabilities and consequently the transition onset.

Optimal control in the framework of laminar or turbulent boundary layers has been utilized in a number of studies. There are numerous studies pertaining the application of optimal control of shear flows (see the review of Gunzburger [7] or a more recent review of Luchini & Bottaro [8], although the latter is in a slightly different context). Many studies have targeted the control of disturbances evolving in laminar or turbulent boundary layers using optimal control algorithms in the incompressible regime (*e.g.* [9, 10, 11, 12, 13, 14, 15]).

2. Governing Equations and Results

The BRE can be derived from the full steady-state compressible N-S equations, where the streamwise coordinate is scaled as $x = \bar{x}/R_\lambda$ while the other two coordinates are the same $y = \bar{y}$, $z = \bar{z}$, and the time is scaled as as $t = \bar{t}/R_\lambda$. Also, the crossflow components of velocity are expected to be small compared to the streamwise component, and variations of pressure are expected to be very small. This suggest the introduction of the scaling of dependent variables as: $u = \bar{u}; v = \bar{v}/R_\lambda; w = \bar{w}/R_\lambda; \rho = \bar{\rho}; p = \bar{p}/R_\lambda^2; T = \bar{T}; \mu = \bar{\mu}; k = \bar{k}$. The BRE are solved efficiently using to the numerical algorithm developed in Es-Sahli et al. [16].

For the optimal control approach, we first write BRE in the generic and more compact form $\mathcal{G}(\mathbf{q}) = 0$, with appropriate initial and boundary conditions, where $\mathcal{G}()$ is the non-linear CBRE differential operator in abstract notation, $\mathbf{q} = (\rho, u, v, w, T)$ is the vector of state variables.

We define an objective (or cost) functional as $\mathcal{J}(\mathbf{q}, \phi) = \mathcal{E}(\mathbf{q}) + \sigma \left(\|\phi_x\|^{\beta_2} + \|\phi\|^{\beta_2} \right)$, where $\mathcal{E}(\mathbf{q})$ is a specified target function to be minimized. A common

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approach to transform a (nonlinear) constrained optimization problem into an unconstrained problem is by using the method of Lagrange multipliers (see, for example, [10], [7], [12]). To this end, we consider the Lagrangian $\mathcal{L}(\mathbf{q},\phi,\mathbf{q}^a) = \mathcal{J}(\mathbf{q},\phi) - \langle \mathcal{G}(\mathbf{q}), \mathbf{q}^a \rangle$, where $\mathbf{q}^a$ is the vector of Lagrange multipliers $(\rho^a, u^a, v^a, w^a, T^a)$, also known as the adjoint vector. In other words, the Lagrange multipliers are introduced in order to transform the minimization of $\mathcal{J}(\mathbf{q},\phi)$ under the constraint $\mathcal{G}(\mathbf{q}) = 0$ into the unconstrained minimization of $\mathcal{L}(\mathbf{q},\phi,\mathbf{q}^a)$ via $\delta\mathcal{L} = \frac{\partial \mathcal{L}}{\partial \mathbf{q}}\delta\mathbf{q} + \frac{\partial \mathcal{L}}{\partial \phi}\delta\phi + \frac{\partial \mathcal{L}}{\partial \mathbf{q}^a}\delta\mathbf{q}^a = 0$ All directional derivatives must vanish, providing different sets of equations: (i) adjoint CBRE equations are obtained by taking the derivative with respect to $\mathbf{q}$, $\frac{\partial \mathcal{L}}{\partial \mathbf{q}} = 0 \Rightarrow \mathcal{G}^a(\mathbf{q}^a) = 0$; (ii) optimality conditions are obtained by taking the derivatives with respect to ϕ, $\frac{\partial \mathcal{L}}{\partial \phi} = 0 \Rightarrow \mathcal{O}(\mathbf{q}^a, \mathbf{q}, \phi) = 0$; (iii) the original CBRE equations are obtained by taking the derivative with respect to $\mathbf{q}^a$, $\frac{\partial \mathcal{L}}{\partial \mathbf{q}_a} = 0 \Rightarrow \mathcal{G}(\mathbf{q}, \psi) = 0$. These equations form the optimal control system that can be utilized to determine the optimal states and the control variables.

The relationship between the state variables and the adjoint variables can be expressed by the adjoint identity, $\langle \mathcal{G}(\mathbf{q}), \mathbf{q}^a \rangle = \langle \mathbf{q}, \mathcal{G}^a(\mathbf{q}^a) \rangle + \mathcal{B}(\phi)$, where the last term, $\mathcal{B}$, represents a residual from the boundary conditions.

The BRE and the associated adjoint BRE and optimality conditions are not included here for brevity (more details will be given in the presentation).

From figure 1, it is clear that the optimal control approach causes a reduction in the the kinetic energy of the flow. In the context of transition from laminar to turbulent flow, the reduction in the kinetic energy of the flow implies that the occurrence of the secondary instabilities leading to transition is delayed, which consequently elongates the laminar region of the boundary layer. The wall shear stress in figure 3 follows a similar pattern. The reduction in the wall shear stress directly translates to a reduction in the boundary layer skin friction. Figure 3 visually shows the effect of the control approach on the centrifugal instabilities of the boundary layer. As more iterations are applied, the shape of the instabilities (especially in the region closest to the wall) is drastically altered and the instabilities gradually flatten out.

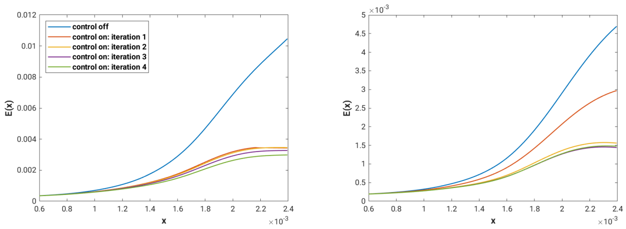

Fig. 1 Vortex energy of the uncontrolled and controlled boundary layer for a) M = 4, c) M = 6.

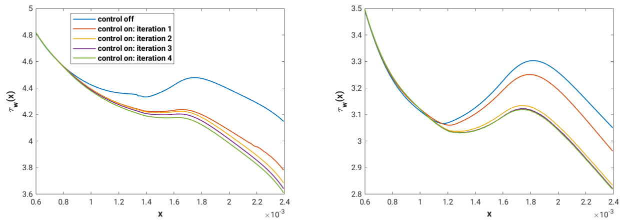

Fig. 2 Wall shear stress of the uncontrolled and controlled boundary layer for M = 4 (left) and M = 6 (right).

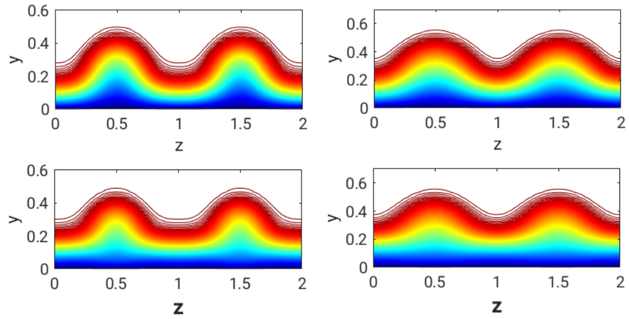

Fig. 3 Streamwise velocity contour plots for M = 4 (left) and M = 6 (right); from top to bottom, contours respectively represent the uncontrolled flow and 4^{th} control iteration.

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