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1 Surfactant transport between foam films

2 Paul Grassia¹ †

3 ¹Department of Chemical and Process Engineering, University of Strathclyde, James Weir Building, 75
4 Montrose St, Glasgow G1 1XJ, UK

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6 Surfactant transport from foam film to foam film is an essential (yet poorly understood) aspect
7 of the viscoplastic yielding behaviour of flowing foam. Recent experimental and modelling
8 work by Bussonnière & Cantat (2021) has however helped to advance understanding of the
9 relevant surfactant transport processes: the significance of that work is described herein.

10 **Key words:** Foams, Thin films, Lubrication theory, Capillary flows

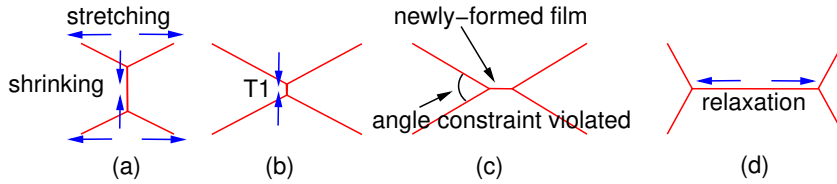
11 1. Introduction

12 Foams are non-Newtonian fluids exhibiting viscoplastic rheology: they only flow once a
13 yield stress is exceeded. On the bubble scale, the yielding behaviour manifests itself via
14 the so called $T1$ topological transformation, which involves bubbles exchanging neigh-
15 bours (Durand & Stone 2006). This transformation is described as follows (see Figure 1).
16 As a foam is deformed, certain foam films shrink whilst others stretch. Eventually as a
17 film shrinks away to nothing, bubbles that were originally in contact lose contact with one
18 another. Other bubbles come into contact in their place and a new film is formed. The new
19 film however is oriented in such a way as to violate geometrical constraints (the so called
20 Plateau's laws) on angles at which films in an equilibrium foam must meet. The system must
21 therefore relax to return close to equilibrium. During this relaxation, the newly-formed film
22 stretches rapidly and films around it shrink. This then is the fashion in which foam yields.

23 Thus far the discussion has focussed on the geometric configuration of foam. This however
24 ignores an important aspect, namely physical chemistry. Foam films are stabilised by the
25 presence of surfactant molecules on film surfaces. Stretching a foam film that is newly formed
26 after a $T1$ depletes the surfactant concentration (per unit area) on its surface. Analogously
27 shrinking neighbouring films causes their surfactant concentration to increase. Surface
28 tension is however a function of surfactant concentration: high surfactant concentration
29 implies low surface tension and vice versa. After a $T1$, Marangoni stresses due to surface
30 tension gradients (Satomi *et al.* 2013) should then drive surfactant from the shrinking (i.e.
31 neighbouring) films to the stretching (i.e. newly-formed) one.

32 Although the physical principle is clear, determining the surfactant flux transported
33 from film-to-film is challenging. There are hypotheses in literature (Durand & Stone 2006;

† Email address for correspondence: paul.grassia@strath.ac.uk

Figure 1: Schematic of a $T1$ topological transformation

34 Satomi *et al.* 2013; Grassia *et al.* 2012) suggesting how a surfactant transport relation might
 35 be formulated, but none of them have been subjected to a rigorous fluid mechanical analysis.
 36 Indeed a recent study (Vitasari *et al.* 2020) modelling surfactant transport within a simple
 37 foam structure, considered only surfactant transport along individual films, but simply
 38 neglected film-to-film surfactant transport for want of a good model. As described below,
 39 recent work by Bussonnière & Cantat (2021) has the potential to change that.

40 2. Experimental study

41 Bussonnière & Cantat (2021) do *not* solve the long standing question of how surfactant is
 42 transported between foam films following a $T1$ transformation. Instead, given that the $T1$
 43 is just one mechanism (amongst other possible mechanisms) by which surfactant fluxes
 44 on and between foam films can be first generated and then relax, Bussonnière & Cantat
 45 (2021) elect to focus attention on surfactant fluxes more generally. Towards this end, they
 46 built an experimental device involving five foam films held in a frame, one central film
 47 and four peripheral films, similar to what is sketched in Figure 1(d). The peripheral films
 48 can however be stretched or compressed via the action of motors (see also Figure 1 within
 49 Bussonnière & Cantat (2021)). A configuration is selected in which films on the left start off
 50 longer than those on the right, but the former are compressed whilst the latter are stretched.
 51 The experiment operates such that the total length of all films is conserved. Effectively what
 52 the experiment does then is drive surfactant from left to right, without changing the average
 53 surfactant concentration when averaged over all the films.

54 Using various optical measurements, Bussonnière & Cantat (2021) determine what hap-
 55 pens to film thicknesses, and hence deduce information about stretching or shrinkage of
 56 individual film elements. They also observe so called Frankel films extracted from the
 57 menisci (usually called Plateau borders in this context) at which three films meet. They can
 58 thereby make a distinction between material originally on a given foam film, and material
 59 recently added to it that has moved around the Plateau border. Deflection of the Plateau
 60 border menisci themselves gives information about surface tensions in the films that meet at
 61 the border. Experimental measurements are made both during a driving phase (when motors
 62 are switched on) and during a subsequent relaxation phase after motors are switched off.

63 3. Modelling study

64 The experimental study of Bussonnière & Cantat (2021) is an impressive achievement in its
 65 own right. However it is also supplemented by an insightful theoretical/modelling analysis.
 66 The analysis highlights the fundamental difficulty faced when attempting to model film-to-
 67 film surfactant transport, namely “geometrical frustration”. Because films meet threefold (an
 68 odd number) at a Plateau border, there is no way that velocities can be uniform along interfaces
 69 and simultaneously uniform across the thickness all films. Shear flow in at least parts of two
 70 or more films must therefore occur. For the particular set up considered (i.e. surfactant driven

71 from left to right with no change the sum total film length), Bussonnière & Cantat (2021)
 72 argue flow is uniform along and across the central film, so geometrical frustration manifests
 73 on those interfaces where peripheral films meet one another.

74 A clever fluid mechanical asymptotic analysis reveals that far from Plateau borders films
 75 are stretched or compressed uniformly via plug-like flows. Very close to Plateau borders
 76 there is a static meniscus and also a dynamic meniscus region (both viscous and capillary
 77 terms retain relevance in the latter): this situation is familiar in any system involving menisci
 78 with fluid motion. The new asymptotic region that Bussonnière & Cantat (2021) identify
 79 however is a sheared film region which occurs intermediate between the uniform far field
 80 region and the dynamic meniscus region.

81 A fluid mechanical model based on lubrication theory is presented for how this sheared
 82 film region must behave. The essence of the model is that viscous shear stress is matched
 83 to Marangoni stress, that surface tension variation is matched to surfactant concentration
 84 variation on the surface, and finally that the surfactant flux is conserved, modulo surfactant
 85 diffusing from one side of a foam film to another. The model's objective is to derive a relation
 86 between the change in the peripheral film tension $\Delta\sigma$ (measured relative to equilibrium film
 87 tension, which continues to apply in the central film) and the velocity U at which material
 88 is moving at the junction between the central film and a peripheral film: this is then what
 89 provides the sought after relation between film-to-film tension difference and surfactant flux.

90 In addition to U , three more characteristic velocity scales are identified, one associated
 91 with capillary effects U_c , one associated with diffusion effects U_d , and one associated with
 92 the Plateau border meniscus U_m (treating the Plateau border meniscus as a possible reservoir
 93 of surfactant). Bussonnière & Cantat (2021) go on to derive scaling laws for various limiting
 94 cases expressed in terms of relations between the above mentioned velocity scales.

95 In one extreme limit, it is considered that the Plateau border cannot act as a reservoir at
 96 all. If the film under consideration is a peripheral film that happens to be compressed, there
 97 cannot be any motion whatsoever at the point where this meets a neighbouring peripheral
 98 film. Any surfactant flux carried from the far field towards the Plateau border meniscus must
 99 therefore be transferred around the Plateau border meniscus wholly on the interface which
 100 connects with the central film. This relies on transferring surfactant diffusively across the
 101 film. A linear scaling law between tension difference $\Delta\sigma$ and velocity U is shown to ensue.

102 Still neglecting any reservoir effect of Plateau border menisci, it is found that the case of
 103 a stretched peripheral film sometimes behaves analogously to a compressed one. A linear
 104 scaling law between $\Delta\sigma$ and U again results. just with the signs of $\Delta\sigma$ and U being opposite
 105 from the compressed case. However the stretched case also admits a second kind of behaviour.
 106 A strongly stretched system with flow being pulled outwards from the point at which two
 107 peripheral films meet will have very low surfactant concentration around that point. The
 108 requirement to have limited surfactant flux at this particular point is then automatically met,
 109 irrespective of the velocity on the film interface. In such a case, the surface tension on one
 110 interface of the stretched peripheral film (where it joins the central film) can remain at an
 111 equilibrium tension σ_0 , but on the opposite interface is $\sigma_0 + E$, where E denotes the so called
 112 Gibbs elasticity. In the model of Bussonnière & Cantat (2021) this is the maximum increase in
 113 surface tension over and above σ_0 , corresponding to a bare surface without surfactant. In this
 114 strongly stretched limit, the film tension (the sum of the two surface tensions) then saturates
 115 at a value E above the equilibrium film tension, regardless of how large U is, and hence
 116 regardless of how much surfactant is being transferred around the Plateau border meniscus
 117 from the central film to the stretched peripheral film. This then breaks the symmetry between
 118 compression and stretching. Note however that any strong stretching here is restricted to just
 119 one side of the peripheral film, i.e. the interface which connects to another peripheral film:
 120 had stretching occurred on both sides, $\Delta\sigma$ could have been twice as large.

121 Another interesting limit occurs when the Plateau border meniscus acts as a very effective
 122 surfactant reservoir. In the compression case there is no longer any need for surfactant to
 123 diffuse from one interface of a film to the other. Compared to the limit discussed earlier,
 124 this means that only half of the surfactant flux flowing along the compressed film far from
 125 the Plateau border manages to find its way around the Plateau border onto the central film.
 126 Diffusive transport within the sheared film turns out to remain an essential feature in the
 127 stretched case however, and the length of the sheared film region grows to accommodate that.

128 Whether or not Plateau border menisci are able to act as reservoirs affects the ratio
 129 between flow velocity on peripheral films and velocity at the junction between the peripheral
 130 and central films at the Plateau border itself. The former velocity will be comparable with the
 131 latter in the presence of reservoir action, but only half as large in the absence of a reservoir.
 132 Bussonnière & Cantat (2021) also point out that in practice a Plateau border's ability to act
 133 as a reservoir is likely to exhaust over time.

134 4. Outlook

135 Toward the end of their work, Bussonnière & Cantat (2021) propose a simple illustrative
 136 model specifically to describe the behaviour of their five-film device informed by the results
 137 they have covered to date. The model describes the stretching of films based on stretching
 138 of the Lagrangian material elements that instantaneously make up any given film, plus
 139 the film-to-film transfer of material around a Plateau border. Owing to the symmetry of
 140 the five-film device (film compression rates opposite and equal to film stretch rates), and
 141 assuming tension differences remain linear in velocity around Plateau border menisci, a
 142 simple *analytical* solution is obtained for how tension difference between films (and by
 143 implication, surfactant flux around the Plateau border menisci) varies with time. Towards the
 144 end of their work, Bussonnière & Cantat (2021) suggest a possible route to “scale up” from
 145 their five-film device results to rheology of a 3-D foam as a whole.

146 The work of Bussonnière & Cantat (2021) does not answer all the open questions about
 147 how film-to-film surfactant transport proceeds following a $T1$ topological transformation nor
 148 between foam films more generally. The work relies on a very specific geometry, which is
 149 similar to (but not the same as) a $T1$ geometry. Moreover the geometry is highly symmetric,
 150 so a number of variables can be eliminated on symmetry grounds: the case of disordered
 151 foam would not retain those symmetries. Nevertheless insights that Bussonnière & Cantat
 152 (2021) provide will likely prove instrumental in future efforts to gain a better understanding
 153 of bubble-scale processes that occur in foams, taking proper account of surfactant physical
 154 chemistry. This will in turn advance understanding of foam rheology in general.

155 **Declaration of interests.** The author reports no conflict of interest.

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