One-dimensional Kronig-Penney superlattices at the LaAlO₃/SrTiO₃ interface

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ABSTRACT

Semiconductor heterostructures [1] and ultracold neutral atomic lattices [2] capture many of the essential properties of one-dimensional (1D) electronic systems. However, fully 1D superlattices are highly challenging to fabricate in the solid state due to the inherently small length scales involved. Conductive atomic-force microscope (c-AFM) lithography applied to an oxide interface can create ballistic few-mode electron waveguides with highly quantized conductance and strongly attractive electron-electron interactions [3]. Here we show that artificial Kronig-Penney-like superlattice potentials can be imposed on such waveguides, introducing a new superlattice spacing that can be made comparable to the mean separation between electrons. The imposed superlattice potential fractures the electronic subbands into a manifold of new subbands with magnetically-tunable fractional conductance. The lowest plateau, associated with ballistic transport of spin-singlet electron pairs [3], shows enhanced electron pairing, in some cases up to the highest magnetic fields explored. A 1D model of the system suggests that an engineered spin-orbit interaction in the superlattice contributes to the enhanced pairing observed in the devices. These findings are an advance in the ability to design new families of quantum materials with emergent properties and the development of solid-state 1D quantum simulation platforms.

Quantum theory provides a unified framework for understanding the fundamental properties of matter. However, there are many quantum systems whose behavior is not well understood because the relevant equations are not able to be solved using known approaches. The idea of “quantum simulation”, first articulated by Feynman [4], aims to exploit the quantum-mechanical properties of materials to compute the properties of interest and gain insight into the quantum nature of matter. There are two main “flavors” of quantum simulation: one based upon the known efficiency of circuit-based quantum computers to solve the Schrödinger equation, and the other based on microscopic control over quantum systems to emulate a given Hamiltonian. The former approach is limited by the capabilities of present-day quantum computers. The latter approach has shown great promise using a variety of methods including ultracold atoms [2, 5, 6], spin systems from ion trap arrays [7], superconducting Josephson junction arrays [8], photonic systems [9], and various solid-state approaches [1, 10, 11, 12]. Platforms capable of quantum simulation of Hubbard models would be of enormous value in condensed matter physics and beyond.

Complex oxides offer new opportunities to create a platform for quantum simulation in a solid-state

environment. Their complexity gives access to important quantum phases of matter, such as superconductivity, where the model Hamiltonians (e.g., 2D Hubbard model) are challenging to understand theoretically. In addition, their nanoscale reconfigurability makes it possible to engineer new forms of quantum matter with extreme nanoscale precision [13, 14].

Here we present experiments which contribute to a solid-state quantum simulation platform based on a reconfigurable complex-oxide material system. Using conductive atomic force microscope (c-AFM) lithography, we create Kronig-Penney-like [15] 1D superlattice structures by spatially modulating the potential of a 1D electron waveguide device at the $\text{LaAlO}_3/\text{SrTiO}_3$ interface. C-AFM lithography has been used to create a variety of devices at the $\text{LaAlO}_3/\text{SrTiO}_3$ interface [16]. A conductive AFM tip, moving in contact with the LaAlO_3 surface and positively-biased with respect to the $\text{LaAlO}_3/\text{SrTiO}_3$ interface, locally creates (“writes”) conducting regions at the interface (Figure 1a), while a negatively biased tip locally restores (“erases”) the interface to an insulating state. This technique achieves nanoscale control, with precision as high as 2 nm [14], over the conductivity of the $\text{LaAlO}_3/\text{SrTiO}_3$ interface and most of its properties.

The work described here concerns $\text{LaAlO}_3/\text{SrTiO}_3$ electron waveguide devices [3] that have been perturbed by a spatially periodic potential, similar to the one first envisioned by Kronig and Penney [15]. Unperturbed waveguides exhibit highly quantized ballistic transport with conductance steps at or near integer values of the conductance quantum e^2/h . The subband structure of $\text{LaAlO}_3/\text{SrTiO}_3$ electron waveguides can be described by a waveguide model which takes into account vertical, lateral, and spin degrees of freedom [3]. Representative orbitals for electron waveguides, subject to parabolic lateral confinement and half-parabolic vertical confinement, are shown in Figure 1b, where $|m, n, s\rangle$ is a state specified by quantum numbers m , n , and s that correspond to the number of lateral (m) and vertical (n) modes of the wavefunction, and the spin s . The complete set of states form a basis for describing extended states along the waveguide direction x . The periodic modulation of the waveguide may couple different vertical modes (for example those highlighted in black in Figure 1b) with the ground state $|0, 0, \uparrow\rangle$. Due to attractive electron-electron interactions, subband energy minima can “lock” together to form electron pairs [3] that also propagate ballistically. Pairing in electron waveguides arises from the same electron-electron interactions that give rise to superconductivity [17]. In some cases, more exotic locking of subbands can occur, e.g., the Pascal conductance plateaus which indicate the binding of $n \geq 2$ electron states [18]. The presence of strong, tunable electron-electron interactions [19] makes these electron waveguide devices an interesting starting point for engineering 1D quantum systems.

1D superlattice devices are created by first writing a conductive nanowire with a constant positive voltage applied to the AFM tip ($V_{\text{tip}} \sim 10$ V). The same path is re-traced along the same direction while applying a sinusoidally varying tip voltage $V_{\text{tip}}(x) = V_0 + V_k \sin(kx)$, to produce a spatially periodic potential modulation. A short unpatterned waveguide is written in series next to the superlattice, which helps to control the chemical potential in the device structure [3]. Four-terminal magnetotransport measurements are carried out in a dilution refrigerator at or near its base temperature in the range $T \sim 15\text{--}50$ mK, unless otherwise indicated. Data is presented for three 1D superlattice devices, Devices A, B1, and B2, and two control devices (straight electron waveguides without superlattice modulation), Devices C1 and C2.

Figure 2a shows the transconductance dG/dV_{sg} as a function of out-of-plane magnetic field B and side-gate voltage V_{sg} for Device A. The transconductance map provides a visual indication of the subband structure. Purple regions, where the transconductance is nearly zero, represent conductance plateaus. Bright colored

(red/yellow/green/blue) regions signify increases in conductance that generally correspond to the emergence of new subbands. White regions indicate negative transconductance, which is associated with non-monotonic behavior in the range $4\text{ T} < |B| < 7\text{ T}$. The transconductance is generally found to be highly symmetric with respect to the applied magnetic field. Figure 2b shows a series of conductance curves versus V_{sg} for a sequence of out-of-plane magnetic fields B , ranging between 0 T (leftmost) to 16 T (rightmost). Curves are offset by $\Delta V_{\text{sg}} \sim 7.5\text{ mV/T}$ for clarity and curves at 1 T intervals are highlighted in black. At low magnetic fields ($|B| \sim 2\text{ T}$), a plateau at around $1.8e^2/h$ develops before bifurcating into two distinct plateaus, one of which decreases in value, while the other increases towards a nearly quantized value of $1.99e^2/h$. The onset of the two plateaus can be seen clearly in the transconductance (Figure 2a) as a minigap that appears in the lowest subband.

In addition to the plateau at $2e^2/h$, many other subband features are readily seen at higher conductance values. Close-ups of three selected areas (Figures 2c-e) reveal that there are several conductance plateaus visible. The conductance increases between these plateaus correspond to new subbands, the so-called “fractured” states, becoming available in the transconductance map. These appear to be fractional subbands as the increase in conductance between the plateaus are fractions of the conductance quanta e^2/h . Figure 2e shows the fractional conductance feature occurring below the $2e^2/h$ plateau in more detail. The feature first appears in the form of a conductance peak then bifurcates forming the $\sim 2e^2/h$ plateau, and a lower fractional conductance feature that evolves with magnetic field.

Data for a second superlattice devices (Device B1, Extended Data Figure 2), yields qualitatively similar behavior. The overall subband structure resembles the subband structure of an electron waveguide device with no superlattice patterning, but the subbands are, like with Device A, fractured into additional manifolds with fractional conductance plateaus, including a plateau at $2e^2/h$ (Extended Data Figure 3). Device B1 also shares the prominent highly quantized conductance plateau at $2e^2/h$. We present a third Device B2 (Extended Data Figure 4) that also exhibits fractional conductance plateaus in the supplementary information.

Finite-bias spectroscopy of Device A (Extended Data Figure 1) reveals a characteristic diamond structure in the transconductance. This feature is associated with clean ballistic transport [20, 21] and is due to unevenly populated subbands at large finite biases which give rise to half-plateaus. The diamond visible in the transconductance corresponds to a fractional conductance feature below the $2e^2/h$ plateau at around $0.5e^2/h$ at zero-bias and about half that value at finite-bias. The presence of this characteristic diamond structure makes it unlikely that the fractional features are due to trivial causes, e.g., reduced transmission due to partial reflection at one end of the nanowire. Finite-bias spectroscopy of Device B2 also features a similar diamond structure, as shown in Extended Data Figure 5.

Unmodulated control devices (two of which are shown in Extended Data Figure 6 and more than ten are published elsewhere [3, 18]) do not exhibit fractional conductances. The characteristic behavior of such control electron waveguide devices is a series of conductance plateaus that are quantized in integer values of e^2/h , i.e., lacking in the fractionalized subbands observed here.

Fractional conductances in 1D transport have been reported in a variety of systems, and the phenomenon generally arises when there are strong electron-electron interactions. The fractional quantum Hall state [22] is perhaps the best known and investigated example. The 0.7 anomaly [23] in quantum point contacts [24, 25] has been extensively investigated. Electron-electron interactions are believed to play a central role in the formation of the conductance plateau observed at $0.7 \times (2e^2/h)$ [26]. Fractional conductances have been observed in

several 1D quantum wire systems such as strained Ge-based hole quantum wires [27] and GaAs-based quantum wires [28].

Shavit et al have considered a theoretical framework [29, 30] in which fractional conductances arise in multichannel 1D quantum wires due to high order back-scattering processes. Such fractional conductances were shown to require strong repulsive electron-electron interactions to be observed, but a recent theory suggests that this could also be the case for strong attractive modulated interactions [31]. For the simplest high-order scattering process involving three particles, this theory predicts a plateau at $1.8e^2/h$ which can become gap-protected for strong modulated interactions. Our observation of a stable conductance plateau near $1.8e^2/h$ near $B = 2$ T, together with its absence in unmodulated waveguides, is consistent with this prediction.

A defining characteristic of the $\text{LaAlO}_3/\text{SrTiO}_3$ system is the prevalence of strong *attractive* electron-electron interactions [3, 17, 18, 19]. Both vertical superlattice devices show signs of (weak) superconductivity at $B = 0$ T (Extended Data Figure 7). Empirically, unperturbed electron waveguides (which possess attractive interactions) do not exhibit fractional conductance plateaus. Devices at the $\text{LaAlO}_3/\text{SrTiO}_3$ interface exhibit electron pairing without superconductivity [3, 17]. In electron waveguides, this interaction causes electron subband energy minima to lock together, either near zero magnetic field or at re-entrant values, resulting in conductance steps of $2e^2/h$. The superlattice modulation of the electron waveguides is empirically linked to enhanced electron pairing fields. The effect is significant: superlattice devices have been observed with pairing fields of $B_p > 16$ T, significantly higher than control devices written in the same area of the sample. The largest pairing field observed in an unmodulated waveguide is $B_p \sim 11$ T [3]. The enhanced pairing strength appears to be influenced by the potential modulation, but a direct correlation is obscured by other variations between devices. Superlattices formed by lateral modulation do not show an enhanced pairing field [32].

Another effect that is correlated with the vertical modulation is a spin-orbit like effect in the device. The lowest subband in Device A (seen in the transconductance map in Figure 2a) bends upward at zero magnetic field, so that the minima of the lowest subband are at a finite magnetic field. This may be due to the engineering of a spin-orbit field, and is not usually observed in quasi-1D electron waveguide devices at the $\text{LaAlO}_3/\text{SrTiO}_3$ interface. As the electrons travel through the device with momentum $\vec{k} = k\hat{x}$ the periodic vertical modulation will create an effective electric field $\vec{E}_{\text{eff}}(x) = E_0 \sin(kx)\hat{z}$ which will yield an effective spin-orbit field $\vec{B}_{\text{SO}} \propto \vec{k} \times \vec{E}_{\text{eff}}$ in the $\hat{y}$ direction. This could result in a coupling between the spin-up and spin-down particles which may also be the mechanism for enhancing the pairing field in these devices. Note that the $\text{LaAlO}_3/\text{SrTiO}_3$ interface is known to exhibit an intrinsic Rashba spin-orbit coupling [33, 34], but theoretical models suggest that a confinement along a nanowire could significantly reduce it [35]. The measurements presented here suggest that a non-negligible value of spin-orbit coupling could be restored when the modulation is present.

We present below a minimal model which provides a basic description of two of the features generated by a modulated waveguide: the fracture of the first conductance line and enhanced spin-orbit coupling. Figure 3a shows a calculated transconductance map for a single-particle model of a straight, unmodulated electron waveguide, described in more detail elsewhere [3]. In this model, the states $|m, n, s\rangle \otimes |k\rangle$ are characterized by single-particle energies $\xi_{mn\sigma k}$ (see Eq. (3) in the SI) where $\sigma = \downarrow (\uparrow)$ for $s = -1/2 (1/2)$. Transconductance lines appear for $\xi_{mn\sigma 0} = 0$, i.e., when the chemical potential reaches the bottom of the bands. The waveguide model shows an overall resemblance to the experimental data for the superlattice (Fig. 2a), except that the subbands

are fractured into a manifold of new states with fractional conductances. These fractures can be accounted for qualitatively by adding a Kronig-Penney potential to the model (see SI). In this model, the potential creates band gaps in each subband single-particle spectrum that appear when the Fermi wavelength of an electron in a given subband corresponds to a fraction of the modulation period. Figure 3b shows the corresponding transconductance map, where the band gaps translate into curved lines above each subband, which notably exhibit negative regions. This interference effect is particularly reminiscent of the fracture observed in the first transconductance line in Fig. 2a, which suggests that it is the basis of the behavior observed in the experiments. This single-particle Kronig-Penney model cannot predict the full fractional conductance plateaus, which are understood to require the presence of strong electron-electron interactions [29, 30, 31].

As noted above, the modulation can also produce an enhanced spin-orbit interaction, and we can investigate the potential consequences of this in a 1D model for the two first orbitals $|0,0,\downarrow\rangle$ and $|0,0,\uparrow\rangle$ with a spin-orbit coupling term of the form $H_{\text{so}} = \alpha_{\text{so}} k \sigma_y$, and, to simplify the calculations, with no spatial dependence of the spin-orbit coupling strength α_{so} . We neglect here the band gaps produced by the modulation to focus on the effect of the spin-orbit coupling alone. Starting from a model with Hubbard interactions, in the mean-field approximation, our Hamiltonian in momentum space [3], reads

$$H = \sum_k \left[\sum_{\sigma} (\xi_{00\sigma k} + \Sigma_{\sigma}) c_{\sigma k}^{\dagger} c_{\sigma k} + \Delta (c_{\uparrow k}^{\dagger} c_{\downarrow -k}^{\dagger} - c_{\uparrow k} c_{\downarrow -k}) + (i\alpha_{\text{so}} k - \chi_{\downarrow\uparrow}) c_{\uparrow k}^{\dagger} c_{\downarrow k} - (i\alpha_{\text{so}} k + \chi_{\uparrow\downarrow}) c_{\downarrow k}^{\dagger} c_{\uparrow k} \right], \quad (1)$$

where $c_{\sigma k}$ is the annihilation operator of an electron of momentum k and spin σ in the transverse mode $|0,0\rangle$ ($\sigma = \downarrow, \uparrow$), and Σ_{σ} , Δ , and $\chi_{\sigma\bar{\sigma}}$ are respectively the Hartree shifts, the pairing order parameter, and the Fock fields defined as

$$\Sigma_{\sigma} = \frac{U}{2\pi} \int \langle c_{\bar{\sigma} k}^{\dagger} c_{\bar{\sigma} k} \rangle dk, \quad \Delta = \frac{U}{2\pi} \int \langle c_{\downarrow -k} c_{\uparrow k} \rangle dk, \quad \chi_{\sigma\bar{\sigma}} = \frac{U}{2\pi} \int \langle c_{\sigma k}^{\dagger} c_{\bar{\sigma} k} \rangle dk, \quad (2)$$

where $U \equiv U(B) < 0$ is an attractive electron-electron interaction (in units of energy $\times$ length) rescaled by the magnetic field, and where $\bar{\sigma}$ denotes the opposite spin of σ (see SI for details). The mean-fields are found self-consistently, and indicate the presence of electrons and paired electrons (though not superconductivity in this 1D model) in the waveguide. Calculating them determines phase diagrams for different values of α_{so} and U . Our results are shown in Figure 4. We find an enhanced pairing area, defined as the region of non-zero Δ , for increasing α_{so} . This minimal model (which could be extended to position-dependent potentials and spin-orbit coupling) thus supports the idea that a spin-orbit coupling engineered by the experimental setup can increase the pairing of the two lowest subbands $|0,0,\downarrow\rangle$ and $|0,0,\uparrow\rangle$ up to higher magnetic field, yielding a first conductance step of $2e^2/h$. A similar effect (i.e., an enhanced spin-orbit coupling) is seen in lateral 1D superlattice devices [32]. However, the engineered spin-orbit coupling in the lateral superlattice devices does not lead to an enhanced pairing. This can be understood since in our framework a lateral modulation creates a spin-orbit field $\mathbf{B}_{\text{SO}}$ along the quantization axis z and hence does not couple directly the spin up and down electrons, in contrast to the case of a vertical modulation.

The Kronig-Penney devices described here also show variations in properties, but the fractional plateaus are not observed in any of the unmodulated quantum devices. SrTiO_3 also has well-known ferroelastic distortions below $T = 105$ K that can impact the electronic structure of unmodulated quantum wires, and the distortions

are believed to play a role in parameter variations between unmodulated nanowires created under nominally identical conditions [18]. Experiments performed on nanoscale crosses [36] show highly reproducible behavior, which is attributed to pinning of ferroelastic domains in the insulating regions by the nanocross structure.

The theoretical analysis presented above suggests that spin-orbit interactions increase the effective pairing strength in the Kronig-Penney nanowires. Because of the natural variation in devices parameters, it is not possible to conclude that this effect is observed experimentally. Future experiments that are able to constrain the ferroelastic domains surrounding the device, coupled with systematic variations in modulation amplitude and spatial frequency, may provide more definitive answers to these questions.

The ability to create new superlattice structures, and modulate interactions in 1D systems, opens new frontiers in the development of quantum matter. The systems created here focus on low-dimensional confined structures, which are challenging to create using other methods. The regular superlattice structure can be replaced with quasiperiodic order, artificially imposed disorder, topological defects, or combined with lateral perturbations, to name just a few possibilities. Unlike the Kronig-Penney description, electron-electron interactions play a defining role in the resulting quantum phases, and future discoveries of emergent phases in this family of 1D systems are highly likely.

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Author Contributions

M.B., P.I., and J. Levy conducted the experiments. F.D., E.M., and A.J.D. made the theoretical model calculations. J.Li and M.H. processed the samples. H.L., J.-W.L., K.E. and C.-B.E. synthesized the thin films and performed structural and electrical characterizations. All authors reviewed the manuscript.

Competing interests

The authors declare no competing interests.

Figure Legends

Figure 1. Schematic of c-AFM writing and 1D superlattice device. **a**, C-AFM writing schematic. A positive bias on the AFM tip protonates the LaAlO_3 surface, locally creating a conducting channel at the $\text{LaAlO}_3/\text{SrTiO}_3$ interface. **b**, Table showing different representative wavefunctions calculated using a single particle model for electron waveguide devices [3]. The imposed vertical superlattice structure may cause mixing of vertical modes of an electron waveguide device, mixing the ground state with modes highlighted in black. **c**, 1D vertical superlattice device schematic. The superlattice is created by first writing the main channel with a positive tip voltage. The same path is then traced while applying a sinusoidal tip voltage to periodically modulate the confining potential of the device. The superlattice is created in series with two highly transparent tunnel barriers.

Figure 2. Magnetotransport characteristics and fractional conductance features. **a**, Transconductance dG/dV_{sg} is plotted as a function of magnetic field B and side gate voltage V_{sg} . Purple regions indicate zero transconductance (conductance plateaus). Bright regions indicate increasing conductance - energies where new 1D subbands become available. Negative transconductance is indicated in white, mainly in two lobes above the $2e^2/h$ plateau near 5 T. Regions highlighted in colored boxes are a guide to the eye to indicate the location of the conductance curves. **b**, Conductance G vs side gate voltage V_{sg} at magnetic fields from $B = 0$ T to 16 T. Curves are offset by $\Delta V_{\text{sg}} \sim 7.5$ mV/T for clarity. Curves at 1 T intervals are highlighted in black. Colored boxes indicate corresponding locations in panels (a),(c-e). **c,d**, Conductance curves showing conductance plateaus which correspond to the “fractured” states in the transconductance map. The conductance jump between the plateaus are fractions of the conductance quanta e^2/h . **e**, Conductance curves highlighting the fractional conductance feature occurring below the $2e^2/h$ plateau. Data shown is for Device A and taken at $T = 15$ mK.

Figure 3. Waveguide models. **a**, Transconductance spectra for non-interacting single-particle electron waveguide model. **b**, Transconductance spectra for an electron waveguide with vertical modulation characterized by a period $\lambda_M = 10$ nm and potential strength $V = 0.075$ meV. Effective broadening energy 0.015 meV. Other parameters for both **a** and **b**: $m_x = m_y = 1.9m_e$, $m_z = 6.5m_e$, $g = 0.62$, $l_z = 8.1$ nm, and $l_y = 26$ nm.

Figure 4. Phase diagrams of the mean-field model. The panels **a**, **b** and **c** show phase diagrams of the waveguide obtained from (1) as a function of the magnetic field B and chemical potential μ for different electron-electron interaction U and spin-orbit coupling α_{SO} . **a**, No interaction ($U(0) = 0$ meV nm) and no spin-orbit coupling ($\alpha_{\text{SO}} = 0$ meV nm). The areas labeled as "vacuum", "1S", and "2S" correspond to an empty phase, a single-particle phase, and a two-single-particle phase, respectively. The phases are delimited by the Zeeman-split single-particle energies $\xi_{\sigma k}(B, \mu)$ of the states $|0, 0, \downarrow\rangle \otimes |k\rangle$ and $|0, 0, \uparrow\rangle \otimes |k\rangle$ (see SI for detailed expressions) satisfying $\xi_{\sigma 0}(B, \mu) = 0$ (solid black lines). The inset shows the associated conductance. **b**, Finite interaction ($|U(0)| = 4.2$ meV nm) and small spin-orbit coupling ($\alpha_{\text{SO}} = 0.05$ meV nm), as to account for a potential intrinsic Rashba spin-orbit coupling. The presence of interactions induces a pairing Δ of the orbitals in some region of parameters (phase "P"), moving the splitting of the two single-particle bands to an higher magnetic field ($B \sim 10$ T). This diagram could correspond to typical unmodulated waveguides with large pairing fields (see Fig. 6 in SI). **c**, Finite interaction ($|U(0)| = 4.2$ meV nm) and larger spin-orbit coupling ($\alpha_{\text{SO}} = 0.25$ meV nm). The enhanced spin-orbit coupling increases the area of the paired phase up to $B \sim 16$ T. This spin-orbit-assisted pairing phenomenon is not sensitive to the specific parameter values.

Methods

3.4 unit cell (u.c.) $\text{LaAlO}_3/\text{SrTiO}_3$ samples were grown using pulsed laser deposition (PLD) (described in more detail elsewhere [37]). Electrical contact was made to the interface by ion milling through the interface and backfilling with Ti/Au. C-AFM writing was performed by applying a voltage bias between the AFM tip and the interface, with a $1 \text{ G}\Omega$ resistor in series. Writing was performed in 30-40% relative humidity using an Asylum MFP3D AFM. Written samples were then transferred into a dilution refrigerator and cooled to a base temperature of ~ 15 mK. Four-terminal measurements were taken using standard lockin techniques at a frequency of 11 Hz with an oscillation amplitude of 1 mV. 4-terminal I-V curves were measured by applying a DC source-drain bias across the device.

Data Availability

Source data are available at the Harvard Dataverse [38]. Other data and code that support the findings of this study are available from the corresponding author upon reasonable request.

Figure 1. Schematic of c-AFM writing and 1D superlattice device. **a**, C-AFM writing schematic. A positive bias on the AFM tip protonates the LaAlO₃ surface, locally creating a conducting channel at the LaAlO₃/SrTiO₃ interface. **b**, Table showing different representative wavefunctions calculated using a single particle model for electron waveguide devices [3]. The imposed vertical superlattice structure may cause mixing of vertical modes of an electron waveguide device, mixing the ground state with modes highlighted in black. **c**, 1D vertical superlattice device schematic. The superlattice is created by first writing the main channel with a positive tip voltage. The same path is then traced while applying a sinusoidal tip voltage to periodically modulate the confining potential of the device. The superlattice is created in series with two highly transparent tunnel barriers.

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Extended Data Figures

Extended Data Figure 1. Finite-bias spectroscopy. **a**, Conductance (G) intensity map as a function of four-terminal voltage V_{4t} and side gate voltage V_{sg} . Pink and blue dashed lines indicate the locations for the vertical linecuts shown in (c). **b**, Transconductance (dG/dV_{sg}) intensity map as a function of four-terminal voltage V_{4t} and side gate voltage V_{sg} . The transconductance map shows the diamond features indicating ballistic transport in the superlattice devices. **c**, Vertical conductance linecuts at $V_{4t} = 0$ and $90 \mu V$. Circles indicate fractional conductance values below the $\sim 2e^2/h$ plateau (corresponding to the lowest diamond features visible in the transconductance map in panel (b)) that become half of their value at a finite bias. Curves are offset for clarity. Data shown is from Device A and taken at $B = 13$ T and $T = 15$ mK.

Extended Data Figure 2. Magnetotransport data for vertical superlattice Device B1. **a**, Transconductance map dG/dV_{sg} as a function of side gate voltage V_{sg} and magnetic field B . Purple regions indicate conductance plateaus, zero transconductance. Red/yellow/green/blue regions indicate increases in conductance when new subbands become available. White regions indicate negative transconductance. Colored boxes are guides to the eye indication the location of highlighted conductance curves. **b**, Plot showing full conductance data. Conductance curves at 1 T intervals are highlighted in black and are offset clarity. **c-e**, Conductance G as a function of side gate voltage V_{sg} curves at different out-of-plane magnetic field B values highlighting some fractional conductance features. $T = 15$ mK.

Extended Data Figure 3. Device B1 1.8 Feature. **a**, Conductance data - same as shown in Figure 2**b**. **b**, Zoom on region highlighting feature at $G \sim 1.8 e^2/h$. Red lines are guides to the eye to show that the feature near 1.6 is in fact $0.2 e^2/h$ below the $2 e^2/h$ plateau, which itself is not fully resolved until the highest magnetic fields.

Extended Data Figure 4. Magnetotransport data for vertical superlattice Device B2. **a**, Transconductance map dG/dV_{sg} as a function of side gate voltage V_{sg} and magnetic field B . Purple regions indicate conductance plateaus, i.e., zero transconductance. Red/yellow/green/blue regions indicate increases in conductance when new subbands become available. Colored boxes are guides to the eye indicating the location of highlighted conductance curves. **b**, Plot showing full conductance data. Each curve is colored according to the transconductance at each side gate value (white indicates a plateau, black indicates rapidly increasing conductance). Curves are offset for clarity. **c**, **d**, Conductance G as a function of side gate voltage V_{sg} curves at different out-of-plane magnetic field B values highlighting some fractional conductance features. $T = 50$ mK.

Extended Data Figure 5. Device B2 Finite Bias Spectroscopy. **a**, Conductance (G) intensity map as a function of four-terminal voltage V_{4t} and side gate voltage V_{sg} . Pink and blue dashed lines indicate the locations for the vertical linecuts shown in **(c)**. **b**, Transconductance (dG/dV_{sg}) intensity map as a function of four-terminal voltage V_{4t} and side gate voltage V_{sg} . The transconductance map shows the diamond features indicating ballistic transport in the superlattice devices. **c**, Vertical conductance linecuts at $V_{4t} = 0$ and $121 \mu\text{V}$. Circles indicate fractional conductance values below the $\sim 2e^2/h$ plateau (corresponding to the lowest diamond features visible in the transconductance map in panel **(b)**) that become half of their value at a finite bias. Curves are offset for clarity. Data shown is from Device B2 and taken at $B = 9 \text{ T}$ and $T = 50 \text{ mK}$.

Extended Data Figure 6. Magnetotransport data for control electron waveguide devices. **a**, Transconductance map dG/dV_{sg} as a function of side gate voltage V_{sg} and magnetic field B for Device C1. Data previously published in Ref. [3]. **b**, Transconductance map for Device C2. $T = 50 \text{ mK}$.

Extended Data Figure 7. Superconductivity in Device A. Conductance map G_{4t} as a function of side gate voltage V_{sg} and four-terminal voltage V_{4t} . A small increase of G_{4t} near $V_{4t} = 0$ μ V is associated with superconductivity. $B = 0$ T and $T = 15$ mK.

Supplementary Information: One-dimensional Kronig-Penney superlattices at the LaAlO₃/SrTiO₃ interface

1 Device writing and measurement parameters

1D vertical superlattice Devices A and B1 were written with the same c-AFM writing parameters, in the same location on the LaAlO₃ surface prior to different cooldowns. The main channel of the device was written with a tip voltage $V_{\text{tip}} = 12$ V at a speed of 50 nm/s. The barriers for the electron waveguide were $L_B = 5$ nm, separated by $L_S = 10$ nm, and created by applying negative voltage pulses of -9 V at a speed of 5 nm/s. The superlattice was created by applying a tip voltage $V_{\text{tip}}(x) = (5 \text{ V}) \sin((\pi/5 \text{ nm}) x)$ with 18 periods. The control device Device C1 consisted of only the electron waveguide section written with the same parameters. Devices B2 and C2 were written on a different canvas and measured in a cryostat with maximum magnetic field $|B| < 9$ T.

Unless noted otherwise, all data was taken at the base temperature of a dilution refrigerator, $T \sim 15$ mK. Transport data for 1D superlattice devices was taken using standard lockin techniques with an oscillation amplitude of 1 mV, 250 μ V, and 25 μ V, and at a reference frequency of 11, 13.46, and 13 Hz for Devices A, B1, and B2, respectively. Data displayed for Device C1 is the zero bias cut of a finite bias spectroscopy. More examples of typical electron waveguide devices can be found elsewhere [3, 18].

1.1 Device B1

The transport in Device B1 did not appear to be as clean as Device A. There was also an issue with the side gate leaking at low side-gate and magnetic field values, causing the data to be distorted.

1.2 Device B2

Device B2 was written on a different canvas and measured in a cryostat with maximum magnetic field $|B| < 9$ T.

2 Theoretical model

The single-particle energies $\xi_{mn\sigma k}$ of the states $|m, n, \sigma\rangle \otimes |k\rangle$ are given by [3]

$$\xi_{mn\sigma k} = \frac{\hbar^2 k^2}{2m_x^*} \left(1 - \frac{\omega_c^2}{\Omega^2} \right) - g\mu_B B s + \hbar\Omega \left(m + \frac{1}{2} \right) + \hbar\omega_z \left((2n+1) + \frac{1}{2} \right) - \mu, \quad (3)$$

where $\omega_c = eB/\sqrt{m_x^* m_y^*}$ is the cyclotron frequency, B the out-of-plane magnetic field, m_x^* (m_y^*) the effective mass of the electron along the x (y)-direction, ω_z ($\Omega = \sqrt{\omega_y^2 + \omega_c^2}$) the trapping frequency along the vertical (lateral) direction, g the Landé factor, μ_B the Bohr magneton, and $s = -1/2$ ($1/2$) for $\sigma = \downarrow$ ($\uparrow$). Figures 3a and b were produced by taking $m_x^* = m_y^* = 1.9m_e$, $m_z^* = 6.5m_e$, $g = 0.62$, $l_z = 8.1$ nm and $l_y = 26$ nm, where m_e is the electron mass.

The transconductance lines in Fig. 3a correspond to $\xi_{mn\sigma 0} = 0$. The lines were broadened by convolution with the thermal broadening function $dn/d\mu$, where n is the Fermi distribution, using a broadening energy of 0.015 meV. Figure 3b was obtained following [15] by calculating the new subband structure in the presence of an infinite potential of the form

$$V(x) = \begin{cases} V/2 & \text{for } x \in \left[\ell\lambda_M, \frac{(2\ell+1)}{2}\lambda_M \right] \\ -V/2 & \text{for } x \in \left[\frac{(2\ell+1)}{2}\lambda_M, (\ell+1)\lambda_M \right] \end{cases} \quad (4)$$

with period $\lambda_M = 10$ nm, strength $V = 0.075$ meV, and $\ell = 1, 2, 3, \dots$. The potential produces gaps in the single-particle energy spectra whose the positions and the sizes depend respectively on λ_M and V . The corresponding conductance map $G(\mu, B)$ was broadened as before, and the transconductance lines obtained through $dG/d\mu$.

Figure 4 was produced by solving iteratively at zero temperature the eigensystem (1) in the electron-hole basis $(c_{\uparrow k}, c_{\uparrow -k}^\dagger, c_{\downarrow k}, c_{\downarrow -k}^\dagger)$ until convergence of the mean-fields, as described in [3]. Here, the interaction strength U has the magnetic field dependence $U(B) = U(0)\omega_y/\Omega$ (in units of energy $\times$ length). Physically, this scaling of U compensates the increase with magnetic field of the range of momenta $k = \sqrt{2m_x^*E/\hbar^2}(\Omega/\omega_y)$ involved in the interactions (it amounts to contract the real-space volume reciprocally associated to the relevant range of k). This has the effect of keeping the mean-fields $\Delta, \Sigma_\sigma \propto U \int \cdot dk$ independent of the effective scaling of the momenta, and allows us to obtain results in qualitative agreement with the experimental data over the whole range $B = 0$ -16 T. Single-particle phases were identified from the number of crossing points with the zero-energy axis (i.e. the number of finite Fermi momenta) of the resulting single-particle spectra. A conductance of e^2/h was associated to each finite Fermi momentum. The pair phase in Figure 4 corresponds to the regime where $|\Delta| > 10^{-3}$ meV $\approx k_B T$ (with k_B the Boltzmann constant and T the temperature), i.e., when the gap of the single-particle spectra is larger than the typical thermal fluctuation energy, and is associated to a conductance of $2e^2/h$. The resulting pairing energy of our model behaves as $\Delta \propto U^2(0)m_x^*$, in agreement with the expected behaviour $\Delta \propto U^2(0)/t$ of a tight-binding model with hopping parameter $t = \hbar^2/(m_x^*a^2)$ and lattice spacing a . The pairs are broken when the Zeeman splitting energy $g\mu_B B$ compensates the pairing energy Δ , which provides the typical scaling of the pairing field $B_p \propto U^2(0)m_x^*/(g\mu_B)$ at which the lines split. Parameters used to produce Figure 5 are $m_x^* = m_y^* = 3.2m_e$, $m_z^* = 6.5m_e$, $g = 0.3$, $l_z = 8.1$ nm, and $l_y = 15$ nm.