This is a peer-reviewed, accepted author manuscript of the following article: Feng, Y., Li, S., Chen, D., & Xiao, Q. (2019). Predictions for combined In-Line and Cross-Flow VIV responses with a novel model for estimation of tension. *Ocean Engineering*, *191*, [106531].
https://doi.org/10.1016/j.oceaneng.2019.106531

Predictions for Combined In-Line and Cross-Flow VIV Responses with a Novel Model for Estimation of Tension

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Abstract

The dynamic responses of slender cylinders with high aspect ratios undergoing vortex-induced vibrations (VIV) are studied. In detail, a three-dimensional model predicting the VIV responses in both the In-Line and Cross-Flow directions of slender cylinders is proposed based on the nonlinear equation governing the dynamic deformation and a wake oscillator. The tension in the cylinder is estimated according to the incoming stream velocities. To predict the VIV responses, the cylinder is discretized into finite segments, and the vibrations of each segment are estimated from solving the governing equation when the excitation forces are modelled using the Van Der Pol's wake oscillator. Considering that the wake oscillator model estimates the excitation forces according to the dynamics of the cylinder, it reveals the interactions between the flow and the dynamics of the cylinder. In order to verify the model calculating the mean tension, the VIV responses, which has been experimentally tested, is numerically studied. The comparison between the numerically predicted and experimentally measured responses shows that, the approach, especially the novel tension model, proposed herein is reliable as the frequency

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of vibrations, dominant mode number and vibration amplitude are all in good agreement with the experimental measurements and results from peer-reviewed publications.

Key words: Vortex-induced vibration; Van der Pol; Tension; Combined inline-crossflow responses

1. Introduction

Cylinders with high aspect ratios are widely used in the field of ocean engineering, especially in the offshore structures installed in deep waters Wu et al. (2012). For example, the risers widely used in the oil and gas exploration in the ocean are considered as slender cylinders in their structural designs. Along with the exploration moving from nearshore areas to deep waters, the aspect ratio of the riser connecting the floating platform and subsea production system increases significantly. In addition, the slender cylinder is also widely used to model the tendons of the tension leg platform (TLP). Given the operation water depth of the TLP (300m~1500m), the aspect ratio of the tendon is normally with an order of 10000. It is commonly acknowledged that the fatigue of the riser and the TLP tendon is the major cause for their structural failure. Considering the cost for maintaining the offshore structures and the subsea systems, the structural failure of the riser or the tendon of the TLP could lead to huge economic losses. Consequently, the vibrations of cylinders with high aspect ratios are of primary concern for their structural designers.

Instability of flow around cylinders induces regular vortex formation and asymmetric vortex shedding, which produces periodic fluctuations in lift forces acting on the cylinders (Bearman et al., 2001). Given fluctuating lift forces, the cylinder vibrates at the frequencies determined by the interaction between the flow and the dynamics of the cylinder. When the frequency Ω_f of vortex shedding, which determines, to a large extent, the vibration frequency of the cylinder, approaches the natural frequency Ω_s of the cylinder, vibration amplitude of the cylinder is enlarged and Ω_f is locked onto Ω_s for a range of stream velocities (Stansby, 1976). Due to the large resonance vibration amplitudes and the lock-in effect, the vibrations induced by the vortex shedding, which is referred to as Vortex-Induced Vibrations (VIV) hereafter, could bring serious damages to the cylinder, and hence the offshore structure in which the slender cylinder is an essential piece. Even if the resonance is avoided, the long-term VIV leads to the fatigue of the riser, which ultimately damages the system. Therefore, VIV of slender cylinders has long been a focus for the academic investigations.

Vortex shedding is a three-dimensional (3D) process, which makes cylinders vibrate in both in-line (IL) and cross-flow (CF) directions. The VIV of slender cylinders could be investigated through simulating the dynamics of elastically mounted short rigid cylinders (low aspect ratio) in experiments. In such experiments, only the vibration in the cross-flow direction is commonly investigated (Dahl et al., 2006). Recent studies, however, have found that long flexible cylinders acquire different dynamic responses from those found in the vibrations of short rigid cylinders. Firstly, long flexible cylinders usually vibrate at higher frequencies and in higher modes comparing to the short rigid ones (Trim et al., 2005). Secondly, both standing wave pattern and traveling wave pattern are observed for long flexible cylinders while only standing waves relate to the vibrations of short rigid cylinders. Because Vandiver et al. (2009) found that most of the VIV energy was concentrated in traveling waves, the difference in vibration patterns could lead to unrealistic estimates of VIV responses for the long flexible cylinder based on experimental results of short rigid cylinders. Thirdly, in the lock-in region, short rigid cylinders vibrate in only one mode because modal frequencies are well separated for the short rigid cylinders (Iwan and Jones, 1987), but vibrations with combined modes are frequently observed for long flexible cylinders. Fourthly, when the excitation frequency ratio (IL vibration frequency $f_{e,IL}$ to CF vibration frequency $f_{e,CF}$) equals the natural frequency ratio (IL natural frequency $\Omega_{s,IL}$ to CF natural frequency $\Omega_{s,CF}$), dual resonance is observed for the long flexible cylinders but not properly simulated in the experiments with short rigid cylinders (Dahl et al., 2006; Dahl et al., 2010). More importantly, some studies suggested that IL vibrations are as important as CF vibrations for long flexible structures. For example, Sarpkaya has found that the combination of IL and CF vibrations in a two degree-of-freedom (i.e. 2DOF) system could produce larger amplitude VIV responses (Sarpkaya, 1995) compared to the VIV considering only CF direction dynamics. Such findings are in agreement with the conclusion drawn by Moe and Wu (1990) and others that large CF amplitude occurred in a wider range of reduced velocities due to energy transforming from IL motions to CF motions (Dahl et al., 2006). Trim et al. (2005) conducted a benchmark experiment investigating the IL and CF VIV responses under uniform and shear flow conditions and found comparable magnitude of fatigue damage in both CF and IL directions. Blevins and Coughran have found that a 2DOF model of the long flexible cylinders has larger velocity entrainment (sometimes referred to as the synchronization, lock-in or lock-on) band than the model containing only CF vibrations (Blevins and Coughran, 2009). That means 2DOF response is larger at constant reduced damping and mass ratio. Therefore, further investigations on the dynamics of the long flexible cylinders, with emphasis on the combined CF and IL vibrations, are necessary for predicting its VIV responses.

There are generally two approaches to numerically predict VIV of slender cylinders, i.e. computational fluid dynamics (CFD) techniques and semi-empirical methods in which forces exerted on the oscillating cylinders are estimated via a semi-empirical model. While the CFD simulation explicitly produces all the details of the flow around the cylinder, which in turn yields reliable estimations of drag and lift forces exerted on the cylinder, the semi-empirical model estimates drag and lift forces according to the data measured in experiments (Wu et al., 2012). Although the CFD simulation produces more accurate and reliable estimations of drag and lift forces, it requires much more computational resources for the case with realistic Reynolds numbers when comparing to the semi-empirical method. In addition, the CFD simulation results are not realistic for predicting VIV for long flexible structures (Xu et al., 2008). Semi-empirical methods, on the other hand, include various wake oscillator models and predicting VIV using the data measured in forced vibration experiments. In general, a wake oscillator model contains a dynamical system to simulate vortex shedding. It therefore helps enhance the understanding of the physics of VIV as the model provides some insights into the physical mechanism governing the flow and vortex shedding. The commercial codes developed based on the semi-empirical methods (such as VIVA, VIVANA and SHEAR7), usually build hydrodynamic coefficient database using the measurements gathered in a series of experiments and predict the VIV using the coefficients kept in the database. It should be noted that the common semi-empirical model only consider the VIV at a limited number of discrete frequencies in the CF direction. Consequently, there is still room for the semi-empirical model to be improved in terms of predicting the VIV response in both the IL and CF directions of slender cylinders.

In the light of estimating the vibrations of cylinders under the excitations of the vortex shedding, Bishop and Hassan have suggested that the wake of the cylinder behaves as a conventional mechanical oscillator (Bishop and Hassan, 1964). Following their suggestion, Hartlen and Currie proposed a wake oscillator model, in which the fluctuating lift coefficient satisfies a Van Der Pol type equation (Hartlen and Currie, 1970). Based on the Hartlen-Currie model, Skop and Griffin devised a modified Van Der Pol equation and developed relations between empirical constants and physically meaningful parameters (Skop and Griffin, 1973). In addition, the work of Skop and Griffin (1973) contains a verification showing that the proposed model predictions are in quantitative agreement with experimental observations. Nayfeh et al. (2003) combined CFD simulation and wake oscillator assuming that drag is the function of lift as their first step in the development of a reduced order model. They found that Van Der Pol equation is suitable to model the lift compared to Rayleigh equation. Facchinetti et al. (2004) investigated three different schemes coupling motions of cylinder segments and the wake oscillator (displacement, velocity and acceleration coupling). It turned out that acceleration coupling yielded the best agreement with experimental measurements. Using the acceleration coupling scheme, Xu et al. (2008) proposed a model for high aspect ratio riser with nonlinear coupling between axial and the CF motions and compared the results with CFD results and experimental data. Violette et al. (2007), in addition, numerically solved the Partial Differential Equations (PDEs) governing the vibrations of the cylinder coupled with the Van Der Pol oscillator and compared the solution with Direct Numerical Simulation (DNS) results and experiment data. Based on the work of Nayfeh et al. (2003), Akhtar et al. (2009) developed the reduced order model (Van der Pol-Duffing model) for flow over elliptic cylinders with different eccentricities. They performed the CFD simulations first, then the CFD results was used to identify the coefficients in the reduced order model. Their model results agree well with the CFD data. Later, Srinil and Zanganeh (2012a) modelled CF and IL vibrations using double structural duffing equations-Van der Pol wake oscillators and introduced cubic and quadratic nonlinear terms to structural equations. Gu et al. (2012) applied the Generalized Integral Transform Technique to predict VIV of the slender cylinder via transforming

PDEs to Ordinary Differential Equations (ODEs). Stabile et al. (2018) proposed a novel reduced order model, which consists a forced Van der Pol oscillator and a linear state-space model to model the CF and IL forces. Their model matches the experimental results well.

It is widely acknowledged that the tensions in the risers, or other structure members with high aspect ratio, influence their VIV responses. In fact, Srinil (2011) has shown that a realistic model estimating the tensions in the slender cylinder is critical for predicting its VIV behaviors. Generally, there are three branches in terms of modelling the tension associated with a slender cylinder. The most common approach is to model the tension as a constant determined purely based on the forces applied at the end of the riser (Mathelin and de Langre, 2005; Sanaati and Kato, 2012). Such an approach is useful in the case where the riser is pre-tensioned, but the variations induced by the deformation of the riser is neglected. In addition, the tension can be modelled as a function of the height (depth), as in the studies of Srinil and Chen, Li et al. (Chen et al., 2012; Srinil, 2011) to account for the losses in tension due to the buoyance. Like the constant tension model, the models employing a vertically varying tension is infeasible to account for the axial deformation found in risers due to the VIV. To model the influence of tensions in risers in a more realistic way, Ge et al. (2009) and Gu et al. (2012) introduced a model calculating the tension according to the cylinder prolongation. More specifically, the axial deformation of the riser is calculated according to the motions of the riser segments. Given that tensions are calculated according to the length of the riser, the dynamic responses of long risers can be estimated in a more realistic manner, which showed different patterns from the vibrations of risers with constant or vertically varying tensions.

Although the study of Gu et al. (2012) included a tension model employing the prolongation of the riser as the independent variable, it focused on the CF vibrations only. In addition, the work of Ge et al. (2009) didn’t evaluate the relation between the tension and flow velocity. Thus, more efforts are needed in terms of contributing a 3D model predicting for both the IL and CF vibrations with dynamically varying tensions.

In the present study, a 3D VIV model is proposed, which includes a new formula calculating the tension of a long flexible cylinder and considering combined IL and CF VIV responses. The formula is validated against the data presented in Trim et al.'s work (Trim et al., 2005) and the model results are compared to the numerical simulations performed by Ge et al. (2009) and Gu et al. (2012). After the introduction, Section 2 presents the proposed 3D model and the formula calculating the tension. Section 3 shows a case study in which the VIV in both IL and CF directions are predicted using the proposed model. Moreover, the results are compared to the experimental data to verify the proposed model in Section 3. The advantages and disadvantages of the proposed model are also discussed in Section 3 based on the similarities and differences between the numerical predictions and the experimental measurements. Conclusions are drawn in Section 4.

2. Model description

2.1 Nonlinear coupled structure and wake oscillator model

The physical system considered herein is a flexible beam with simple supports, modelling a riser with diameter D subjected to a uniform flow with the stream velocity of *U*. The deflection of the beam is expressed by the Euler-Bernoulli beam equation. The riser is free to oscillate both in IL direction (x-axis) and CF direction (y-axis) as shown in Fig.1. The motion governing equation for x and y displacements at any time t > 0 and position 0 < z < L along the riser's length is expressed as

$$M\frac{\partial^2 r}{\partial t^2}+R\frac{\partial r}{\partial t}+\frac{\partial^2}{\partial z^2}\left(EI\frac{\partial^2 r}{\partial z^2}\right)-\frac{\partial}{\partial z}\left(T\frac{\partial r}{\partial z}\right)=F\ , \qquad (1)$$

where

$$r=x+iy\ ,\ \ F=F_x+iF_y\ ,\ \ M=m_s+\frac{\pi}{4}C_a\rho D^2\ ,R=R_s+R_f=2\xi_d M\Omega_s+\gamma\rho D^2\Omega_f\ ,\Omega_s=\sqrt{\frac{\Omega_n^2}{1+\dfrac{C_a}{m_s/(\frac{\pi}{4}\rho D^2)}}}\ .(2)$$

The meaning of the symbols appeared here and in the following equations can be found in APPENDIX: LIST of SYMBOLS

In Equations (1) and (2), displacement *r* and excitation force *F* are represented by complexes and *i* is the imaginary unit. *x* and *y* are the displacements in the IL and CF directions, respectively. And excitation force *F* is the fluid force exerted on the riser, decomposing into F_x and F_y. The external force in the x direction F_x is the sum of the mean f_D and fluctuating drag force f_D' as shown in

$$F_x = f_D + f_D' . \qquad (3)$$

In Equation (1), the total mass M includes structural mass m_s and added mass. ρ is the fluid density and D is the diameter of the riser. According to Song et al., the added mass coefficient C_a varies along the riser's length significantly in both IL and CF directions (Song et al., 2016). In the present study, C_a is taken as a constant and its variation is neglected. Damping R is decomposed into structural damping R_s and added damping R_f as well. ξ_d is the damping ratio, γ is the empirical parameter and Ω_f is the Strouhal frequency. The representative natural frequency Ω_s will be the natural frequency in water due to low mass ratio of the riser (Bearman et al., 2001). Ω_n is the natural frequency in air of the riser. E is the Young's modulus. The rigidity EI, on the other hand, is assumed as constant along the length of the cylinder.

For the original structural motion equation (equation (1)), tension has been already included as a dedicated term ($\frac{\partial}{\partial z}(T\frac{\partial y}{\partial z})$). Considering the tension essentially influences the stiffness of the riser, the tension calculated according to the term in the equation would modify the dynamics of the riser, and eventually the resulting deformation of the riser. Consequently, directly involving the tension term as in the original equation leads to an iterative process in estimating the VIV at every time step. Understandably, iterations within a single time step is at high computational cost for a semi-empirical model to estimate the VIV. In order to make the simulation more efficient, tension is assumed to be a constant $T\frac{\partial^2 y}{\partial z^2}$ in various studies (Furnes and Sørensen, 2007; Gao et al., 2018; Gao et al., 2019; Ge et al., 2009; Ge et al., 2011). However, Gu et al. (2013) found that the prolongation, and also tension, increases with the increasing velocity, which agrees with the experimental results. In addition, Lee and Allen (2010) found that the increase of the vibration frequency with flow speeds is strongly related to the rise of the axial tension. Based on their studies, tension should not be simply modeled as a constant. In the present study, a new model calculating the tension according to the flow velocity is proposed, which will be elaborated in subsection 2.2.

In estimating the external forces, Vandiver modified the empirical relation for predicting the total drag coefficient $\overline{C}_D$ as

$\bar{C}_D = C_{D0}\left[1+1.043\left(2Y_{RMS/D}\right)^{0.65}\right]$, where C_{D0} is the mean drag coefficient of a stationary rigid cylinder and $Y_{RMS/D}$ is the root mean square of the antinode displacement (Vandiver, 1983). Following the suggestion of Rosetti and Nishimoto et al., $\bar{C}_D$ is taken as $C_{D0}\left(1+Kq^2\right)$ (Rosetti et al., 2009) where K is amplification factor and q is the cross flow variable which wille be discussed later. Thus, mean drag force is calculated as

$$f_D = \frac{1}{2}\bar{C}_D \rho D U^2 = \frac{1}{2} C_{D0}\left(1+Kq^2\right)\rho D U^2 . \tag{4}$$

The variables p and q, as shown in

$$f_D^{'} = \frac{1}{2} C_{Di} \rho D U^2 , \tag{5}$$

$$F_y = \frac{1}{2} C_L \rho D U^2 , \tag{6}$$

$$C_{Di} = C_{Di0}\frac{p}{2}, \ \ C_L = C_{L0}\frac{q}{2}, \tag{7}$$

are introduced to model the fluctuating nature of drag and lift forces.

Here, C_{Di} and C_L are the vortex shedding drag and lift coefficients, C_{Di0} is the amplitude of vortex shedding drag coefficient and C_{L0} is the lift coefficient of a stationary rigid cylinder. The fluctuating drag and lift forces are periodic and self-limited, satisfying Van Der Pol nonlinear oscillator equations, which are expressed as

$$\frac{\partial^2 p}{\partial t^2} + 2\varepsilon_x \Omega_f (p^2-1)\frac{\partial p}{\partial t} + 4\Omega_f^2 p = \frac{A_x}{D}\frac{\partial^2 x}{\partial t^2}, \tag{8}$$

$$\frac{\partial^2 q}{\partial t^2} + \varepsilon_y \Omega_f (q^2-1)\frac{\partial q}{\partial t} + \Omega_f^2 q = \frac{A_y}{D}\frac{\partial^2 y}{\partial t^2}, \tag{9}$$

where $\Omega_f = 2\pi S_t \dfrac{U}{D}$. (10)

Here, ε_x, ε_y, A_x and A_y are the empirical parameters which will be stunned by the experiments The acceleration coupling scheme is adopted in the present study to link structural motion equations and Van Der Pol oscillator as suggested by Facchinetti et al. (2004).

A number of studies (Kurushina et al., 2018; Ogink and Metrikine, 2010; Postnikov et al., 2016) indicated that hydrodynamic forces rely on the relative velocity between cylinder motion and free stream velocity and some studies (Ge et al., 2009; Ge et al., 2011; Srinil and Zanganeh, 2012b; Wang et al., 2003) considered force decomposition. In this study, force decomposition technique is adopted and the excitation forces are expressed as

$$\begin{aligned} F_x &= f_D + f_D^{'} - f_L \dot{y} \quad U \quad \frac{1}{2} C_{D} \quad Kq \quad DU \quad \frac{1}{2} C_{Di} \frac{p}{2} \quad DU \quad \frac{1}{2} C_{L} \frac{q}{2} \quad DU\dot{y} \\ F_y &= f_L + f_D^{'} \dot{y} \quad U \quad \frac{1}{2} C_{L} \frac{q}{2} \quad DU \quad \frac{1}{2} C_{Di} \frac{p}{2} \quad DU\dot{y} \end{aligned} \tag{11}$$

where the dot represents the derivative of the time, i.e. $\dot{y}$ is the velocity in the y direction.

2.2 Determination for mean tension

Lee and Allen have conducted a series of experiments and have found that for tension-dominated riser, top tension increases with the increasing stream velocity (Lee and Allen, 2010). In the subsection, mean tension force is derived as a function of the stream velocity.

Mean tension force can be computed according to the Hooke's law showing

$$T = T_{ini} + EA\frac{\Delta L}{L}, \tag{12}$$

where $\Delta L = S - L$. (13)

In Equations (12-13), the mean tension T is modelled proportional to the prolongation of the cylinder. T_{ini} is the initial tension, A is the cross-sectional area, L is the initial length of the riser, S is the instantaneous length and $\triangle L$ is the prolongation of the riser. In the literature, the instantaneous length of the cylinder S is calculated as

$$S = \int_0^L \sqrt{1 + \left(\frac{\partial y}{\partial z}\right)^2 + \left(\frac{\partial x}{\partial z}\right)^2} dz. \tag{14}$$

For a long flexible cylinder, the prolongation is resulted from the deflections forced by the drag and lift forces experienced by the cylinder. Considering that the force is consist of the mean drag, fluctuating drag and lift, the deflection contains corresponding three parts. Among them, the deflection x_{mean} induced by mean drag force is typically greater than the deflection resulted from fluctuating drag and lift forces. More importantly, in a long term simulation of VIV, the prolongation induced by deflections corresponding to fluctuating drag and lift does not have lasting influence. More specifically, the deflections corresponding to fluctuating drag and lift vary temporally and therefore has only impacts on limited temporal and spatial scales. So, $\left(\frac{\partial y}{\partial z}\right)^2$ in Equation (14) is neglected. In the case that x_{mean} is sufficiently small and its derivative is assumed to be small, the instantaneous length S could be simplified as

$$S=\int_0^L\sqrt{1+\left(\frac{\partial x_{mean}}{\partial z}\right)^2}dz=\int_0^L\sqrt{1+x'_{mean}}dz\approx\int_0^L 1+\frac{1}{2}x'^2_{mean}dz\,, \tag{15}$$

where x'_{mean} represents the derivative of x_{mean} with respect to z.

Simplifying Equations (13-15) yields

$$\Delta L\approx\frac{1}{2}\int_0^L x'^2_{mean}dz\,. \tag{16}$$

For a pin-ended beam, an analytical solution to Equation (16) exists if the deflection shape is expressed as a sine function (Gu et al., 2012) showing,

$$x_{mean}=\frac{P}{EI\frac{\pi^4}{L^4}+T\frac{\pi^2}{L^2}}\sin\left(\pi\frac{z}{L}\right), \tag{17}$$

where P is the external force per unit length approximated as,

$$P=\frac{1}{2}\rho DU^2C_D\,. \tag{18}$$

After some manipulations of Equations (12-13,16-18), the mean tension is obtained,

$$T=T_{ini}+\frac{EA}{16\pi^2}\left(\frac{\rho DU^2C_DL^3}{EI\pi^2+TL^2}\right)^2. \tag{19}$$

Based on the tension calculated according to Equation (19), the influence of tensions on the vibrations in both the CF and IL directions are investigated in the present study. Then the formula, i.e. Equation (19), is verified.

2.3 Boundary and initial conditions

Since the long flexible cylinder is assumed to be pinned at the ends, the displacements and moments at the ends should hence be kept zero during vibrations:

$$r(0,t)=r(L,t)=0 \qquad \text{(t > 0)},$$

$$\frac{\partial^2 r}{\partial z^2}(0,t)=\frac{\partial^2 r}{\partial z^2}(L,t)=0 \qquad \text{(t > 0)}.$$

The initial conditions in solving for the displacements of the cylinder and the corresponding wake variables are, given as,

$$r(z,0)=0, \qquad \frac{\partial r}{\partial t}(z,0)=0, \qquad \text{(0 < z < L)};$$

$$p(z,0)=q(z,0)=2, \qquad \frac{\partial p}{\partial t}(z,0)=\frac{\partial q}{\partial t}(z,0)=0, \qquad \text{(0 < z < L)}.$$

Given the boundary and initial conditions, the time history of displacements along the cylinder are then simulated through numerically solving the governing equations of cylinder dynamics. More specifically, the standard central finite difference scheme is employed to discretize the equations in both space and time domains. The cylinder is separated into *N* segments by *N-1* points and by the distance *h*. Let the displacement *r*, at point *i* ($3 \le i \le N-2$) be denoted as r_i. The superscript refers to the time step. The approximations of Equations (1, 8-9) are

$$M\left(\frac{r_i^{(t+1)}-2r_i^{(t)}+r_i^{(t-1)}}{(\Delta t)^2}\right)+(R_f+R_s)\left(\frac{r_i^{(t+1)}-r_i^{(t-1)}}{2\Delta t}\right)+EI\left(\frac{r_{i+2}^{(t)}-4r_{i+1}^{(t)}+6r_i^{(t)}-4r_{i-1}^{(t)}+r_{i-2}^{(t)}}{h^4}\right)-T\left(\frac{r_{i+1}^{(t)}-2r_i^{(t)}+r_{i-1}^{(t)}}{h^2}\right)$$
$$=F_i^{(t)}, \tag{20}$$

$$\frac{p_i^{(t+1)}-2p_i^{(t)}+p_i^{(t-1)}}{(\Delta t)^2}+2\varepsilon_x\Omega_f(p_i^{2(t)}-1)\frac{\left(p_i^{(t+1)}-p_i^{(t-1)}\right)}{2\Delta t}+4\Omega_f^2 p_i^{(t)}=\frac{A_x}{D}\left(\frac{x_i^{(t+1)}-2x_i^{(t)}+x_i^{(t-1)}}{(\Delta t)^2}\right), \tag{21}$$

$$\frac{q_i^{(t+1)}-2q_i^{(t)}+q_i^{(t-1)}}{(\Delta t)^2}+\varepsilon_y\Omega_f(q_i^{2(t)}-1)\frac{\left(q_i^{(t+1)}-q_i^{(t-1)}\right)}{2\Delta t}+\Omega_f^2 q_i^{(t)}=\frac{A_y}{D}\left(\frac{y_i^{(t+1)}-2y_i^{(t)}+y_i^{(t-1)}}{(\Delta t)^2}\right), \tag{22}$$

as done by Ge et al. (2009) and Gosse and Barsdale (1969).

With the solution for the above equations, the displacements, as a function of the z coordinate and time, are obtained showing the vibrations of the cylinder.

3. Case study

In this section, the experimental test case conducted by Trim et al. (2005) is simulated using the proposed numerical model to validate that Equation (19) is reliable to model the mean top tension for a long flexible cylinder. Main parameters defining the cylinder are listed in Table 1. The free-stream velocity in the simulation varies from 0.3m/s to 2.4m/s with an increment of 0.1m/s.

Table 1 Riser characteristics (Trim et al., 2005)

Outer diameter	D=0.027m	Young's modulus of elasticity	E=36.2×10^{9}N/m^2
Inner diameter	d=0.021m	Axial tension	T_{ini}=4-6kN
Wall thickness	t_w=0.003m	Aspect ratio	1407
Length	L=38m	Mass ratio	1.6
Structure mass	m_s=0.939kg/m	Density	ρ=1025kg/m^3

In the present study, the environmental parameters are given (See Table 2) following the suggestions in the literature (Furnes and Sørensen, 2007; Ge et al., 2009; Rosetti et al., 2009).

Table 2 Environmental parameters

Ca	1	St	0.17
C_{D0}	1.2	C_{Di0}	0.1
C_{L0}	0.3	γ	0.45
ε_x	0.3	ε_y	0.3
A_x	12	A_y	12

Figure 2 shows the variation of mean tension with stream velocity in the present simulation and in the simulation reported by Gu et al. (2012). In addition, the influence of mean tensions on the natural frequency of the cylinder, or the riser, is also presented in Figure 2. It is evident from the figure that the mean tension increases from 5kN at U=0.3m/s to 15.9kN at U=2.4m/s as predicted by Equation (19). In the simulation reported by Gu et al. (2012), on the other hand, the mean tension increases from 6kN to 10.6kN. Moreover, Figure 2 implies a linear relationship between the natural frequency and the square root of the mean tension as in experimental studies (Lee and Allen, 2010). In fact, Figure 2 substantiates that the natural frequency of the cylinder is not an inherent property of tensioned structural component but a variable reflecting the influence of external loads. The fact that the numerical simulation reported in the present study support a linear relationship between the natural frequency and the square root of the mean tension substantiates that the proposed numerical model is applicable to simulate the overall dynamics of the cylinder under the influence of tensions and environmental flows.

Besides the illustrative comparisons shown in Figures 2, Figure 3 and Figure 4 present the comparison of dominant mode numbers and vibration frequencies obtained in the present simulation and extracted from the manuscripts of Ge et al. (2009), Gu et al. (2012) and Trim et al. (2005). More specifically, Figure 3 shows that the dominant mode number increases with the stream velocity, and the number corresponding to the IL vibration is almost twice as large as the number corresponding to the CF vibration. Comparisons presented in Figure 3 indicate that the CF dominant mode numbers obtained from the present study agree with the experimental measurements and numerical results reported by Ge et al. (2009) and Gu et al. (2012). The maximum deviation of mode number, from the prediction of the proposed model to the experimental results, in the IL direction is 5, smaller than the value reported by Ge et al. (2009). The maximum deviation in the CF direction, in addition, is 2 comparing to 3 as reported by Ge et al. (2009) and 4 as reported by and Gu et al. (2012). In the IL direction, Figure 3 shows that the proposed model is able to capture the vibration with high mode numbers in the high-speed streams. The predictions given by Ge et al. (2009), on the other hand, considerably deviate from the experimental data. It should be noted that the IL vibrations are not included in the simulation conducted by Gu et al. (2012), and therefore no data is available for the comparison. When the free-stream velocity is within the range of 1.7-2.2 m/s, the mode numbers predicted by both the proposed model and the model suggested by Ge et al. (2009) are lower than the experiment results reported by Trim et al. (2005). In the CF direction, both the proposed model and the model suggested by Gu et al. (2012) predict vibrations with a higher mode number, in contrast to the results reported by Ge et al. (2009), when the stream velocity is in the range of 2 to 2.4m/s. When the flow velocity is 1.5-2m/s, the proposed model predicts a larger value of the mean tension than the model suggested by Gu et al. (2012), resulting in a higher mode number. Consequently, the dominant mode number predicted by the proposed model is in better agreement with the experimental data given by Trim et al. (2005). When the stream velocity is in the range of 1.5-2 m/s, the dominate mode number predicted by the proposed model is, however, lower than the Trim's experimental (Trim et al., 2005) results in the IL direction, and higher than Trim's results (Trim et al., 2005) in the CF direction. Such a finding implies that the proposed model still needs improvements.

Figure 4 shows that frequencies of cylinder vibrations increase with stream velocities. In addition, Figure 4 reveals that the frequencies corresponding to the IL vibration is twice as large as the frequencies of the vibrations in the CF direction. For the sake of being illustrative, the vortex shedding frequency calculated according to the Strouhal relation and the doubled shedding frequency are plotted in Figure 4. It is well reported in the previous investigations that the measured/simulated frequencies of the cylinder vibration are different from the vortex shedding frequency of the fixed cylinder (Trim et al., 2005). Such differences are also observed in Figure 4. In fact, Figure 4 indicated that the vibration frequencies predicted by the proposed model are in better agreement with the experimental data when comparing the numerical results given by Ge et al. (2009). The root mean square (RMS) of deviation is 2.10 in the IL direction compared to 2.45 reported by Ge et al. (2009) while it is 1.13 in the CF direction compared to 1.43 and 1.15 reported by Ge et al. (2009) and Gu et al. (2012), respectively. As the dynamic responses of the cylinder are in associations with the stiffness of the structure, which is in turn influenced by the tension, the agreement presented in Figure 5 validates the proposed model in modelling the mean tension and its influence on the dynamics of the cylinder (Equation 19).

As introduced in Section 2.3, both IL and CF displacements are the function of space and time. In the verification, the maximum of displacement standard deviation ξ, which is defined as in Equation (23), is used to quantitatively assess the deviation of the prediction from the proposed model to the experimental data.

$$\xi = \max\left(S_i\right),\ S_i = \frac{std(r_i(t))}{D} = \frac{1}{D}\sqrt{\frac{\sum\left(r_i^{(t)} - \overline{r}_i\right)}{Nt-1}},\ 1 \le i \le N, \tag{23}$$

where N and Nt are total numbers of nodes and time steps, respectively, S_i represents displacement standard deviation at node i and $\overline{r}_i$ is mean value of displacements at node i. It should be noted that data is taken after the simulation is stable, i.e. periodic vibration with almost constant amplitude. Figure 5 shows the maximum of displacement standard deviation ξ in CF and IL directions varying with the stream velocity. It is shown in the figure that ξ for CF and IL directions are around 0.9D and 0.2D, respectively. In addition, Figure 6 implies that the amplitude predicted by the proposed model for CF vibrations is larger than the experimental data, which makes the predictions conservative in the assessment of the safety of the cylinder as a riser. Moreover, Figure 5 substantiates that the predictions of the proposed model in IL direction are close to Trim's experimental results (Trim et al., 2005). The mean squared deviation of the displacements from the proposed model to the experiment data is 0.11 for the IL direction and 1.26 for the CF direction.

The inaccuracy in predicting ξ is mainly attributed to the inaccuracy of the damping model, including the estimation of the hydrodynamic damping coefficient λ and structural damping ratio ξ_D. λ usually takes the value of 0.8 as reported in the literature, but the simulation results suggest that the value of λ should be around 0.45 for the prediction of the proposed model to better match the experimental data.

In order to directly illustrate the cylinder vibrations simulated by the proposed model, the RMS of displacements in both IL and CF directions at U=1m/s are presented in Figures 6. It is apparent that the 13^{th} and 7^{th} modes are predominant for the IL and CF vibrations of the cylinder, respectively. Such findings are in line with the numerical and experimental investigations on the vibrations of the long flexible cylinders. In fact, the predominant mode number shown in Figures 6 indicates that the IL vibrations are with the frequency twice as large as the frequency of the CF vibrations. The same mode number is obtained as that from Gu et al. (2012) except the difference between the amplitudes. In addition, the IL vibrations are not included in the simulation conducted by Gu et al. (2012), and therefore not shown in Figure 6.

Figure 7 gives the time histories of non-dimensional displacements of y/D and x/D at different locations, with z/L equal to 0.84, 0.67, 0.5, 0.33 and 0.16 at U=1m/s, and corresponding response spectra. It is evident that the displacement follows a precisely periodic trend and the segment at different elevations vibrates at the same frequency. With a specific case shown in Figure 7, the cylinder vibration frequencies, regardless of the location along its length, in CF and IL directions are 6.29Hz and 12.5Hz, respectively. As reported in Vandiver et al.'s study (Vandiver et al., 2009), there are two harmonic components in IL or CF vibrations and their intensities are different at different locations. More specifically, the harmonic component with higher frequencies weakens approaching to the end of the cylinder. The reason why only a harmonic vibration with a single frequency is produced in the numerical simulation reported in the present and similar investigations could be the variation in structural properties of the cylinder. More specifically, the experiment employs a cylinder that unavoidably contains flaws in the manufacturing, which results in variations in structural properties along the length of the cylinder.

Figure 8 and 9 show the trajectories of the vibration of cylinder segments at different elevations at U=1m/s and evolutions of the non-dimensional displacement at U=1.5m/s. In Figure 8, the trajectories shifted back and forth between the traditional "figure 8" and the typical "crescent" shapes. It is argued that many factors could impact trajectory patterns, such as the cylinder scale, the ratio of IL natural frequency to CF natural frequency, mass ratio and the IL and CF frequencies (Kang et al., 2016). The trajectories shown in Figure 8 are the evidences that IL and CF waves are not phase locked. Such feature, i.e. the figure-of-eight trajectory, is also observed by Srinil and Zanganeh (2012b). In Figure 9, a traveling wave is observed as reported in the literature and its propagation direction is arbitrary (Violette et al., 2007). Compared to Ge et al.'s results (Ge et al., 2009), the predicted amplitudes of the proposed model are higher, and closer to Trim's experimental results (Trim et al., 2005). In addition, the same dominant mode numbers for the vibrations in both IL and CF directions are observed. The similarities and differences in amplitude and dominant mode numbers are consistent with the findings shown in Figure 2 and 4 for the two numerical models. With the increase in flow velocities, dominant wave pattern shifted from standing wave to traveling wave.

Conclusions

A three-dimensional model predicting the VIV in both the IL and CF directions, coupled with a set of modified Van Der Pol equations, is presented. Fluid forces, including the lift and drag forces, due to the vortex-shedding are modelled by the flow variables the same as that in other semi-empirical models which contain a wake oscillator. A new tension formula is proposed to account for the variations in tensions due to the prolongations occurring in the cylinder. The proposed model (especially the tension formula) is validated by comparing to the available experimental data and numerical results. The comparison shows that the present model is capable of simulating VIV of long flexible cylinders in both the CF and IL directions under the influence of uniform incoming flow. Since the tension is modelled as a function of the stream velocity, the dynamics of cylinder VIV is more realistically simulated in this study. In fact, it is found that the proposed model outperforms the model proposed by Ge et al. (2009) and Gu et al. (2012) in predicting the variations for the vibration amplitudes and frequencies and dominant mode number with the stream velocity. Most importantly, this is the first attempt, to the best of the author's knowledge, to propose a model containing dynamically determined tension to predict VIV in both the CF and IL directions.

Some aspects of long slender cylinders undergoing VIV can be reproduced qualitatively and quantitatively, such as dominant mode number, vibration frequency, amplitude and traveling wave phenomenon. Future research will be focused on the physical meaning of the model parameters.

Declarations of interest: none.

Acknowledgements

The authors would like to express their gratitude towards following organizations for financially supporting the work described in the present paper, which includes National Natural Science Foundation of China (Grant No. 51608302, Grant No. 51579227)

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APPENDIX: LIST OF SYMBOLS

M	the sum of structure mass m_s and added fluid mass per unit length
F	hydrodynamic force per unit length, with components Fx and Fy
r	the deflection, a vector, with components x and y
EI	bending stiffness
E	Young's modulus
T	axial tension
C_a	added mass coefficient
ρ	density of seawater
D	diameter of the riser
R	damping coefficient due to hydrodynamic damping and structural damping
R_f	damping coefficient due to hydrodynamic force
R_s	damping coefficient due to structure force
γ	parameter determined through experiments
Ω_f	vortex shedding frequency
Ω_n	natural frequency of the riser in air
Ω_s	natural frequency of the riser in water
ξ_D	damping ratio
T_{ini}	tension force before deflection
A	cross section area of the riser
L	length of the riser
$\triangle L$	prolongation of the riser

F_x external force exerted perpendicularly on the model in the x direction

F_y external force exerted perpendicularly on the model in the y direction

f_D mean drag force per unit length

f_D' fluatuating drag force per unit length

f_L lift force per unit length

$\bar{C}_D$ total drag coefficient during vortex-shedding

C_{D0} mean drag coefficient of a stationary rigid cylinder

$Y_{RMS/D}$ the amplitude of vibration in the CF direction

C_{Di} vortex shedding drag coefficient

C_{Di0} the amplitude of vortex shedding drag coefficient

C_{L0} lift coefficient of a stationary rigid cylinder

C_L lift coefficient

p in line variable

q cross flow variable

ε_x, ε_y, A_x, A_y non-dimensional parameters estimated through experiments

S_t Strouhal number