Impedance Modelling and Stability Analysis of Diode-Rectifier based HVDC Connected Offshore Wind Farms

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Abstract—This paper investigates the stability of diode rectifier based HVDC (DR-HVDC) connected offshore wind farms. The impedance model of the DR-HVDC connected offshore wind farms in dq frame is analytically developed and is validated by comparing its impedances to those obtained from frequency scanning of time domain EMT model in PSCAD/EMTDC. Based on the impedance model, the impacts of the sizes of DR-HVDC DC smoothing reactors and AC filters on offshore system stability are analyzed. It shows reduced DC smoothing reactance degrades the d-axis stability margin and can potentially lead to system instability. Higher bandwidth of offshore AC voltage control of wind turbine line side converter is proposed to minimize such instability potential. The analysis also shows that reduced AC filters size helps to increase q-axis stability margin and leads to a more stable system. Moreover, stability analysis considering the interaction between WTs is conducted and it reveals that the WT P - V control has a significant impact on the stability of the DR-HVDC connected WTs.

Index Terms—diode rectifier based HVDC, grid forming wind turbine control, impedance modelling, interaction between offshore wind farms and HVDC, stability analysis.

I. INTRODUCTION

Many large offshore wind farms are being developed and more have been planned. The rapid development of offshore wind power generation requires efficient power transmission technology [1]. Comparing with high voltage alternating current (HVAC), high voltage direct current (HVDC) offers significant advantages for integrating large offshore wind farms over long distance, such as lower losses and cost [2]. However, offshore wind farms connected with HVDC lead to a converter-dominated network, with no rotating mass which physically links the generated power with the frequency and no resistive loads which provide sufficient damping [3, 4]. In such isolated offshore network, the interaction between wind turbine (WT) converters and HVDC offshore converter may pose challenges to its stable operation. For instance, oscillation was reported when voltage source converter based HVDC (VSC-HVDC) is used to transmit the offshore wind power in North Sea, resulting in the shut-down of wind farms for several months [5]. The small signal impedance-based analysis is now regarded as an effective way to clarify the interaction between WTs and HVDC systems [6, 7]. When the detailed parameters of wind turbines are difficult to obtain and only a black box WT model is provided by the manufactory, the impedance-based analysis can be readily used to determine the system stability

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after obtaining the output impedance of the wind farms. With this approach, the feasibility/effectiveness of stability analysis for such a system can be significantly improved. In [6], the dq impedances of the WT and two level VSC-HVDC are derived and the bandwidth ratio of the subsystem is suggested to improve system stability. In [7], the modular multilevel converter (MMC) impedance is presented and the impact of circulating current control on the offshore system stability is investigated.

Recently, diode rectifier based HVDC (DR-HVDC), where an uncontrolled diode rectifier is used at the offshore station and an MMC is used at the onshore station, has received increased attentions, especially for large and remote offshore wind farm connection [8-10]. When replacing the offshore VSC with the diode rectifier, it provides the advantages of lower investment, lower space requirement, higher efficiency and improved robustness. It has the potential to reduce the transmission loss by 30%, offshore substation topside volume by 80% and total cost by 30% [11]. However, the use of DR poses challenges for the start-up of the offshore wind farms due to its unidirectional characteristic. To overcome this issue, [11, 12] propose a scheme where umbilical AC cables operate in parallel with the DR-HVDC link. In [13, 14], additional battery storage is installed in the WTs to power the auxiliaries during the initial energization. In addition, this transmission technology requires the WTs to take more control responsibilities, e.g. the WT line side converters (LSCs) need to provide grid-forming capability rather than work on grid-following mode. References [8, 9] propose a WT grid forming control, where the current used to establish the offshore voltage and frequency of point of common coupling (PCC) is shared among all the WT LSCs. However, the need for measuring the voltage and current at PCC requires high-speed communications among the WTs. Reference [15] propose a fixed frequency control based on global positioning system (GPS) to provide a common angular reference for all the WTs. Such control solution avoids the need for remote measurements but can potentially increase the control system cost and reduce system reliability. A decentralized WT control is proposed in [16, 17], where a reactive power frequency droop control is adopted to regulate the WT phase angle, but such control without the use of PLL may pose challenges during initial start-up, as each WT has to synchronize with the offshore network before de-blocking. In [14, 18, 19], a distributed PLL based WT control is proposed, which enables each WT to form the offshore network based on local variables and behave as distributed AC voltage sources with synchronization capability.

However, the stability analysis of DR-HVDC connected offshore wind farms has not been thoroughly conducted. As the

offshore network in such systems is made up of uncontrolled diode rectifier and grid forming WT LSCs, the interaction between them is very different from the interaction between MMC-HVDC and grid following WT LSCs. Therefore, the findings obtained from the research on MMC-HVDC system cannot be applied to the DR-HVDC connected offshore wind farm system. Thus, this paper carries out the impedance modelling and stability analysis of DR-HVDC connected offshore wind farms. The main contributions of this paper are summarized as:

- A small signal impedance model of DR-HVDC is analytically developed. Comparing with the small signal model presented in [19], the model developed in this paper reflects the frequency dependent overlap angle dynamics. Impedance comparison between the developed model and time domain model shows the modelling accuracy is significantly improved.
- As the sizes of the DR-HVDC DC smoothing reactor and AC filters play a decisive role in determining the weight, volume, and cost of the offshore station, the impacts of reduced DC smoothing reactors and AC filters on the offshore system stability are analyzed using the impedance models.
- Stability analysis considering the interaction between WTs is conducted and it reveals that the WT P - V control has a significant impact on the stability of the DR-HVDC connected WTs.

The rest of this paper is organized as follows. Section II presents the structure of DR-HVDC connected offshore wind farms. The impedances of DR-HVDC and the offshore wind farms are developed in Section III and IV, respectively. Section V presents the stability analysis based on the developed impedance model, considering the variations of DC smoothing reactors and AC filters, and Section VI draws conclusions.

II. STRUCTURE OF DR-HVDC CONNECTED OFFSHORE WIND FARMS

Fig. 1 presents the diagram of 1000 MW DR-HVDC connected offshore wind farms, which consist of 10 strings. Each string is made up of 10 permanent magnet synchronous generators (PMSG) based WTs. Each WT is rated at 10 MW and connected to the collection system through a step-up transformer from 690 V to 66 kV. The offshore HVDC station includes a 12-pulse diode rectifier with star-star and star-delta connected transformers, AC filters and DC smoothing reactors. The onshore HVDC station uses modular multilevel converter (MMC), which controls the DC voltage of the HVDC system. As this paper focuses on the analysis of interaction between offshore WT LSCs and DR-HVDC, the WT generation side converters are simplified as DC sources [8, 9]. References [20, 21] demonstrate that the back-to-back converters in PMSG-WT decouples the generator side and line side and the states of the generator side do not actively participate in the offshore network oscillatory modes. Meanwhile, the WTs are represented as an aggregated LSC and the onshore MMC is simplified as a DC source.

Fig. 1 DR-HVDC connected offshore wind farms.

III. IMPEDANCE MODELLING OF DR-HVDC

A. DR-HVDC impedance modelling

For the diode rectifier, it can be considered as a current source on the AC side and a voltage source on the DC side [22]. Thus, with AC voltage and DC current disturbances (Δv_d , Δv_q and ΔI_{dc}), the small signal DC voltage (ΔV_{dc}), and AC current of the diode rectifier (Δi_d and Δi_q) in dq domain can be represented in a generic form as

$$\begin{bmatrix} \Delta i_d \\ \Delta i_q \\ \Delta V_{dc} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{bmatrix} \Delta v_d \\ \Delta v_q \\ \Delta I_{dc} \end{bmatrix} \quad (1)$$

The DC side dynamics can be considered using the equivalent DC impedance as

$$\Delta V_{dc} = \Delta I_{dc} Z_{dc} \quad (2)$$

Thus, the diode rectifier AC dynamics in dq domain are described as $\begin{bmatrix} \Delta v_d \\ \Delta v_q \end{bmatrix} = \mathbf{Z}_{DR} \begin{bmatrix} \Delta i_d \\ \Delta i_q \end{bmatrix}$, where the equivalent AC impedance is given as

$$\mathbf{Z}_{DR} = \mathbf{Y}_{DR}^{-1} = \begin{bmatrix} Y_{DRdd} & Y_{DRdq} \\ Y_{DRqd} & Y_{DRqq} \end{bmatrix}^{-1} \quad (3)$$

and

$$\begin{aligned} Y_{DRdd} &= K_{11} + \frac{K_{13}K_{31}}{Z_{dc} - K_{33}} & Y_{DRdq} &= K_{12} + \frac{K_{13}K_{32}}{Z_{dc} - K_{33}} \\ Y_{DRqd} &= K_{21} + \frac{K_{23}K_{31}}{Z_{dc} - K_{33}} & Y_{DRqq} &= K_{22} + \frac{K_{23}K_{32}}{Z_{dc} - K_{33}} \end{aligned} \quad (4)$$

The following subsections present how the elements of K_{mn} in (1) are derived.

1) Diode rectifier DC voltage dynamics

As the DC voltage of a 6-pulse diode rectifier consists of six repeated segments in each fundamental cycle, it is sufficient to only consider one segment (i.e. $\pi/3$ interval). Here, the segment when the conduction mode D_6D_5 (D_6 and D_5 conducting) is commuted to D_6D_1 (D_6 and D_1 conducting) is studied, as shown in Fig. 2. The DC voltage (V_{dcp} and V_{dcn}) and AC current (i_a) during and after the commutation in this segment are presented in Fig. 3.

Fig. 2 Diode rectifier commutation from conduction mode D_6D_5 to D_6D_1 .

Fig. 3 DC voltage and current during and after the commutation.

During the commutation from D_6D_5 to D_6D_1 , the three-phase AC current dynamics are expressed as

$$L_c \frac{di_a}{dt} = v_a - V_{dcp}, \quad L_c \frac{di_b}{dt} = v_b + V_{dcn}, \quad L_c \frac{di_c}{dt} = v_c - V_{dcp} \quad (5)$$

where

$$i_a + i_c = -i_b = I_{dc} \quad (6)$$

Thus, the DC voltages of the diode rectifier during and after the commutation are given respectively as

$$V_{dc}^c = \frac{v_a + v_c}{2} - v_b - \frac{3}{2} L_c \frac{dI_{dc}}{dt} \quad (7)$$

$$V_{dc}^{nc} = v_a - v_b - 2L_c \frac{dI_{dc}}{dt} \quad (8)$$

Representing the AC voltage in dq domain, the DC voltages during and after commutation are expressed as

$$V_{dc}^c = \frac{3}{2} [v_d \cos(\varphi + \frac{\pi}{3}) - v_q \sin(\varphi + \frac{\pi}{3})] - \frac{3}{2} L_c \frac{dI_{dc}}{dt} \quad (9)$$

$$\text{when } -\frac{\pi}{3} < \varphi < -\frac{\pi}{3} + \mu$$

$$V_{dc}^{nc} = \sqrt{3} [v_d \cos(\varphi + \frac{\pi}{6}) - v_q \sin(\varphi + \frac{\pi}{6})] - 2L_c \frac{dI_{dc}}{dt} \quad (10)$$

$$\text{when } -\frac{\pi}{3} + \mu < \varphi < 0$$

The average DC voltage over one segment is

$$V_{dc} = \frac{3}{\pi} \left[\int_{-\frac{\pi}{3}}^{-\frac{\pi}{3} + \mu} V_{dc}^c d\varphi + \int_{-\frac{\pi}{3} + \mu}^0 V_{dc}^{nc} d\varphi \right] \quad (11)$$

$$= \frac{3\sqrt{3}}{2\pi} [v_d(1 + \cos \mu) - v_q \sin \mu] - s \left(2 - \frac{3\mu}{2\pi} \right) L_c I_{dc}$$

Linearizing (11) yields the DC voltage deviation as

$$\Delta V_{dc} = \frac{3\sqrt{3}}{2\pi} [\Delta v_d(1 + \cos \mu_0) - v_{d0} \sin \mu_0 \Delta \mu - \Delta v_q \sin \mu_0 - v_{q0} \cos \mu_0 \Delta \mu] - s \left(2 - \frac{3\mu}{2\pi} \right) L_c \Delta I_{dc} \quad (12)$$

Thus,

$$K_{31} = \frac{\Delta V_{dc}}{\Delta v_d} = \frac{3\sqrt{3}}{2\pi} [1 + \cos \mu_0] + \frac{3\sqrt{3}}{2\pi} (-v_{d0} \sin \mu_0 - v_{q0} \cos \mu_0) \frac{\Delta \mu}{\Delta v_d} \quad (13)$$

$$K_{32} = \frac{\Delta V_{dc}}{\Delta v_q} = -\frac{3\sqrt{3}}{2\pi} \sin \mu_0 + \frac{3\sqrt{3}}{2\pi} (-v_{d0} \sin \mu_0 - v_{q0} \cos \mu_0) \frac{\Delta \mu}{\Delta v_q} \quad (14)$$

$$K_{33} = \frac{\Delta V_{dc}}{\Delta I_{dc}} = -s \left(2 - \frac{3\mu}{2\pi} \right) L_c + \frac{3\sqrt{3}}{2\pi} (-v_{d0} \sin \mu_0 - v_{q0} \cos \mu_0) \frac{\Delta \mu}{\Delta I_{dc}} \quad (15)$$

B. Commutation overlap angle dynamics

As can be seen in (13)-(15), the small signal DC voltage variation depends on the dynamics of the commutation overlap angle. This subsection describes how the input disturbance affects the commutation overlap angle.

According to (5) and (6), the phase 'a' current i_a is

$$2L_c \frac{di_a}{dt} - L_c \frac{di_{dc}}{dt} = v_a - v_c \quad (16)$$

Considering the impact of phase 'a' current by the DC current and AC voltages separately, i_a is represented as two parts

$$i_a = i_{1a} + i_{2a} \quad (17)$$

where i_{1a} and i_{2a} represent the parts of the current determined by the DC current I_{dc} and the AC voltage, respectively, as

$$2L_c \frac{di_{1a}}{dt} = L_c \frac{dI_{dc}}{dt} \quad (18)$$

$$2L_c \frac{di_{2a}}{dt} = v_a - v_c \quad (19)$$

According to (17), i_a at the end of commutation, i.e. when $\varphi = \varphi_f + \mu$, is given as

$$i_a(\varphi_f + \mu) = i_{1a}(\varphi_f + \mu) + i_{2a}(\varphi_f + \mu) \quad (20)$$

where

$$i_{1a}(\varphi_f + \mu) = I_{dc}(\varphi_f + \mu) \quad (21)$$

$$i_{1a}(\varphi_f + \mu) = \frac{1}{2} [I_{dc}(\varphi_f + \mu) - I_{dc}(\varphi_f)] \quad (22)$$

$$i_{2a}(\varphi_f + \mu) = i_{2d}(\varphi_f + \mu) \cos(\varphi_f + \mu) - i_{2q}(\varphi_f + \mu) \sin(\varphi_f + \mu) - [i_{2d}(\varphi_f) \cos(\varphi_f) - i_{2q}(\varphi_f) \sin(\varphi_f)] \quad (23)$$

Linearizing (20) yields

$$\frac{1}{2} [\Delta I_{dc}(\varphi_f + \mu) + \Delta I_{dc}(\varphi_f)] = \Delta i_{2d}(\varphi_f + \mu) \cos(\varphi_f + \mu) - \Delta i_{2q}(\varphi_f + \mu) \sin(\varphi_f + \mu) - i_{2d}(\varphi_f + \mu) \sin(\varphi_f + \mu) \Delta \mu - i_{2q}(\varphi_f + \mu) \cos(\varphi_f + \mu) \Delta \mu - [\Delta i_{2d}(\varphi_f) \cos(\varphi_f) - \Delta i_{2q}(\varphi_f) \sin(\varphi_f)] \quad (24)$$

Transforming (24) to Laplace domain, and also transforming the variables with angle at $\varphi_f + u$ to the variables at φ_f [23], as

$$L[\Delta i_{2dq}(\varphi_f)] = e^{-su/\omega_0} L[\Delta i_{2dq}(\varphi_f + u)] \quad (25)$$

equation (24) can be represented as

$$-\frac{1}{2} \Delta I_{dc} (1 + e^{-s\mu_0/\omega_0}) + \Delta i_{2d} \left[\cos(u_0 - \frac{\pi}{3}) - \frac{1}{2} e^{-s\mu_0/\omega_0} \right] - \Delta i_{2q} \left[\sin(u_0 - \frac{\pi}{3}) + \frac{\sqrt{3}}{2} e^{-s\mu_0/\omega_0} \right] - \Delta \mu \left[i_{2d0} \sin(u_0 - \frac{\pi}{3}) + i_{2q0} \cos(u_0 - \frac{\pi}{3}) \right] = 0 \quad (26)$$

As i_{2a} is determined by the AC voltage shown in (19), i_{2dq} can be described as

$$\begin{bmatrix} \Delta i_{2d} \\ \Delta i_{2q} \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} \Delta v_d \\ \Delta v_q \end{bmatrix} \quad (27)$$

where

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \frac{1}{4L_c(s^2 + \omega_0^2)} \begin{bmatrix} 3s - \sqrt{3}\omega_0 & \sqrt{3}s + 3\omega_0 \\ -\sqrt{3}s - 3\omega_0 & 3s - \sqrt{3}\omega_0 \end{bmatrix} \quad (28)$$

Combining (26) and (27), the commutation overlap deviations $\frac{\Delta \mu}{\Delta v_d}$, $\frac{\Delta \mu}{\Delta v_q}$ and $\frac{\Delta \mu}{\Delta I_{dc}}$ are obtained. The details are given in Appendix A.

C. Diode rectifier AC current dynamics

For the diode rectifier AC side current, during the commutation interval, it is described as

$$i_a = i_a, \quad i_b = -I_{dc}, \quad i_c = I_{dc} - i_a \quad (29)$$

Through Park transformation, the d-axis AC current during the commutation i_{di}^c is expressed as

$$i_{di}^c = \frac{2\sqrt{3}}{3} \left[i_a \sin\left(\varphi + \frac{\pi}{3}\right) - I_{dc} \sin\varphi \right] \quad (30)$$

where

$$i_a = i_{2d}(\varphi) \cos\varphi - i_{2q}(\varphi) \sin\varphi + \frac{1}{2} I_{dc}(\varphi) \\ - i_{2d}(\varphi_f) \cos(\varphi_f) + i_{2q}(\varphi_f) \sin(\varphi_f) - \frac{1}{2} I_{dc}(\varphi_f) \quad (31)$$

The average d-axis current i_d^c during the commutation is

$$i_d^c = \frac{3}{\pi} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}+\mu} i_{di}^c d\varphi = \frac{2\sqrt{3}}{\pi} \left[-\frac{1}{2} \cos 2\mu - \frac{\sqrt{3}}{2} \sin 2\mu + \frac{1}{2} + \sqrt{3}\mu \right. \\ \left. - \frac{1}{2} \sin 2\mu + \frac{\sqrt{3}}{2} \cos 2\mu - \frac{\sqrt{3}}{2} + \mu \right] i_{2q} + \frac{1}{2} I_{dc} (1 - \cos \mu) \quad (32) \\ - \frac{1}{2} \frac{\omega_0 m}{s^2 + \omega_0^2} i_{2d} - \frac{\sqrt{3}}{2} \frac{\omega_0 m}{s^2 + \omega_0^2} i_{2q} - \frac{1}{2} \frac{\omega_0 m}{s^2 + \omega_0^2} I_{dc}$$

where

$$m = \omega_0 - (\omega_0 \cos \mu_0 + s \sin \mu_0) e^{-s\mu_0/\omega_0} \quad (33)$$

On the other hand, during the non-commutation period, the AC current of the diode rectifier is

$$i_a = I_{dc}, \quad i_b = -I_{dc}, \quad i_c = 0 \quad (34)$$

Thus, the d-axis AC current i_{di}^{nc} and its average i_d^{nc} during non-commutation period are expressed as

$$i_{di}^{nc} = -\frac{2\sqrt{3}}{3} I_{dc} \sin\left(\varphi - \frac{\pi}{3}\right) \quad (35)$$

$$i_d^{nc} = \frac{3}{\pi} \int_{-\frac{\pi}{3}+\mu}^0 i_{di}^{nc} d\varphi = \frac{2\sqrt{3}}{\pi} I_{dc} \left[\frac{1}{2} + \cos\left(\mu + \frac{\pi}{3}\right) \right] \quad (36)$$

After linearization of (32) and (36), K_{11} , K_{12} and K_{13} can be obtained as detailed in Appendix B. Similarly, K_{21} , K_{22} and K_{23} can be obtained as presented in Appendix B.

D. DR-HVDC impedance modelling validation

To validate the derived DR-HVDC impedance model, frequency scanning is conducted in PSCAD/EMTDC, where a specific frequency signal is injected into the DR-HVDC time domain model and the output signal with the same frequency is recorded for impedance/admittance calculation [18]. The model parameters are shown in Appendix C. The diode rectifier AC filters are also included during the frequency scanning, their dynamics can be found in [19] and the admittance of total filter is the sum of the admittances of DR reactive power compensation, low pass filter and high pass filter. Fig. 4 presents the magnitudes and phases of the DR-HVDC admittances from the model developed in this paper, admittances from the small signal model in [19], and admittances from time domain model. The red solid lines are the results from the developed analytical model in this paper,

the black dash lines are the results from the model in [19] and the blue dots represent the frequency scanning results from the time domain model. As can be seen, the frequency responses from the model developed in this paper are in excellent agreements with those from time domain frequency scanning, while the frequency responses from the model presented in [19] show some errors, especially the Y_{dq} and Y_{qq} responses.

Fig. 4 Admittances of derived model (red solid), admittances of model in [19] (black dash) and frequency scanning results (blue dot).

IV. IMPEDANCE MODELLING OF WT LSC

A. WT LSC impedance modelling with inner current and outer voltage control

Fig. 5 (a) presents the WT LSC with both the inner current control and outer voltage control [14], and the corresponding small-signal impedance model is shown in Fig. 5 (b). Thus, its impedance is expressed as

$$\mathbf{Z}_{vi} = \mathbf{v}_f / \mathbf{i}_s = -\mathbf{Z}_C (\mathbf{I} - \mathbf{Y}_L \mathbf{G}_{del} \mathbf{G}_{dei})$$

$$\left(\mathbf{I} - \mathbf{Y}_L \mathbf{G}_{del} \mathbf{G}_{dei} + \mathbf{Z}_C \mathbf{Y}_L + \mathbf{Y}_L \mathbf{G}_{del} \mathbf{G}_{ci} \right. \\ \left. - \mathbf{Z}_C \mathbf{Y}_L \mathbf{G}_{del} \mathbf{G}_{ci} \mathbf{G}_{dev} + \mathbf{Z}_C \mathbf{Y}_L \mathbf{G}_{del} \mathbf{G}_{ci} \mathbf{G}_{cv} - \mathbf{Z}_C \mathbf{Y}_L \mathbf{G}_{del} \right)^{-1} \quad (37)$$

where the WT inductance admittance $\mathbf{Y}_L$, WT capacitance impedance $\mathbf{Z}_C$, current control matrix $\mathbf{G}_{ci}$, current control decoupling $\mathbf{G}_{dei}$ term, time delay term $\mathbf{G}_{del}$ caused by the PWM delay (T_{delay}), voltage control matrix $\mathbf{G}_{vi}$ and voltage control decoupling $\mathbf{G}_{dev}$ term are expressed as

$$\mathbf{Y}_L = \mathbf{Z}_L^{-1} = \begin{bmatrix} Z_{Ldd} & Z_{Ldq} \\ Z_{Lqd} & Z_{Lqq} \end{bmatrix}^{-1} = \begin{bmatrix} Ls & -\omega L \\ \omega L & Ls \end{bmatrix}^{-1} \quad (38)$$

$$\mathbf{Z}_C = \mathbf{Y}_C^{-1} = \begin{bmatrix} Y_{Cdd} & Y_{Cdq} \\ Y_{Cqd} & Y_{Cqq} \end{bmatrix}^{-1} = \begin{bmatrix} Cs & -\omega C \\ \omega C & Cs \end{bmatrix}^{-1} \quad (39)$$

$$\mathbf{G}_{ci} = \begin{bmatrix} k_{pi} + k_{ii}/s & 0 \\ 0 & k_{pi} + k_{ii}/s \end{bmatrix}, \quad \mathbf{G}_{dei} = \begin{bmatrix} 0 & -\omega_0 L \\ \omega_0 L & 0 \end{bmatrix} \quad (40)$$

$$\mathbf{G}_{del} = \begin{bmatrix} 1/(1 + T_{delay}s) & 0 \\ 0 & 1/(1 + T_{delay}s) \end{bmatrix} \quad (41)$$

$$\mathbf{G}_{cv} = \begin{bmatrix} k_{pv} + k_{iv}/s & 0 \\ 0 & k_{pv} + k_{iv}/s \end{bmatrix}, \quad \mathbf{G}_{dev} = \begin{bmatrix} 0 & -\omega_0 C \\ \omega_0 C & 0 \end{bmatrix} \quad (42)$$

Fig. 5 (a) grid forming converter with voltage and current control; (b) small-signal impedance converter model with voltage and current control.

B. WT LSC impedance modelling considering PLL based frequency control

To ensure distributed control of offshore WTs, a distributed PLL based WT frequency control was proposed in [20] by the authors, where an additional PLL based frequency loop is used for each WT to regulate the q -axis voltage reference v_{fqref} , as shown in Fig 6 (a). Fig. 6 (b) presents the small-signal impedance model when the PLL based frequency control is considered, and the corresponding output impedance is

$$Z_{vf} = -Z_C (I - Y_L G_{del} G_{dei})$$

$$\left(\begin{array}{c} I - Y_L G_{del} G_{dei} + Z_C Y_L + Y_L G_{del} G_{ci} \\ -Z_C Y_L G_{del} G_{ci} G_{dev} + Z_C Y_L G_{del} G_{ci} G_{ev} - Z_C Y_L G_{del} \\ + G_{ci} G_{del} Y_L Z_C G_{ev} + G_{ci} G_{del} Y_L Z_C G_{PLL} G_{cf} G_{ev} \end{array} \right)^{-1} \quad (43)$$

where the PLL control matrix G_{PLL} and the frequency control matrix G_{cf} are expressed as

$$G_{PLL} = \begin{bmatrix} 0 & 0 \\ 0 & k_{ppll} + k_{ipll}/s \end{bmatrix} \quad G_{cf} = \begin{bmatrix} 1 & 0 \\ 0 & k_f \end{bmatrix} \quad (44)$$

Fig. 6 (a) PLL based frequency control; (b) small-signal impedance model of the converter with PLL based frequency control, outer voltage control and inner current control.

C. WT LSC impedance modelling considering power control

Fig. 7 (a) presents the power control of WT LSC, where P - V and Q - f control is used to regulate the active and reactive power, respectively [14]. Although P - f and Q - V control is traditionally adopted for the synchronous generators in power system, these generators work on the load-following mode. During the load change, the generators adjust the power output to ensure the power balance between the sources and loads. In such way, the grid voltage and frequency are maintained in an acceptable value. However, for the DR-HVDC connected offshore system, all the generators (i.e. the offshore WTs) work on the maximum-power-point-tracking mode instead of load-following mode. There are no load-following sources in this offshore system. If P - f and Q - V control is still adopted for these

WTs, the loads are suggested to work on the source-following mode (i.e. the loads vary according to the generated power). Such source-following mode can ensure the power balance between the generators and loads, meanwhile more importantly, ensure the offshore voltage and frequency are not significantly affected during the wind power change. However, when considering the DR-HVDC link as a 'special load', it is more like a constant load that cannot be easily adjusted to increase or decrease. Under such case, if P - f and Q - V control is still adopted for the WTs, the frequency of this isolated offshore system is the only variable that is used to regulate the active power absorbed by the DR-HVDC. Such scheme will lead to unacceptable offshore frequency change during the wind power increase from 0 pu to 1 pu. On the other hand, if P - V control is used for the WTs, during the wind power change, the offshore PCC voltage magnitude varies between 0.9 pu to 1 pu [8, 14]. Such voltage variation is acceptable for the offshore system. Thus, P - V control is adopted for these DR-HVDC connected WTs. As the voltage magnitude is used for the WT active power regulation, Q - f control is used to ensure the reactive power sharing among all the WTs [14, 19]. The corresponding small-signal impedance model after including the power control is shown in Fig. 7 (b) and its impedance is expressed as

$$Z_{PQ} = -Z_C (I - Y_L G_{del} G_{dei} + G_{ci} G_{del} Y_L G_{PQi} G_{cPQ} G_{cf} G_{ev})$$

$$\left(\begin{array}{c} I - Y_L G_{del} G_{dei} + Z_C Y_L + Y_L G_{del} G_{ci} \\ -Z_C Y_L G_{del} G_{ci} G_{dev} + Z_C Y_L G_{del} G_{ci} G_{ev} - Z_C Y_L G_{del} \\ + G_{ci} G_{del} Y_L Z_C G_{ev} + G_{ci} G_{del} Y_L Z_C G_{PLL} G_{cf} G_{ev} \\ + G_{ci} G_{del} Y_L G_{PQi} G_{cPQ} G_{cf} G_{ev} + G_{ci} G_{del} Y_L G_{PQv} G_{cPQ} G_{cf} G_{ev} \end{array} \right)^{-1} \quad (45)$$

where the power control matrix G_{cPQ} and power calculation small signal matrix G_{PQv} and G_{PQi} are expressed as

$$G_{cPQ} = \begin{bmatrix} k_{pp} + k_{ip}/s & 0 \\ 0 & k_q \end{bmatrix} \quad (46)$$

$$G_{PQv} = 1.5 \begin{bmatrix} I_{wd0} & I_{wq0} \\ I_{wq0} & -I_{wd0} \end{bmatrix} \quad G_{PQi} = 1.5 \begin{bmatrix} V_{fd0} & V_{fq0} \\ -V_{fq0} & V_{fd0} \end{bmatrix} \quad (47)$$

Fig. 7 (a) WT active power and reactive power control; (b) small-signal impedance model of the converter considering power control.

D. WT LSC impedance modelling validation

To validate the derived WT LSC impedance model, frequency scanning is also conducted in the time domain model in PSCAD/EMTDC. Fig. 8 compares the magnitudes and phases of the WT impedances (the WT transformer impedance is also included), where the red lines are the results from the established analytical model and the blue dots represent the frequency scanning results. As can be seen, the impedances from the analytical model are in excellent agreements with those from PSCAD/EMTDC simulation, validating the

accuracy of the WT LSC analytical impedance modelling.

Fig. 8 WT LSC impedance modelling validation.

V. STABILITY ANALYSIS

This section conducts stability analysis based on the derived impedance models, considering the change of DC smoothing reactors and AC filters (system parameters are presented in Appendix C). For the offshore station, the DC smoothing reactors and AC filters significantly affect the weight, volume, and cost of the system, and are preferred to have small values from economic consideration [9, 24, 25]. In [25], the reactance of the total DC smoothing reactors is suggested to be 0.12 H for a 1200 MW DR-HVDC link, which is one tenth of 1.2 H in the 1000 MW Cigre-benchmark LCC link model. In [9, 24], the AC filters are reduced from 446 MVar to 89 MVar for a 1000 MW DR-HVDC link, with the benefits of no filter switching, reduced installation costs and increased offshore station reliability. Meanwhile, [9, 24] also analyze the impacts of the reduced filter on offshore harmonics and reactive power balance, and conclude that such filter reduction has negligible impact on the harmonic distortion and reactive power can be properly balanced with the support from WT LSC. However, how does such design with significantly reduced DC reactors and AC filters affect the stability of the DR-HVDC connected offshore wind farms has not been thorough analyzed. In addition, this section also conducts the stability analysis considering the interaction between WTs.

A. Principle of impedance-based stability analysis

The small-signal impedance model of DR-HVDC connected WT is presented in Fig. 9, where Z_{WT} is the WT impedance (including the WT transformer impedance) and $Y_{DR-HVDC}$ is the admittance of the DR-HVDC (including the AC filters). With this small signal model, the current injected into the DR is expressed as

$$I_{DR} = \frac{V_{WT}}{Z_{DR-HVDC} + Z_{WT}} = \frac{V_{WT}}{Z_{DR-HVDC}} \frac{1}{1 + Z_{WT} Y_{DR-HVDC}} \quad (48)$$

Fig. 9 Small-signal impedance model of DR-HVDC connected WT.

Considering the following conditions are usually satisfied: 1) the grid forming WT LSC is stable when not connected to DR; 2) the DR-HVDC is stable when connected to an ideal

voltage source, the stability of this interconnected system is determined by $Z_{WT} Y_{DR-HVDC}$. If and only if $Z_{WT} Y_{DR-HVDC}$ satisfies the Nyquist stability criterion, the current flowing into the DR is stable [26].

B. Stability analysis with different DC smoothing reactors

Fig. 10 shows the impacts of different DC smoothing reactance on the DR-HVDC output admittances (excluding DR AC side filters), with system operating at rated active power. As can be observed, when the DC reactance reduces from 0.6 H to 0.12 H, the phase of Y_{dd} sees a significant change within 0-100 Hz, while smaller DC reactance makes slower phase drop of Y_{dd} to -90° . On the other hand, the variation of DC reactance has negligible impact on Y_{qq} .

Fig. 10 DR-HVDC admittances with different DC smoothing reactance.

Fig. 11 presents the Nyquist plot of the d- and q-axis impedance ratios when the DC reactance is reduced from 0.6 H to 0.12 H. As can be observed, the d-axis impedance ratio moves towards the point $(-1, j0)$ when the DC reactance initially reduces, while the q-axis impedance ratio does not have a large change and always satisfies the general Nyquist criteria. This phenomenon agrees with the observation from Fig. 10, as the DC reactance variation mainly affects the d-axis admittance. When the DC reactance is reduced to 0.26 H, the d-axis impedance ratio intersects with the unit circle at 30 Hz, with phase margin reducing to 1° . With further reduction of the DC reactance to 0.12 H, the d-axis impedance ratio encircles the point $(-1, j0)$, resulting in instability of the interconnected system. Fig. 12 shows the WT generated active power and PCC current from time domain simulation for DC smoothing reactance of 0.26 H. Fourier analysis confirms that the WT active power experiences oscillation at around 30 Hz, as predicted from the impedance model. Further time domain simulation also confirms that the system is unstable when the DC reactance is reduced to 0.12 H.

Fig. 11 Nyquist plot of the d- and q-axis impedance ratios when the DC reactance is reduced from 0.6 H to 0.12 H.

Fig. 12 WT generated active power and PCC current from time domain simulation with 0.26 H DC smoothing reactance.

To ensure the stability of the interconnected system when the DC reactance is reduced, the WT output impedances are reshaped through the modification of its control system (as it is difficult to modify the DR-HVDC output impedance due to its uncontrollability). Fig. 13 shows the Nyquist plots of the impedance ratios when the WT LSC dq voltage control bandwidth is increased from the initial value of 75 Hz to 92 Hz, and then to 127 Hz, with DC reactance of 0.12 H. As can be seen, when the voltage loop bandwidth is increased, the d-axis stability margin is significantly increased. Fig. 14 shows the time domain simulation results when the voltage control bandwidth is increased from 75 Hz to 127 Hz at 0.2 s, demonstrating the interconnected system can operate stably with voltage control bandwidth at 127 Hz with a small DC reactance of 0.12 H. The stability of the interconnected system with increased voltage bandwidth is further investigated at other operating points (WT generates 0.5 pu and 0.1 pu active power, respectively) with 0.12 H DC reactance. Nyquist plot in Fig. 15 demonstrates the system can operate stably at all these operation points, which is also confirmed by the time domain simulation.

Fig. 13 Nyquist plot of the d- and q-axis impedance ratio when voltage bandwidth is increased from 75 Hz to 127 Hz.

Fig. 14 WT generated active power and PCC current from time domain simulation when voltage control bandwidth is increased from 75 Hz to 127Hz.

Fig. 15 Nyquist plots of the impedance ratios with increased voltage bandwidth at different operating point.

C. Stability analysis with different AC filters

Fig. 16 shows the DR-HVDC admittance when the AC

filters reduce from 400 MVar to 100 MVar with system operating at rated active power (DC reactance is 0.12 H and voltage control bandwidth is 127 Hz). The reactive power required by the DR will be automatically provided by the WTs due to the use of Q - f droop control. As can be observed, the AC filter reduction has a minor impact on Y_{dd} below 100 Hz, indicating the d-axis stability margin is largely not affected (as the d-axis impedance ratio intersects with the unit circle at low frequency range, which is the critical frequency range for stability consideration). On the other hand, the reduction of AC filters has a significant impact on the phase of Y_{qq} above 50 Hz.

Fig. 16 Admittance with different AC filters (400 MVar, 200MVar and 100MVar respectively).

Fig. 17 (a) shows the Nyquist plot of the d- and q-axis impedance ratios when the AC filters are decreased from 400 MVar to 100 MVar at rated active power generation. As can be observed, with the decrease of AC filters, the q-axis impedance ratio moves away from the point $(-1, j0)$, indicating the increase of stability margin. On the other hand, the d-axis impedance ratio does not see a significant change around its intersection point. Fig. 17 (b) shows the Nyquist plot of the d- and q-axis impedance ratios with the same three filter sizes when the system operates at half rated active power generation. As seen, when the AC filters are 400 MVar at this operating point (241 MVar excessive reactive power is absorbed by the WT LSC), the q-axis impedance ratio intersects with the unit circle at 81 Hz, with the phase margin at 8° . When the AC filters are at 200 MVar or 100 MVar, the q-axis stability margin is significantly increase. Fig. 18 shows the corresponding WT generated active power and PCC current from time domain simulation, when the filters are reduced from 400 MVar to 200 MVar at 0.2 s. As can be seen, the WT oscillation is eliminated after the AC filter decrease.

Fig. 17 Nyquist plots of d- and q-axis ratios when filters are reduced from 400 MVar to 100 MVar: (a) at rated active power; (b) at half rated active power.

Fig. 18 WT generated active power and PCC current when filters are decreased from 400 Mvar to 200 Mvar at half the rated active power generation.

The impact of filters change on the system stability is further investigated when the WT generates 0.1 pu active power. Fig. 19 shows the Nyquist plot of the impedance ratios at this operation point, when the AC filters are at 400, 200 and 100 MVar, respectively. As seen, when the AC filters are 400 MVar (WT LSC absorbs 350 MVar reactive power), the q-axis impedance ratio encircles the point $(-1, j0)$, demonstrating an unstable WT-HVDC system. Time domain simulation also confirms the system is unstable when 400 Mvar filter is connected at this operating point. When the AC filters are 200 MVar or 100 MVar, both the Nyquist plot and time domain simulation show the WT-HVDC interconnected system operates stably.

Fig. 19 Nyquist plot of the d- and q-axis impedance ratios when the AC filters are decreased from 400 MVar to 100 MVar at 0.1 p.u. active power generation.

According to the analysis conducted in subsection B and this subsection, it can be concluded that

- Smaller DC reactance can contribute to reducing the weight, value and cost of offshore station, but pose challenges on d-axis stability of the WT-HVDC interconnected system. Faster offshore voltage control of WT LSC is a way to minimize such instability potential.
- In contrast to that small DC reactance degrades d-axis stability margin, small sizes of AC filters lead to increased q-axis stability margin. Thus, when considering the stability of WT-HVDC interconnected system, and offshore station weight/volume/cost, small sizes of AC filters are suggested, if the harmonic filtering and reactive power compensation requirements are satisfied.

D. Stability analysis considering the interaction between WTs

This subsection conducts the stability analysis considering the interaction between WTs. The offshore wind farms are thus modelled as: ten 10 MW WTs representing one detailed WT string, an aggregated 300 MW WT and an aggregated 600 MW WT, as shown in Fig. 20. The studied offshore wind farms are considered as two subsystems from the connection point C_{12} , i.e. WT1 as the first subsystem while the other WTs and DR-HVDC as the second subsystem. The corresponding small-signal impedance model of the system is presented in Fig. 21. The stability of this interconnected system is thus determined by $Z_{WT1}Y_{eq}$, where Y_{eq} is the equivalent admittance of the other

WTs and DR-HVDC (Note that Y_{WTi} ($i=2,3,\dots,10,300,600$) are the admittances after transferring from the local dq frame to the global dq frame). If and only if $Z_{WT1}Y_{eq}$ satisfies the Nyquist stability criterion, the current of WT1 is stable.

Fig. 20 DR-HVDC connected offshore wind farms.

Fig. 21 Small-signal impedance model of WT1 and other WTs connected to DR-HVDC.

Fig. 22 shows the d- and q-axis impedances of WT1 which operates at rated power. It demonstrates that the WT has a low-frequency resonance point in the d-axis impedance. As the $P-V$ control plays an important role in determining the impedance in such frequency range, the impact of the WT $P-V$ control on the system stability is analyzed. Fig. 23 shows the Nyquist plot of the d- and q-axis impedance ratios when the proportional gain of the $P-V$ control k_{pp} decreases from 0.3043 pu to 0.3002 pu. As observed, with the decrease of k_{pp} , the d-axis impedance ratio moves towards to the point $(-1, j0)$, indicating the decrease of stability margin. When k_{pp} is decreased to 0.3015 pu, the d-axis impedance ratio intersects with the unit circle at 15.9 Hz, with the phase margin at 1° . Fig. 24 shows the corresponding time-domain WT1 generated active power and WT1 current, when k_{pp} is 0.3021 pu during 0-0.5 s. Fourier analysis confirms that the WT active power experiences oscillation at around 16 Hz, as predicted from the impedance model. When the proportional gain of the $P-V$ control k_{pp} is increased to 0.3043 pu at 0.5 s, the WT oscillation is effectively eliminated, as shown in Fig. 24.

Fig. 22 WT1 d- and q-axis impedance.

Fig. 23 Nyquist plot of the d- and q-axis impedance ratio when k_{pp} decreases from 0.3043 pu to 0.3002 pu.

Fig. 24 WT1 generated active power and current when k_{pp} are increased from 0.3021 pu to 0.3043 pu at 0.5 s.

To further investigate the interaction between WTs, stability analysis is conducted considering different operating points. Fig. 25 shows the Nyquist plot of the d- and q-axis impedance ratios when the WTs generate 1 pu, 0.5 pu and 0.2 pu active power (the proportional gain of the active power control k_{pp} is set as 0.3043 pu). As can be seen, with the decrease of generated active power, the d-axis impedance ratio moves towards to the point $(-1, j0)$, demonstrating the decrease of stability margin, while the q-axis impedance ratio sees a little change. When the WTs generate 0.2 pu active power, the d-axis impedance ratio encircles the point $(-1, j0)$, representing the instability of the system. Fig. 26 shows the WT1 generated active power from time domain simulation, when it is increased from 0.2 pu (2 MW) to 0.5 pu (5 MW) during 1-2.5 s. As seen, the WT oscillation is eliminated gradually when the WT operates at 0.5 pu, which is in agreement with the small signal analysis.

Fig. 25 Nyquist plot of the d- and q-axis impedance ratio when the WTs generate 1 pu, 0.5 pu and 0.2 pu active power.

Fig. 26 WT1 generated active power.

Fig. 27 shows the Nyquist plot of the d- and q-axis impedance ratios when the proportional gain of the active power control k_{pp} increases from 0.3043 pu to 0.3136 pu (with all WTs operating at 0.2 pu active power). As can be seen, when k_{pp} is increased to 0.3118 pu, the d-axis impedance ratio intersects with the unit circle at 16.3 Hz, with the phase margin at 6° . When k_{pp} is further increased to 0.3136 pu, the phase margin of the d-axis impedance ratio is increased to 80° . On the

other hand, the q-axis impedance ratio almost keeps unchanged and always satisfies the general Nyquist criteria during the variation of the k_{pp} . Time-domain simulation confirms that WTs operate stably when k_{pp} is increased to 0.3136 pu.

Fig. 27 Nyquist plot of the d- and q-axis impedance ratio when k_{pp} increases from 0.3118 pu to 0.3136 pu.

VI. CONCLUSION

This paper develops an impedance model of DR-HVDC connected offshore wind farm. The analytical impedance is verified by the frequency scanning results from time-domain simulation model in PSCAD/EMTDC. Based on the derived impedance model, stability analysis is conducted considering the variations of the sizes of DC smoothing reactors and AC filters. It shows that reduced DC smoothing reactance degrades the stability of the interconnected system. Faster voltage control is a potential way to mitigate the problem and increase its stability margin under such cases. In addition, the analysis also shows that reduced AC filters help to increase the system stability margin, although the selection also needs to consider the harmonic filtering and reactive power compensation requirements. Moreover, stability analysis considering the interaction between WTs reveals that the WT P - V control has a significant impact on the stability of the DR-HVDC connected WTs. Increased proportional gain of WT P - V control can help improve system stability.

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APPENDIX

A. Commutation overlap angle dynamics

$$\frac{\Delta\mu}{\Delta v_d} = G_{11} \frac{\Delta\mu}{\Delta i_{2d}} + G_{21} \frac{\Delta\mu}{\Delta i_{2q}} = \frac{\omega_0 \left(-s \sin \mu_0 + \omega_0 \cos \mu_0 - \omega_0 e^{-s\mu_0/\omega_0} \right)}{(s^2 + \omega_0^2) \left(v_{d0} \sin \mu_0 + v_{q0} \cos \mu_0 \right)} \quad (\text{A.1})$$

$$\frac{\Delta\mu}{\Delta v_q} = G_{12} \frac{\Delta\mu}{\Delta i_{2d}} + G_{22} \frac{\Delta\mu}{\Delta i_{2q}} = \frac{\omega_0 \left(-s \cos \mu_0 - \omega_0 \sin \mu_0 + s e^{-s\mu_0/\omega_0} \right)}{(s^2 + \omega_0^2) \left[v_{d0} \sin(\mu_0) + v_{q0} \cos(\mu_0) \right]} \quad (\text{A.2})$$

$$\frac{\Delta\mu}{\Delta I_{dc}} = \frac{(1 + e^{-s\mu_0/\omega_0}) \omega_0 L_c}{\sqrt{3} \left[v_{d0} \sin(\mu_0) + v_{q0} \cos(\mu_0) \right]} \quad (\text{A.3})$$

B. Diode rectifier AC current dynamics

$$K_{11} = \frac{\Delta i_d}{\Delta v_d} = \frac{\Delta i_d^c + \Delta i_d^{nc}}{\Delta v_d} = G_{11} \frac{\Delta i_d}{\Delta i_{2d}} + G_{21} \frac{\Delta i_d}{\Delta i_{2q}} = \frac{3}{4\pi L_c (s^2 + \omega_0^2)} [-s \sin 2\mu + 2\mu s + \omega_0 \cos 2\mu - \omega_0] + \frac{4\omega_0^3 - 4\omega_0^2 (\omega_0 \cos \mu_0 + s \sin \mu_0) e^{-s\mu_0/\omega_0}}{(s^2 + \omega_0^2)} \quad (\text{A.4})$$

$$K_{12} = \frac{\Delta i_d}{\Delta v_q} = G_{12} \frac{\Delta i_d}{\Delta i_{2d}} + G_{22} \frac{\Delta i_d}{\Delta i_{2q}} = \frac{3}{4\pi L_c (s^2 + \omega_0^2)} [-s \cos 2\mu + s - \omega_0 \sin 2\mu + 2\omega_0 \mu] - \frac{4\omega_0^2 s - 4\omega_0 s (\omega_0 \cos \mu_0 + s \sin \mu_0) e^{-s\mu_0/\omega_0}}{(s^2 + \omega_0^2)} \quad (\text{A.5})$$

$$K_{13} = \frac{\Delta i_d}{\Delta I_{dc}} = \frac{2\sqrt{3}}{\pi} \left[\frac{1 + \cos \mu}{2} - \frac{\omega_0^2 - \omega_0 (\omega_0 \cos \mu_0 + s \sin \mu_0) e^{-s\mu_0/\omega_0}}{2(s^2 + \omega_0^2)} \right] \quad (\text{A.6})$$

$$K_{21} = \frac{\Delta i_q(s)}{\Delta v_d} = G_{11} \frac{\Delta i_q}{\Delta i_{2d}} + G_{21} \frac{\Delta i_q}{\Delta i_{2q}} = \frac{3}{4\pi L_c (s^2 + \omega_0^2)} [-s \cos(2\mu) + s - \omega_0 \sin(2\mu)] - 2\mu\omega_0 + \frac{4\omega_0^2 s - 4\omega_0^2 (s \cos \mu - \omega_0 \sin \mu) e^{-s\mu_0/\omega_0}}{(s^2 + \omega_0^2)} \quad (\text{A.7})$$

$$K_{22} = \frac{\Delta i_q(s)}{\Delta v_q} = G_{12} \frac{\Delta i_q}{\Delta i_{2d}} + G_{22} \frac{\Delta i_q}{\Delta i_{2q}} = \frac{3}{4\pi L_c (s^2 + \omega_0^2)} [s \sin(2\mu) + 2\mu s - \omega_0 \cos(2\mu)] + \omega_0 - \frac{4\omega_0 s^2 - 4\omega_0 s (s \cos \mu - \omega_0 \sin \mu) e^{-s\mu_0/\omega_0}}{(s^2 + \omega_0^2)} \quad (\text{A.8})$$

$$K_{23} = \frac{\Delta i_q(s)}{\Delta I_{dc}(s)} = \frac{2\sqrt{3}}{\pi} \left[-\frac{\sin \mu}{2} - \frac{\omega_0 s - \omega_0 (s \cos \mu - \omega_0 \sin \mu) e^{-s\mu_0/\omega_0}}{2(s^2 + \omega_0^2)} \right] \quad (\text{A.9})$$

C. Parameters of the tested DR-HVDC system

Table I Parameters of the Tested DR-HVDC System

Parameters		Values
DR-HVDC	Rated Power & DC voltage	1000 MW; ± 320 kV
	DR Transformer	66 kV/261.8 kV/261.8 kV; 0.18 pu
	DC reactors	0.6 H / 0.26 H/ 0.12 H
	Filters	400 MVar
		C ₁ :58.489 μ F
		C ₂ :116.98 μ F; C ₃ :1300 μ F;
		L ₁ :7.8 mH; R ₁ :1.70 Ω ; R ₂ : 14.97 Ω
		C ₄ :116.98 μ F; L ₂ :0.78 mH; R ₃ : 4.76 Ω
		200 MVar
		29.2445 μ F
		58.49 μ F; 650 μ F;
		15.6 mH; 3.40 Ω ; 29.94 Ω
		58.49 μ F; 1.55 mH; 9.52 Ω
		100 MVar
		14.6222 μ F
		29.2445 μ F; 325 μ F;
		31.2 mH; 6.80 Ω ; 59.88 Ω
		29.24 μ F; 3.11 mH; 19.05 Ω
WT LSC	Rated power	1000 MW
	WT Transformer	0.69 kV/66 kV; 0.08 pu
	Line reactance	0.1 pu
	Filter capacitor	0.05 pu
WT LSC control	PQ control	k_{pp} : 0.334 pu; k_{ip} : 18.56 pu; k_q : 0.032 pu;
	PLL based frequency control	k_{ppll} : 0.6 pu; k_{ipll} : 28.3 pu; k_f : 0.077 pu;
	Voltage control	k_{pv} : 0.12 pu; k_{iv} : 22.62 pu bandwidth: 75 Hz
	Current control	k_{pi} : 0.4 pu; k_{ii} : 0 pu; bandwidth: 300 Hz