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Rent dissipation in share contests

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Abstract

This article investigates rent dissipation—the total costs of rent seeking in relation to the value of the contested rent—in share contests. We consider preferences that are more general than usually assumed in the literature, which allow for contestants to have diminishing marginal utility. With sufficiently concave preferences the equilibrium will feature over-dissipation if the rent is small, and under-dissipation if the rent is large: if contestants have strong diminishing marginal utility and they are contesting a small rent they are highly sensitive to changes in their allocation of the rent so are relatively effortful in contesting it; by contrast when the rent is large they are less effortful relative to the size of the rent. Thus, the inclusion of diminishing marginal utility allows us to reconcile the Tullock paradox—where rent-seeking levels are observed to be relatively small compared to the contested rent—with observed over-dissipation of rents in, for example, experimental settings. We also propose a more general rent dissipation measure that applies to any contest and is suitable for general preferences.

Keywords: Rent seeking; rent dissipation; share contests.

JEL Classification: C72; D72; H40

1 Introduction

Contests are a common phenomenon within a multitude of economic contexts. These scenarios involve agents investing sunk effort to appropriate a contestable rent, such as political rent seeking, litigation, and violent conflict, to name but a few. Due to the frequency of such activities, attention has focused on understanding the incentives to engage in contests as well as the associated costs and impacts. One fundamental concept within the study of contests is the notion of rent dissipation: analyzing the total costs of rent seeking in relation to the value of the contested rent. Explaining the degree of rent dissipation can help us to estimate and anticipate the severity of possible social losses from such activities, or how a rent holder can maximize effort in their favor. Consequently, rent dissipation has been central to the discussion of contests since the first formal analysis was introduced (Tullock, 1980).

Contest theory has been developed and extended to provide a tractable explanation of the drivers of rent-seeking activity, and the associated rent dissipation. Within this analysis, there have been two separate focal points of interest: the (i) under-dissipation; and (ii) over-dissipation of rents. Under-dissipation—also known as the so-called '*Tullock Paradox*'—is where the costs of rent seeking are far lower than the value of what is being contested. Since Tullock (1989), it has been questioned why real-world situations (e.g., lobbying) often involve very limited expenditures in contesting highly valuable rents. Scholars also inquired whether over-dissipation can occur, i.e., if the costs of rent seeking could be larger than the value of the rent. If so, this would have fundamental consequences for the understanding of the social cost of rent seeking.

In the early literature (e.g., Tullock, 1989) attention focused on the real-world dissipation of rent being much lower than what is theoretically predicted. Numerous theories have been proposed in the context of standard contest models to explain this 'Tullock Paradox', including risk aversion (Hillman and Katz, 1984), heterogeneity in valuations (Hillman and Riley, 1989), uncertain number of contestants (Myerson and Wärneryd, 2006; Münster, 2006; Lim and Matros, 2009; Kahana and Klunover, 2015), and group rent seeking (Ursprung, 1990).[1] The literature has also considered whether over-dissipation can be explained, with contributions appealing to contestants being risk loving (Jindapon and Whaley, 2015), and contest settings in which the only Nash equilibrium is in mixed strategies where over-dissipation can occur in incidence (but not in expectation) (Baye et al., 1999).

This literature has focused on winner-take-all contests in which it is assumed the contested rent is indivisible and a single contestant receives the entire rent with all other contestants

[1]See Hillman and Long (2019) for a recent survey.

receiving nothing. This strict indivisibility is often an imperfect modeling assumption for many real-world environments. An alternative approach is to consider that the rent is perfectly divisible and each contestant receives a share of the rent dependent on their engagement with the contest relative to that of other contestants. Share contests are arguably more appropriate for modeling rent seeking over the divisions of public funds (Mauro, 1998; Hodler, 2007), government policy (Epstein and Nitzan, 2007; MacKenzie, 2017; Duggan and Gao, 2020) and numerous other contexts such as remuneration rewards (Singh and Masters, 2018) and the distribution of pollution permit allowances (MacKenzie and Ohndorf, 2012).[2]

It has recently been shown that in all but the simplest settings, share contests and winner-take-all contests are not strategically equivalent and therefore command separate study (Dickson et al., 2018). The majority of the literature has exploited features of winner-take-all contests in an attempt to explain the patterns of dissipation. However, this literature cannot help in understanding the patterns of dissipation in share contests precisely because these features take the analysis beyond the simple settings and the equivalence between winner-take-all and share contests breaks down. In this article we therefore fill this gap presented by the literature and investigate dissipation within share contests.

In our study of share contests we allow for contestants to have diminishing marginal utility over the contest outcome—a very natural assumption to make—and demonstrate that sufficiently strong diminishing marginal utility can reconcile both under- and over-dissipation of rents. In particular, when the contested rent is small—and so diminishing marginal utility means contestants are highly sensitive to changes in the spoils they are awarded from the contest—they are relatively effortful in contesting the rent and this results in the monetary cost of rent seeking exceeding the monetary value of the rent, leading to over-dissipation. By contrast, if the rent is large and contestants are less sensitive to changes in their allocation of the rent they will be relatively less effortful leading to under-dissipation. Thus, when we account for diminishing marginal utility we can elegantly, and very intuitively, reconcile the Tullock paradox (when the stakes are large) and over-dissipation (when the stakes are small).

Experimental evidence of Tullock contests (in which the stakes are arguably small) routinely highlights that subjects expend more effort than otherwise predicted from the pure-strategy Nash equilibrium (Fallucchi et al., 2013; Shupp et al., 2013; Chowdhury et al., 2014; Dechenaux et al., 2015; Cason et al., 2010, 2020). This holds for both the winner-take-all designs

[2]Indeed, even in the early literature, many of the rent-seeking applications can be interpreted as share contests. For example Tullock (1989) provides examples of rent seeking of dairy farmers to obtain a share of government funds as well as lobbying for Korean steel import bans, which can be viewed as rent-seeking for an increased market share by domestic producers.

of the Tullock contest as well as the share approach. For example, Cason et al. (2020) finds that within share contests there is approximately 30% more effort than the prediction from the Nash equilibrium. Behavioral aspects such as non-monetary utility from winning, systematic mistakes by contestants, and impulsivity have been suggested for this occurring (Sheremeta, 2018; Hillman and Long, 2019). While many of these provide partial explanations (and some reasoning only applies to the winner-take-all context), there remains an active debate as to how we can reconcile the empirical findings of contests with the theoretical predictions. In this article we advance the literature by providing a simple framework that can account for under- and over-dissipation of rent with an intuitive reasoning for such activity.

We begin our analysis in the simplest possible setting in which we can incorporate diminishing marginal utility: a 'simple' Tullock contest in which each player receives a share of the rent proportional to their effort. If x_i represents the effort of contestant $i = 1, \ldots, n$ the Tullock 'contest success function', that in a share contest determines the share of the rent contestant i receives, is given by $\frac{x_i^r}{x_i^r + \sum_{j \neq i} x_j^r}$ and in a simple Tullock contest the contest technology parameter r equals 1. In this setting we demonstrate that when preferences over the contest outcome are sufficiently concave there is always a level of the contested rent below which there will be over-dissipation, and a level above which under-dissipation will occur. We then extend the analysis to consider $r > 0$ more generally. Here we have to be more careful about the existence of a pure-strategy Nash equilibrium, but we demonstrate that incorporating diminishing marginal utility into contests relaxes the restriction on the parameter r required for a pure-strategy Nash equilibrium to exist, and under this restriction our results on the over- and under-dissipation of the rent extend easily to these settings.[3]

Thus, our analysis allows us to elegantly and intuitively reconcile observations of the Tullock paradox, where the monetary cost of rent seeking falls short of the monetary value of the rent, and observations, for example in experimental settings, of over-dissipation where the monetary cost of rent seeking exceeds the monetary value of the rent. Yet, when we explicitly consider share contests and that contestants derive utility from the contest outcome, we question whether comparing the monetary value of effort to the monetary value of the rent is the appropriate ratio to capture the essence of what we mean by dissipation. To remedy this limitation, we thus propose a more general measure of dissipation, encompassing previous dissipation ratios that have been proposed by scholars (e.g., Chung, 1996; Hurley, 1998; Baye et al., 1999; Alcalde and Dahm, 2010). Our novel measure evaluates the relative *utility loss* in a contest compared to the first-best solution, thus constituting a well-designed tool for welfare

[3] In contrast to the literature on winner-take-all contests, we show that in symmetric n-player (2-player) contests our results hold (under conditions) for values of r larger than $\frac{n}{n-1}$ (2), respectively.

analysis. Unsurprisingly, this adapted dissipation measure is always (in expectation) bounded by 1, thus revealing that while over-dissipation under the standard definition may well happen at equilibrium, the efficiency loss will never exceed the maximum achievable welfare.

The remainder of the article is organized as follows. In Section 2 the model is outlined. Section 3 generalizes the model and Section 4 provides a generalized dissipation ratio for general preferences. Section 5 provides some concluding remarks.

2 The model

Consider a contest in which a set of agents $N = \{1, \ldots, n\}$ expend effort to obtain a share of a perfectly divisible rent Z. Agent $i \in N$ selects their effort $x_i \geq 0$ in order to capture a share of the rent which is determined according to

$$\phi(x_i, \mathbf{x}_{-i}) = \begin{cases} \frac{x_i^r}{x_i^r + \sum_{j \neq i \in N} x_j^r} & \text{if } \sum_{k \in N} x_k > 0 \text{ or} \\ \frac{1}{n} & \text{if } \sum_{k \in N} x_k = 0, \end{cases}$$

where $\mathbf{x}_{-i} = \{x_j\}_{j \neq i}$ is the vector of other players' effort choices. The rent apportioned to contestant i is then given by $z_i \equiv \phi(x_i, \mathbf{x}_{-i})Z$. This is the standard Tullock (1980) contest success function, and for our initial exposition we focus on so-called 'simple' Tullock contests in which $r = 1$: contestants receive a proportional share of the rent according to their efforts. Later in the article we extend the analysis to consider more general contest technologies where $r \neq 1$.

In share contests it is almost unanimously assumed that contestants have a linear valuation of their allocation of the rent from the contest, z^i.[4] By contrast, we consider that contestants have an additively separable utility function given by

$$u_i(z_i, x_i) = v_i(z_i) - c_i(x_i)$$

where $v_i(0) = 0$, $v_i' > 0$ and $v_i'' \leq 0$, allowing us to capture that contestants have diminishing marginal utility over the contest outcome. $c_i(\cdot)$ is the monetary cost of effort, measured on the same scale as the contested rent, and we assume $c_i(0) = 0$, $c_i' > 0$ and $c_i'' \geq 0$.

Incorporating diminishing marginal utility in the analysis of share contests is, we believe, important. For instance, in a contest over public funds it is intuitive to consider that agents may experience large marginal utility gains for initially redirected public funds but the gains to utility reduce as their captured public funds increase. Equally, for a rent-seeking game over the determination of a government policy, large gains in utility may exist when government policy

[4]An exception is Dickson et al. (2018) that considers a general formulation of utility over a share contest.

moves in an agent's favorable direction, but these marginal gains will reduce as the policy is more distant to the agent's most desirable policy, that is, when the contested policy space is larger (e.g., Persson and Tabellini, 2000). As we have become accustomed in microeconomic analysis, incremental gains that improve one's lot from a relatively poor position are worth more than those that improve it from a relatively good position.

Contestants simultaneously choose their effort to maximize their utility in a game of complete but imperfect information, and we look for a Nash equilibrium in pure strategies. Each contestant can be seen as solving the problem

$$\max_{x_i \geq 0} v_i(z_i) - c_i(x_i) \text{ s.t. } z_i = \frac{x_i}{x_i + \sum_{j \neq i} x_j} Z$$

taking the effort choices of others as given. Our assumptions on value and cost functions imply this optimization problem is globally concave and so the first-order condition is both necessary and sufficient for identifying the contestant's best response. Letting $X_{-i} \equiv \sum_{j \neq i} x_j$, we denote this best response $\hat{x}_i(X_{-i}) = \max\{0, x_i\}$, where x_i is the solution to

$$l_i(x_i, X_{-i}) \equiv v_i'(z_i) \frac{X_{-i}}{[x_i + X_{-i}]^2} Z - c_i'(x_i) = 0.$$

For ease of exposition we follow the existing literature and assume all players are symmetric ($v_i(\cdot) = v(\cdot)$ and $c_i(\cdot) = c(\cdot)$ for all $i \in N$), and focus on symmetric Nash equilibria in which $x_i = x^*$ for all $i \in N$, and therefore $z_i^* = Z/n$ for all $i \in N$. Note that the results in this article are easily extended to incorporate asymmetric contests—using the tools of aggregate games—but are omitted for the sake of brevity and because the underlying mechanisms are neatly captured in a symmetric setup. Equilibrium effort in an interior symmetric Nash equilibrium satisfies

$$v'(Z/n) \frac{n-1}{n^2} \frac{Z}{x^*} - c'(x^*) = 0.$$

Given strict global concavity of the objective function (along with the fact that the payoff with zero effort is zero), the symmetric Nash equilibrium will be unique. It will involve strictly positive effort levels if each player's marginal payoff with zero effort when everyone else is using the candidate equilibrium effort is positive, i.e., $l(0, [n-1]x^*) > 0$, otherwise the rent will be insufficient relative to the costs of rent seeking and all contestants will optimally respond with zero effort.

It is convenient to re-write the condition defining effort in an interior symmetric Nash equilibrium as

$$x^* c'(x^*) = \frac{n-1}{n^2} v'(Z/n) Z. \tag{1}$$

We want to investigate the nature of the dissipation ratio, measuring the total rent-seeking outlays in relation to the contested rent, which is given by

$$D \equiv \frac{nc(x^*)}{Z} = \frac{n-1}{n}\frac{v'(Z/n)}{\eta(x^*)}, \tag{2}$$

where $\eta(x) \equiv \frac{xc'(x)}{c(x)}$ is the elasticity of the cost function.

To illustrate ideas, we consider in the first part of our analysis that costs are linear: $c(x) = cx$. In this case there is an explicit solution for equilibrium contest effort given by

$$x^* = \frac{n-1}{n^2}v'(Z/n)\frac{Z}{c},$$

which is strictly positive so the unique symmetric Nash equilibrium is indeed interior. We also have an explicit expression for the equilibrium dissipation ratio

$$D = \frac{n-1}{n}v'(Z/n)$$

since for a linear cost function $\eta(x) = 1$ for all $x \geq 0$.

From this expression it is transparent that both under- and over-dissipation can emerge in the contest equilibrium depending on whether $v'(Z/n)$ is less than, or exceeds $\frac{n}{n-1}$. If $v(\cdot)$ is linear (as is assumed in the literature on share contests) then $v'(\cdot) = 1$ which is, of course, less than $\frac{n}{n-1}$ implying under-dissipation regardless of the size of the contested rent. However, if $v(\cdot)$ is strictly concave (implying $v'(Z/n)$ is decreasing in Z) this raises the possibility that $v'(Z/n) > \frac{n}{n-1}$ when Z is small enough.

Proposition 1. *Assume $v'(z) \to 0$ as $z \to \infty$. If $v'(0) > \frac{n}{n-1}$ then there exists a $\tilde{Z} \equiv nv'^{-1}\left(\frac{n-1}{n}\right)$ such that $D > (<)1 \Leftrightarrow Z < (>)\tilde{Z}$, so the contest exhibits over-dissipation for $Z < \tilde{Z}$. By contrast, if $v'(0) \leq \frac{n}{n-1}$ the contest never exhibits over-dissipation.*

Proof. Suppose $v'(0) \leq \frac{n}{n-1}$. Then $\frac{n-1}{n}v'(0) \leq 1$ and concavity of $v(\cdot)$ implies $D = \frac{n-1}{n}v'(Z/n) \leq 1$ for all $Z > 0$. By contrast, if $v'(0) > \frac{n}{n-1}$ then since $v'(z) \to 0$ as $z \to \infty$ the intermediate value theorem implies there is a $\tilde{Z} > 0$ such that $\frac{n-1}{n}v'(\tilde{Z}/n) = 1$. Concavity of $v(\cdot)$ then implies $D \equiv \frac{n-1}{n}v'(Z/n) > (<)1 \Leftrightarrow Z < (>)\tilde{Z}$. □

Thus, if preferences are 'sufficiently concave'—here measured by $v'(0)$ being larger than $\frac{n}{n-1}$—there will be over-dissipation in contests in which the rent is relatively small, while there will be under-dissipation in contests with large rents. We illustrate with a worked example.

Example 1. *Consider a contest in which there are n contestants each with $v^i(z) = \gamma z^\alpha$ where $\alpha \in (0,1)$, $\gamma \in (0,1]$, and $c^i(x) = cx$ with $c > 0$. Then effort in the symmetric Nash equilibrium is given by*

$$x^* = \frac{n-1}{n}\frac{\alpha\gamma}{c}[Z/n]^\alpha$$

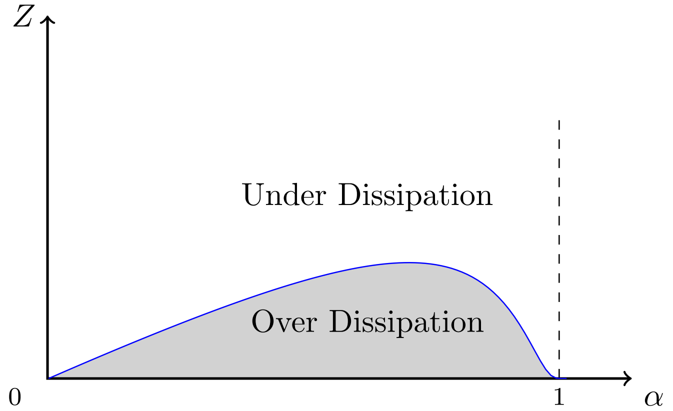

Figure 1: This illustrates the critical threshold of rent, $\tilde{Z}$, under which over-dissipation of the rent will occur, plotted as a function of α which controls the degree of concavity of preferences.

and the dissipation ratio takes the form

$$D = \frac{n-1}{n} \frac{\alpha\gamma}{[Z/n]^{1-\alpha}}.$$

It follows that $D \gtreqless 1 \Leftrightarrow Z \lesseqgtr \tilde{Z}$ *where* $\tilde{Z} = n \left[\frac{n-1}{n}\alpha\gamma\right]^{\frac{1}{1-\alpha}}$.

In this example, when preferences are sufficiently concave (i.e., α is small enough) the contest equilibrium exhibits over-dissipation if the contested rent is small enough, and in the limit where the contested rent becomes infinitesimally small, $D \to \infty$. Consider a particular value for α and consider reducing the contested rent in a contest. Then as the contested rent reduces, the effort of each contestant reduces (as in this example equilibrium effort is monotonically increasing in Z), but the reduction becomes smaller relative to the reduction in the rent, so as the rent gets sufficiently small (i.e., below $\tilde{Z}$) the dissipation ratio exceeds 1. Because of the concavity of the utility function contestants increasingly care about further reductions in their allocation of the rent and so become *relatively* more effortful in contesting it. By contrast, when the contested rent is relatively large the contest exhibits under-dissipation, and inspection of the expression for D reveals $D \to 0$ as $Z \to \infty$.

Having studied the linear cost case, we now return to the general cost case where the analysis is somewhat more nuanced as contest effort is only implicitly defined by (1).

Proposition 2. *Assume preferences are such that* $v'' < 0$ *and they satisfy the Inada conditions* $\lim_{z\to 0} v'(z) = \infty$ *and* $\lim_{z\to\infty} v'(z) = 0$*; and the cost function is such that* $\eta(x) \in (0,\infty)$ *for all* $x > 0$, $\lim_{x\to 0}\eta(x) = \underline{\eta} \in (0,\infty)$ *and* $\lim_{x\to\infty}\eta(x) = \bar{\eta} \in (0,\infty)$. *Then there is a unique interior symmetric Nash equilibrium in which:*

1. *$D \to 0$ as $Z \to \infty$, implying there is a $\bar{Z}$ such that in a contest with $Z > \bar{Z}$ under-dissipation occurs; and*

2. *$D \to \infty$ as $Z \to 0$, implying there is a $\underline{Z}$ such that in a contest with $Z < \underline{Z}$ over-dissipation occurs.*

Proof. Recall that

$$x^* c'(x^*) = \frac{n-1}{n^2} v'(Z/n) Z \text{ and} \tag{1}$$

$$D \equiv \frac{nc(x^*)}{Z} = \frac{n-1}{n} \frac{v'(Z/n)}{\eta(x^*)} \text{ where } \eta(x) \equiv \frac{xc'(x)}{c(x)}. \tag{2}$$

The assumption $\lim_{z \to 0} v'(z) = \infty$ implies $l(0, X_{-i}) > 0$ for all $X_{-i} > 0$ and therefore (1) will identify the unique interior symmetric Nash equilibrium.

First consider the large Z limit in case 1, and let $\lim_{Z \to \infty} v'(Z/n)Z = \bar{y}$. From (1), $\lim_{Z \to \infty} x^* c'(x^*) = \frac{n-1}{n^2} \bar{y}$ (*). There are three cases to consider. a) if $\bar{y} = 0$ then (*) implies that $\lim_{Z \to \infty} x^* = 0$. Our assumption that $\lim_{x \to 0} \eta(x) > 0$ combined with $\lim_{z \to \infty} v'(z) = 0$ can be used in (2) to conclude that $\lim_{Z \to \infty} D = 0$. b) if $\bar{y} \in (0, \infty)$ then (*) implies that $\lim_{Z \to \infty} x^* \in (0, \infty)$ and so it follows that $\lim_{Z \to \infty} \eta(x) \in (0, \infty)$. Then the assumption that $\lim_{z \to \infty} v'(z) = 0$ can be used to deduce from (2) that $\lim_{Z \to \infty} D = 0$. c) if $\bar{y} = \infty$ then (*) implies $\lim_{Z \to \infty} x^* = \infty$. Then our assumption that $\lim_{x \to \infty} \eta(x) \in (0, \infty)$ along with $\lim_{z \to \infty} v'(z) = 0$ can be used in (2) to deduce that $\lim_{Z \to \infty} D = 0$.

Next we consider the small Z limit (case 2), where we write $\lim_{Z \to 0} v'(Z/n)Z = \bar{\bar{y}}$ and note from (1) that $\lim_{Z \to 0} x^* c'(x^*) = \frac{n-1}{n^2} \bar{\bar{y}}$ (**). There are again three cases. a) if $\bar{\bar{y}} = 0$ then (**) implies $\lim_{Z \to 0} x^* = 0$. Then the assumption that $\lim_{x \to 0} \eta(x) < \infty$ combined with $\lim_{z \to 0} v'(z) = \infty$ can be used in (2) to deduce that $\lim_{Z \to 0} D = \infty$. b) if $\bar{\bar{y}} \in (0, \infty)$ then (**) implies $\lim_{Z \to 0} \in (0, \infty)$ and therefore $\lim_{Z \to \infty} \eta(x) \in (0, \infty)$. The assumption that $\lim_{z \to 0} v'(z) = \infty$ can then be used in (2) to deduce that $\lim_{Z \to 0} D = \infty$. c) if $\bar{\bar{y}} = \infty$ then (**) implies $\lim_{Z \to \infty} x^* = \infty$. Then our assumption that $\lim_{x \to \infty} \eta(x) < \infty$ along with $\lim_{z \to 0} v'(z) = \infty$ can be used in (2) to deduce that $\lim_{Z \to 0} D = \infty$. □

This proposition tells us that for 'sufficiently concave' preferences (which in this general case is interpreted as $v(\cdot)$ satisfying the Inada conditions) there will be contests, in which the rent is small enough, that exhibit over-dissipation: the fact that preferences exhibit high marginal utility over small allocations from the contest means that when contestants are fighting over small allocations they fight for them relatively hard, and the monetary cost of rent seeking outweighs the monetary value of the prize being contested. This is not universally true for all contests, however: when the contest involves a rent that is large enough we get the usual result of under-dissipation.

Our model therefore helps us to reconcile the so-called Tullock paradox—that rent-seeking efforts are relatively small in situations where large rents are being contested—with more recent findings of over-dissipation. Our extension to the theory, which accounts for diminishing marginal utility over the allocation of the contested rent, allows for the incentives to engage in rent seeking to differ depending on the size of the rent being contested. When the rent is large the marginal incentives to further increase rent seeking effort are low and so the monetary cost of effort is low relative to the value of the rent; by contrast, when the rent is small the marginal incentive to engage in rent seeking, to improve one's allocation of the spoils from a low level, is large leading contestants to be relatively effortful and implying the monetary cost of effort is large relative to the value of the contested rent, leading to over-dissipation.

3 More general Tullock share contests

In the previous section we initially modeled a simple contest success function and showed under what circumstances over and under rent dissipation can occur. In this section, we consider a more general Tullock share contest in which the contest success function is determined by:

$$\phi(x_i, \mathbf{x}_{-i}) = \begin{cases} \frac{x_i^r}{x_i^r + \sum_{j \neq i} x_j^r} > 0 & \text{if } \sum_{k \in N} x_k > 0, \text{ or} \\ \frac{1}{n} & \text{if } \sum_{k \in N} x_k = 0 \end{cases} \tag{3}$$

with $r > 0$.

With this more general contest success function the issue of the existence of a pure strategy Nash equilibrium becomes pertinent, as for permissible values of r the payoff function might not be globally strictly concave. As with a simple Tullock contest, if r is such that the payoff function is globally strictly concave then there will be a unique interior symmetric Nash equilibrium so long as the marginal payoff is strictly positive with zero effort. By contrast, if the payoff function is not globally concave a local strict concavity check is required, combined with checking that contestants' payoffs in the candidate equilibrium are positive.

Investigating this issue in a symmetric 2-player linear contest (in which there is an explicit solution), Baye et al. (1994)'s calculations show that for $r \leq 1$ payoff functions are globally strictly concave, whereas for $r > 1$ they are not. However, for $1 < r \leq 2$ the payoff function is locally strictly concave around a candidate equilibrium, and the candidate equilibrium payoffs are non-negative. By contrast, for $r > 2$ the candidate equilibrium payoffs are negative where the first-order condition is satisfied and so there is no pure strategy Nash equilibrium and the only equilibrium in the contest is in mixed strategies. A simple extension of these calculations reveals that for symmetric linear n-player contests, $r \leq 1$ remains the condition required for global strict concavity; and local strict concavity alongside strictly positive payoffs requires $r \leq \frac{n}{n-1}$.

We now want to investigate the nature of the restrictions required for global strict concavity with non-linear preferences that satisfy the Inada conditions (so that the marginal payoff is strictly positive with zero effort), in which case there will be a unique pure strategy Nash equilibrium involving strictly positive levels of effort. In a Nash equilibrium in pure strategies each agent can be seen as choosing their effort to maximize their payoff taking the effort choices of others as fixed:

$$\max_{x_i \geq 0} v_i(z_i) - c_i(x_i) \text{ s.t. } z_i = \frac{x_i^r}{x_i^r + \sum_{j \neq i} x_j^r} Z.$$

The first-order condition is

$$v_i'(z_i)\frac{\partial z_i}{\partial x_i} - c_i'(x_i) = 0, \tag{4}$$

where $\frac{\partial z_i}{\partial x_i} = \frac{r x_i^{r-1} \sum_{j \neq i} x_j^r}{[x_i^r + \sum_{j \neq i} x_j^r]^2} Z$, and the second-order condition implying global strict concavity (suppressing the arguments of functions) is

$$v_i'' \left[\frac{\partial z_i}{\partial x_i}\right]^2 + v_i' \frac{\partial^2 z_i}{\partial x_i^2} - c_i'' < 0 \tag{5}$$

for all $x_i > 0$. Note that, after some simplification,

$$\frac{\partial^2 z_i}{\partial x_i^2} = \frac{r x_i^{r-2} \sum_{j \neq i} x_j^r [[r-1][x_i^r + \sum_{j \neq i} x_j^r] - 2r x_i^r]}{[x_i^r + \sum_{j \neq i} x_j^r]^3} Z.$$

If $r \leq 1$ this term is globally negative. If $v(\cdot)$ is linear then this is the only term in the second-order condition necessitating $r \leq 1$ to ensure global concavity. However, when we account for a concave evaluation of the contest outcome there is an additional negative term in the second-order condition (even if costs are linear), the magnitude of which depends on the degree of concavity of the payoff function. This raises the possibility that the second-order condition can be globally satisfied when $r > 1$, or even when $r > \frac{n}{n-1}$, the conditions for which we now investigate.

Suppose costs are linear so the final term in the second-order condition is zero.[5] Focusing on the first two terms in (5), to obtain a globally concave payoff function (we assume here $\frac{\partial^2 z_i}{\partial x_i^2}$ is positive, for otherwise the second-order condition is *de facto* satisfied), we require

$$v_i'' \left[\frac{\partial z_i}{\partial x_i}\right]^2 + v_i' \frac{\partial^2 z_i}{\partial x_i^2} < 0,$$

[5]By considering linear costs we are studying the most stringent scenario since allowing for convex costs 'helps' in terms of the payoff function being globally concave.

which can be expressed as

$$\epsilon_i(z_i)\frac{\frac{\partial z_i}{\partial x_i}}{z_i\frac{\partial^2 z_i}{\partial x_i^2}}\frac{\partial z_i}{\partial x_i} < -1 \tag{6}$$

where $\epsilon_i(z_i) \equiv \frac{z_i v''}{v'}$ is the z_i-elasticity of the marginal utility. Thus there exists a unique symmetric pure-strategy Nash equilibrium as long as this condition holds, which requires preferences to be sufficiently concave.

Proposition 3. *A unique symmetric pure-strategy Nash equilibrium exists in the Tullock share contest if* (6) *holds, so even if $r > \frac{n}{n-1}$ a pure strategy Nash equilibrium can exist if preferences are sufficiently concave.*

We illustrate Proposition 3 with the worked example introduced in Section 2.

Example 1′. *Consider the same setting as in Example* 1*, but where $\phi(x_i, \mathbf{x}_{-i})$ is given by* (3) *with $r > 0$. Each contestant's marginal payoff is given by*

$$\alpha\gamma\left[\frac{x_i^r}{\sum_{j\in N} x_j^r}Z\right]^{\alpha-1}\frac{r x_i^{r-1}\sum_{j\neq i} x_j^r}{[\sum_{j\in N} x_j^r]^2}Z,$$

and consequently the optimization problem is globally concave if and only if:

$$\alpha\gamma r Z^\alpha \frac{\left(\sum x_j^r\right)^\alpha \sum_{j\neq i} x_j^r x_i^{\alpha r-2}}{\left(\sum x_j^r\right)^{2(\alpha+1)}}\left[(\alpha r-1)\sum x_j^r - (\alpha+1) r x_i^r\right] < 0.$$

A sufficient condition for global concavity is therefore that $r < \frac{1}{\alpha}$, so when the evaluation of the contest outcome is concave ($\alpha < 1$), the payoff function can be strictly concave even when r exceeds 1 (so long as it is not too large). Put differently, for any value of $r > 1$ there always exists a non-empty set of concavity parameters $\alpha \in (0, \frac{1}{r}]$ such that a unique pure-strategy Nash equilibrium exists.

We now turn to investigate the nature of the dissipation ratio in these more general contest settings. From the first-order condition (4), in the symmetric Nash equilibrium

$$x^* c'(x^*) = \frac{n-1}{n^2} r v'(Z/n) Z, \tag{7}$$

and the dissipation ratio is given by

$$D \equiv \frac{nc(x^*)}{Z} = \frac{n-1}{n} r \frac{v'(Z/n)}{\eta(x^*)}. \tag{8}$$

Since these expressions are only slight modifications to the simple contest case in which $r = 1$, our results are also similar. In the case of linear costs where we have an explicit solution, the analog of Proposition 1 for the more general case is as follows.

Proposition 4. *Assume* $v'(z) \to 0$ *as* $z \to \infty$. *If* $v'(0) > \frac{1}{r}\frac{n}{n-1}$ *then there exists a* $\tilde{Z} \equiv nv'^{-1}\left(r\frac{n-1}{n}\right)$ *such that* $D > (<)1 \Leftrightarrow Z < (>)\tilde{Z}$, *so the contest exhibits over-dissipation for* $Z < \tilde{Z}$. *By contrast, if* $v'(0) \leq \frac{1}{r}\frac{n}{n-1}$ *the contest never exhibits over-dissipation.*

Adapting the worked example introduced in Section 2 helps to further illustrate this result.

Example $1'$ **(continued).** *Consider the same setting as in Example* $1'$. *Effort in the symmetric Nash equilibrium is given by:*

$$x^* = \frac{n-1}{n}\frac{\alpha\gamma r}{c}[Z/n]^{\alpha},$$

and the dissipation ratio takes the form

$$D = \frac{n-1}{n}\frac{\alpha\gamma r}{[Z/n]^{1-\alpha}}.$$

It follows that $D \gtreqless 1 \Leftrightarrow Z \lesseqgtr \tilde{Z}$ *where* $\tilde{Z} = n\left[\frac{n-1}{n}\alpha\gamma r\right]^{\frac{1}{1-\alpha}}$.

In the case of non-linear cost functions, Proposition 2 holds *mutatis mutandis* for more general contests where r is such that the payoff function is globally strictly concave.

4 General preferences and rent dissipation

Our analysis so far reveals that under- and over-dissipation can both occur with general preferences when considering the standard definition of dissipation, namely the ratio of players' aggregate cost of contest effort to the contested rent. Yet, while our model helps rationalize real-world regularities that scholars have only be able to explain so far with the help of behavioral biases (e.g., risk-loving preferences), the realization (as opposed to the expectation) of mixed strategies, or uncertainty (e.g., the number of contestants), it also reveals the limitations of the dissipation measures considered so far by the literature. We thus revisit the rent dissipation concept and propose a general measure that encompasses existing ones, while also accommodating the more general setting proposed in this paper, as well as different frameworks involving probabilistic contests and all-pay auctions.

In the context of share contests, we consider any model where players are seen as optimizing the following problem

$$\max_{x_i \geq 0} u_i(z_i, x_i) \text{ s.t. } z_i = \frac{p_i(x_i)}{\sum_{j\in N} p_j(x_j)}Z. \tag{9}$$

The unique Nash equilibrium in pure strategies is known to exist under certain conditions (see Dickson et al., 2018).

Let $\bar{\mathbf{z}}$ be vector of shares (or expected benefit) such that $\bar{\mathbf{z}} = (\bar{z}_1, \bar{z}_2, ..., \bar{z}_n)$, $\sum_{i=1}^{n}\bar{z}_i = Z$, and

$$\bar{U}(\bar{\mathbf{z}}, 0) \equiv \max_{\bar{\mathbf{z}}=(\bar{z}_1,\bar{z}_2,\ldots,\bar{z}_n)} \sum_i^n U_i(\bar{z}_i, 0). \tag{10}$$

As such, $\bar{\mathbf{z}}$ defines the first-best solution, and $\bar{U}(\bar{z}, 0)$ is obviously single-valued. Our general dissipation measurement aims at measuring the inefficiency of non-cooperative play in the current strategic setting, by capturing the utility loss arising from non-cooperative play. We therefore define the general dissipation ratio as:

$$\mathcal{D} = \frac{\bar{U}(\bar{\mathbf{z}}, 0) - \sum_{i=1}^n U_i(z_i^*, x_i^*)}{\bar{U}(\bar{\mathbf{z}}, 0)}. \tag{11}$$

It is immediate to observe that $\mathcal{D} \in [0, 1]$: the ratio is indeed bounded from below at 0 by construction, while it can only exceed 1 if players obtain (on average) negative payoffs, which cannot happen in a complete information setting where players can always secure a payoff of 0 by refraining from investing in effort.

Notice that this general dissipation ratio encompasses previous measures considered by scholars. Importantly, the specific case where the valuation of the rent and the cost of effort are linear and additive, i.e., $u_i(z_i, x_i) = z_i - x_i$, our proposed measure reduces to $\mathcal{D} = \frac{\sum_i x_i}{Z}$, the standard dissipation ratio considered elsewhere in the literature. Chung (1996) adapts the rent dissipation measure to non-linear endogenous production of rents. By imposing the restriction that $u_i(z_i, x_i) = z_i(x_i) - x_i$, our $\mathcal{D}$-measure exactly reproduces the dissipation ratio considered in Chung (1996). Lastly, Hurley (1998) proposes the contest-efficiency concept CE, relating aggregate payoffs to the first-best allocation of the prize, i.e., an inverse measure of rent dissipation since $D = 1 - CE$. Once again, in contexts featuring probabilistic contests and asymmetric valuations of the prize as in Hurley (1998), one obtains that $\mathcal{D} = 1 - CE$.

5 Concluding remarks

In this article we have reconciled two apparent paradoxes in economics: the real world under-dissipation of rents, i.e., the Tullock paradox; and the observed over-dissipation of rents in experimental settings. Arguments in the literature so far used to rationalize the former are inconsistent with the latter, explanations of which have appealed to behavioral ideas. Our approach, based on share contests, simply recognizes that contestants may have diminishing marginal utility over the contest outcome. If they do, and this is sufficiently strong, then we can explain both over-dissipation of rents when they are small (as they arguably are in experimental settings) and under-dissipation of rents when they are large (as is arguably the case in Tullock's observations).

The intuition is simple: with sufficiently strong diminishing marginal utility, when the contested rent is small contestants are highly sensitive to changes in the spoils they are awarded from the contest and so they are relatively effortful in contesting the rent, resulting in the monetary cost of rent seeking exceeding the monetary value of the rent leading to over-dissipation; by contrast, if the rent is large the contestants are less sensitive to changes in their allocation of the rent, and they will be relatively less effortful leading to under-dissipation.

Our theory thus expands the literature on rent seeking by providing a rational explanation for observed phenomena, and one that relies on the backbone of economic theory, namely diminishing marginal utility. Yet, upon closer observation, the standard dissipation ratio could fail to accurately measure inefficiencies when players have non-linear preferences. To remedy this limitation, we thus also propose in our article a more general measure of dissipation that evaluates the relative utility loss in a contest compared to the first-best solution, thus constituting a well-designed tool for welfare analysis.

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