A re-evaluation of the hull girder shakedown limit states

Shen Li*, Simon D. Benson

Marine, Offshore and Subsea Technology Group, School of Engineering, Newcastle University, Armstrong Building, Queen Victoria Road, Newcastle upon Tyne, NE1 7RU, UK

*Corresponding email address: s.li37@newcastle.ac.uk

Shen Li is a PhD student at Newcastle University. He is now researching on the dynamic collapse of ship hull girders and is aiming to develop a novel hydro-elastoplasticity method for predicting this phenomenon. The analyses presented in this paper were performed by Shen Li and he is the person drafting this paper.

Simon D. Benson currently holds the position of Lecturer in Naval Architecture at Newcastle University. He is also the academic director of the Marine Propulsion Laboratory and undergraduate admission tutor for marine technology. The present paper is contributed by Simon Benson with his important discussion and critical insight. He also contributes to the revision of the manuscript.

Abstract: This paper investigates the use of shakedown limit state in the assessment of longitudinal strength of ship hull girders. The consideration of shakedown limit state is related to the fact that a structural system subjected to cyclic loadings may suffer from plastic collapse even when the loading magnitude is less than the instantaneous collapse load of single excursion. For ships sailing in an extreme sea state, large magnitude cyclic load might be experienced. This suggests that a monotonic ultimate bending strength assessment may overestimate the capacity of ship hull girders under longitudinal bending. This paper first elucidates the difference between shakedown limit state and conventional ultimate limit state using the procedure proposed by Jones (1975) where geometric linearity is assumed. In addition, the effect of the inherent geometrical nonlinearity is evaluated with the aid of the nonlinear finite element method. Cyclic elastoplastic large deflection finite element analyses are performed to investigate the structural behaviours of a box girder model under six different loading protocols, which are regarded as unsafe in the shakedown limit state evaluation. The rationality of a shakedown limit state is discussed and an energy-based characterization of limit state is suggested. Lastly, a recommendation for future work is given. The study shows that, whilst Jones' original method may be overly conservative, the safety margin based on ultimate limit state approach might be considerably reduced.

Keywords: shakedown limit state; ultimate strength; ship hull girder; cyclic loading; ductile collapse.

1. Introduction

A ship hull girder undergoes significant longitudinal bending due to the imbalanced distribution of its weight and the buoyancy (still-water bending moment) as well as the external wave actions (wave-induced bending moment). Therefore, the longitudinal strength of a ship hull girder becomes one of the most fundamental aspects for the ship structural integrity (Yao et al., 2016). Two philosophies are commonly employed in the design of ship structures, namely the allowable stress principle and the limit state design. In the former, the determination of structural scantlings are based on the criterion that maximum resultant stress under a prescribed loading does not exceed the allowable stress specified by relevant stakeholders. Although the allowable stress principle provides practical design guidelines, it is not able to determine the true safety margin against extreme conditions for ship structures. To this regard, a limit state design philosophy can be employed where all of the possible failure modes are explicitly taken into account in the estimation of structural capacity (Paik and Thayamballi, 2003). Four limit states are commonly identified for ship structures, namely serviceability limit state, ultimate limit state, fatigue limit state and accidental limit state.

However, Jones (1975) argued that a shakedown limit state associated with the failure caused by cyclic loading should be considered instead of the ultimate limit state. Generally, the forms of collapse of ship hull girders can be categorized as 1) single excursion failure; 2) cyclic failure (14th ISSC, 2000). When it comes to ultimate collapse strength calculation of ship hull girders, static monotonic response are normally referred to, which is applicable to the single excursion failure related with the incorrect operation of loading and unloading of cargos, such as the failure of *Energy Concentration*. However, a ship hull might also experience multiple excursions with extreme magnitudes that results in a catastrophic overall collapse in a severe sea state, such as a series of storm waves. The peak cyclic load may cause alternating loading direction from hogging to sagging and vice versa, or may cause alternating maximum and minimum bending moments in a single direction. The former may be more common on general cargo ships which may sail in either hogging or sagging condition and alternate due to wave load. The latter may be more common in ships designed for specific loading, such as a container ship, which is usually expected to operate in a hogged condition only throughout its lifetime. In both cases, the cyclic failure may occur and cause a reduction in the capacity. It is suggested that structural capacity under cyclic loading can be significantly lower than the instantaneous ultimate strength under a single load excursion (15th ISSC, 2003). Reversal and accumulation of plastic deformation will degrade the structural resistance against subsequent loading and consequently result in an irreversible collapse.

Recent ship wrecking accidents of *MSC Napoli* and *MOL Comfort* may indicate that the overall failure of ship hull girders do not occur after single excursion and there is a need to carry out an investigation on cyclic failure as well as the assessment of shakedown limit states. Since Jones, research activity on shakedown limit state of ship hull girders is scarce until more recently Zhang et al. (2015) presented an approximate method to estimate the elastic shakedown limit of an oil tanker with consideration of buckling using ISFEM. One of the insights of this study is that the local buckling and local failure of structural components are highly important, as the applied load may not result in an instantaneous collapse of the entire hull girder, but it is very likely that some parts of the ship hull have been failed. With a further loading, the hull girder may experience an incremental collapse where the local failure is propagated to the whole hull girder.

This paper aims to re-evaluate the use of shakedown limit state in the safety assessment of ship hull girder. A case study to predict the shakedown limit state of a box girder model is performed in accordance with the procedure described by Jones and with the aid of nonlinear finite element method. From the results of this case study, an energy-based characterization of limit state is proposed together with a recommendation for further study.

2. Background

The state-of-the-art ultimate limit state design is reviewed in this section. With monotonically increased bending, the compressive side of a ship hull girder will fail due to the elastoplastic buckling while the tensile side will suffer from the gross yielding. The loss of load-carrying capacity or ultimate limit state is generally characterized by the overall structural effectiveness, or the bending stiffness in this case, becoming zero. A flow chart of this process is given in Figure 1, while Figure 2 shows the stress distribution and elemental tangent stiffness of a box girder cross section in the pre-ultimate, ultimate and post-ultimate states.

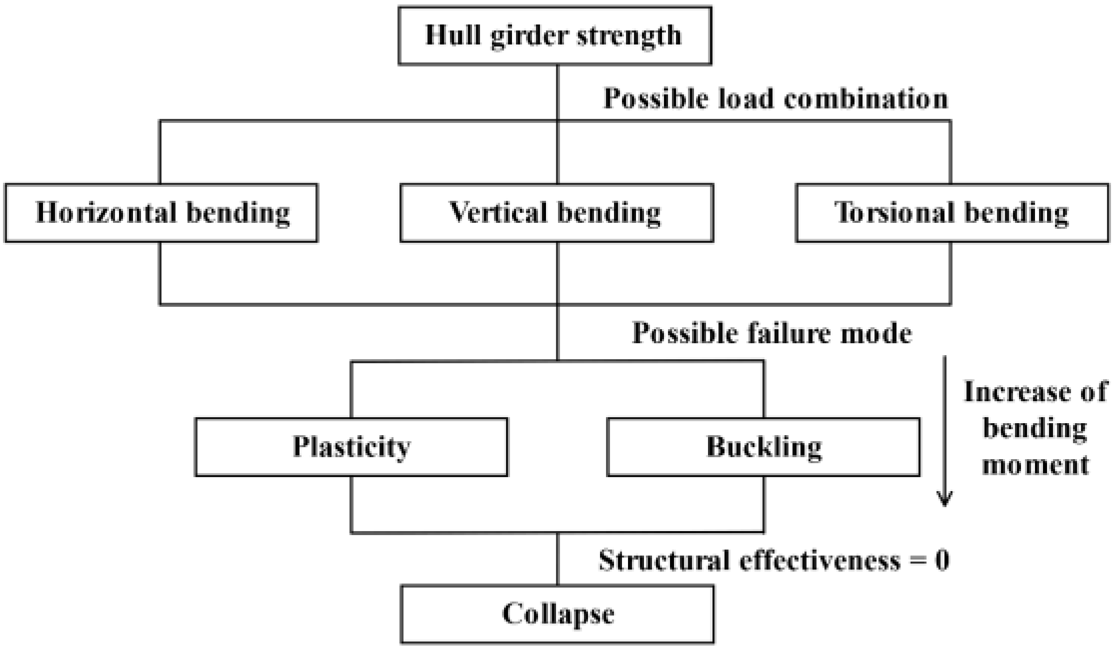


Figure 1. Flow chart of the collapse mechanism of ship hull girders under monotonic bending

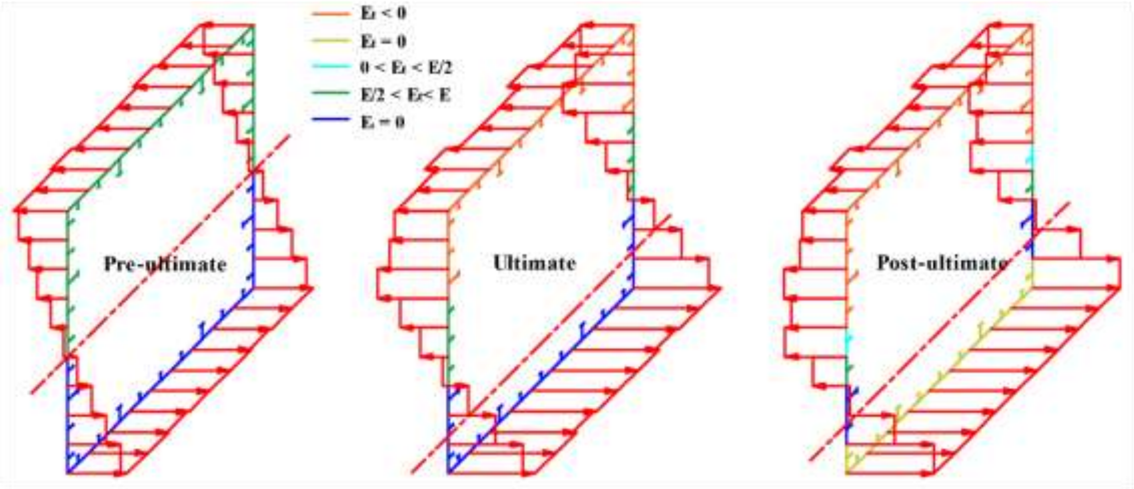


Figure 2. Stress distribution and tangent stiffness of each element of a box girder cross section under monotonic bending

Substantial efforts have been made in the development of theoretical methodology to predict the ultimate bending strength of ship hulls subjected to monotonic loading, which can be categorized into five groups:

- Presumed stress distribution-based method

- Empirical formula
- Simplified progressive collapse method (Smith's method)
- Idealized structural unit method
- Nonlinear finite element method

Caldwell (1965) made the first attempt to provide a solution on this subject by assuming an idealized stress distribution at the collapse state while the effect of buckling was catered using a strength reduction factor. The ultimate longitudinal strength is calculated by taking the first moment of the stress distribution. But the rationality of this method is challenged by the fact that structural members within the cross section of a hull girder normally do not attain their ultimate strengths at the same time due to the linearly distributed bending strain along the depth of the cross section. Paik and Mansour (1995) presented another presumed stress distribution-based method where the stress distribution at collapse state is characterized by plastic/buckling failure at the outer region of compressive side and gross yielding at the outer region of tensile side, whereas the middle region around the neutral axis remains elastic.

In addition to presumed stress distribution-based method, a simple empirical formula was given by Frieze and Lin (1991) for a rapid estimation, in which the ultimate longitudinal strength of ship hull girder was formulated as a function of the ultimate strength of compressive flange. Although the aforementioned methods can reasonably predict the ultimate strength of ship hull girders, the progressive collapse behaviour of ship hull girders including pre-ultimate and post-ultimate regimes cannot be simulated. This problem was solved by Smith (1977) who proposed a simplified method that enabled progressive collapse analysis on the prismatic cross section of a hull girder. A validation exercise was conducted by Dow (1991) where a 1/3-scale frigate model was tested under sagging and the Smith's method prediction gave a high correlation with the experimental measurement. Benson (2013) removed the inter-frame buckling failure assumption embedded in the original Smith's method and proposed a compartment-level progressive collapse method to account for the failure induced by an overall grillage instability. The most important issue in Smith's method might be the subdivision of individual elements and the derivation of average stress-average strain relations of these elements under in-plane loading. In the original Smith's method, that is now incorporated in Common Structural Rules by IACS, a plate-stiffener combination element was utilized while its average stress-average strain relationship can be derived by empirical formula-based approach, analytical approach or nonlinear finite element analysis. For example, an analytical approach of deriving average stress-average strain relationship for individual plate-stiffener element was developed by Yao et al. (1991) based on the elastic large deflection analysis and the rigid plastic mechanism analysis.

In almost the same time as the first proposal of Smith' method, an idealized structural unit method (ISUM) was developed by Ueda et al. (1974, 1984). The intention of this method was to tackle the problem in conventional finite element analysis that too many numbers of elements and degrees of freedom are required. Hence, ISUM elements with larger sizes and less degrees of freedom are reasonably developed. For instance, a stiffened panel can be considered as one unit or one element. Although the computing capability is increasingly improved nowadays, nonlinear finite element method still imposes significant difficulty in obtaining converged solution for a large-scale model, such as a ship hull girder. Therefore it still remains an art of trial and error, which might not be suitable for the routine design and analysis.

3. Shakedown limit state

3.1 Structural response to cyclic loading

Under a specified cyclic loading, a structure may respond in one of four different modes, depending upon the magnitude of repeated loading:

- Pure elastic;
- Elastic shakedown;
- Plastic shakedown, cyclic collapse, low-cycle-high-strain fatigue, or alternating plasticity (yielding);
- Incremental collapse, ratchetting.

In the first case, the structure experiences a purely elastic response since the magnitude of the loading is lower than the elastic limit. No plastic flow is produced in this case. The potential failure of the structure might be a high-cycle fatigue. The characteristics of elastic shakedown, cyclic collapse and incremental collapse can be illustrated with reference to Figure 3. When the magnitude of repeated loading exceeds the elastic limit, certain plastic flows are produced in the structure. However if the magnitude of repeated loading is smaller than a certain level, the structure might ‘shake down’ to behave purely elastically (red line in Figure 3). No more plastic strain will be produced in the subsequent loading. This certain level is termed elastic shakedown limit, being coined by Prager (1948). The benefit of this elastic shakedown on the relaxation of welding-induced residual stress was investigated by Paik et al. (2005) and Gannon et al. (2013). A considerable amount of welding residual stress can be reduced indicated from both of these studies.

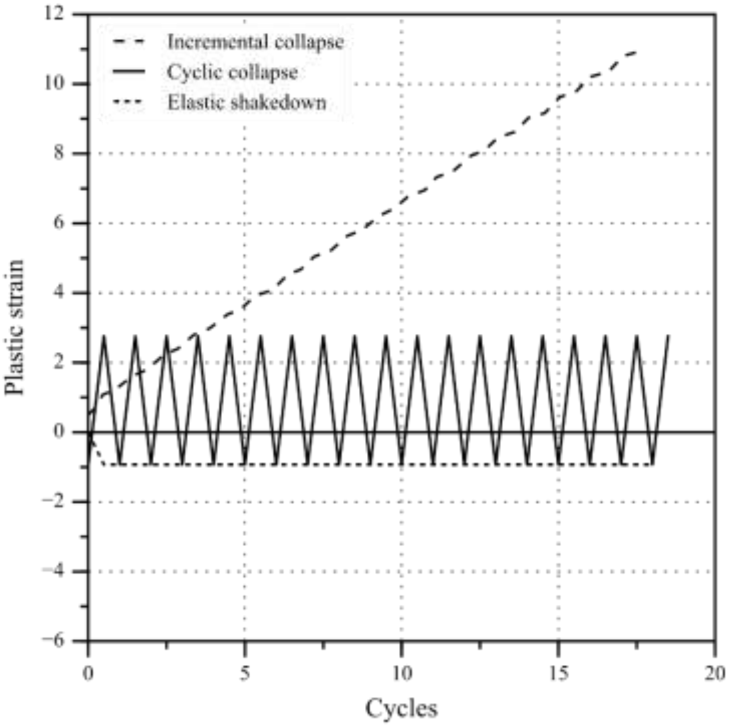


Figure 3. Characteristics of plastic strain accumulation under cyclic loading (Hodge, 1959)

As for cyclic collapse, the structures exhibit a constant but alternating plastic deformation (blue line in Figure 3). Although there is no net increase of plastic deformation at each cycle, the material is damaged by the alternating plastic flow. The physical explanation of this behaviour involves the consideration of the crystallographic basis of plasticity. In essence, the internal structure of the metal is changed due to the reversed plastic flow. This change will weaken the structure and/or make it brittle. Although the macroscopic configuration might remain intact in the first few cycles, it will render the structure unserviceable after sufficient cycles. In the last case, a net plastic deformation is produced at the end of each cycle. As long as the structure is kept loaded, the total plastic deformation will be increased without bound and eventually the structure might lose its designated function, causing an incremental collapse (Neal and Symonds, 1950-1951). It should be noted that this phenomenon might occur even if the magnitude of repeated loading is lower than the instantaneous collapse load.

3.2 Shakedown theorem

Under a specified repeated loading, it might be able to examine whether or not the structure will shake down. For the general case, however, the loading is unknown in advance. Therefore a shakedown theorem was devised to attack this problem. In the application of the longitudinal strength of a ship hull girder, the shakedown theorem is formulated in terms of bending moments. An elastic perfectly plastic hull girder idealized as a beam will approach a shakedown state, provided that the following inequalities are satisfied at every cross section (Hodge, 1959):

$$M_r + M_{max} \leq M_p \tag{1}$$

$$M_r + M_{min} \geq -M_p \tag{2}$$

$$M_{max} - M_{min} \leq 2M_y \tag{3}$$

where M_r is the residual bending moment, M_p is the plastic collapse bending moment, M_y is the initial yield bending moment, M_{max} is the maximum applied bending moment and M_{min} is the minimum applied bending moment.

Jones (1975) reduced the inequalities (1) to (3) into inequalities (4) to (6) or the non-dimensional form (7) to (9), with the aid of the conversion by equations (10) to (12) since the self-equilibrium residual bending moment M_r must vanish.

$$M_{max} \leq M_p \tag{4}$$

$$M_{min} \geq -M_p \tag{5}$$

$$M_{max} - M_{min} \leq 2M_y \tag{6}$$

$$p \leq 1 \tag{7}$$

$$q \geq -1 \tag{8}$$

$$p - q \leq 2/\alpha \tag{9}$$

$$p = M_{max}/M_p \tag{10}$$

$$q = M_{min}/M_p \tag{11}$$

$$\alpha = M_p/M_y \tag{12}$$

The inequalities (7) to (9) can be graphically illustrated using an interactive diagram, as shown in Figure 4. Only the region ACD is considered since $p \geq q$ by definition. Lines AC and AD correspond to the equalities from (7) and (8) which safeguard the structure from incremental collapse. Line BE corresponds to the equality from (9) which prevents the ship hull girder from cyclic collapse. In short, the ship hull girder will not shake down to behave elastically when the combination of maximum bending moment and minimum bending moment lies within the region ABE. If only considering the instantaneous collapse load, the ship hull girder is regarded as safe within the region ACD. However, the safety margin of a ship hull girder is reduced by the amount represented by region ABE when elastic shakedown limit is of concern, which depends upon the value of shape factor α in the original formulation derived by Jones (1975).

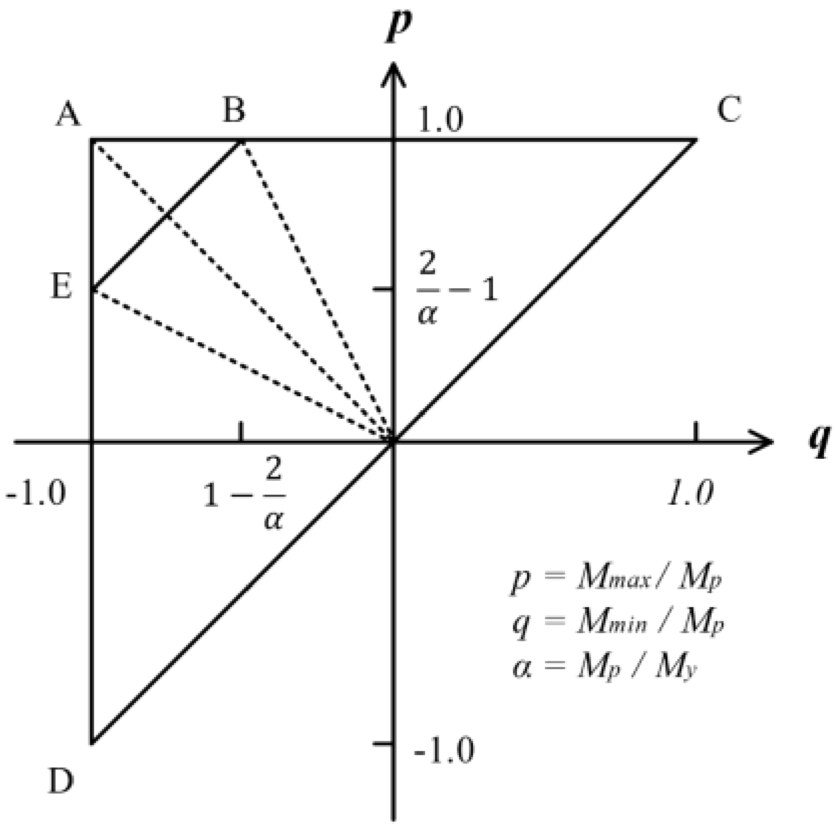


Figure 4. Interactive diagram to determine the shakedown limit state (Jones, 1975).

In Jones' method, the geometric nonlinearity is not considered and the ship hull girder is simply treated as a beam. For the actual cross section of a ship hull girder, which is an assembly of various stiffened panel, a complete analytical solution of shakedown limit is difficult to obtain. To this regard, Zhang et al. (2015) described an approximate method for assessing the shakedown limit of a ship hull girder with consideration of buckling. Its principle is summarised as follows:

- Perform the progressive collapse analyses under sagging and hogging respectively
- Determine the ultimate sagging and hogging bending moments $M_{u/sag}$ and $M_{u/hog}$

- Elastically unload the cross section from ultimate sagging/hogging state and subsequently reload it into the opposite direction until reaching the elastic limit. Obtain the initial yield bending moments M_1 and M_2
- The shakedown limit state is determined as $M_{u/sag} \leq M \leq M_2$ or $M_1 \leq M \leq M_{u/hog}$

4. Case study

A doubly-symmetric box girder model MST-3 is adopted from the testing performed by Nishihara (1983). Its principal particulars are shown and summarised in Figure 5 and Table 1. The estimation of shakedown limit state is carried out using the procedure derived by Jones (1975) while two cases are investigated. The first case assumes the geometric linearity in accordance with Jones' assumption. The second case accounts for the effect of buckling with the aid of nonlinear finite element analysis.

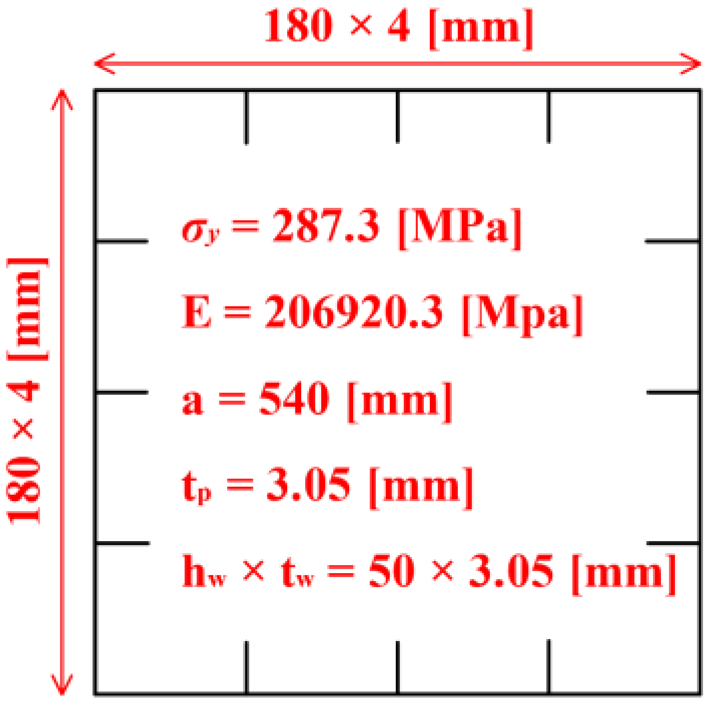


Figure 5. Schematics of the case study box girder

4.1 Shakedown limit state without buckling

In this case, the problem reduces to calculate the fully plastic section modulus and elastic section modulus at the flange of the hull girder. Plastic section modulus can be given by Equation (13) while the elastic section modulus at the flange is given by Equation (14) to (16). The calculated plastic section modulus and elastic section modulus as well as the shape factor are summarized in Table 2. Based on the calculated results and inequalities (7) to (9), an interactive diagram can be drawn (Figure 6). The safety margin of the box girder is reduced by 3.1%, which is calculated as the ratio between the area representing the cyclic failure and the total area.

$$Z_p = \sum_{j=1}^{n} a_j \left| y_j - y_{EAA} \right| \qquad (13)$$

$$I = \sum_{j=1}^{n} \left(a_j y_j^2 + i_j \right) - A y_{NA}^2 \qquad (14)$$

$$y_{NA} = \frac{\sum_{j=1}^{n} a_j y_j}{\sum_{j=1}^{n} a_j} \qquad (15)$$

$$Z = \frac{I}{y_{max}} \qquad (16)$$

where Z_p is the plastic section modulus, Z is the elastic section modulus, a_j is the cross section area of j^{th} member, y_j is the distance from the baseline to the neutral axis of j^{th} member, y_{EAA} is the distance from the base line to the equal area axis, I is the moment of inertia of the ship hull girder cross section, i_j is the moment of inertia of j^{th} member, A is the area of the cross section and y_{NA} is the distance from the baseline to the cross section neutral axis.

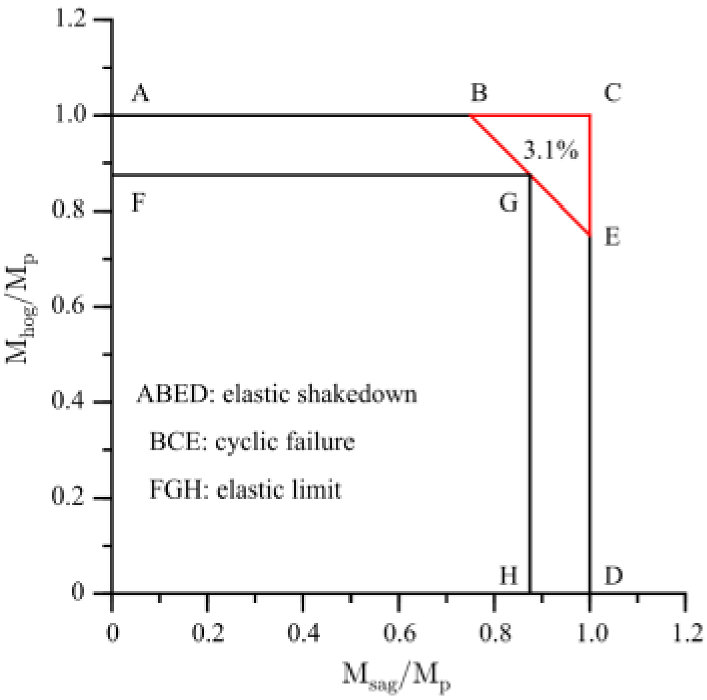


Figure 6. Interactive diagram of the box girder under sagging and hogging without buckling

4.2 Shakedown limit state with buckling

Due to the inherent geometric nonlinearity caused by buckling, the actual instantaneous collapse bending moment is lower than the fully plastic bending moment. In this section, nonlinear finite element analysis with a dynamic implicit solver is performed. In the finite element modelling, a four-node shell element is adopted with a characteristic element size of $10mm \times 10mm$. Initial deflection is applied on the finite element model by mean of the direct node translation. The deflection shape and its magnitude are determined by Equation (17) to (19). No residual stress is considered. The rotational control method is employed for the loading application. A reference point is arbitrarily created at one end of the model, in which the forced-rotation is applied. The opposite end of the model is fixed in six degrees of freedom. This boundary condition can account for the translation of the instantaneous neutral axis.

$$w_0 = A_0 sin\left(\frac{m\pi x}{a}\right) sin\left(\frac{\pi y}{b}\right) \qquad (17)$$

$$A_0 = \min(0.1\beta^2 t, 6) \qquad (18)$$

$$m = integer, m - 0.7 < \frac{a}{b} \leq m + 0.3 \qquad (19)$$

where w_0 is the initial deflection, A_0 is the maximum deflection, m is the number of half wave in the longitudinal direction, β is the plate slenderness ratio, a is the length of the plate and b is the width of the plate.

The criterion for shakedown limit state evaluation derived by Jones (1975) is adapted to make use of the instantaneous collapse bending moment M_u (blue point in Figure 7) instead of fully plastic bending moment M_p, while the initial yield bending moment M_y (red point in Figure 7) is computed by FEA as well. A stress distribution contour plot corresponding to the initial yield status and the ultimate collapse status is given in Figure 8. From the interactive diagram of Figure 9, a 27% reduction of safety margin is predicted.

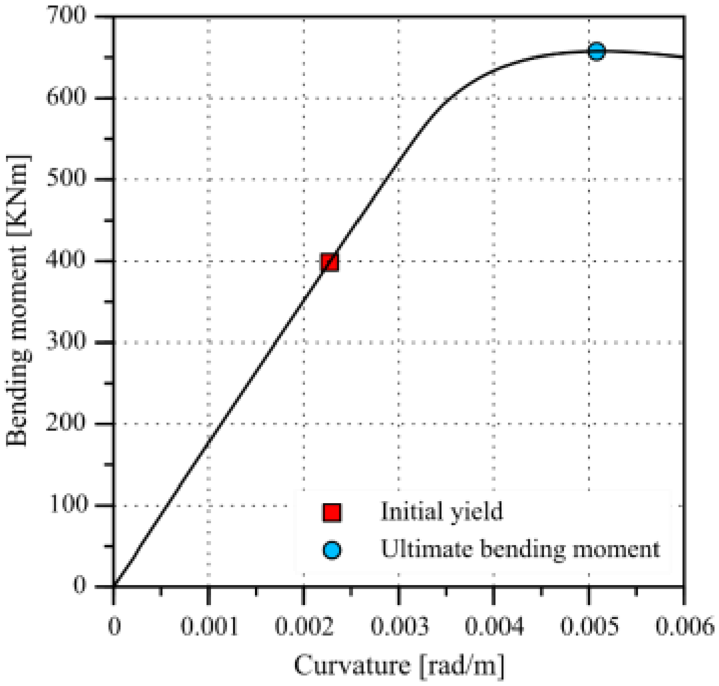


Figure 7. Moment-curvature relationship of the box girder under monotonic bending.

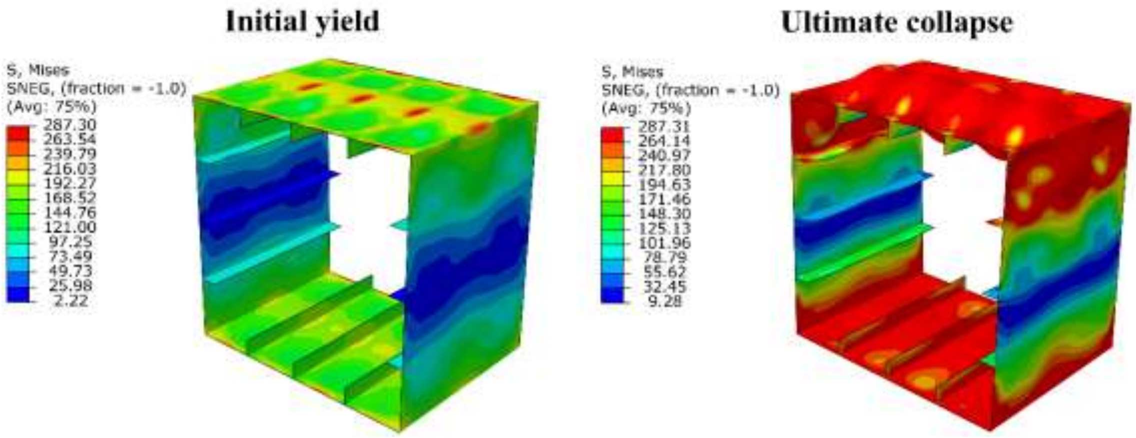


Figure 8. Stress distribution contour plot. Left: initial yield; Right: ultimate collapse.

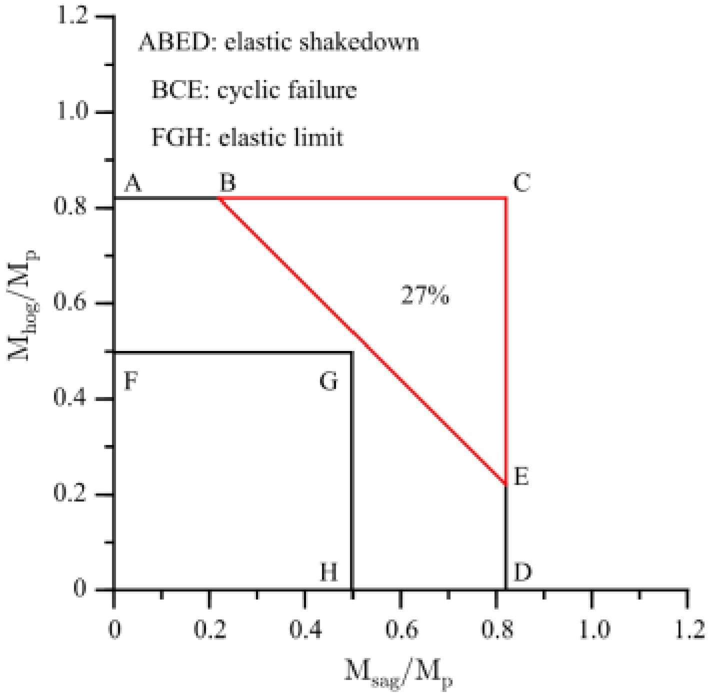
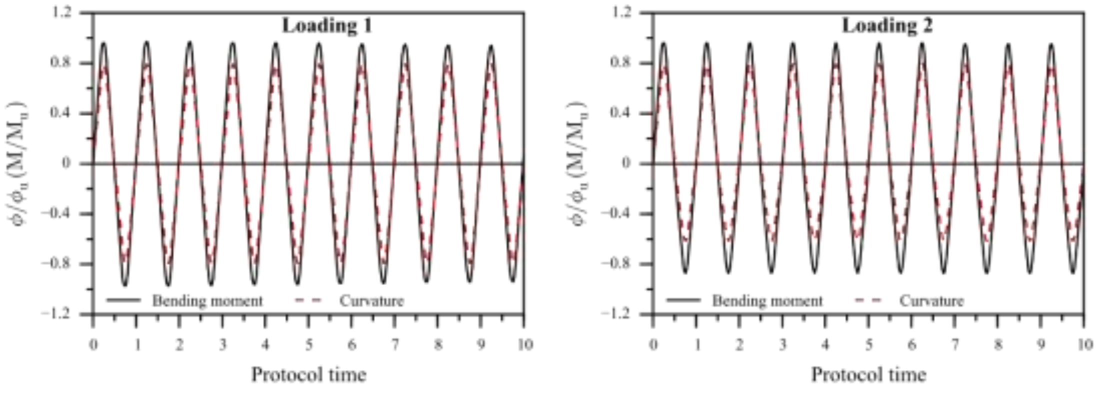


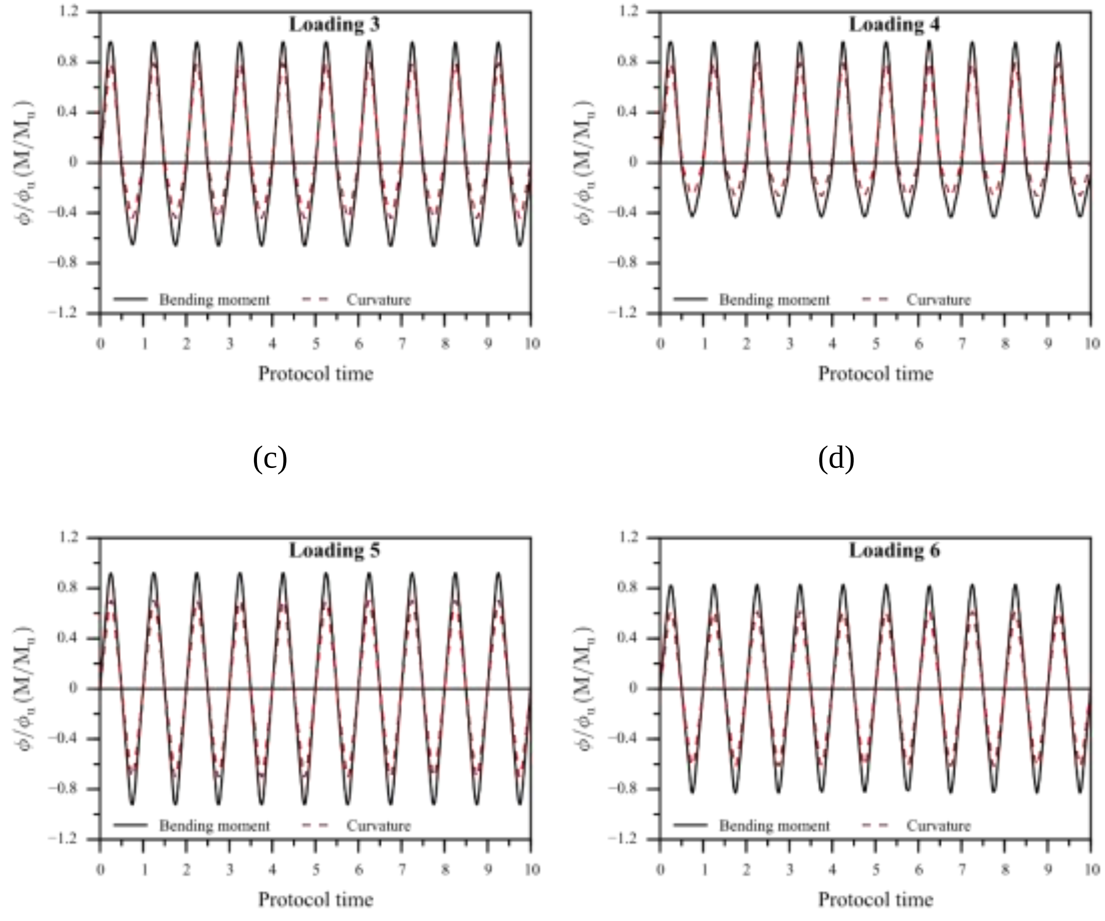
Figure 9. Interactive diagram of the box girder under sagging and hogging with buckling.

4.3 Cyclic elastoplastic large deflection finite element analyses

In addition to the prediction of a shakedown limit state, which is based on the information provided by the analysis with monotonic loading and shakedown theorem, six cyclic elastoplastic large deflection analyses using the finite element method are performed. These analyses are aimed to examine the cyclic structural responses under different loading protocols, which are considered as unsafe according to the evaluation of shakedown limit state in previous case study. For simplicity, an elastic-perfectly plastic material is assumed in this study. However it is very crucial to consider the role of cyclic hardening and Bauschinger effect, which remains as a future study. The dynamic implicit solver is adopted and a displacement (rotation) control is employed to implement the loading application in finite element analysis. The time-dependent loading is described by its peak values and zero-crossing values, assuming a same time interval between each adjacent input signal (30 seconds). However, it is found in the initial study that the input signals cannot be fulfilled where the output rotational response is smaller than the desired value. Hence some preliminary tests are performed to calibrate the actual input value. Six tested loading protocols in terms of the applied curvature as well as the resultant bending moment are plotted in Figure 10(a) - (f), each of which corresponds to different points in the interactive diagram (Figure 10g). Bending moment-curvature relationships are shown in Figure 11(a) – (f). From these diagrams, no obvious strength reduction can be observed. But it should be noted that a relatively large amount of permanent curvature is formed in loading protocols 1 to 4.

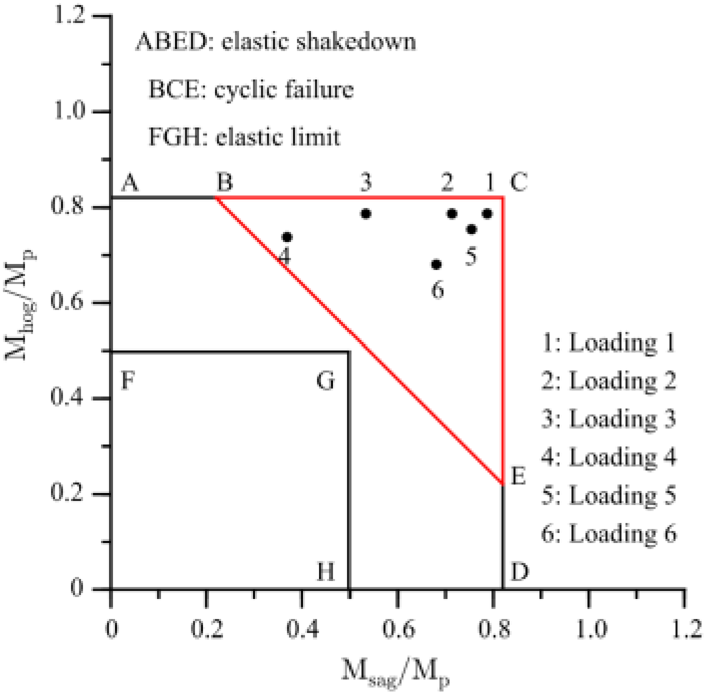


(a) (b)


Loading 3
Loading 4
Loading 5
Loading 6
φ/φ_u (M/M_u)
Protocol time
Bending moment
Curvature
1.2
0.8
0.4
0
−0.4
−0.8
−1.2
0
1
2
3
4
5
6
7
8
9
10

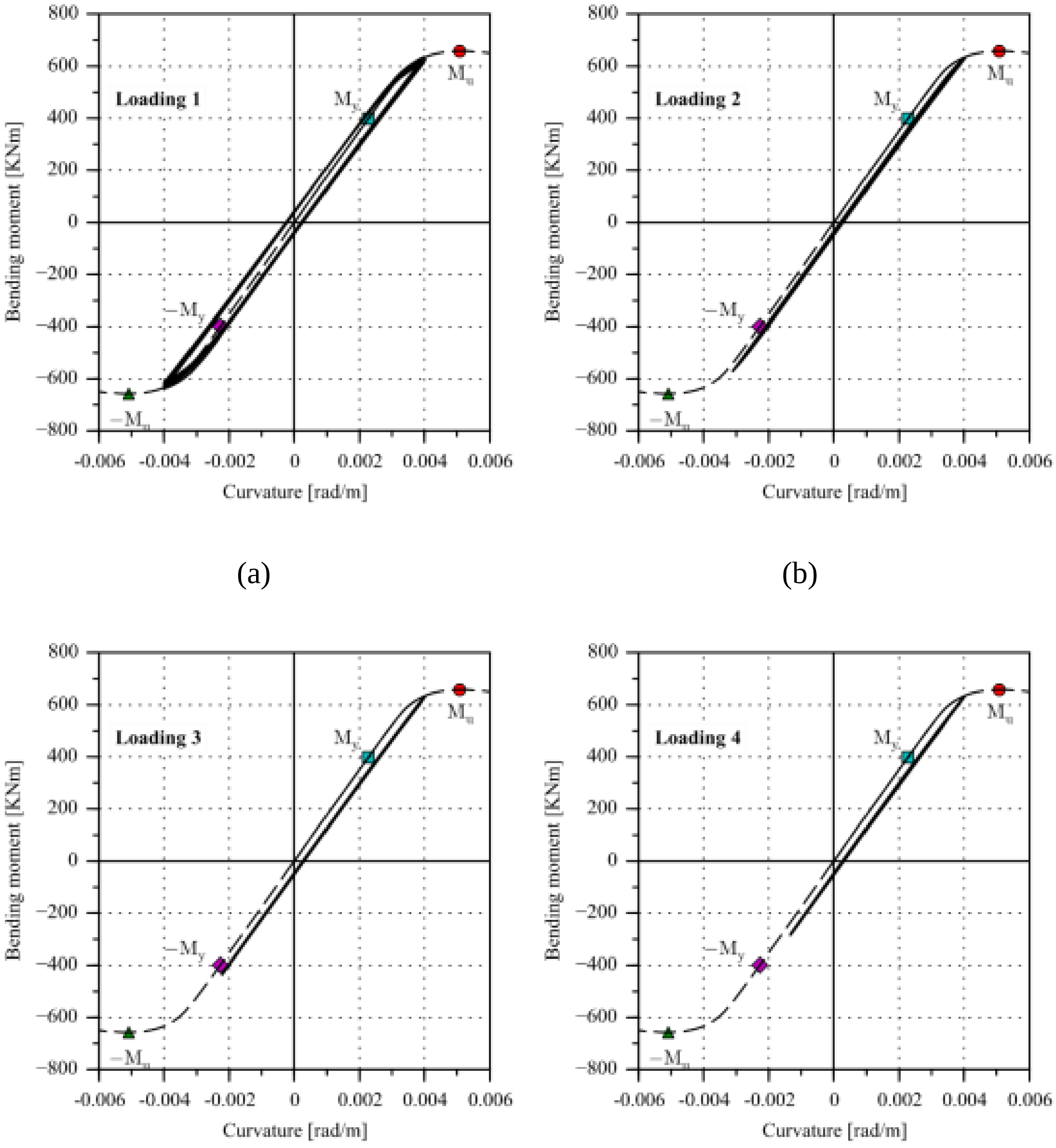
(c) (d)

(e) (f)


ABED: elastic shakedown
BCE: cyclic failure
FGH: elastic limit
A
B
C
D
E
F
G
H
1
2
3
4
5
6
1: Loading 1
2: Loading 2
3: Loading 3
4: Loading 4
5: Loading 5
6: Loading 6
M_hog/M_p
M_sag/M_p
0
0.2
0.4
0.6
0.8
1.0
1.2

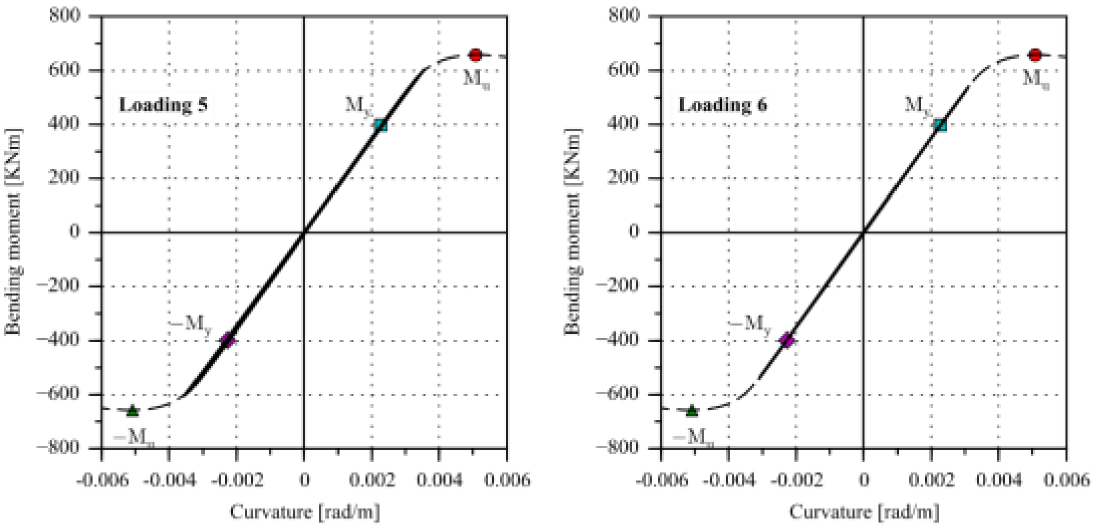
(g)

Figure 10. Loading protocols and resultant responses. (a) Loading protocol 1; (b) Loading protocol 2; (c) Loading protocol 3; (d) Loading protocol 4; (e) Loading protocol 5; (f) Loading protocol 6; (g) The corresponding position of each loading protocol in the interactive diagram.



(a) (b)

(c) (d)



(e) (f)

Figure 11. Bending moment-curvature relationship of cyclic bending tests. (a) Loading protocol 1; (b) Loading protocol 2; (c) Loading protocol 3; (d) Loading protocol 4; (e) Loading protocol 5; (f) Loading protocol 6.

5. Discussion

The reason why the shakedown limit state has not gained enough attention among designers and researchers might be those indicated by Faulkner (1976) in his comment to the paper by Jones (1975). They are summarized as follows:

- The consideration of shakedown seems to boil down to a problem dependent on the magnitude and temporal distribution of cyclic loading, which is essentially a problem of probability. Faulkner (1976) argued that this probability is of a low order in practical ship structure based on his research outcome (Mansour and Faulkner, 1973).
- No information is contained in the shakedown theorem that how many cycles are required for a ship hull girder eventually attaining an unserviceable limit state, making this theory relatively obscure.

In response to the first argument, one query might be raised. The wave load calculation methodology adopted by Mansour and Faulkner (1973) was a linear strip theory which might not be capable of capturing some highly nonlinear wave loads arising from an extreme sea state, such as slamming load and its induced whipping response. In more recent research, it is suggested that slamming and its induced whipping response might lead to several cycles with high magnitudes of bending moment. Slamming-induced whipping might lead to an increase of longitudinal bending moment acting on the ship hull girders, which can result in the collapse of ship hull girders when it exceeds the limit state. With reference to the most general form of the limit state function (Equation 20), in the case of slamming-induced whipping, the demand certainly would be increased by an indefinite amount; however the capacity of a ship hull girder might also suffer from a reduction with respect to its designated capacity estimated by a monotonic loading approach. The combination of increase in demand and decrease in capacity might be the actual reason of hull girder collapse.

$$Demand \leq Capacity \qquad (20)$$

Regarding to Faulkner's second argument, the present application of shakedown theorem on ship structures indeed has an obvious limitation. This is an important issue to be resolved, as it seems that that any structure is bound to collapse when it is loaded with a sufficiently high number of cycles. In the conventional ultimate limit state evaluation of ship hull girders, both demand and capacity are formulated in terms of bending moment. It may become difficult to evaluate the structural limit state through the interpretation of the predicted structural strength when loading direction is alternating, at least for the case study in present study. Thus, other characteristic measurements should be used.

In addition, Jones (1975) suggested that the ultimate limit state assessment should be replaced by the shakedown limit state for a safer design. However, it is more sensible to generalize the shakedown limit state as a part of ultimate limit state assessment, which has been well-recognised and a relevant measure for ship structural strength. More importantly, both limit state concerns with the structural failure involving elastoplastic buckling under a relatively large magnitude load, but different loading application (monotonic and cyclic). Therefore, the shakedown limit state can become an adjustment for the ultimate limit state where the response under cyclic load is assessed.

6. Energy-based characterization of the ultimate limit state

Recalling to Figure 1, the process of approaching ultimate limit state of ship hull girders is described using bending moment and bending stiffness, which is essentially a strength-based characterization. Alternatively, this process might also be viewed as the external work input into the structure and the limit state might be identified as the point where the input energy reaches a threshold value. Introduced by Housner (1956) for the application of earthquake engineering, the energy-based method is now a common approach for seismic design in which energy is used as a measurement of structural demand (Yang et al., 2018). Analogous to this idea where the building structure is designed to resist extreme ground motion, the energy-based method can be applied to ship structure where it is designed to resist extreme fluid motion. For a structural system subjected to external loading or prescribed displacement, which can be regarded as the input mechanical energy, its energy balance can be schematically shown in Figure 12, given by Yang et al (2018). The input energy are balanced by the energy being stored in the structural and the dissipated energy due to plasticity, which is considered to be irrecoverable and causes damage to the structure. Hence it is proposed in the present study that the plastic dissipation energy is used as the measurement for limit state characterization in a cyclic loading scenario.

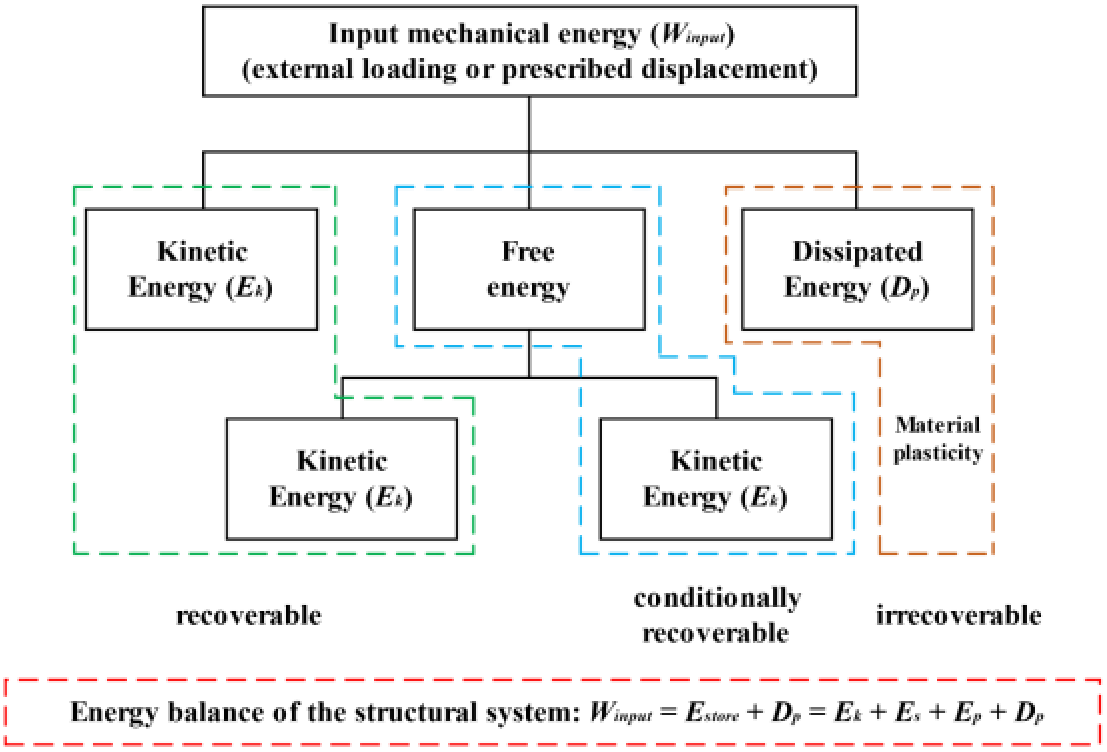


Figure 12. Energy balance of a structural system under external loading (Yang et al., 2018)

A tentative procedure to estimate the limit state under cyclic loading with the aid of energy-based characterization is summarised as following:

- Perform a progressive collapse analysis with monotonic loading for target object (ship hull girder, stiffened panel or unstiffened plate);
- Estimate the plastic dissipation energy gained at the ultimate limit point which is identified using the conventional criterion;
- For a specified cyclic loading case, the limit state is determined as the position where the dissipated plastic energy exceeds the threshold value estimated in progressive collapse analysis.

Using this simple procedure, monotonic ultimate limit state can be generalized to cyclic ultimate limit state and the problem is transformed to predict the plastic dissipation energy for the structure under a specified loading. The monotonic ultimate limit state is still regarded as a limit state, but it may be more appropriate to use the phrase *ultimate energy* rather than *ultimate strength.*

This concept is applied to the cyclic tests performed in section 4.3 and the results are illustrated in Figure 13. Bearing in mind that it is plotted in a logarithm scale, cases under loading protocol 1, loading protocol 2 and loading protocol 5 have much larger increasing rates compared to the remaining cases and exceed the threshold plastic dissipation energy obtained by the monotonic loading case within ten cycles. If assuming that plastic dissipation energy is increased at a constant rate, which could be estimated using the average increase in the ten-cycle test, the required number of loading cycle to exceed the threshold value for cases under loading 3, loading 4 and loading 6 can be predicted, summarized in Table 3. A fairly large number of cycles are required for the cases under loading 4 and 6 to attain limit state, while it is estimated that a limit state will be attained after 24 cycles under loading of case under loading 3. From this comparison, it is suggested that cases under loading 1, 2 and 5 might be problematic; however it is probably too conservative to judge that the remaining cases are unsafe. This might violate the observation using strength-based characterization, but it should be noted that no damage model is incorporated in these analyses.

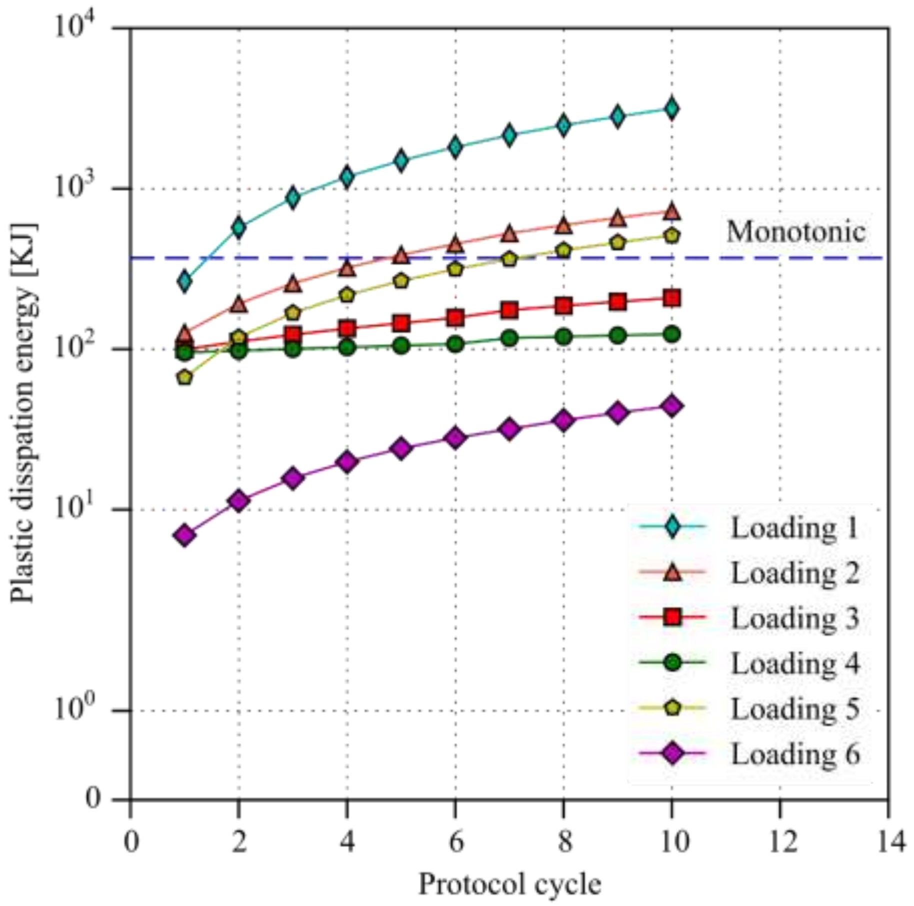


Figure 13. Plastic dissipation energy versus the number of loading cycles

7. Further recommendation

One of the most outstanding advantages of the state-of-the-art progressive collapse method is that the overall behaviour up to collapse can be predicted, but ironically it appears that this has not been given any consideration in a strength assessment. Most of the concerns are still related to the ultimate bending moment. In future development, it might be valuable to consider the overall behaviour and the accumulative loss of the resistance capacity. That is to say, the assessment could be carried out for a given loading scenario that is considered to be of high risk, which in the same time requires a parallel development of the hydrodynamic analysis to identify such severe scenarios. However, it will be extremely difficult to identify the exact cyclic loading protocol applied on the ship hull girder. An idealized and deterministic loading protocol can be used to start off the investigation. It is expected that some probabilistic approaches should be incorporated if an exact cyclic loading protocol is of concern.

In addition, the identification of limit state should be further rationalized. The energy-based concept proposed in this study purely attempts to tentatively provide an answer to Faulkner's comment. In the same time, there is perhaps a need to clarify several terminologies, such as *ultimate limit state*, *ultimate strength* and *collapse*. It seems that these terms are inter-changeable in the literatures. However the well-known *ultimate strength* is restricted to an idealized monotonic load case, whereas *ultimate limit state* and *collapse* should have a more general implication.

Moreover, a simplified calculation approach should continue to be developed, since the nonlinear finite element analysis for cyclic loading simulation is highly time-consuming. The methods outlined in section 2 could be the basis for future development. But one of the challenges is how to incorporate this with the energy-based characterization proposed in present study or equivalent, since all of these simplified methods are dedicated to predict the strength-based moment-curvature relations.

The welding-induced residual stress is neglected in the present study, the role of which is normally recognised in it could deteriorate the structural performance, such as decreasing the ultimate strength and stiffness of a stiffened panel under in-plane compression. However, the residual stress can also be considerably relaxed by cyclic loading even with small amplitude, as shown by Gannon et al. (2013). The interaction between these two factors and their combined influence to cyclic shakedown require a further investigation. One possible investigation can be numerically carried out for unstiffened steel plates with and without residual stresses subjected to a series of cyclic loading with various constant amplitudes. A comparison is made between these two cases on the smallest loading amplitude that results in the onset of plastic strain. A parametric study on this basis might be able to quantify the effect of residual stress on the elastic shakedown limit state.

It is also necessary to investigate the effect of loading sequence. In general, there are two types of cyclic loading protocols, namely the constant amplitude loading and the varied amplitude loading. The structures might rapidly adapt to the constant amplitude loading, in which shakedown takes place; however, a more realistic varied amplitude loading might prevent this adaptation and consequently the occurrence of shakedown.

Whilst it is important to investigate the influence of welding-induced residual stress and loading sequence, a proper selection of the material model is even more critical, as it has a direct impact on the investigation of the former two. An elastic perfectly-plastic material model was used for all the analyses presented in this paper. However, the influence of Bauschinger effect and cyclic hardening need to be clarified. The former can be modelled by a kinematic hardening rule and the latter can be described by an isotropic hardening rule. To account for both of these effects, the Chaboche hardening model can be utilized, which consists of an isotropic component and a kinematic component. In the formulation of this hardening model, eight more parameters in addition to the conventional yield stress and Young's modulus need to be calibrated from experiments, including the maximum change of the size of yield surface, the rate of the yield surface change, initial kinematic hardening modulus and the rate of the change of kinematic hardening modulus. The present authors are carrying out a numerical investigation using the finite element method. The Chaboche material model is incorporated in the FEA, with relevant material constants collected from literature that are originally dedicated for earthquake engineering applications. The primary aim of this on-going numerical investigation is to explore the difference that a combined hardening model will bring to the predicted structural responses, specifically the load-shortening curves, compared with a simple perfectly plastic model.

8. Conclusion

This paper re-evaluates the use of shakedown limit state in the assessment of longitudinal strength of ship hull girders, which was originally proposed by Jones (1975). A case study is performed in accordance with the procedure proposed by Jones. Two cases are considered, namely geometric linearity and geometric nonlinearity. Six cyclic finite element analyses are conducted and an energy-based limit state characterization is proposed. The following insights are concluded:

- The safety margin based on the state-of-the-art ultimate limit state approach might be considerably reduced if buckling is considered in the estimation of shakedown limit state;
- The prediction using Jones' original method is conservative as suggested from the cyclic finite element analyses;
- The future development of structural assessment of ship hull girders under longitudinal bending can be placed on assessing the overall behaviour and accumulative loss of resistance capacity during a given loading scenario;
- For the identification of limit states in a cyclic loading scenario, alternative characteristic measurement should be adopted. The present study proposes an energy-based characterization using plastic dissipation energy;
- A systematic investigation of the hull girder and structural components under large loading and unloading is needed;

- The simplified progressive collapse methodology other than finite element method should continue to be developed for predicting the cyclic response of a ship hull girder.

Reference

14th ISSC, 2000. Ultimate Strength. ISSC Committee III.1. Nagasaki, Japan.

15th ISSC, 2003. Ultimate Strength. ISSC Committee III.1. San Diego, United States.

Caldwell J.B., 1965. Ultimate longitudinal strength. Trans., RINA, 107, 411-430.

Benson, S., Downes, J., Dow, R.S., 2013. Compartment level progressive collapse analysis of lightweight ship structures. Marine Structures 31, 44–62.

Dow, R.S., 1991. Testing and analysis of a 1/3 scale frigate model. In: Advances in Marine Structures 2. Dunfermline, Scotland: Elsevier, 749–773.

Faulkner D., Betts C.V., 1976. Comments on ‘On the shakedown limit of a ship’s hull girder’. Journal of Ship Research, 20:61.

Frieze, P.A., Lin, Y.T., 1991. Ship longitudinal strength modelling for reliability analysis. In: Proceeding of Marine Structural Inspection, Maintenance and Monitoring Symposium, Ship Structure Committee and the Society of Naval Architects and Marine Engineers, Arlington, Virginia, March, III.B.1 - III.B.19.

Gannon, L.G., Pegg, N.G., Smith, M.J., Liu, Y., 2013. Effect of residual stress shakedown on stiffened plate strength and behaviour. Ships and Offshore Structures 8, 638–652.

Hodge, P.G., 1981. Plastic analysis of structures, Rev. ed. 1981 with new material. ed. R.E. Krieger Pub. Co, Malabar, Fla.

Housner, G.W., 1956. Limit design of structures to resist earthquake. In: Proceedings of 1st world conference on earthquake engineering, Berkeley, California. 5-1, 5-13.

Jones N., 1975. On the shakedown limit of a ship’s hull girder. Journal of Ship Research, 19(2):118–121.

Lewis, E.V., Zubaly, R.B., 1975. Dynamic loadings due to waves and ship motions. Ship structure symposium, Washington D.C., United State.

Mansour, A.E., Faulkner D., 1973. On applying the statistical approach to extreme sea loads and ship hull strength. Trans. RINA, 115.

Neal, B.G., Symonds, P.S., 1950-1951. A method for calculated the failure load for a framed structure subjected to fluctuating loads. Journal of institution of civil engineers, 35, 186-197.

Nishihara, S., 1983. Analysis of Ultimate Strength of Stiffened Rectangular Plate (4th Report). Journal of the Society of Naval Architects of Japan 1983, 367–375.

Paik, J.K., Thayamballi, A.K., 2003. Ultimate limit state design of steel plated structures. J. Wiley, Chichster, England ; Hoboken, NJ.

Paik, J.K., Hughes, O.F., Renaud, C., 2005. Ultimate limit state design technology for aluminium multi-hull ship structures. Trans., SNAME 113, 1–37.

Paik, J.K., Mansour, A.E., 1995. A simple formulation for predicting the ultimate strength of ships. Journal of Marine Science and Technology 1, 52–62.

Prager, W., 1948. Problems types in the theory of perfectly plastic materials. Journal of aeronautic sciences, 15, 337-341.

Smith, C. S. 1977 Influence of local compressive failure on ultimate longitudinal strength of a ship's hull, Proceedings, International Symposium on Practical Design of Ships and others Floating Structures (PRADS), October 18–20, Tokyo, Japan.

Ueda, Y., Rashed, S.M.H., 1974. An ultimate transverse strength analysis of ship structure. Journal of the Society of Naval Architects of Japan, Tokyo, 136, 309-324 (in Japanese).

Ueda, Y., Rashed, S.M.H., 1984. The idealized structural unit method and its application to deep girder structures. Computer and Structures, 18(2), 277-293.

Yao, T., Fujikubo, M., 2016. Buckling and ultimate strength of ship and ship-like floating structures. Elsevier ; Butterworth-Heinemann, Amsterdam ; Boston.

Yang, H., Sinha, S.K., Feng, Y., McCallen, D.B., Jeremić, B., 2018. Energy dissipation analysis of elastic–plastic materials. Computer Methods in Applied Mechanics and Engineering 331, 309–326.

Zhang, X., Paik, J.K., Jones, N., 2015. A new method for assessing the shakedown limit state associated with the breakage of a ship's hull girder. Ships and Offshore Structures, 1–13.

Table 1. Scantlings and material properties of the tested box girder.

σ_y	(MPa)	287.3
E	(MPa)	206920.3
a	(mm)	540
t_p	(mm)	3.05
h_w	(mm)	50
t_w	(mm)	3.05

Table 2. Section modulus and shape factor of the box girder.

Plastic section modulus $Z_p(mm^3)$	2778749.013
Elastic section modulus $Z(mm^3)$	2430302.004
Shape factor α	1.14

Table 3. The required number of cycles to attain limit state.

ID	Numbers of loading cycles
Loading 1	2
Loading 2	5
Loading 3	24
Loading 4	87
Loading 5	8
Loading 6	89

Figure 1. Flow chart of the collapse mechanism of ship hull girders under monotonic bending.

Figure 2. Stress distribution and tangent stiffness of each element of a box girder cross section under monotonic bending.

Figure 3. Characteristics of plastic strain accumulation under cyclic loading (Hodge, 1959).

Figure 4. Interactive diagram to determine the shakedown limit state (Jones, 1975).

Figure 5. Schematics of the case study box girder.

Figure 6. Interactive diagram of the box girder under sagging and hogging without buckling.

Figure 7. Moment-curvature relationship of the box girder under monotonic bending.

Figure 8. Stress distribution contour plot. Left: initial yield; Right: ultimate collapse.

Figure 9. Interactive diagram of the box girder under sagging and hogging with buckling.

Figure 10. Loading protocols and resultant responses. (a) Loading protocol 1; (b) Loading protocol 2; (c) Loading protocol 3; (d) Loading protocol 4; (e) Loading protocol 5; (f) Loading protocol 6; (g) The corresponding position of each loading protocol in the interactive diagram.

Figure 11. Bending moment-curvature relationship of cyclic bending tests. (a) Loading protocol 1; (b) Loading protocol 2; (c) Loading protocol 3; (d) Loading protocol 4; (e) Loading protocol 5; (f) Loading protocol 6.

Figure 12. Energy balance of a structural system under external loading (Yang et al., 2018).

Figure 13. Plastic dissipation energy versus the number of loading cycles