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Pantograph Arc Location Estimation using Resonant Frequencies in DC Railway Power Systems

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***Abstract*—Pantograph arcing in electrified railway systems not only reduces the power collection quality of a locomotive but can also damage pantograph strips and overhead lines (OHLs). Most research detects pantograph-to-OHL arcs based on on-board voltage/current measurements, pantograph cameras, etc. The use of on-board voltage/current data, though being cost-effective, rarely reflects arc locations along OHLs. This paper develops an arc positioning method which matches the position-dependent resonant frequency (RF) of an OHL with the RF extracted from voltage measurements in a pantograph arc event. A particular 20 km DC railway line supplied by two substations is first modelled in Matlab/Simulink, with the model effectiveness being assessed based on voltage measurements in an arc event. Then the OHL-related RFs estimated by the model are validated by Tableau formula, and discussed alongside impacts on RFs based on line models, locomotive locations and line lengths. These evaluations permit the generation of an RF curve that links OHL-related RFs with arc locations. The arc positioning method is tested based on the pantograph arc events presumed at various positions along the 20 km line, showing errors within 0.2 km at certain locations. The ability to determine arc locations will permit periodic inspections to be performed on determined line sections.**

***Index Terms*—DC traction power supply system, overhead line, pantograph arc location estimation, resonant frequency.**

I. Introduction

THE electrification of rail transport has played a major role in decarbonisation through increased energy efficiency and a reduced reliance on fossil fuels [1]-[3]. In a DC railway power system, the electrical power of upstream AC grids is rectified into DC power at traction substations (TSS) and then transferred to an electrical locomotive via an overhead line (OHL) suspended along the track or an electrical third rail mounted at the track level; the locomotive collects the power through its pantograph (or contact shoe) which contacts with the OHL (or third rail) [4], [5]. Therefore, the power collection of a locomotive is significantly affected by the quality of the sliding contact between the pantograph and the OHL.

The increasing speed of trains makes it difficult to maintain a regular pantograph-to-OHL contact due to the enlarged impact of dynamic components on contact force [6]. Contact force falling below a certain level will cause a contact loss, leading to an undesired pantograph arc, especially when a heavy current passes through the point of contact [7], [8]. The energy dissipated in the pantograph arc not only decreases the energy efficiency of the railway system but may also result in electrical erosion and fusion for both the pantograph and the OHL, accelerating any deterioration [9] and in turn causing more arcs. This will progressively degrade the quality of pantograph-to-OHL sliding contact and power collection, and may even lead to failure of the overhead contact system [7].

A minimum pantograph-to-OHL contact force is normally required in practice to avoid pantograph arc occurrences caused by OHL irregularity [10], [11] and wear [12]. The contact force is generally monitored at a frequency limited to 20 Hz, though the arc duration can reach 5 ms (i.e. equivalent to 200 Hz) [13] such that high-frequency contact losses cannot be completely reflected by measurements. The pantograph-OHL interaction performance at high frequencies has been investigated in [14] to capture high-frequency contact losses that associate with pantograph arcing so as to maintain the dynamic performance of sliding contact as required [15]. Some research has also been devoted to developing methods for arc detection which supply a basis for predictive maintenance, providing a cost-effective solution to maintenance schedules [16]. By means of a camera installed on the roof of a train, a recorded pantograph video can be analysed by signal and image processing methods to detect the pantograph arc occurrence [16]-[19]. For example, continuous frame-by-frame images decomposed from a pantograph video could be converted into binary images from which the ratio of white pixels to black pixels is evaluated as an arc parameter for the frame [17]. In addition, some arc detection techniques install sensors to monitor physical quantities emitted from pantograph arcing such as (i) significant temperature changes at the contact point recorded by thermal cameras combined with image processing algorithms [20]; (ii) ultraviolet emissions detected by phototubes [7], [21] or predefined light wavelengths for particular metal materials

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monitored by light detectors [22]; and (iii) the electromagnetic field radiation captured by an antenna [23], [24] and processed to determine the possible characteristic radiated frequencies [23]. However, these approaches suitable for the one-time diagnostic require relevant monitoring devices. In order to avoid investment in additional sensors, some algorithms have been developed to extract useful arcing information from measurements of pantograph voltage and current (V-I) which are already available on-board trains and can be implemented for daily monitoring by operating trains. Traction current data was decomposed in [25] by using wavelet multi-resolution analysis from which the arc occurrence was estimated. The low-frequency oscillation characteristics of pantograph current associated with arcing in DC railway systems were exploited in [26]. In [27] and [28], support vector machine learning models were trained based on the V-I data combined with the presence or absence of arc events detected by phototubes, and then applied to V-I measurements to isolate healthy and arc occurring conditions, correctly identifying about 80% of the events. Without the phototube signals based training, an unsupervised classification technique was applied in [29] to separate pantograph current measurements into different clusters which indicated the presence and the average magnitude of an arc event.

The V-I based methods proposed in [25]-[29] have shown the capability of arc detection. However, they are unable to determine the location of an arc event unless the time synchronisation of the arc event and the position of the locomotive are both known. This is important to determine the line section requiring inspections and thus prevent extended interruptions of transportation services due to the inspection of a complete line. To assist in locating the position of arc events, this paper proposes an enhancement of V-I based methods to capture the arc location from information embedded in the voltage waveforms based on the fact that the train movement changes the network topology and thus the resonant frequency (RF) related to the line. The arc positioning method proposes that the location is achieved by matching the OHL-related RF determined for different locomotive positions with the RF estimated from the voltage measurements during a measured pantograph arc event.

The paper first models in Matlab/Simulink a 3 kV DC traction power supply system supplied by two TSS. The measured distortion of the voltage waveform incurred by an arc event is then replicated in order to assess the model effectiveness. Then the RFs related to the OHL split at different locomotive positions are determined by the use of the Simulink model, and validated by the modified Tableau formula where the line is modelled by a finite number of cascaded π-sections with lumped parameters. The OHL-related RFs simulated by different numbers of π-sections and a distributed parameter line model are compared to illustrate the necessity of modelling the line with distributed parameters for more accurate response. Based on the distributed parameter line model, the RFs related to the line are estimated for varying locomotive locations. This evaluation creates an RF curve linking the OHL-related RF with the arc position. The RF curve is then tentatively employed to estimate the presumed arc event positions along a 20 km line. A flow chart describing the process of the proposed arc positioning method and its preliminary tests is shown in Fig. 1. The pantograph arc location estimation method proposed here, starting from the perspective of RFs in DC railway power systems, exploits the relationship of the OHL-related RF with the locomotive position along an OHL. It could provide a new idea for determining pantograph arc locations and line sections requiring inspections based only on the on-board V-I data and encourage the development of on-board monitoring devices that permit capturing the OHL-related RFs from V-I measurements.

This paper is structured as follows: Section II describes the modelling of the 3 kV DC railway traction power supply system by Matlab/Simulink and Tableau formula; Section III compares the OHL-related RFs estimated by different approaches and line models; Section IV generates the RF curve for the arc location estimation and preliminarily tests the arc positioning method; Section V discusses the potential application of the proposed method with respect to double-pantograph modes, requirements on monitoring devices and data processing as well as line impedance uncertainties. Conclusions and recommendations for further work are presented in Section VI.

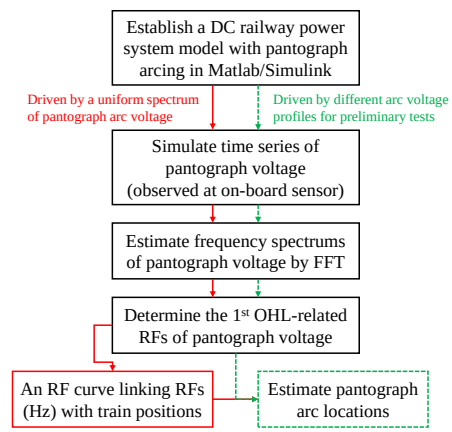

Fig. 1. Process of the RF based pantograph arc location estimation method and preliminary tests.

II. Modelling of DC Traction Power Supply System

A. DC Traction Power Supply System Model in Simulink

This work studies the 3 kV DC traction power supply system supporting a single-track railway line between Pisa and Collesalvetti in Italy (see Fig. 2) [30]. The railway line is fed by two TSS separated by 20 km. Each TSS consists of a rectifier to convert the AC grid voltage into 3.7 kV DC as well as an L-C second-order low-pass filter which is connected at the TSS output to smooth the voltage ripple. The traction system of the train ETR600 is configured to cascade two L-C low-pass filters at the input stage [30] which attenuate the ripple in the collected current and mitigate harmonic emissions and incoming disturbances. The pantograph voltage and current are monitored by a calibrated resistive-capacitive voltage divider, with the output signals being sent to a National Instrument PXI module through an analog/digital/analog optical link and a data acquisition module with a sampling rate of 50 kHz [30].

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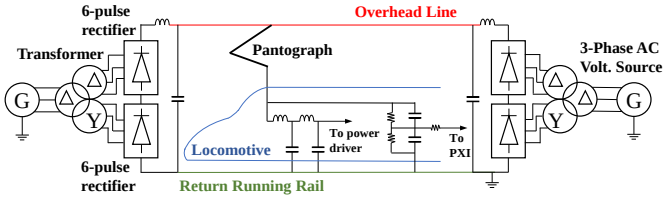

Fig. 2. The scheme of the locomotive in the DC traction power supply system.

Fig. 3 shows the simulation model of the Pisa-Collesalvetti DC railway power system developed in Matlab/Simulink [31]. The TSS is modelled by a combination of a three-phase AC voltage source and a transformer rectifier [32], followed by an L-C low-pass filter. Each segment of the 20 km OHL split by the locomotive is modelled by a distributed parameter line [33]. A controlled voltage source (CVS) is placed between the OHL and the train input to simulate the profile of the pantograph arc voltage V_{arc} which will be adjusted to replicate field measurements of the pantograph voltage V_p, while the arc resistance in the order of 10-100 mΩ [34] being small compared to the overall OHL impedance [35] is generally negligible in the system simulation [30], [35]. The data acquisition system (DAS) following the voltage divider is simulated by an L-C low-pass filter with an RF of about 3.3 kHz [30]. The train is modelled as a constant resistor Rt = 2.37 Ω. The values of circuital components are tabulated in Table I.

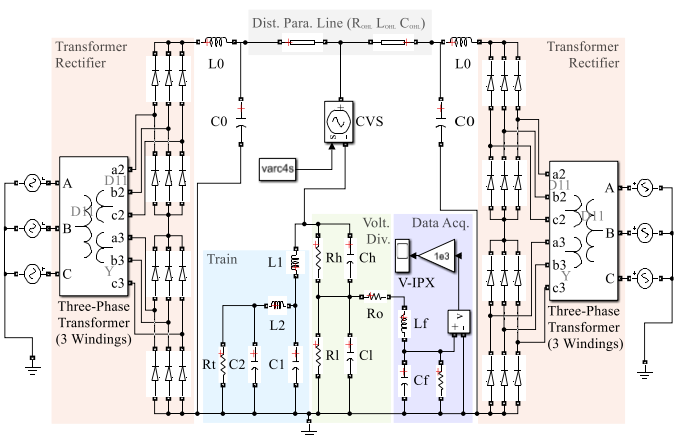

Fig. 3. Simulation model of the Pisa-Collesalvetti line with the locomotive.

TABLE I
VALUES OF KEY CIRCUITAL COMPONENTS [30].

L0	6 mH	L1	45 mH	Rh	6 MΩ	Ro	50 Ω
C0	350 μF	C1	5 mF	Rl	5.9 kΩ	R_{OHL}	0.17 Ω/km
Lf	20 mH	L2	507 μH	Ch	790 pF	L_{OHL}	0.57 mH/km
Cf	116 nF	C2	4 mF	Cl	787 nF	C_{OHL}	20 nF/km

The field measurements of V_p over a particular 4 s period [30] provided by Trenitalia are plotted in Fig. 4 where a significant disturbance was caused by a pantograph arc event which lasted about 4 ms from 2.1525s to 2.1565s. In order to replicate the V_p measurements, different V_{arc} shapes derived from the V_{arc} profile optimised in [30] (i.e. Fig. 16 in [30]) have been tested to improve the consistency between the simulated and measured V_p. Fig. 5 shows a fine-tuned profile of V_{arc} and the resulting V_p waveform when the locomotive is positioned at the middle of the line. The disturbance of V_p is shown to follow the V_{arc} shape, with oscillations after the ignition and extinction of pantograph arcing. These significant oscillations are found to be mainly associated with the L-C components at the TSS output which have RFs of around 110 Hz through an examination of the V_p waveforms after removing different components within the circuit. Although the simplification of the actual V_{arc} profile and the residual electrical elements in the network contribute to the difficulty of completely replicating the field measurements of V_p, the simulated V_p is reasonably matched with the measured arc event signal and illustrates the effectiveness of the railway power system model.

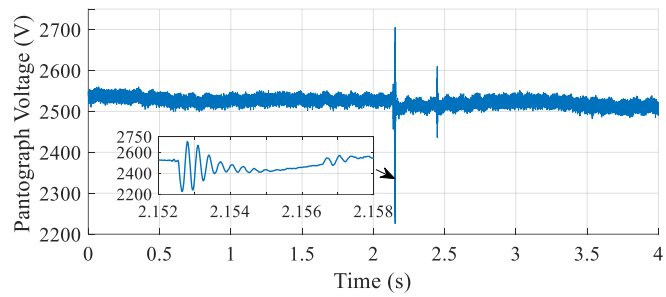

Fig. 4. Measurements of pantograph voltage during a particular 4 s interval.

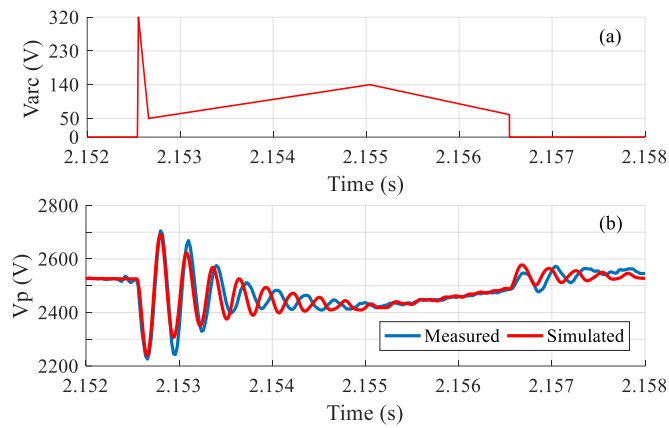

Fig. 5. (a) The fine-tuned profile of pantograph arc voltage V_{arc} and (b) the resulting simulation of pantograph voltage V_p against the measured V_p.

B. *Simplified Simulink Model for RF Estimation*

Given that the modelling focuses on the transient event where V_{arc} is the only applied voltage, the complete Simulink model in Fig. 3 is simplified by replacing each TSS with a resistor R0 = 0.1 Ω that represents the internal resistance of the TSS. Hence, the only voltage source left in the model is the CVS. It has been examined that the complete and simplified models produce almost the same distortion of V_p with a slight periodic difference of around 600 Hz (i.e. equivalent to 1.667 ms) as shown in Fig. 6(a). This is because the simplified model cannot reflect the harmonics caused by the 12-pulse rectifiers (i.e. integer multiples of the 12th harmonics of 50 Hz), though it generates the same OHL-related RFs as the complete model as shown in Fig. 6(b). The relatively higher amplitudes of resonances associated with the L-C filters at the TSS output in Fig. 6(b) can also explain the significant oscillations of V_p in Fig. 5(b).

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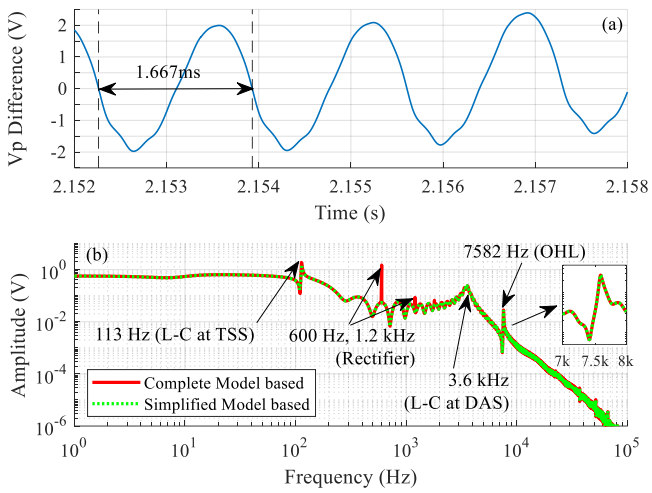

Fig. 6. (a) The differences (V) between pantograph voltage V_p waveforms simulated by the complete and simplified models in an arc event and (b) their respective frequency spectrums.

In order to obtain more general results and avoid the impact that the V-I monitoring device has on the RFs, the voltage divider and the DAS are additionally removed, with the voltage at the input stage of the train being monitored directly, as shown in Fig. 7. The values of circuital components in the simplified model in Fig. 7 are equal to those with the same symbols in the complete model in Fig. 3. A chirp signal [36] which generates a sine wave with frequency linearly increasing from 0 to 500 kHz in 2 s is used as the input signal of the CVS in the simplified model (see Fig. 7). Then the fast Fourier transform [37] is applied to the simulated V_p and the chirp signal respectively to form their spectrums. Since the frequency component (FC) of the chirp signal fluctuating at lower and higher frequency levels (see Fig. 8) may be misleading with regard to the RF estimation (especially at the lower end), the spectrum of V_p is calibrated through dividing its FCs by the corresponding FCs of the chirp signal. The calibrated V_p spectrum can be regarded as that obtained when the CVS input signal has a uniform frequency spectrum, and will be used to estimate the RFs related to the OHL, i.e. the locations of the peaks in the spectrum beyond 1 kHz. The process of determining the OHL-related RFs (RF_{OHL}) is formulated by (1).

$$RF_{OHL} = \text{findpeaks}\left(\frac{FFT(V_p)}{FFT(V_{ch})}; \geq 1\ kHz\right) \tag{1}$$

where $FFT(\cdot)$ is the fast Fourier transform that determines the spectrums of V_p and the chirp signal V_{ch}; and the operator $\text{findpeaks}(\cdot\,; \geq 1\ kHz)$ represents the operation that detects the peak locations in the calibrated spectrum beyond 1 kHz.

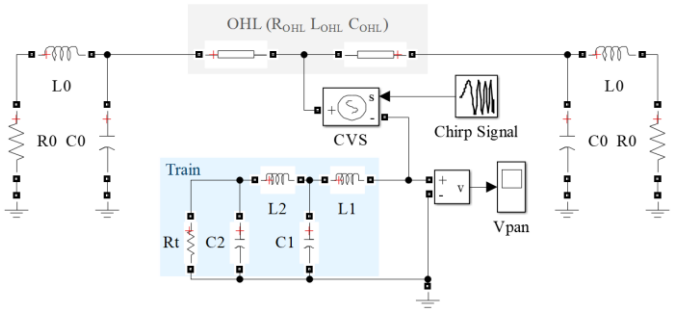

Fig. 7. Simplified model of the Pisa-Collesalvetti line with the locomotive.

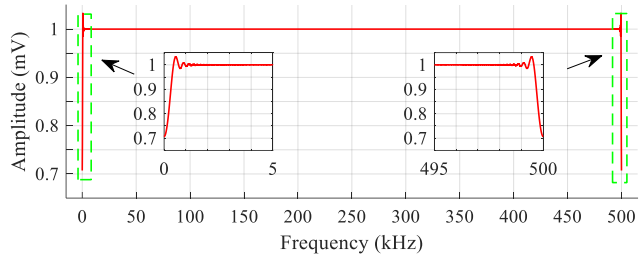

Fig. 8. The frequency spectrum of the chirp signal.

C. Distributed and Lumped Parameter Line Models

A distributed parameter line (DPL) model assumes that the resistance, inductance and capacitance are uniformly distributed along an OHL [38] where the shunt conductance is usually neglected due to a considerably small leakage current [39]. The OHL segment on the left or right side of the pantograph can be simulated by a single DPL in the Simulink models in Figs. 3 and 7. However, the DPL model having an infinite number of states, is not suitable for the state-space representation [40]. To validate the Simulink model based RFs by the Tableau formula [41] that requires a finite number of states, the OHL segment on each side of the pantograph can be approximated by a lumped parameter line (LPL) model that cascades a number of identical π-sections [42]. There are finite states in the LPL model, enabling the state-space representation at the cost of a decreased accuracy in the transient response simulation. The maximum frequency range f_{max} simulated by K π-sections can be approximated by [40]:

$$f_{max} = (K \times v)/(8 \times l_{OHL}) \tag{2}$$

where v and l_{OHL} denote the propagation speed (km/s) and the length (km) of the line. The approximation of the OHL can be improved by using more π-sections, which in turn increases the network complexity and computational burden. This results in a trade-off in the selection of the number of π-sections between the model complexity and the frequency range of interest.

Fig. 9(a) shows a simplified railway system model with each OHL segment being simulated by two cascaded π-sections. The capacitor at the end of a π-section is parallel connected with the capacitor of its neighbouring π-section or the capacitor (C0) in the low-pass filter at the output stage of the TSS. The parallel connected capacitors can then be regarded as a single equivalent capacitor having the sum of the individual capacitances, which will further simplify the representation of the OHL system, as shown in Fig. 9(b). It is noted that the capacitor of a π-section

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parallel connected with C0 is considerably smaller and negligible compared with C0. Therefore, only three equivalent capacitors remain in the OHL circuit in Fig. 9(b). When modelling each OHL segment by K π-sections, the number of equivalent capacitors in the OHL circuit will be $(2K-1)$. The number of π-sections used and the locomotive position that determines the length of each OHL segment will affect the network topology and the values of circuital components, and thus the RFs related to the OHL.

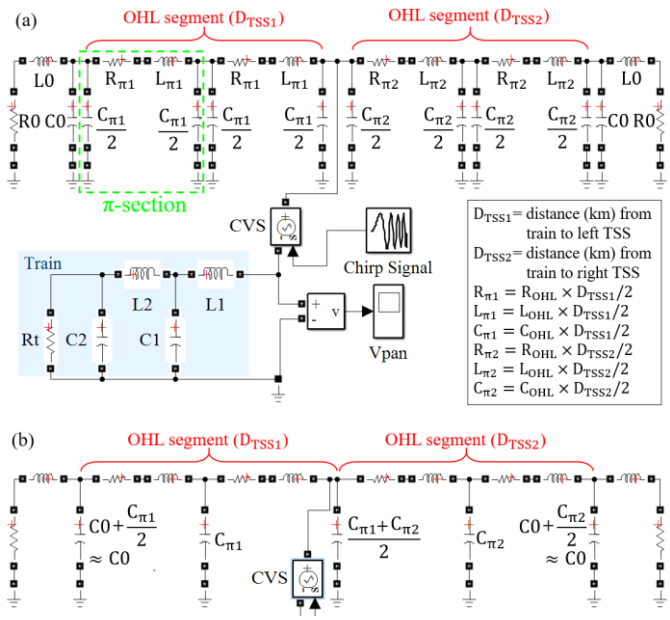

Fig. 9. (a) Simplified DC railway power system model with each OHL segment being modelled by two π-sections and (b) the OHL circuit simplified by merging the parallel connected capacitors.

D. Modified Tableau Formulation

The LPL-based simplified railway power system model that has finite states can be expressed by the Tableau formula which determines the topology of the network by Kirchhoff's current and voltage laws (KCL and KVL) and describes the physics of circuital components by constitutive equations [41]:

$$\begin{bmatrix} A & 0 & 0 \\ 0 & 1 & -A^T \\ Z & Y & 0 \end{bmatrix} \cdot \begin{bmatrix} I \\ U \\ V \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ W \end{bmatrix} \tag{3}$$

where Z and Y are matrices of impedance and admittance; I, U, and V denote vectors of branch currents, branch voltages, and nodal voltages; the nodal incidence matrix and its transpose are denoted by A and A^T respectively; and the term W is the vector of independent sources. The matrix of branch voltages U can be eliminated by substituting $U = A^T V$ into $ZI + YU = W$, giving a modified Tableau formula [43]:

$$\begin{bmatrix} YA^T & Z \\ 0 & A \end{bmatrix} \cdot \begin{bmatrix} V \\ I \end{bmatrix} = \begin{bmatrix} W \\ 0 \end{bmatrix} \tag{4}$$

The independent source W that represents the CVS, i.e. the only voltage source in the simplified model (see Fig. 9), is set to 1 V. Then the voltage at the input stage of the locomotive (i.e. the right node of L1 branch in Fig. 9(a)) is estimated based on (4) where Z and Y are computed for a particular frequency varying from 0 to 500 kHz. This forms the spectrum of V_p from which the OHL-related RFs are determined. Eq. (5) presents an example of the modified Tableau formula for the simplified network of Fig. 9(b). For simplicity each OHL segment is represented by one π-section, $\mathrm{R0} = \mathrm{R}_{\pi1} = \mathrm{R}_{\pi2} = 0$ as well as $\mathrm{L1} = \mathrm{L2} = \mathrm{C1} = \mathrm{C2} = 0$. This reduces the 39x39 matrix of the simplified network of Fig. 9(b) to a 13x13 matrix which is shown by its sub-matrices below.

$$YA^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -s\mathrm{C0} & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -s\left(\frac{\mathrm{C}_{\pi1}+\mathrm{C}_{\pi2}}{2}\right) \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -s\mathrm{C0} & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad W = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} s\mathrm{L0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & s\mathrm{L}_{\pi1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathrm{Rt} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & s\mathrm{L}_{\pi2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s\mathrm{L0} \end{bmatrix} \tag{5}$$

Solving this set of equations for V and I provides complex nodal voltages and branch currents dependent on the complex frequency s. Unlike the Simulink model-based approach that extracts RFs from the waveform of V_p simulated in the time domain, the Tableau formula focuses on the frequency-domain calculation directly, and is expected to provide a more efficient solution to the RF estimation. However, the Tableau formula can be applied to the LPL model only, which must have sufficient π-sections to represent the frequency range required.

III. Assessment of Modelling Approaches

In this section, the RFs of the simplified DC traction power supply system will be estimated by the Simulink model and the modified Tableau formula based on a DPL model and/or LPL models with different numbers of π-sections respectively. The consistency of the Simulink based and the Tableau based approaches and that of different line models are investigated.

A. Consistency of Simulink Model and Tableau Formula

Since the Tableau formula can be applied to a network with a finite number of states only, the consistency of the Simulink based and the Tableau based approaches is assessed here by use of the LPL based simplified railway system model (see Fig. 9).

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When the locomotive splits the 20 km OHL into two segments with lengths of 9 km and 11 km, spectrums of V_p estimated by the two approaches using K π-sections are compared in Fig. 10. As expected, the two approaches are in high agreement. The relative differences between the RF levels (Hz) estimated by the two approaches are found to increase with the RF level, from smaller than 0.02% for the 1st OHL-related RFs of around 7.5 kHz to about 0.5% for the RFs of around 40 kHz. Since the 1st OHL-related RF is to be adopted for the arc positioning, the two approaches on the basis of LPL models are considered to be consistent with a difference smaller than 1.5 Hz for the 1st RF related to the 20 km OHL. In all scenarios, the amplitude increases to a relative maximum at 113 Hz which is close to the RF of the L-C filter at the output stage of each TSS. The number of the RFs exceeding 7 kHz which are related to 9 km and 11 km line segments, equals $(2K-1)$, which is the same as the number of the equivalent capacitors in the OHL circuit as was noted in Section II.C.

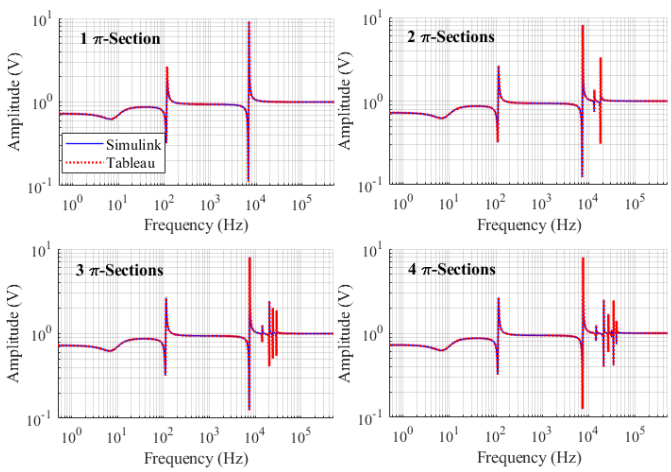

Fig. 10. Spectrums of V_p simulated by the Simulink model and Tableau formula using 1-4 π-sections to represent 9 km and 11 km OHL segments respectively.

B. *Consistency of DPL and LPL Models*

Though the modified Tableau formula directly focusing on the frequency-domain calculation uses less computation time for the RF estimation compared to the Simulink approach, it relies on LPL models and would need a sufficient number of π-sections to improve the accuracy of LPL models in transient response simulation. To determine the value of K that would be required in LPL models to produce the same RFs as DPL models, the V_p frequency spectrums estimated based on DPL and LPL models with different K π-sections are compared. Fig. 11 shows the V_p spectrums around the 1st OHL-related RFs simulated by the Simulink model based on DPL and LPL models with 5, 10, and 30 π-sections for three particular locomotive positions respectively. The consistency of the DPL and LPL models in the simulation of the 1st OHL-related RFs is largely improved by increasing K from 5 to 10. However, a further increase to K=30 gives a relatively slight improvement in their consistency at the cost of the network complexity and computation burden. It is found that there always exists a deviation between the 1st RFs simulated by the DPL and LPL models even if more π-sections are employed to represent each OHL segment due to the lumped parameter approximation [38]. Therefore, the investigation into the characteristics of the OHL-related RFs must be implemented by using DPL models in conjunction with the Simulink model so as to improve the accuracy of the 1st OHL-related RF simulation.

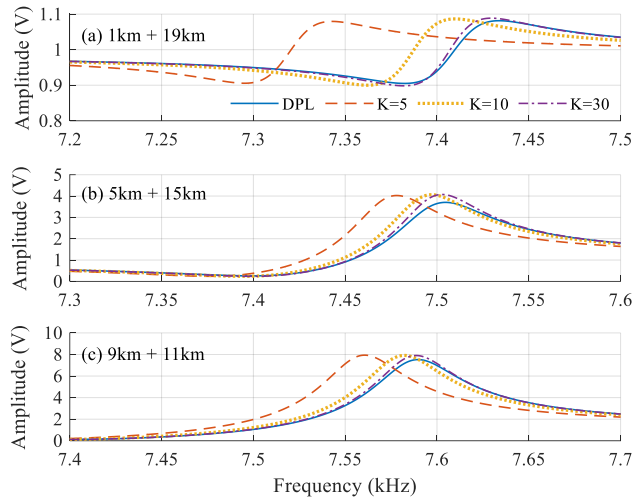

Fig. 11. Spectrums of V_p around the 1st RF related to the 20 km line simulated by DPL and LPL models with different numbers (K) of π-sections for three particular locomotive positions.

It is noted that in Fig. 11 there is a variation in the error of the RF as computed using 5 π-sections compared to the DPL. In Fig. 11(a) the deviation of the RF of K=5 to the RF of the DPL is around 90 Hz. In Figs. 11(b) and 11(c), the same deviation is about 25 Hz. This variation is due to the fact that the same number of π-sections is applied to the 19 km segment of the line (Fig. 11(a)) as well as to the 11 km segment of the line (Fig. 11(c)). This leads to a different effective resolution of π-sections per kilometre of the track and leads to the error being dependent on the locomotive position. Appropriate modelling of the π-sections (e.g. a consistent number of π-sections per kilometre of the track) can eliminate this effect.

IV. RF Curve for Arc Location Estimation

Using the Simulink based approach integrated with the DPL model, the influences of locomotive positions, total line lengths, and the V-I monitoring architecture on the RFs are investigated in this section. An RF curve linking the OHL-related RF with the arc position is then created and tentatively used to estimate the presumed arc location along the 20 km line.

A. *Impacts of Locomotive Position and Line Length on RFs*

The train moving along a line between two TSS changes the location at which the line is split and thus the network topology and associated RFs. The influences of locomotive positions on resonances and anti-resonances of the circuit impedance were explored in [35], [44] and [45] where the equivalent pantograph impedance Z_p made of the left and right OHL segments was formulated by:

$$Z_p = Z_c \cdot \frac{\tanh(\gamma \cdot D_{TSS1}) \cdot \tanh(\gamma \cdot D_{TSS2})}{\tanh(\gamma \cdot D_{TSS1}) + \tanh(\gamma \cdot D_{TSS2})} \tag{6}$$

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where D_{TSS1} and D_{TSS2} are distances (km) from the locomotive to the two TSS and terms Z_c and γ denote the characteristic impedance (Ω) and propagation constant (km^{-1}) of the line respectively. The RFs (Hz) of Z_p have been found to be approximately independent of the locomotive position while the anti-resonant frequencies will vary with the locomotive position [44], [45]. Since this paper deals with the signals of V_p across the train input which are inversely associated with Z_p, the OHL-related resonance in the V_p frequency spectrum will correspond to the anti-resonance of Z_p and thus vary with the train movement. The 1st resonance related to the 20 km line in the V_p spectrum changing with the locomotive location is shown in Fig. 12(a) where its amplitude and location both increase with the TSS-locomotive distance. Furthermore, the OHL-related anti-resonance in the V_p spectrum is approximately fixed for different locomotive locations. Similar findings can be also obtained when the L-C filter at each TSS output is removed, as shown in Fig. 12(b), meaning that the existence of L-C filters at TSS alongside the OHL circuit would not affect the variation of the OHL-related resonance with the TSS-locomotive distance.

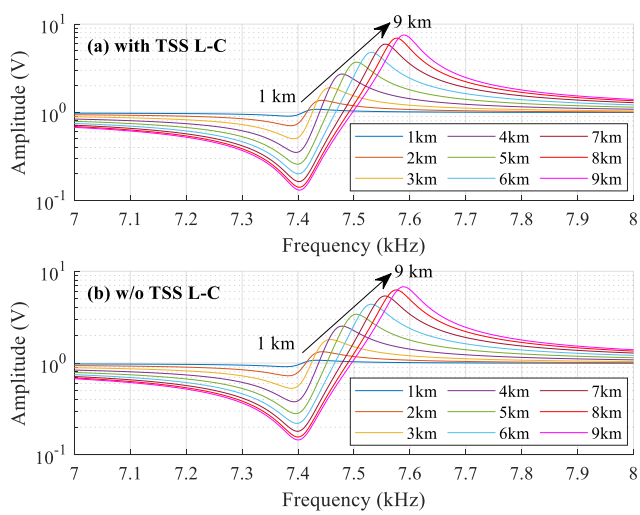

Fig. 12. Frequency spectrums of V_p around the 1st OHL-related resonance simulated for different TSS-locomotive distances (km) along a 20 km line when L-C filters at TSS are (a) kept or (b) removed.

Fig. 13 plots levels (kHz) and amplitudes (V) of OHL-related RFs estimated by the DPL based Simulink model for different locomotive positions along the first half of the 20 km line. Though the OHL simulated by a DPL model has an infinite number of RFs, its 1st – 4th RFs are presented here. When the locomotive is near a particular TSS, the amplitudes of OHL-related RFs are dampened by the influence of the L-C filter at the TSS output [46]. When heading to the middle of the line, only the 1st RF monotonically increases to its maximum in both level and amplitude while the other RFs present periodic changes. These periodic changes arise from the symmetry of the segmented line. When the locomotive is exactly in the middle of the line it forms two identical segments which are in parallel thus eliminating half of the RFs, specifically all the even RFs. This can be seen in Fig. 13 where the 2nd and 4th RFs disappear at 10 km. Similarly, the 3rd RF cannot be detected at 6.67 km or 1/3 of the total line length while the 4th RF disappears at all multiples of 1/4 of the total line length, i.e. at 5 km, 10 km and 15 km (the last is not shown). Fig. 14 shows the V_p spectrum around the 4th RF for the locomotive positions between 3.5 km and 6.5 km where the resonance disappears at 5 km. Compared with higher-order RFs, the 1st RF that can be always deduced from spectrums and monotonically varies along each half of the line, is expected to be a suitable indicator for the locomotive location.

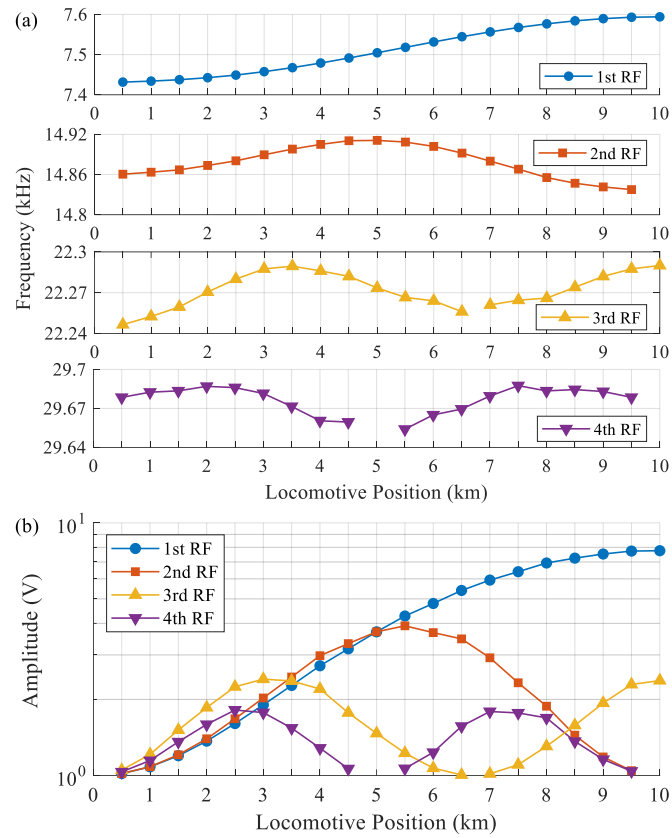

Fig. 13. (a) Levels (kHz) and (b) amplitudes (V) of the 1st – 4th RFs of a 20 km OHL split at different locomotive positions (km away from a particular TSS).

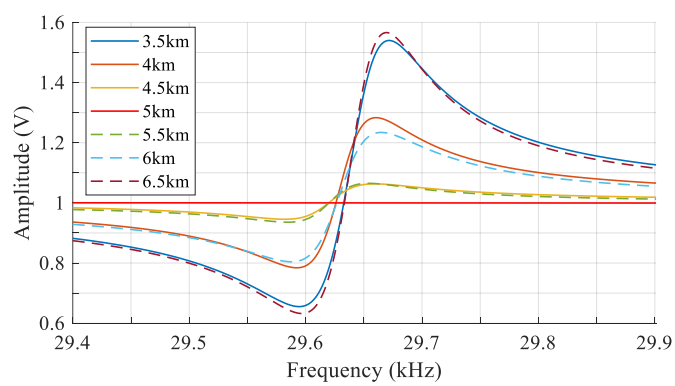

Fig. 14. V_p spectrums around the 4th RFs related to a 20 km OHL split by the locomotive that is 3.5 km – 6.5 km away from a particular TSS.

In addition to the locomotive location, the total length of the OHL affects its related RFs. Fig. 15 plots the 1st RFs related to 30 km and 40 km OHLs split by the locomotive at different positions along the first half of the OHL. With the line length

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increasing from 20 km to 30 km or 40 km, the range of the 1st RFs is shown to decrease from 7.4-7.6 kHz (Fig. 13(a)) to 4.95-5.15 kHz (Fig. 15(a)) or 3.7-3.9 kHz (Fig. 15(b)) respectively. Therefore, the bandwidth of the V-I monitoring device required to capture the 1st RFs from V_p measurements needs to increase for a reduced line length. This is because the amplitude of the 1st RF will be largely dampened if the 1st RF exceeds the bandwidth.

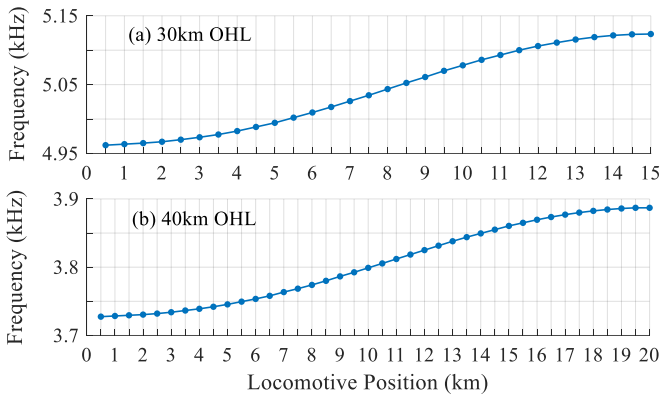

Fig. 15. Levels (kHz) of the 1st RFs related 30 km and 40 km lines split at different locomotive positions (km away from a particular TSS).

It is noted that the monotonic increase of the 1st RF (Hz) with the TSS-locomotive distance along each half of a line is only available in the scenario where the line is supplied by a TSS at each terminal. When the line is supported by a single TSS at either terminal, the 1st RF will monotonically decrease with the TSS-locomotive distance [47].

B. Impacts of V-I Monitoring Architecture on RFs

It should be noted that the aforementioned analysis of OHL-related RFs is conducted based on the V_p simulated at the input stage of the locomotive (see Fig. 7). In practice, an on-board V-I monitoring device must be used to measure V_p, such that the monitoring architecture inclusive of a voltage divider and a DAS (see Fig. 3) may affect the estimated RF. In order to assess the impact of the monitoring architecture on RFs, a chirp signal is injected at the pantograph (i.e. the CVS) in the complete simulation model in Fig. 3. The V_p spectrum including RFs is estimated from the waveform of V_p as observed at the output of the V-I monitoring device. As was noted in Section IV.A, the DAS needs a bandwidth of at least the frequency of the 1st RF, here 7.6 kHz for the 20 km OHL. The existing DAS represented by the 3.3 kHz L-C filter in Fig. 3 would reduce the amplitude of the 1st RF, making it difficult to be detected in practice. An alternative voltage transducer [48] applicable for 3 kV DC measurements can provide up to 13 kHz bandwidth. In order to change the model of Fig. 3 to represent this new transducer, each component in the L-C filter representing the DAS is adjusted to 25% of the original value in Table I. The resulting RF of the filter is about 13.2 kHz. Fig. 16 compares the 1st RFs related to the 20 km OHL determined from the V_p simulated at the monitoring device and those at the input stage of the locomotive without connecting the monitoring device. It shows that in the main the monitoring device does not affect the availability and monotonic characteristics of the estimated 1st RFs along each half of the line, but slightly reduces their levels by around 10 Hz when the locomotive is close to the middle of the OHL. This is because the existence of the monitoring device increases the circuit impedance and thus reduces the levels of OHL-related RFs. When approaching a particular TSS, the differences between the estimated 1st RFs are gradually eliminated since the impedance increase becomes smaller relative to the circuit impedance without a monitoring device.

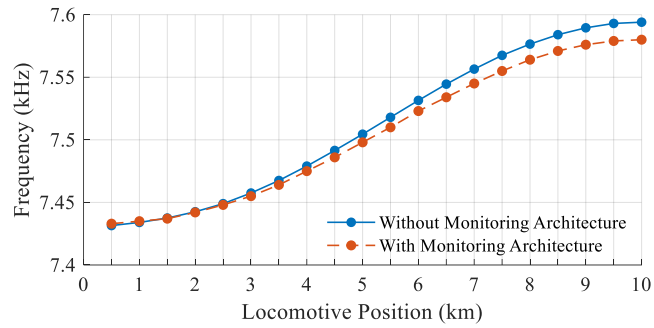

Fig. 16. The 1st RFs (kHz) related to the 20 km line estimated for different locomotive positions (km away from a particular TSS) based on the V_p simulated at the monitoring device and those at the input stage of the train without connecting the sensor.

C. Use of RF Curve for Arc Location Estimation

The RF curve (see Fig. 16) that links the 1st OHL-related RF with the locomotive position taking into account the monitoring device is tentatively used here to estimate the location of a detected pantograph arc event along the first half of the 20 km line. Three arc events with the voltage profile fine-tuned as shown in Fig. 5(a) are presumed to occur at three particular positions, i.e. 1 km, 5 km, and 9 km away from the left TSS. The profile of the arc voltage V_{arc} is taken into the CVS in the complete railway power system model (see Fig. 3) to simulate the waveform of the resulting V_p at the monitoring device for each pantograph arc location respectively, as shown in Fig. 17. The deviations among three V_p waveforms are caused by the varying arc position which changes the network topology and RFs related to the OHL.

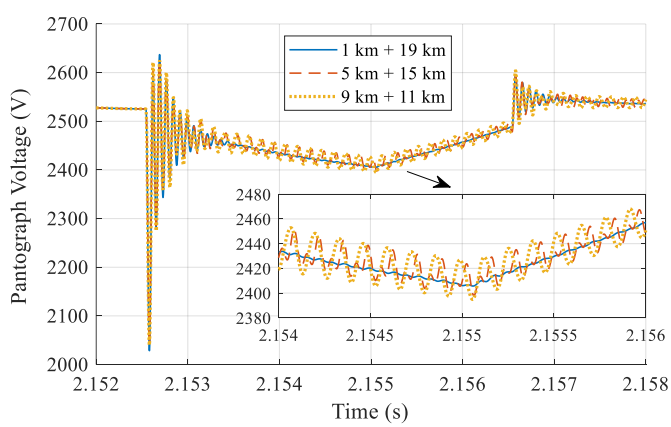

Fig. 17. Time series of V_p simulated at the V-I monitoring device based on the fine-tuned voltage profile of a pantograph arc event occurring at three particular positions along the first half of the 20 km OHL.

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In order to detect the 1st OHL-related RF that implies the arc location, the three V_p waveforms simulated in the time domain are converted into the frequency domain by the FFT, as shown in Figs. 18(a)-(c). It is noted that, unlike the previous V_p spectrums that are calibrated based on FCs of a chirp signal, these V_p spectrums reflect the FCs of V_{arc} (see Fig. 18(d)) since the exact V_{arc} profile is not available to calibrate the V_p spectrum in practice. Each V_p spectrum shows resonances at around 113 Hz and 13.3 kHz which are associated with the filter placed at the TSS output and the filter representing the DAS, respectively. In addition, the use of 12-pulse rectifiers (see Fig. 3) results in harmonics at integer multiples of the 12th harmonic of the nominal 50 Hz, i.e. 600 Hz [49].

Since the amplitude of the 1st RF related to the OHL split into 1 km and 19 km segments is quite small (see Fig. 13(b)), it is difficult to determine the resonance related to the OHL in Fig. 18(a) where the V_p spectrum around the 1st OHL-related RF mainly depends on the FC of the arc voltage (FC_{arc}). With the arc position being closer to the middle of the line, the 1st OHL-related RF has a higher amplitude such that the OHL-related resonance can be detected, i.e. around 7497 Hz and 7578 Hz for the arc events occurring at 5 km and 9 km from the left TSS as shown in Figs. 18(b) and 18(c) respectively. According to the OHL-related RF curve (see Fig. 16) which is formed on the basis of the calibrated V_p spectrums, the detected resonances indicate that the arc events may occur at around 4.95 km and 9.33 km from the left TSS, with errors of 0.05 km and 0.33 km respectively (i.e. differences between the presumed and estimated locations).

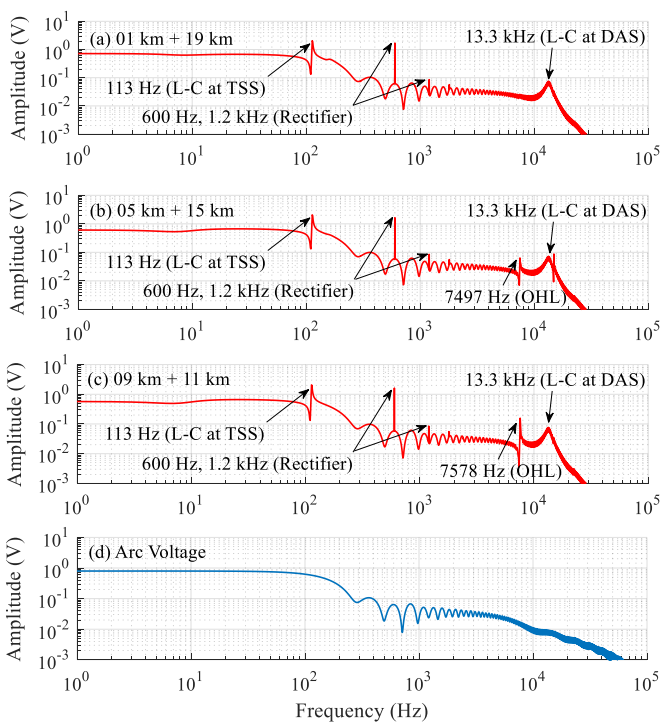

Fig. 18. Spectrums of V_p simulated at the monitoring device for the presumed arc events occurring at (a) 1 km, (b) 5 km, and (c) 9 km away from the left TSS and (d) the spectrum of the fine-tuned arc voltage profile.

The use of the RF curve for arc positioning is also examined for other pantograph arc events that are presumed to occur along the first half of the line at different positions greater than 2.5 km from the left TSS (below which the OHL-related resonances are difficult to be detected in the V_p spectrums). In addition to the fine-tuned V_{arc} profile in Fig. 5(a) where the duration of the first pulse is 120 μs, different V_{arc} profiles with a longer first pulse of 500 μs or 2 ms or a single pulse of 4 ms or 8 ms, as shown in Fig. 19, are used here to test the arc positioning method. The differences between the presumed positions and their respective estimates are plotted in Fig. 20 where the V_{arc} profiles tested here with varying amplitudes and pulse durations are shown to have similar arc location estimation errors. Relatively smaller errors (e.g. within ±0.2 km) are observed for the presumed arc locations between 4 km and 8 km. This is because when the locomotive locates at around 4-8 km, the deviation (Hz) of the FC_{arc} affected RF from the calibration based RF leads to a relatively smaller change in the locomotive position, as shown in the RF curve which has a higher gradient for the TSS-locomotive distances of about 4-8 km in Fig. 16.

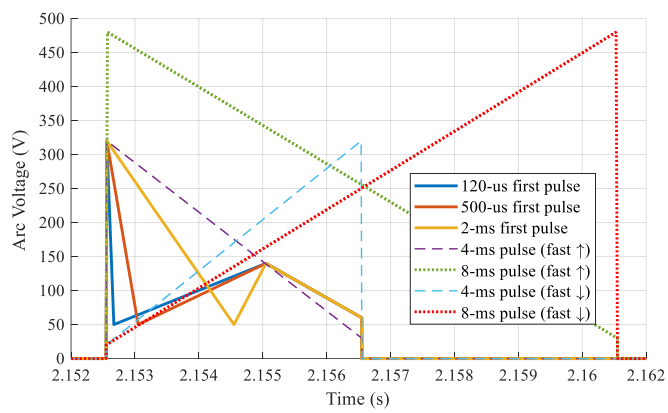

Fig. 19. Presumed pantograph arc voltage profiles with varying pulse durations and amplitudes for the test of the arc positioning method.

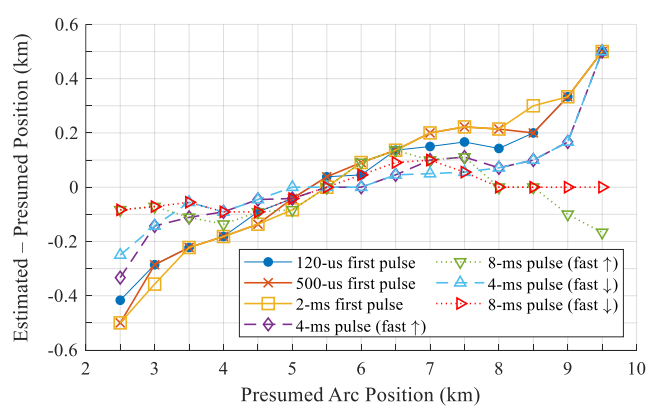

Fig. 20. Deviations (km) between the estimated and presumed arc locations that are 2.5 km – 9.5 km away from the left TSS.

V. Discussions

Considering that the wires making up OHLs in railway traction systems are generally around 1.5 km long [50], the arc position estimate with an error of up to 0.4 km in most cases is expected to correctly indicate the wire where the arc has

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occurred in this work. If repeated arcs are detected approximately at the same position, the suspected OHL section will need an inspection. However, it should be pointed out that further work needs to deal with the limitations that the arc positioning method has in application: (i) it could be difficult to detect the RF when an arc event is very close to a TSS due to the dampening effect of the filter at the TSS output; (ii) the method infers arc positions along either half of a line that is supplied by two TSS at both terminals, without indicating which half may be in question; and (iii) an arc position cannot be determined if the FC_{arc} affected RF is greater than the maximum level of the RF curve (e.g. the RF of 7582 Hz estimated for an arc at the middle of the 20 km line exceeding the maximum of 7580 Hz in the RF curve disables the arc positioning method).

The pantograph arc positioning method developed here can be extended to double-pantograph current collection modes that are normally adopted by multiple-unit trains [51]. To generate an RF curve for the double-pantograph operation, the complete simulation model in Fig. 3 needs to be modified by placing an additional set of train components in parallel with the existing set and adding an additional OHL segment between the two train sets. Given the distance between pantographs (or the additional OHL segment length) being 150 m or 200 m, the 1st OHL-related RFs observed at the left pantograph are estimated for different locomotive positions (i.e. distances from left TSS to left pantograph), as shown in Fig. 21. (The OHL-related RFs at the right pantograph can be inferred from Fig. 21). The 1st RFs varying between 7.44 kHz and 7.74 kHz in the double-pantograph mode have a wider range than those when a single pantograph is simulated. In addition, a 50-m increase of the distance between pantographs results in a slight growth of the 1st OHL-related RFs. When pantograph arcing is determined at a particular pantograph, the RF curve produced for double-pantograph modes in Fig. 21 can be compared with the RFs captured from on-board measurements to estimate the pantograph arc location along the OHL.

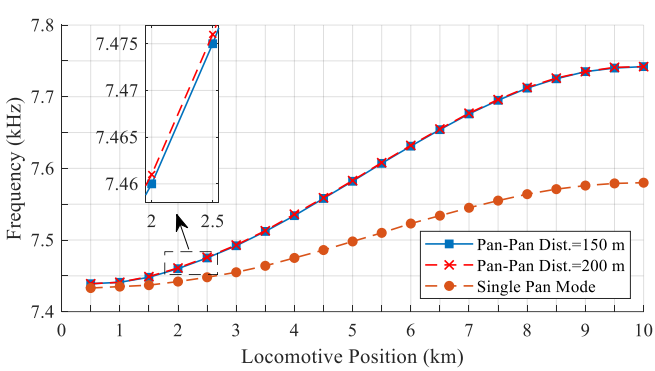

Fig. 21. The 1st RFs related to the 20 km line observed at the left pantograph in double-pantograph mode for different locomotive positions (i.e. distances from left pantograph to left TSS) when two pantographs are 150 m or 200 m apart.

In order to capture the 1st OHL-related RFs from V_p signals, the bandwidth of on-board V-I monitoring devices should be several times greater than the maximum level of the 1st RFs so as to avoid the amplitude of the 1st RF being largely dampened if the 1st RF exceeds the bandwidth. This requires determining the range (kHz) of the 1st RFs in advance based on the line length and available values of circuital components including line impedance parameters. In addition, the sampling frequency of V-I monitoring devices must be greater than twice the maximum level of the 1st OHL-related RF so that the frequency range of the FFT spectrum can cover the 1st RFs according to Nyquist criterion [52]. For example, the 20 km line modelled here would require a sampling frequency of at least 16 kHz. The frequency resolution of the FFT spectrum is determined by the ratio of the sampling frequency to the number of V_p measurement points [52] or the reciprocal of the length of V_p measurements in time domain. Though a higher resolution can be achieved by applying the FFT to a longer measurement window size, the TSS-locomotive distance would vary so much in a long time period that the accuracy of the 1st RF associated with the arc location may be degraded. Considering a trade-off between the frequency resolution and the train travel distance, a 1-Hz resolution (i.e. applying FFT to a 1-s window of V_p measurements) is expected to be a reasonable option due to its performance assessed in Fig. 20 and that a 250 km/h train ETR600 [30] travelling less than 70 m over 1 s is found to incur the 1st RF deviation within 1 Hz.

The RF curves embedded within the arc positioning method have been generated here based on known OHL impedance parameters which, however, may be not with good precisions in practice. A sensitivity analysis is conducted here to investigate impacts of the line impedance parameter uncertainty on RFs. Fig. 22 compares the 1st OHL-related RFs evaluated for the TSS-locomotive distance of 1 km, 5 km or 10 km based on different combinations of line inductance and capacitance parameters which are assumed to vary within 10% of their respective values in Table I. A 2% underestimation in the inductance or capacitance is shown to overestimate the 1st RFs by about 60 Hz – 70 Hz in this case, which would affect the arc positioning method performance. Therefore, it is necessary to validate the impedance parameters of OHL sections based on filed measurement campaigns [53]-[55] in order to improve the accuracy of the generated RF curve. In addition, the Simulink DPL blocks [33] used here do not accurately represent the frequency dependency of line impedance parameters (e.g. skin effects in conductors). The frequency-dependent DPL blocks [56] could be suitable substitutes to perform more accurate simulations for a wide frequency range (typically from DC to 100 kHz or more). Furthermore, considering that the validated and frequency-dependent line impedance parameters may be still subject to uncertainties, their possible errors can be translated into the uncertainty of the RF curve by some statistical models (e.g. the Monte Carlo simulation [57]), permitting a probabilistic estimation of the pantograph arc position. This would require embedding multiple RF curves evaluated based on various combinations of line impedance parameters within the arc positioning method.

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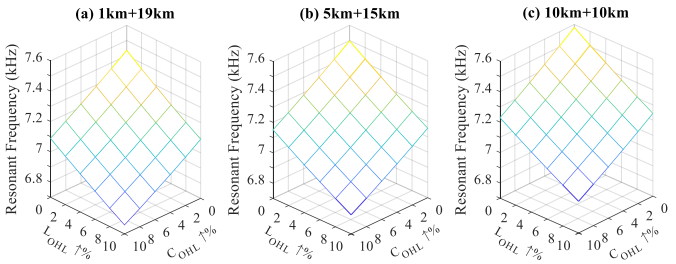

Fig. 22. The 1st OHL-related RFs simulated for TSS-locomotive distances of (a) 1 km, (b) 5 km and (c) 10 km given the line inductance L_{OHL} and/or capacitance C_{OHL} increasing by 0% - 10%.

VI. Conclusions and Future Work

This paper has proposed a V-I measurement based method to determine the pantograph arc location along an overhead line (OHL) supported by two traction substations (TSS) in a DC railway traction power supply system. The proposed method relies on a resonant frequency (RF) curve that links the OHL-related RF with the pantograph arc location at which the line is split. The line simulated by a distributed parameter model has an infinite number of RFs, from which the 1st RF is selected to produce the RF curve. This is because the 1st RF monotonically increases with the TSS-locomotive distance along each half of the line while the higher-order RFs present periodic changes. The monotonic characteristics of the 1st OHL-related RFs have been validated for 20 km, 30 km, and 40 km OHLs, with their levels (Hz) decreasing with the total length of the line.

The proposed arc positioning method has been tested on the waveforms of pantograph voltage V_p simulated at the on-board monitoring device based on the fine-tuned arc voltage profiles that are presumed to occur at different positions along the first half of a 20 km line. Most of the presumed arc positions have been estimated with errors smaller than 0.4 km; for pantograph arc events occurring at around 4-8 km from the TSS where the RF curve has a higher gradient, the position estimation errors should be within 0.2 km. When an arc event is very close to a TSS (e.g. a TSS-locomotive distance smaller than 2.5 km in this study), the amplitudes of OHL-related RFs are largely dampened by the filter at the TSS output, making it difficult to detect the OHL-related resonance from the V_p spectrum.

Building on the present work, the proposed arc positioning method should be developed further to determine the half of the line in question and deal with the exceedance of an RF estimate over the range of an RF curve. Furthermore, the method will be enhanced to simulate the frequency-dependent impedance parameters and model the propagation of OHL impedance uncertainties to the uncertainty of the RF curve. In addition, the arc positioning method will require to be evaluated on more arc voltage profiles produced by pantograph arcing models [58], [59] that could better describe transient dynamics of arc voltage as well as on field measurements of V_p available on-board trains. This requires the monitoring device having a bandwidth several times greater than the maximum level of the 1st RFs related to the OHL of interest so as not to attenuate amplitudes of the 1st RFs. The field data in the measured pantograph arc events will be explored to validate the findings in this paper regarding the change of the 1st RF with pantograph arc location along the OHL, and also determine the accuracy of the captured RFs. It will also be necessary to examine the benefit of adopting a higher sampling rate monitoring system which could increase the RF detection accuracy or even eliminate the need for a low-pass filter at the data acquisition system, without having to accurately characterise the filter response to correct RF curves.

Acknowledgment

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