

Theoretical and experimental studies on the optimization of finger seal

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Abstract

For the finger seal, the impact of structural parameters on its performance is complicated, and it is difficult and contradictory to lower its leakage and wear synchronously. Therefore, the problems relating to decision-making of optimal results and selection of reference variables still confuse the optimization of finger seal. In this paper, the performance (leakage and wear) of finger seal are calculated with finite element simulation, and a sensitivity analysis is carried out based on grey correlation method to obtain the influence degrees of structural parameters on the performance of finger seal; then a multi-objective optimization model is proposed by the combination of genetic algorithm, BP neural network and Nash equilibrium game theory; finally, the experiments of finger seals with various optimal structures are carried out to verify the theory and method established in this paper. The theoretical and experimental results indicate that the finger seal optimized with the model proposed in this paper has a better synthetical performance, which proves its feasibility. The theory and method established in this paper can be applied to enhance the synthetical performance of finger seals effectively.

Keywords : Finger seal, Optimization, Leakage, Wear, Sensitivity analysis

1. Introduction

Finger seal is an innovative seal with flexible structures. As shown in Fig. 1, the finger seal is assembled with several pieces of finger elements, a forward cover plate, an aft cover plate and several rivets which is used to clamp the finger elements with the cover plates; in each finger element, there are a series of flexible fingers distributing along the circumferential direction symmetrically, and a series of gaps are also produced between the adjacent fingers; thus, the finger elements rotate a small degree relative to each other to cover the gaps.

As the finger's flexibility, the contact pressure between finger feet and rotor can be lowered, that makes the finger seal can accommodate the radial displacement and thermal expansion of rotor much better. Proctor et al., (2002, 2006) compared the leakage and power loss of various seals by experiments, they found the leakage and power loss of finger seal are quite lower than these of labyrinth seal; and compared with the brush seal, their leakages are quite approximate, but the finger seal only costs half of the brush seal, and can avoids potential problems caused by brush wire breakage. Chen et al., (2014, 2015, 2016) did several works to study the relationships between the structural parameters and the performance of finger seal, and established an equivalent dynamic model for finger seal. Liu and Wang et al., (2003, 2006) searched more appropriate materials and coatings to reduce wear for the finger seal. Su et al., (2007) optimized the finger seal by weighted optimal method to transform the multi-objective optimization into a single objective one. Chen et al., (2007, 2008) optimized the finger seal to minimize its hysteresis leakage based on Stackelberg game theory. Zhang et al., (2011) established a multi-objective optimal model for finger seal by Nash equilibrium game theory.

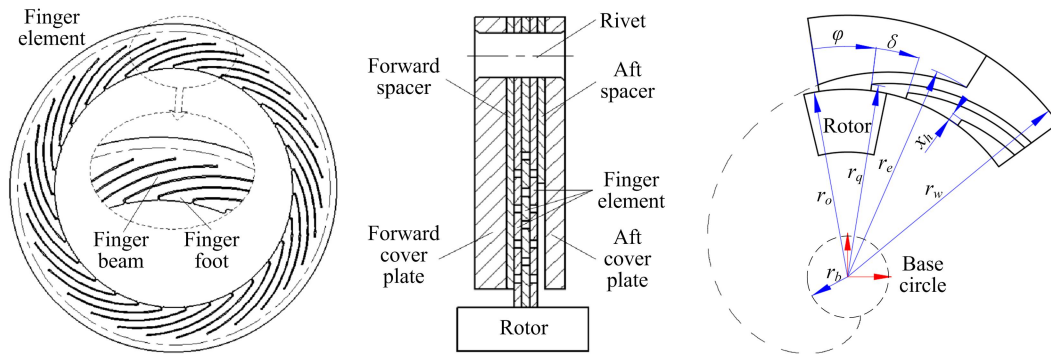


Fig. 1. Structure and parameters of finger seal

When the finger seal is functioning, its finger feet could move up and down under the action of contact force and the elastic restoring force of fingers. When the finger feet drop down, they could not return to their original positions rapidly due to the frictions between the adjacent elements or cover plates which is caused by pressure differential between upstream and downstream; this leads to the hysteresis leakage. To reduce the hysteresis leakage, a straightforward approach is enhancing the stiffness of fingers to enlarge the restoring force. However, this method will also enlarge the contact pressure between the finger feet and rotor, and then cause more severe wear. Therefore, it is contradictory to find the optimal parameters of the fingers to reduce the leakage and wear synchronously. In general, the optimization of finger seal is a typical multi-objective optimization problem with contradictoriness.

In this paper, a performance calculation model of finger seal is established by finite element simulation primarily; and the influence degrees of structural parameters on its performance is obtained by sensitivity analysis based on the grey correlation method; then a model for multi-objective optimization of finger seal is proposed by a combination of genetic algorithm, BP neural network and Nash equilibrium game theory (GA-BP-Nash); finally, the experiments for finger seals with various optimal structures are carried out to verify the feasibility of theoretical models and methods proposed in this paper.

2 Theoretical models and method

2.1 Performance calculation model of finger seal

In this paper, the performance (leakage rate and wear rate) calculation of finger seal includes two steps: 1) Obtain the hysteresis leakage gap and the contact pressure between the finger feet and the rotor via finite element simulation. 2) Establish the relationship between leakage rate and leakage gap, and the relationship between wear rate and contact pressure; the leakage rate and wear rate of finger seal can then be obtained with the relationships.

According to the cyclic symmetrical structure in circumferential direction, and the cyclic superimposed structure in axial direction, a simplified model (as shown in Fig. 2) is established to simulate the motion of finger seal. The model includes part of the rotor and aft cover plate, two complete fingers in circumferential direction, and three layers of finger elements in axial direction. The following boundary conditions are imposed:

- The external circular surfaces of finger elements and the back face of aft cover plate are fixed.
- The sections of rotor, finger elements and aft cover plate are imposed symmetric cyclic boundary conditions; the rotor is imposed axial constraint.
- The pressure differential between upstream and downstream Δp is imposed at the front face of the first finger element.
- Contact pairs are imposed at the contact area between finger feet and rotor, the contact area between the last finger element and aft cover plate, and the contact areas between adjacent finger elements.

In this paper, the materials of finger element, rotor and aft cover plate are 1Cr13 alloy steel, 40CrNiMo alloy steel and 2Cr13 alloy steel respectively, their materials properties are shown in Table 1.

Table 1. Materials properties of finger element, rotor and aft cover plate

| Name | Materials | Elasticity modulus (GPa) | Poisson's ratio |
|-----------------|-----------|--------------------------|-----------------|
| Finger element | 1Cr13 | 206 | 0.3 |
| Rotor | 40CrNiMo | 210 | 0.3 |
| Aft cover plate | 2Cr13 | 200 | 0.3 |

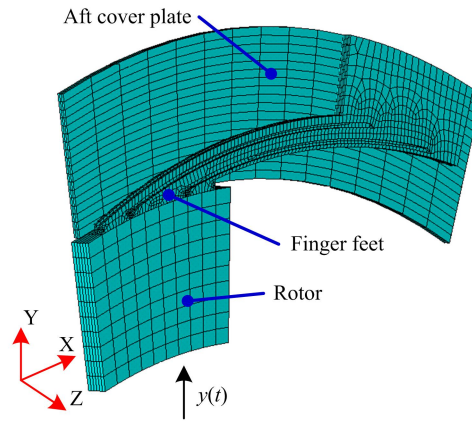


Fig. 2. Finite element model of finger seal

The simulation analysis is carried out by two load steps. At the first load step, a radial displacement Δr is imposed on the rotor to simulate its radial run-out, and the fingers are deformed under the action of contact force; at the end of this step, the contact pressure of all nodes on the feet can be obtained; the average of these contact pressures could be defined as the feet/rotor contact pressure p . While at the second load step, the rotor is moved to its original position, due to the hysteresis, there is a distance between finger feet and rotor. The distances of all nodes are measured and the average value of these distances could be defined as the height of leakage gap h .

Ignoring the axial deformation of finger element, the tilting and the axial run-out of rotor, the flow in leakage gap can be simplified as pure pressure flow (Chen G. D. et. al, 2014). Therefore, the leakage rate of finger seal can be obtained with formula below,

$$Q = \pi D h \left(\frac{\lambda l}{2h} + 1.5 \right)^{-0.5} \sqrt{2P\rho} \quad (1)$$

Where, P is the pressure loss while the gas flows through the leakage gap; D is the rotor diameter; ρ is the gas density; h, l, λ are the height, axial length and drag coefficient of the leakage gap respectively.

The wear between finger feet and rotor is mainly adhesive under the stable working condition. According to the Archard model for adhesive wear (Wang X., 2006), the volume wear rate of the finger seal is,

$$V = K_w \frac{p b n_r r_0^2 \pi (2\pi - \delta N)}{90 \sigma_s} \quad (2)$$

In which, K_w is the proportionality constant; p is the feet/rotor contact pressure; n_r is the rotating speed of rotor; N is the number of fingers; σ_s is the yield limit of material processing the finger seal.

Substitute the height of leakage gap h and feet/rotor contact pressure p calculated by finite element simulation into formula (1) and (2), the leakage rate and wear rate of finger seal can be obtained.

2.2 Sensitivity analysis between structural parameters and performance of finger seal

Based on grey correlation method, a sensitivity analysis is produced to obtain the impacts degree of structural parameters on the performance of finger seal.

In the sensitivity analysis, five structural parameters (including the angle of finger foot width φ , the thickness of finger element b , the height of finger foot x_h , the angle of finger gap δ and the finger base radius r_b) are selected as the reference factors. The leakage rate and wear rate of finger seal are selected as the objective factors. The matrixes of reference and objective factors are,

$$\begin{aligned}
 X &= \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} x_1(1) & x_1(2) & \cdots & x_1(m) \\ x_2(1) & x_2(2) & \cdots & x_2(m) \\ \vdots & \vdots & \ddots & \vdots \\ x_n(1) & x_n(2) & \cdots & x_n(m) \end{bmatrix} \\
 Y &= \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_r \end{bmatrix} = \begin{bmatrix} y_1(1) & y_1(2) & \cdots & y_1(m) \\ y_2(1) & y_2(2) & \cdots & y_2(m) \\ \vdots & \vdots & \ddots & \vdots \\ y_r(1) & y_r(2) & \cdots & y_r(m) \end{bmatrix}
 \end{aligned} \tag{3}$$

Where, n is the number of reference factors, and $n=5$; r is the number of objective factors, and $r=2$; m is the number of samples for grey correlation analysis, the samples are obtained by quadratic orthogonal rotation experiment design, and $m=27$ in this paper.

To minimize the error caused by dimensional difference, the nondimensionalization is adopted for formula (3), and it could be written as,

$$\begin{aligned}
 X' &= [X'_1, X'_2, \dots, X'_n]^T \\
 Y' &= [Y'_1, Y'_2, \dots, Y'_r]^T
 \end{aligned} \tag{4}$$

Then, the difference matrix could be calculated as formula (5),

$$\Delta Y_{jk} X_{hk} = |Y'_j(k) - X'_h(k)| \tag{5}$$

Where, $j=1,2,\dots,r$; $h=1,2,\dots,n$; $k=1,2,\dots,m$.

Finally, the gray correlation degree (Liu S. F., 2004) between reference and objective factors can be calculated in terms of formula (6),

$$R_{Y_{jk} X_{hk}} = \frac{1}{k} \sum_{k=1}^m \left(\frac{\Delta_{\min} + K \Delta_{\max}}{\Delta Y_{jk} X_{hk} + K \Delta_{\max}} \right) \tag{6}$$

Where, Δ_{\min} and Δ_{\max} are the minimum and maximum in the difference matrix respectively; K is difference identify factor, which is used to improve the significance of the difference. Normally $K \in [0,1]$, and $K=0.5$ in this paper.

The calculation results of gray correlation degrees between the five structural parameters and performance of finger seal at various operating conditions are obtained are listed in Table 2 and Table 3. However, the gray correlation degrees at various operating conditions are discrete. To evaluate the influence degree for these parameters more straightforwardly, the averages of gray correlation degrees between each structural parameter and each performance parameter at various operating conditions are calculated and represented by \bar{R} in the tables. According to the value of \bar{R} , the sequence of influence degree for the five structural parameters are also given in the tables. As shown in the tables, the top two important/sensitive structural parameters for the leakage performance of finger seal are the angle of

finger foot width and the angle of finger gap, and for the wear performance are the finger base radius and the angle of finger foot width.

Table 2. Correlations between structural parameters and leakage rate (gray correlation degrees, average gray correlation degrees and their sequence)

| Δp (MPa) | Δr (mm) | φ | b | x_h | δ | r_b |
|---------------------|--------------------|-----------|-------|-------|----------|-------|
| | 0.003 | 0.805 | 0.695 | 0.775 | 0.797 | 0.772 |
| 0.3 | 0.006 | 0.796 | 0.686 | 0.782 | 0.787 | 0.780 |
| | 0.009 | 0.805 | 0.676 | 0.798 | 0.797 | 0.806 |
| | 0.003 | 0.766 | 0.654 | 0.730 | 0.759 | 0.728 |
| 0.5 | 0.006 | 0.787 | 0.668 | 0.768 | 0.776 | 0.764 |
| | 0.009 | 0.819 | 0.695 | 0.809 | 0.811 | 0.814 |
| | 0.003 | 0.723 | 0.615 | 0.687 | 0.750 | 0.689 |
| 0.7 | 0.006 | 0.771 | 0.645 | 0.746 | 0.764 | 0.740 |
| | 0.009 | 0.797 | 0.668 | 0.782 | 0.784 | 0.785 |
| | \bar{R} | 0.785 | 0.667 | 0.764 | 0.781 | 0.764 |
| | Sequence | 1 | 5 | 4 | 2 | 3 |

Table 3. Correlations between structural parameters and wear rate (gray correlation degrees, average gray correlation degrees and their sequence)

| Δp (MPa) | Δr (mm) | φ | b | x_h | δ | r_b |
|---------------------|--------------------|-----------|-------|-------|----------|-------|
| | 0.003 | 0.669 | 0.659 | 0.662 | 0.618 | 0.720 |
| 0.3 | 0.006 | 0.672 | 0.627 | 0.663 | 0.6335 | 0.762 |
| | 0.009 | 0.673 | 0.606 | 0.662 | 0.6121 | 0.769 |
| | 0.003 | 0.679 | 0.673 | 0.668 | 0.6292 | 0.729 |
| 0.5 | 0.006 | 0.674 | 0.624 | 0.658 | 0.6244 | 0.752 |
| | 0.009 | 0.675 | 0.609 | 0.660 | 0.6072 | 0.765 |
| | 0.003 | 0.687 | 0.684 | 0.675 | 0.6437 | 0.737 |
| 0.7 | 0.006 | 0.680 | 0.630 | 0.662 | 0.6236 | 0.753 |
| | 0.009 | 0.681 | 0.610 | 0.664 | 0.6108 | 0.768 |
| | \bar{R} | 0.677 | 0.636 | 0.664 | 0.623 | 0.751 |
| | Sequence | 2 | 4 | 3 | 5 | 1 |

2.3 Optimization of finger seal by GA-BP-Nash

To solve the contradictoriness and decision-making for the optimization of finger seal, a model for multi-objective optimization is proposed with the combination of genetic algorithm, BP neural network and Nash equilibrium game theory (called GBN for short),

2.3.1 Nash Equilibrium Game Optimization of Finger Seal

The Nash equilibrium game theory was proposed by J.F. Nash in 1951. In the game, for a multi-objective optimization, each objective is controlled by a player, and each player has its own strategy set and criterion to search its own best strategy in the search space until no one can further improve its criterion. Because each player's strategy is the best response to others at each game circulation, the final strategy in Nash equilibrium game will be the best one suitable for all players. Therefore, The Nash equilibrium game theory can be used to solve the contradiction and decision-making for the optimization of finger seal in theory.

For the optimization of finger seal, there are two optimization objectives, leakage rate Q and wear rate V .

According to the Nash equilibrium game theory, the optimal model can be expressed as,

$$\mathbf{G} = \{\mathbf{M}, \mathbf{W}_{ijkl}, f_{M_i}\} \quad (7)$$

Where, \mathbf{M} is the player set, for the optimization of finger seal $\mathbf{M} = \{Q, V\}$; \mathbf{W}_{ijkl} ($i=1$ or 2) is the strategy set of each player; f_{M_i} ($i=1$ or 2) is each player's payoff function, it represents the relationship between strategy and payoff of each player.

According to the results of sensitivity analysis, we select the angle of finger foot width and the angle of finger gap as the strategy factors for the leakage rate, and select the finger base radius and the height of finger foot for the wear rate, then,

$$\begin{aligned} \mathbf{W}_{1jkl} &= \{w_{1jk}, w_{2ls}^*\} = \{(\varphi_j, \delta_k), (r_{bl}^*, x_{hs}^*)\} \\ \mathbf{W}_{2jkl} &= \{w_{1jk}^*, w_{2ls}\} = \{(\varphi_j^*, \delta_k^*), (r_{bl}, x_{hs})\} \end{aligned} \quad (8)$$

Where, w_{1jk} and w_{2ls} are the two players' strategy respectively; φ_j is the j th value in its search space, and δ_k , r_{bl} and x_{hs} have the similar meaning. The superscript * represents the strategy is fixed while another player is searching strategy.

Then, the payoff functions can be defined as,

$$\begin{aligned} f_{M_1} &= f_Q(w_{1jk}, w_{2ls}) = 1 - Q_{jkl} / Q_{\max} \\ f_{M_2} &= f_V(w_{1jk}, w_{2ls}) = 1 - V_{jkl} / V_{\max} \end{aligned} \quad (9)$$

In which, Q_{\max} and V_{\max} are the maximum of leakage rate and wear rate in the search space respectively; Q_{jkl} and V_{jkl} are leakage rate and wear rate at the unite strategy $((\varphi_j, \delta_k), (r_{bl}, x_{hs}))$ respectively.

The calculation process of Nash equilibrium game is shown in Fig. 3. In the game (the optimization process), each player tries to search its own best strategy in the search space to maximize its payoff while another player's strategy and payoff are fixed. After the two players complete a gaming circulation, they exchange the best strategy with each other, and start next circulation. When the two players' strategy satisfy formula (10), the Nash equilibrium state is reached, and the best unite strategy for the two players is obtained,

$$\begin{aligned} f_Q(w_{1jk}^{**}, w_{2ls}^{**}) &= \inf_{w_{1j} \in \mathcal{S}_1} f_Q(w_{1jk}, w_{2ls}^{**}) \\ f_V(w_{1jk}^{**}, w_{2ls}^{**}) &= \inf_{w_{2l} \in \mathcal{S}_2} f_V(w_{1jk}, w_{2ls}) \end{aligned} \quad (10)$$

Where, \mathcal{S}_1 and \mathcal{S}_2 are the search space of each player respectively. $(w_{1jk}^{**}, w_{2ls}^{**})$ is the unite strategy in Nash equilibrium state.

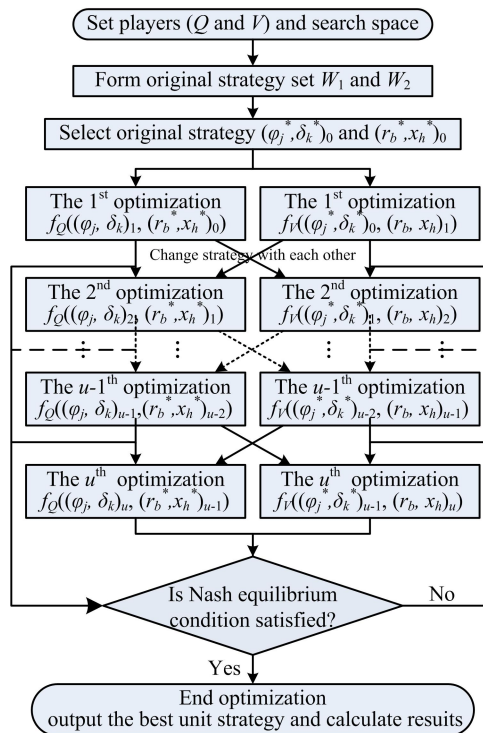


Fig. 3. Calculation process of Nash equilibrium game

2.3.2 BP neural network between strategy and payoff

According to the Nash equilibrium game theory, it is necessary to obtain the players' payoff functions, which means the functions between the four structural parameters and the performance of finger seal must be known. But there are still no explicit functions for the relationships, and it is quite clear that finite element simulation is not an efficient and feasible method to obtain the relationships. Therefore, BP neural network is used to establish the relationship between structural parameters and performance of finger seal in this paper.

The arrangement of the four structural parameters is obtained by quadratic orthogonal rotation experiment design, and the performances are calculated with the calculation model established in this paper. The arrangement and calculation results constitute the samples for BP neural network. With the training of samples, two reliable BP neural networks between structural parameters and performance are obtained, which can be used to calculate the two players' payoff in Nash equilibrium game.

In this paper, the configuration of the BP neural network is 4-12-1. With this network, several trainings are carried out with various numbers of training samples, the results are shown in Table 4. As shown in the table, after the number of training samples is up to 65, the changes of average errors of both the two networks are very little. Therefore, the numbers of training samples for the two networks are 65.

Table 4. Errors of BP Neural network with various numbers of training samples

| Number of training samples | Average error | |
|----------------------------|---------------|---------------|
| | Network for Q | Network for V |
| 25 | 0.025560 | 0.02862 |
| 35 | 0.020230 | 0.02132 |
| 45 | 0.012060 | 0.01538 |
| 55 | 0.009042 | 0.00834 |
| 65 | 0.005254 | 0.006631 |
| 75 | 0.005337 | 0.006429 |
| 85 | 0.005468 | 0.006164 |

2.3.3 Update of strategy set by genetic algorithm

In the Nash equilibrium game, the strategy set contains all the strategies that one player can select, and distributes in the search space. The strategy set should be updated after a game circulation; if not, the result is only a local optimization (Cai, H. S., 2004). Therefore, to obtain the global optimization and enhance the efficiency of optimization, the GA is used to update the two players' strategy sets after a game circulation.

In the update of strategy set by GA, for each player, each strategy is expressed as a chromosome by floating codes, after a game circulation, according to the criterion of GA, the update of strategy set depends on the value of player's payoff, the greater value of player's payoff, the greater probability that the strategy is held. The strategies with greater payoff can be held, while the strategies with poorer payoff are recombined and transformed by the operations of cross and mutation to generate new strategies. Therefore, the updated strategy set has the potential to bring greater payoff for the players. The parameters of GA used in this paper are shown in Table 5.

Table 5. Parameters of GA

| Parameter | Value |
|----------------------------|-------|
| Size of initial population | 25 |
| Cross probability | 0.2 |
| Mutation probability | 0.001 |
| Choose probability | 0.4 |
| Number of generations | 400 |

2.3.4 Optimization of finger seal by GA-BP-Nash

With the GBN optimization model, the structure of finger seal is optimized. And as a contrast, the optimization of finger seal is also carried out by weighted optimal method (called WOM for short). The optimization results of the two methods are all shown in Table 6. As shown in this table, the performance of finger seal after optimization is enhanced more or less. Compared with the WOM structures, it is clear that the comprehensive performance of GBN structure is better. The reason is the different characteristics of the two optimal models. In GBN, the two players collaborate to search their best payoff; it presents a mutual benefit and win-win result. While in WOM, the weight of impact for each structural parameter on performance of finger seal is fixed; it could not consider the requirement of leakage and wear synchronously in this constraint. Furthermore, there is only one optimization result for the GBN, but four optimization results for the WOM. In fact, ten optimizations are carried out with ten different original strategy sets. Because of the global optimization capability of GA and the characteristic of Nash equilibrium game, the optimization based on GBN always gives the only result, while the optimization based on WOM gives thirty various results. This also indicates the advantage of GBN on the decision-making of optimal result.

Table 6. The optimization results of finger seal

| Structure | $\varphi(^{\circ})$ | x_h (mm) | $\delta(^{\circ})$ | r_b (mm) | Q (g/s) | $V(10^{-13}m^3/s)$ |
|--------------------|---------------------|------------|--------------------|------------|-----------|--------------------|
| Original structure | 9.75 | 0.8 | 0.4 | 7.4 | 1.9039 | 4.4491 |
| GBN structure | 9.75 | 0.6 | 0.4 | 5.0 | 1.6842 | 2.7863 |
| WOM structures | 10.85 | 0.9 | 0.4 | 6.6 | 1.7301 | 4.0260 |
| | 8.6 | 0.8 | 0.4 | 6.1 | 1.8617 | 3.6037 |
| | 10.85 | 0.5 | 0.4 | 5.5 | 1.5896 | 3.2257 |
| | 9.00 | 0.8 | 0.4 | 5.8 | 1.8419 | 3.5224 |

3 Experimental facilities and method

In this paper, the finger seals with four different structures (the first four structures shown in Table 6) are tested at various pressure differentials and rotating speeds. As shown in Fig. 4, The test rig is mainly comprised with motor, speed increaser, test unit and mass flow meters; with the motor (its rated speed is 5000rpm) and speed increaser (its transmission ratio is 1:6), the rotating speed in the experiments can reach 30000rpm. The tested finger seals are assembled in the test unit, which divide the interior of the test unit into three chambers (a high pressure chamber and two low pressure chambers); high pressure gas is pumped into the high pressure chamber to produce pressure differential between the high pressure chamber and the two low pressure chambers; due to the leakage of the tested finger seals, the gas leaks into the two mass flow meters through the two vents at the bottom of the low pressure chambers, then their leakage rates are measured.

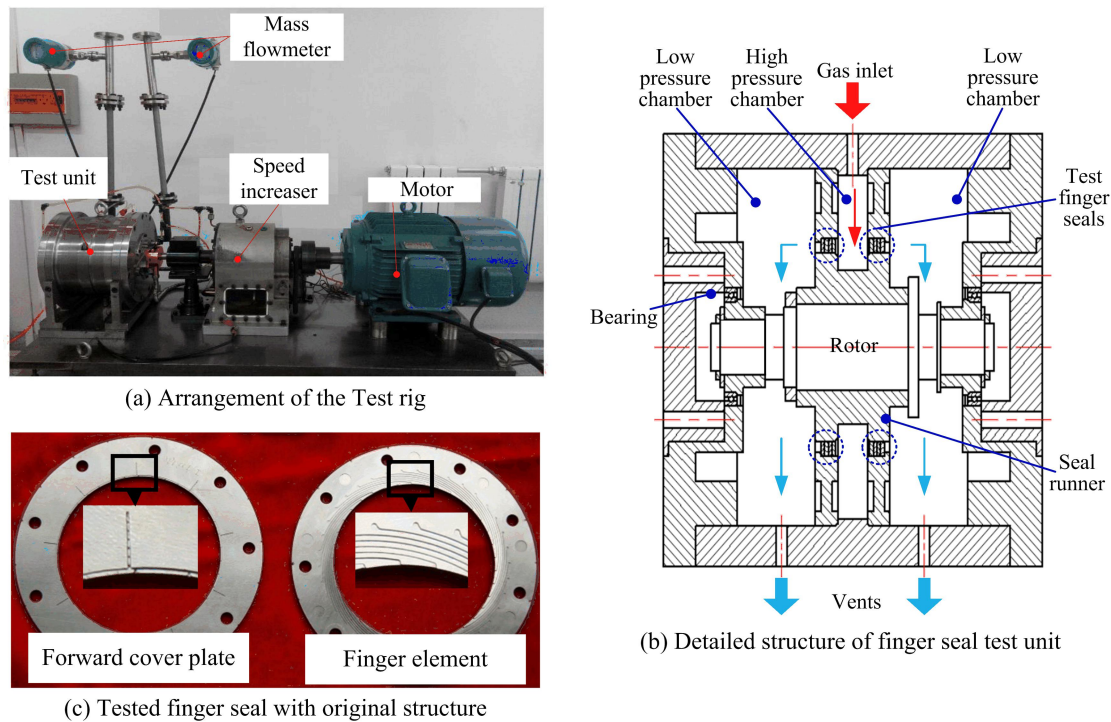


Fig. 4. Test rig and tested finger seal

The other structural parameters of the four kinds of tested finger seals are same, and shown in Table 7; they are all made of 1Cr13. The seal runner is made of 40CrNiMo and coated with hard chrome.

Table 7. Main structural parameters of tested finger seals

| Parameter | Value |
|--|-------|
| Thickness of finger element b (mm) | 0.2 |
| Number of layers m | 8 |
| Outside radius of finger seal r_w (mm) | 47 |
| Inside radius of finger seal r_o (mm) | 33 |

The leakage tests are carried out at four pressure differentials (0.1MPa, 0.2MPa, 0.3MPa and 0.4MPa). In each pressure differential, the rotating speed is stepped up and down as follows: 0rpm, 3000rpm, 5000rpm, 7000rpm, 9000rpm, 12000rpm, 9000rpm, 7000rpm, 5000rpm, 3000rpm and 0rpm, as shown in Fig. 5(a).

For the wear of the finger seal, considering the stability and safety of the test rig, the test at each condition doesn't operate for a long time. After the leakage tests are completed at all conditions, the scanning electron microscope (SEM) and energy disperse spectroscopy (EDS) analysis are carried out to evaluate the wear of the finger seals with various structures. To enhance the accuracy of the wear evaluation, for each structure, four finger feet at various regions are

selected to take SEM photos; as shown in Fig. 5(b), the point 1 and point 3 locate at the left and right finger feet of the upstream finger element along the horizontal direction (X direction in the figure), point 2 and point 4 locate at the left and right finger feet of the downstream finger element along the horizontal direction (X direction in the figure). Besides, the EDS analysis is carried out on the finger foot at point 2 for each structure.

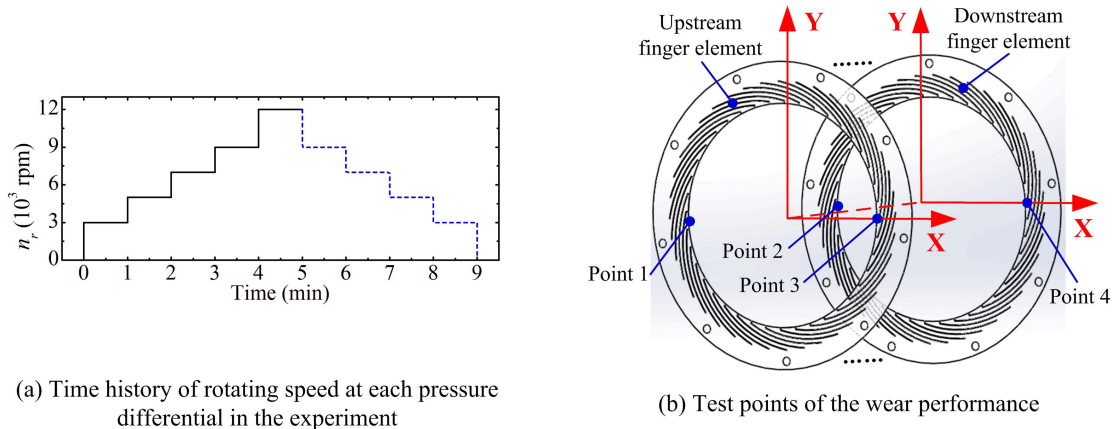


Fig. 5. Time history of rotating speed and test points in the experiment

4 Results and discussion

4.1 Leakage performance results

Fig. 6 shows the experimental results of the leakage rate. It is clear that the finger seal with the GBN structure has the lowest leakage rate, which proves the GA-BP-Nash is feasible for the optimization of finger seal.

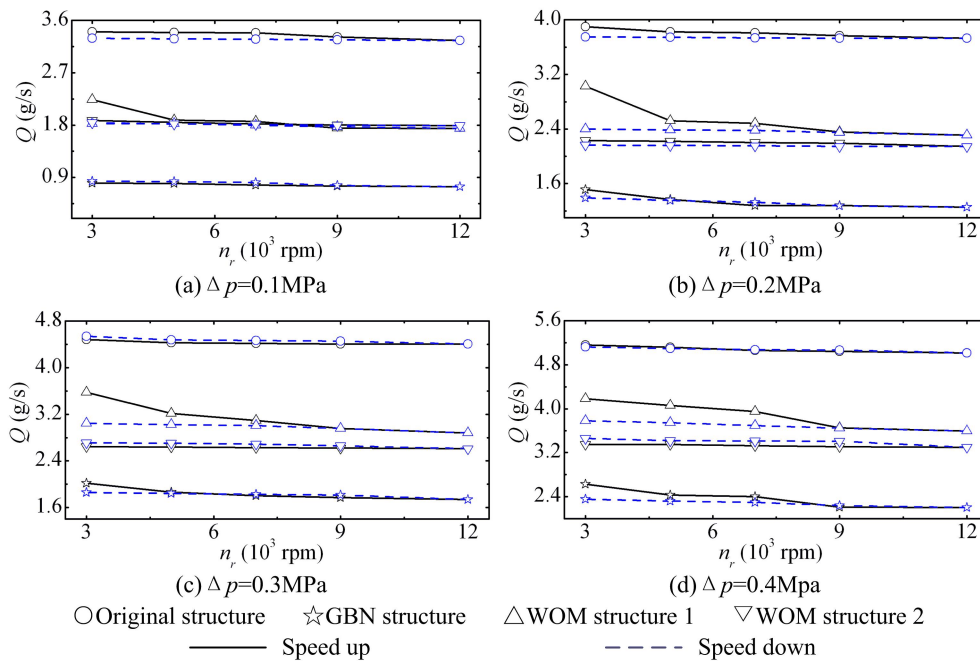


Fig. 6. Experimental results of leakage rate

4.2 Wear performance results

The SEM photos of finger seals are given in Fig. 7. It can be seen that the surfaces of finger seal with GBN structure are with some shallow dimples and grooves, and a bit of material transfer is also observed. The observations indicate the wear of finger seal with GBN structure is at the initial stage, and not severe. As a contrast, the surfaces of

finger seals with WOM structures are of deeper grooves, and with obvious material transfer; even at point 1 or point 3, some small pits caused by material transfer can be observed; these indicate the form of wear of finger seals with the WOM structures is a kind of adhesive wear, which is agree with the conclusions in references of Liu and Wang et al., (2003, 2006); and also show that the wear of finger seals with the WOM structures is more severe than that of GBN structure. And for the original structure, the material transfer is the most obvious, especially at point 2 and point 4; and more severely, much larger and deeper pits appears at point 1 and point 3. Obviously, the wear of finger seal with original structure is the most severe.

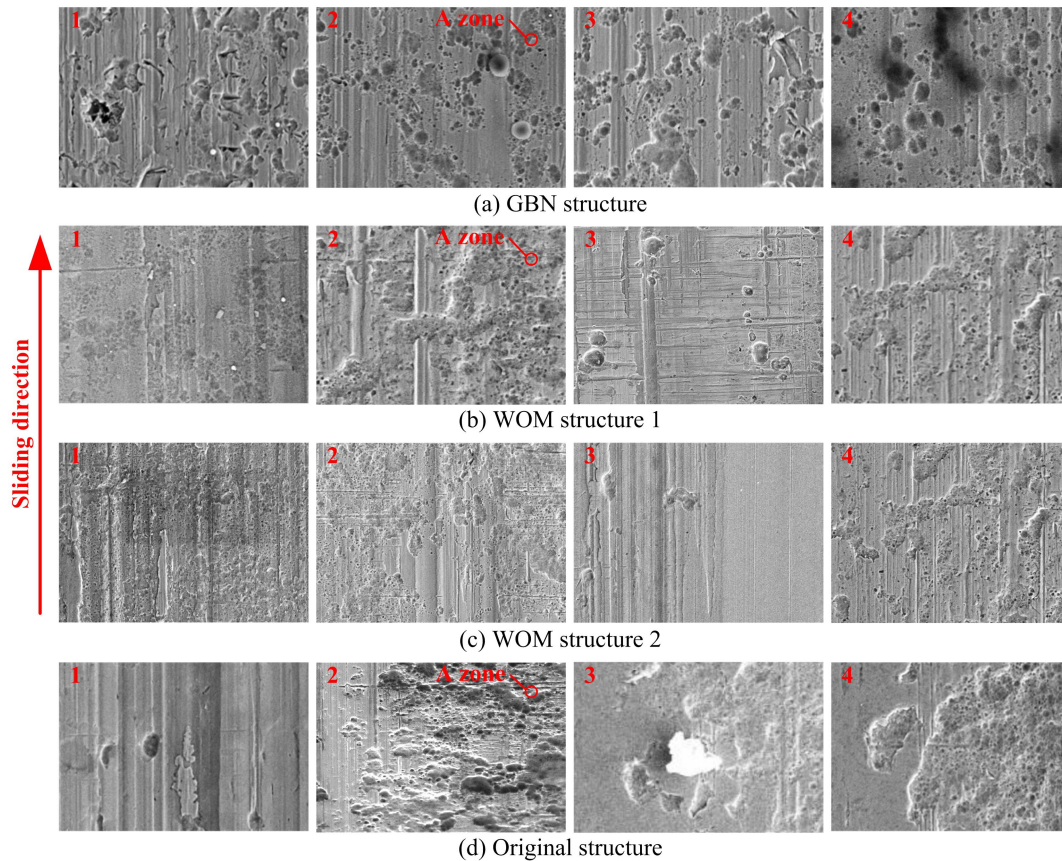


Fig. 7. SEM photos of finger seal with different structures ($\times 600$)

The EDS analysis results of finger seals are shown in Fig. 8 (the EDS analysis is carried out at A zone marked in Fig. 6 with the red circle). As it is known, the weight of Cr for 1Cr13 steel is 11.5%~13.0%. According to the EDS analysis results, after the tests, the weight of Cr is 12.19% for the finger seal with GBN structure, 15.26% for the WOM structure 1, and 20.66% for the original structure. The contents of Fe and Mn are basically the same. It reveals that, because of the adhesive wear between finger feet and seal runner, the material transfer appears at the finger seal with the WOM structure and original structure obviously, especially for the original structure, while the finger seal with GBN structure only transferred a little material from the seal runner, or even no material is transferred. The EDS analysis results show great agreement with the results of SEM photos, which also prove the wear performance of finger seal with GBN structure is the best.

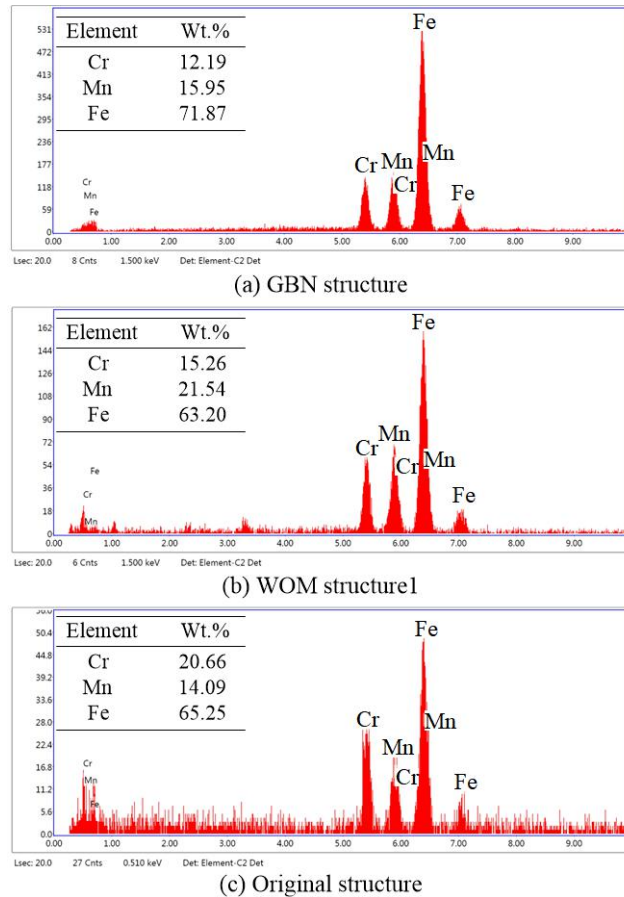


Fig. 8. EDS analysis of finger seal with various structures

5. Conclusions

This paper carried out the analysis and optimization of finger seal with the combination of finite element analysis, grey correlation method, genetic algorithm, BP neural network and Nash equilibrium game theory. According to the theoretical and experimental results, the followed conclusions are drawn:

- 1) The top two important/sensitive structural parameters for the leakage of finger seal are the angle of finger foot width and the angle of finger gap; while the counterparts for its wear are the finger base radius and the angle of finger foot width.
- 2) The GA-BP-Nash optimal model established in this paper can solve the problems relating to contradiction and decision-making for the optimization of finger seal effectively.
- 3) The theoretical and experimental results show the finger seal with the GBN structure has the best leakage and wear performances, which proves the feasibility and application value of the models proposed in this paper.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (Grant No. 51305343 and No.52075436), Science and Technology Achievements Transfer and Promotion Plan project of Shaanxi Province in China (Grant No. 2018XNCG-G-11), Special plan for Local Service of Shaanxi Education department in China (Grant No. 19JC030), China Postdoctoral Science Foundation (Grant No. 2019M653879XB), and Shaanxi Basic research plan of Natural Science (Grant No. 2019JQ-307)

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