

# Optimal design of electric vehicle charging stations for commercial premises

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## Summary

Influx of plug-in electric vehicles (PEVs) creates a pressing need for careful charging infrastructure planning. In this paper, the primary goal is to devise a closed-form expression for the PEV charging station capacity problem. Two types of commercial charging stations are considered. The first problem is related to the calculation of the optimal service capacity for charging lots located at workplaces where PEV parking statistics are given a priori. The second problem, on the other hand, is related to the optimisation of arrival rates for a given station capacity. In the second part, the mathematical models are expanded for the case where multiple charger technologies serve customer demand. This time the goal is to calculate the optimal customer load for each charger type according to its rate. Calculations are carried out for both social and individual optimality cases. Markovian queues are used to model the charging station system to capture the complex interactions between customer load, service waiting times, and electricity cost. The related optimisation problems are solved using convex optimisation methods. Closed-form expressions of station capacity and optimal arrival rates are explicitly derived. Both analytical calculations and discrete-event simulations are carried out and the results show that 60% of the waiting times and 42% of the queue length can be reduced by optimal capacity planning.

## KEYWORDS

charging station design, electric vehicles, queueing theory, smart grid

## 1 | INTRODUCTION

There is a global motivation to decarbonise the transport sector to address pressing environmental challenges. Electrification of transportation is an ideal solution to lower carbon emissions and improve air quality. In developed countries (eg, UK, EU), ambitious targets have been set to achieve net-zero emissions by 2050 and bold actions are taken to transform the transportation

sector.<sup>1,2</sup> To increase the plug-in electric vehicle (PEV) adoption rates, a number of government-led policies were introduced.<sup>3</sup> Some of the most promising policies include phasing out the sale of internal combustion engine vehicles (ICEV) (eg, Denmark by 2030, UK by 2035, and France by 2040), providing financial incentives to purchase PEVs, and declaring low-carbon zones in major city centres (eg, London, Berlin, etc.). In developing countries such as India and China, residents, particularly

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in megacities, suffer from traffic congestions, air pollutions, and climate change.<sup>4</sup> The adoption of PEVs for urban mobility eliminates tailpipe emissions and offers affordable transportation mode for the most vulnerable segments of the society. However, the success of such a major shift is linked to the capability of electric grids to provide the necessary coverage and capacity to fuel large collections of PEVs with minimal grid reinforcements.

Electric power grids have been ageing and were designed to operate as near real-time supply-demand systems. The increasing integration of renewables has increased the level of uncertainties at the generation side and continues to create stability and reliability challenges. Moreover, a number of studies show that uncontrolled PEV charging could have disruptive impacts on the grid.<sup>5</sup> For instance, at the distribution level, concurrent PEV charging could lead to premature ageing of transformers,<sup>6</sup> deteriorate power quality,<sup>7</sup> and trigger supply shortages.<sup>8</sup> At the power generation side, a number of studies show that uncontrolled PEV charging could require sizable capacity expansion. In California, for example, the sales of ICEV will be banned by 2035 and the additional PEV demand is expected to increase by 25%.<sup>9</sup> In Reference 10, additional electricity generation requirement to charge PEVs in Germany by 2050 is studied. It is shown that uncontrolled charging would increase the peak consumption by 20%. On the other hand, controlled charging methods have the potential to reduce this requirement by half.

At present, PEVs can be charged by one of the following options.<sup>5,11</sup> The first type is slow charging (level 1) which is often rated up to 1.92 kW using existing electrical circuits. Considering the current PEV landscape (20+ kWh), level 1 charging is suitable for residential use cases due to long charging sessions. The second possibility, however, uses level 2 chargers (typically rated around 6 kW) and is largely deployed at public parking places such as universities, park and ride facilities, and airports. The third option is DC fast charging and typically rated at 22, 50, or 150 kW. Similar to the previous case, fast chargers are located at public locations to serve customers who are in need of urgent need.<sup>11</sup> A detailed overview of charging types, costs, power rating, and locations are shown in Table 1. In all charging cases, concurrent PEV charging increases peak electricity demand and sustainable charging facilities and intelligent demand scheduling policies are required to (a) ensure power grid

reliability, (b) minimise required grid reinforcements, and (c) provide convenient service to customers. Therefore, there is a pressing need to design and operate public charging facilities in line with growing PEV penetration. Moreover, public charging facilities are becoming instrumental for vehicle-to-grid (V2G) applications in which a cluster of PEVs, coordinated by an aggregator, can join ancillary energy markets to stabilize power grids. In return, PEV owners get paid for services rendered<sup>12</sup> which reduces the cost of PEV ownership. With the growing number of PEVs, V2G applications have gained momentum and supported by a number of field studies and actual implementations.<sup>13</sup>

Optimal capacity planning of charging stations is a complex problem as it involves stochastic interaction between customer types, arrival patterns, service durations, and waiting times and existing methods (planning based on peak demand) lead to underutilisation of stations.<sup>5</sup> The novelty of this paper is to devise a closed-form expression to calculate optimal station capacity and demand in terms of charger rates, electricity cost, and waiting times to capture the trade-off between station capacity and cost. We consider two design problems for PEV charging lots. In the first one, the primary assumption is that the charging lot is located at a workplace and PEV arrival statistics are acquired in advance (eg, stay parked during work hours). The goal is to calculate the electric power or the size of the transformer needed to serve customers. In the second problem, we consider a public parking lot which is located in a busy area and station is fed by a fixed capacity. In this case, the goal is to calculate the optimal arrival rate that maximises a revenue function composed of financial parameters such as waiting and electricity costs. Next, we expand the second case and assumed that the charging facility employs multiple charger technologies and the goal is to calculate optimal demand assignment for each charger to minimise waiting times. The optimisation problem is solved both for social and individual optimality cases. Note that multiple charger technologies are considered only for the second case because PEVs stay parked shorter than workplace parking and customer needs are likely to be more diverse than in the first case.

Over the last decade, a large and growing body of literature has addressed modelling, optimisation, and

**TABLE 1** Electric vehicle charger type and costs<sup>14</sup> and power ratings<sup>15</sup>

Charger type	Unit cost range	Unit installation costs	Power rating (EU)	Location
Level 1	\$300-\$1500	0-\$3000	3.7	Primarily home
Level 2	\$400-\$6500	\$600-\$12 700	3.7-22	Home, workplace, public
DC fast	\$10 000-\$40 000	\$4000-\$51 000	>22	Public and intercity

planning of PEV charging infrastructures. The modelling approaches aim to approximate PEVs' arrival, departure, and system usage patterns such as charger occupancy and the electricity demand profiles. In majority of the published literature, Poisson process (homogenous and non-homogenous) has been the primary adoption to model PEV behaviour.<sup>16-18</sup> The optimisation of PEV demand refers to the economic scheduling of aggregate PEV demand to minimise peak electricity usage while meeting customer demand.<sup>19-21</sup> Planning of charging facilities is related to calculating the capacity of station resources such as the size of the transformer or the capacity of a possible on-site storage to serve peak customer demand.<sup>5</sup> This paper lies at the intersection of the first and the third topics advances PEV modelling approaches by proposing a capacity optimisation framework for PEV charging stations located at commercial premises. Next, we present a detailed comparative analysis.

Optimisation of PEV demand has been the focus of a number of studies where chargers are deployed on an existing electrical network, the primary goal is to coordinate charging of PEVs in a way that power system constraints are not violated. In Reference 22, a method to solve an online optimisation problem for charging and discharging PEVs in a distribution network is presented. In Reference 23, a distributed algorithm is developed to solve multi-period optimal power flow problem with networks serving large numbers of PEVs. In Reference 24, a coordinated charging algorithm is presented to minimise the distribution system losses by minimizing the load variance on the network. In Reference 2, a simulation study was conducted to estimate power quality deterioration caused by PEV charging. On the other hand, this study differs from the enumerated ones because charging facilities at commercial premises operate with dedicated distribution level equipments. In this case, the goal of the planning studies is optimal sizing and siting of chargers and serving customer demand. In related literature, stochastic modelling and queueing theory is widely used to capture customer demand characteristics. Some of the related modelling studies are as follows. In Reference 16, using actual PEV charging data (level 2 chargers) collected from a university campus in California, a Markovian model was developed to represent the charging habits of drivers. It was concluded that PEV drivers do not fully charge their cars, therefore, charging durations are less than the expected. In this paper, we follow a similar approach and assumed that customer behaviour is modelled with an M/M/1 queue. In Reference 25, an optimisation framework is developed to lower the peak electricity demand of a university by optimally charging and discharging parked PEVs on campus. In Reference 26, PEV parking loads are modelled as demand-response agents, and an optimal charging coordination problem is

solved to maximise the parking lot's monetary benefits. In our previous work, (presented in Reference 27), we proposed a capacity planning framework for a large scale PEV charging lot using an effective bandwidth approach. We have shown that due to statistical gains, charging resources needed to allocate per charger is significantly less than the peak demand. On the other hand, the previous model does not capture the financial aspects of a typical charging network such as waiting times and electricity prices. The optimisation model presented in this paper, however, captures the trade-off between station capacity and aforementioned financial considerations. Queueing-based optimisation approaches for PEV charging stations have been adopted in References 28 and 29

Other related literature is listed as follows. In Reference 30, the role of PEV parking lots to improve power reliability is studied. In References 31 and 20 actual data sets for parking lots located at university campuses are presented. For instance, the analysis in Reference 20 shows that nearly 50% of the charging sessions have more than 5 hours of flexibility, which is the difference between session duration and minimum charging time. In addition, nearly 65% of the arrival occurs between 8 AM and 10 AM. It is noteworthy that the working hours at universities are relatively flexible when compared to most workplaces. Hence, it is reasonable to assume that for regular office jobs, most of the system arrivals and departures would be around work starting and ending times. In Table 2, publicly available datasets related to PEV charging and testing are presented. Moreover, a number of literature surveys discuss preceding topics in detail (see References 32 and 17). Next, we present two design problems and present case studies to provide insights.

## 2 | OPTIMAL CHARGING STATION DESIGN-SINGLE CHARGER TECHNOLOGY

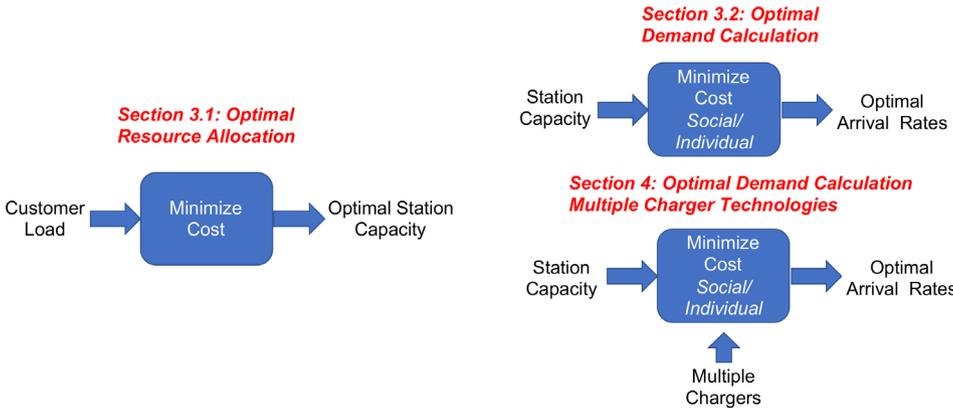
In this section, we consider capacity planning problems for PEV charging lots equipped with single charger technology. Moreover, in the last part of the section, social and individual optimality cases are presented. Preceding optimality cases are further discussed in the next section and the problem formulation is expanded to the multiple charger technology case. An overview of the design problems is presented in Figure 1.

### 2.1 | Optimal resource calculation

We consider a PEV parking lot located at a workplace premise dedicated for employees and visitors arrivals. It is

Name/location	Type	Size/duration	Reference
ElaadNL, NL	Public charging	2011-15, 1M+ data	33
My Electric Avenue, UK	Residential charging	100+ EVs, 2012-2015	34
Caltech, USA	Campus charging	32 000+ sessions	20
Idaho, USA	Charger profile	6 different PEVs	35
Dundee, UK	Public charging	Selected days	36
Boulder, USA	Public charging	City wide during 2018	37
UCSD, USA	Campus charging	2011-14, 6800 sessions	31

**TABLE 2** An overview of publicly available datasets for PEV charging statistics



**FIGURE 1** Overview of addressed design problems in Sections 2 and 3 [Colour figure can be viewed at wileyonlinelibrary.com]

assumed that the number of vehicles visiting the parking lot can be estimated from parking permits or historic arrival patterns. In this case, the goal of the first design problem is to calculate the size of the total power demand feeding a number of vehicles. We assume the station operates under the following stochastic assumptions. PEVs arrive to the charging lot according to the Poisson process with rate  $\lambda$ . This is a common assumption in the literature, see References 5 and 18. It is noteworthy that service durations are independent of the arrivals and charging services follow an exponential amount of time with mean service rate of  $\mu$ . Note that  $\lambda$  PEVs per unit time arrive at the station, while the station can charge  $\mu$  PEVs in unit time. As there is only one feeder providing power to the lot, the station is modelled as an M/M/1 queueing system. It is further assumed that  $\mu$  is the decision variable, while  $\lambda$  (eg, hourly arriving customers) is known in advance.

In this model, two types of costs occur. Let  $c_p$  denote a service cost rate that is related to the amount of the power drawn from the grid. Moreover, let  $c_w$  denote a waiting-cost which is related to the delay in charging service and measured by cost per unit time per customer. It is noteworthy that average long-term cost is equal to the expected steady state cost, which holds if and only if  $\mu > \lambda$  (otherwise system costs approaches to  $\infty$ ).<sup>38</sup> Hence, without loss of generality we assume that  $\mu > \lambda$ . To that end, the objective is to minimise long term average cost represented by

$$C(\mu) = c_p\mu + c_w W(\mu), \tag{1}$$

where  $W(\mu)$  is the expected number of PEVs in the parking lot. It is further assumed that PEVs are charged in a first in first out (FIFO) principle, therefore, from Reference 39, the expected number of PEVs can be written as

$$W(\mu) = \frac{\lambda}{\mu - \lambda}. \tag{2}$$

To that end, the optimisation problem can be written as

$$\min_{\mu: \mu > \lambda} C(\mu) = c_p\mu + c_w \frac{\lambda}{\mu - \lambda}. \tag{3}$$

It is important to note that the second derivative of the objective function with respect to parameter  $\mu$  is

$$C''(\mu) = \frac{2c_w\lambda}{(\mu - \lambda)^3} > 0, \forall \mu > \lambda, \tag{4}$$

hence,  $C(\mu)$  is convex in service rate  $\mu \in (\lambda, \infty)$ . Furthermore, the objective function given in Equation (3) has the following relation:  $C(\mu) \rightarrow \infty$  as  $\mu \uparrow \infty$  and as  $\mu \uparrow \lambda$ . Therefore, the optimal values can be calculated by taking

the derivative of the cost function and setting it equal to zero. That is,

$$C'(\mu) = c_p - \frac{c_w \lambda}{(\mu - \lambda)^2} = 0. \quad (5)$$

From the above equation, it is easy to see that the unique optimal service rate  $\mu^*$  can be written as

$$\mu^* = \lambda + \sqrt{\left(\frac{c_w \lambda}{c_p}\right)}. \quad (6)$$

By plugging the above optimal value into the cost function (given in Equation (3)), the optimal objective function is given by

$$C(\mu^*) = c_p \lambda + \sqrt{\lambda c_p c_w} + \sqrt{\lambda c_p c_w}.$$

The above result can be interpreted as follows. The first cost component,  $c_p \lambda$  denotes a fixed costing of supplying the minimal required level of charging service, that is,  $\lambda = \mu$ . The next cost terms, denote the charging service and the waiting costs related with the optimal surplus service level, respectively. In Figure 2, we present a case study of the proposed model with the following parameter settings:  $\lambda = 10$ ,  $c_p = 2$ , and  $c_w = 4$ . The optimal service rate is found as 14.48 kW and the minimum cost is attained at 37.88. The presented results further show the convexity of the problem. It can be noticed that as the service rate increases, PEVs get charged faster. Therefore, the waiting time approaches to zero. It is

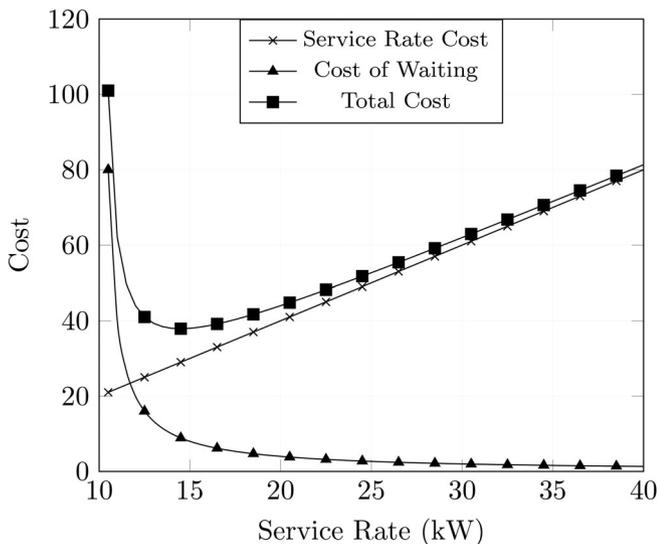


FIGURE 2 Total system cost as a function of charging rate

important to note that waiting cost could be a nonlinear function. In this case, the convexity assumption can be valid for certain ranges of service rate.

## 2.2 | Optimal demand calculation

In this section, we present a design problem in which the charging facility has a fixed charging rate and the goal is to calculate optimal arrival rates such that the station revenue is maximised. Let  $c_r$  denote reward per an arriving customer, while the waiting cost is represented by  $c_w$ . Let  $B(\lambda)$  denote the per unit time expected net benefit and written as

$$B(\lambda) = \lambda c_r - c_w W(\lambda), \quad (7)$$

where  $W(\lambda)$  is given in Equation (2).

Similar to the previous case, without loss of generality it is assumed that the station operates at a stable region and  $\lambda < \mu$ . To that end, the optimisation problem is given as below

$$\begin{aligned} \max_{\lambda} \quad & B(\lambda) \\ \text{s.t} \quad & 0 \leq \lambda < \mu \end{aligned} \quad (8)$$

This optimisation problem is more complicated than the one presented in the previous section due to the constraint  $\lambda \geq 0$ . Notice that,  $B(\lambda)$  approaches to  $\infty$  as the arrival rate of PEVs gets closer to the service rate,  $\lambda \uparrow \mu$ , hence, the upper limit of the feasible limit is not a major concern. We continue our analyses, similar to the previous case, by differentiating the benefit function twice with respect to arrival rate as below

$$B''(\lambda) = \frac{2c_w \mu}{(\mu - \lambda)^3} < 0, \forall \mu > \lambda. \quad (9)$$

Note that  $B(\lambda)$  is differentiable and concave. The optimal arrival rate  $\lambda^*$  is a unique solution to the followings:

$$\begin{cases} \text{Case 1 } \lambda = 0, & B'(0) \leq 0 \\ \text{Case 2 } \lambda > 0, & B'(\lambda) > 0 \text{ and } B'(0) > 0 \end{cases} \quad (10)$$

To that end, using Equation (7), the optimal unique solution  $\lambda^* \in [0, \mu)$  can be written for the two cases as:

$$\begin{cases} \text{Case 1 } \lambda^* = 0, & \text{if } c_r \leq \frac{c_w}{\mu} \\ \text{Case 2 } \lambda^* = \mu - \sqrt{\mu c_w / c_r}, & \text{if } c_r > \frac{c_w}{\mu} \end{cases} \quad (11)$$

It is noteworthy that  $\mu > \sqrt{\mu c_w/c_r}$  if and only if  $c_r > c_w/\mu$ , hence both cases given in Equation (11) can be combined and rewritten as

$$\lambda^* = \left(\mu - \sqrt{\mu c_w/c_r}\right)^+, \quad (12)$$

where  $a^+ := \max\{0, a\}$ . Next, we present two cases studies, using the same set of parameter settings given in the preceding section, to calculate optimal arrival rates given in above. Results for both cases are presented in Figures 3 and 4. Note that in Figure 3 the net benefit is negative. This is because the revenue gained by charging PEVs is less than the cost of customer waiting. Hence, the optimal arrival rate is  $\lambda = 0$  and there is no economic

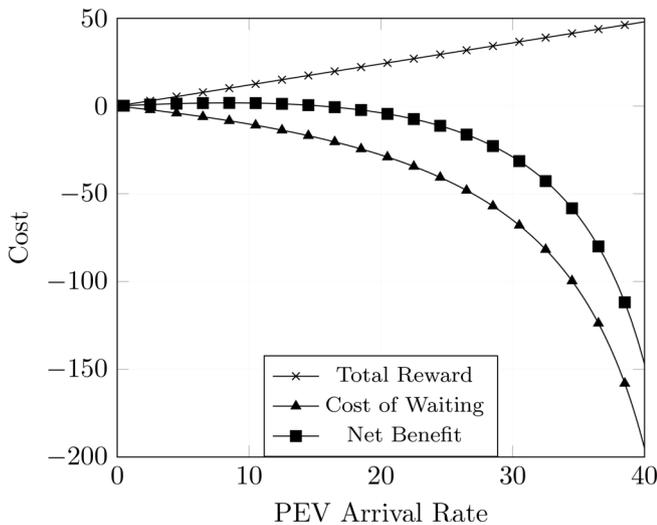


FIGURE 3 Optimal arrival rate for case 1 ( $c_r \leq c_w/\mu$ )

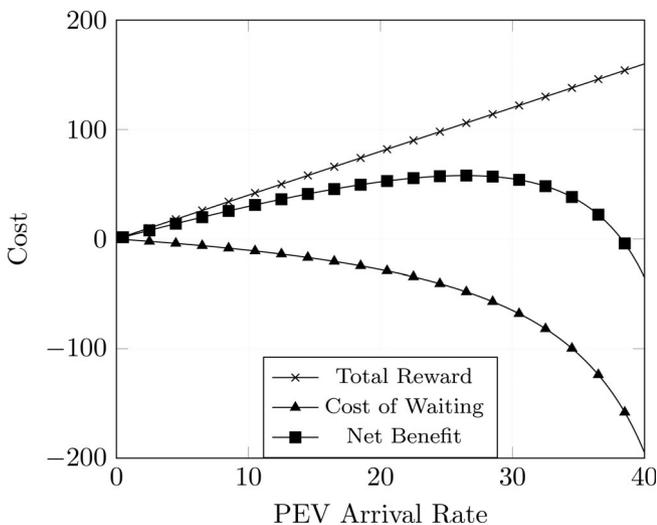


FIGURE 4 Optimal arrival rate for case 2 ( $c_r > c_w/\mu$ )

incentive for the charging station to admit any PEV. On the other hand, for the second case presented in Figure 4, the optimal arrival rate is attained at  $\lambda^* = \mu - \sqrt{\mu c_w/c_r}$ .

### 2.2.1 | Social and individual optimality

In the previous model, the arrival rate is chosen by the station operation and the goal is to maximize the number of EVs that can be charged based on the benefit that they bring ( $c_r$ ) minus the cost to the facility ( $c_w$ ). In this section, we defined two alternatives for the decision-maker who admits or rejects customers to the station. Let us assume that the customer arrival process follows a Poisson process with rate  $\Lambda$ . This customer is admitted to the station with probability  $p$  and rejected with probability  $1 - p$ . It is noteworthy that customer admittances are independent of the number of customers in the station and mutually independent from successive decisions. According to the superposition property of Poisson processes, the mean customer demand becomes  $\lambda = p\Lambda$  and the expected benefit equal to

$$p(c_r - c_w W(\lambda)) + (1 - p)0 = p(c_r - c_w W(\lambda)).$$

In optimising customer demand in shared facilities, *social welfare* maximization is used in addition to facility optimality. Social welfare is often described as the sum of all customer's net benefits.<sup>40</sup> In this case, the decision-makers are all customers and cost/benefit functions refer to individual PEVs. The objective cost function, similar to the above case given in Equation (7), for social optimality becomes  $B(\lambda) = \lambda(c_r - c_w W(\lambda))$  and customers are accepted with probability  $p = \lambda/\Lambda$ . To that end, the socially optimal arrival rate, denoted by  $\lambda^s$  is written by

$$\lambda^s = \left(\mu - \sqrt{\mu c_w/c_r}\right)^+. \quad (13)$$

Then, the station operator can achieve social optimality by making acceptance probability  $p^s$  equal to  $\lambda^s/\Lambda$ . We proceed to explain the second optimality approach, known as the *individual optimality*, and compare it with socially optimal arrival rates. Each PEV arriving to the station with rate  $\lambda$  experiences a net benefit of  $c_r - c_w W(\lambda)$ . If a customer is rejected (with probability  $(1 - p)$ ), then no benefit is obtained. For the individual optimality case, the choice of acceptance probability  $p$  is purely selfish and does not concern with overall system performance. In this case, the optimising customer has the following expected net benefit,

$$p(c_r - c_w W(\lambda)) + (1 - p)0,$$

where similar to the above case  $p$  takes values between 0 and 1. Given the expected net benefit, probability distribution takes the following values

$$\begin{cases} p=0, & \text{if } c_r < c_w W(\lambda) \\ p=1, & \text{if } c_r > c_w W(\lambda) \\ 0 \leq p \leq 1 & \text{if } c_r = c_w W(\lambda) \end{cases}$$

Notice that in the first case, the customers do not enter the facility since the net benefit is negative. In the second case, PEVs enter the facility since there is a positive benefit. In the third case, however, the customer will enter the station with indifferent probability among all  $0 \leq p \leq 1$ . Next, we define an individually optimal arrival rate (also called as *Nash equilibrium*) by the property that no individual customer has any incentive to deviate from the equilibrium arrival rate  $\lambda^e$ . Based on the preceding discussion, it can be concluded that the Nash equilibrium for the first case would be  $\lambda^e = 0$  (and  $p^e = 0$ ). For the second case ( $c_r > c_w W(\lambda)$ ), the unique equilibrium point would be  $\lambda^e = p^e \Lambda$  such that  $c_r = c_w W(\lambda)$ . To that end, the relationship between three different optimality can be written as below

$$\begin{cases} 0 < \lambda^* = \lambda^s < \lambda^e, & \text{if } c_r > c_w / \mu \\ 0 = \lambda^* = \lambda^s = \lambda^e, & \text{if } c_r \leq c_w / \mu \end{cases} \quad (14)$$

### 3 | OPTIMAL DEMAND FOR MULTIPLE CHARGER TECHNOLOGIES

In this section, we extend the model presented in Section 2.2 and assume that a parking lot can employ multiple charger technologies with different charging rates (eg, most common rates in Great Britain include 7 kW, 22 kW, 43 kW). We consider a parking lot that hosts  $n$  different chargers with service rates  $\mu_j$  and arrival rates  $\lambda_j$  where  $j = 1, \dots, n$ . Note that  $\mu_j$  are fixed technological constraints and the  $\lambda_j$  are decision variables. The goal is to minimise waiting times while the aggregate PEV arrival rate should add up to a fixed  $\lambda$  level. Hence, the optimisation problem becomes optimal splitting of incoming traffic among  $n$  parallel chargers. The problem is formulated as

$$\begin{aligned} \min_{\lambda_j} \quad & \sum_{j=1}^n \frac{\lambda_j}{\mu_j - \lambda_j} \\ \text{s.t.} \quad & \sum_{j=1}^n \lambda_j = \lambda \\ & 0 \leq \lambda_j < \mu_j, j = 1, \dots, n \end{aligned} \quad (15)$$

Similar to the discussion in Section 2.2.1, demand splitting is done in a probabilistic fashion independently of the present and past states of the system. An arriving customer is directed to the charger  $j$  with probability  $p_j = \lambda_j / \lambda$  such that arrival rate for charge  $j$  becomes  $\lambda_j$ . Using the convexity of Equation (15), we introduce the Lagrange multiplier and rewrite the optimisation problem as:

$$\begin{aligned} \min_{\lambda_j} \quad & \sum_{j=1}^n \frac{\lambda_j}{\mu_j - \lambda_j} - \alpha \sum_{j=1}^n \lambda_j, \\ \text{s.t.} \quad & 0 \leq \lambda_j < \mu_j, j = 1, \dots, n \end{aligned} \quad (16)$$

where  $\alpha$  is the Lagrange multiplier. Note that the above optimisation problem is segregable and the optimisation problem can be solved for each charger (eg,  $\min(\lambda_1 / \mu_1 - \lambda_1) - \alpha \lambda_1$ ). Therefore, for a single charger  $j$ , the optimisation problem takes the form of the one given in Equation (8) with  $c_w = 1$  and  $c_r = \alpha$  and the socially optimal solution, from Equation (12), is

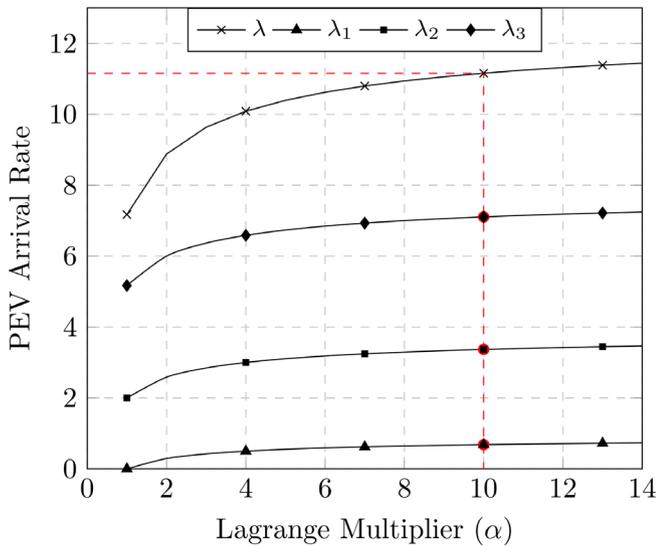
$$\lambda_j = \lambda^s(\alpha) := \left( \mu_j - \sqrt{\mu_j / \alpha} \right)^+, \quad \forall j. \quad (17)$$

The above solution is further subject to the choice of  $\alpha$  that satisfies  $\sum_j \lambda_j^s(\alpha) = \lambda$ . Let  $L(\cdot)$  denote the objective function given in Equation (16). Then, the following conditions are met for all chargers:

$$\begin{cases} L'_j(\lambda(j)) = \frac{\mu_j}{(\mu_j - \lambda_j)^2} = \alpha, & \text{if } \lambda_j > 0, \forall j \\ L'_j = \frac{1}{\mu_j} \geq \alpha, & \text{if } \lambda_j = 0, \forall j \end{cases}$$

Optimal arrivals rates can be computed in two ways: *graphical method* and *explicit solution*. In the first method, the optimal solution of Equation (16) is plotted with respect to a range of Lagrange multiplier  $\alpha$ . Then, for a specific arrival rate  $\lambda$ , corresponding  $\lambda_j^s(\alpha)$  are found by drawing a horizontal line from the  $y$ -axis until it reaches  $\lambda$  and drawing a vertical line to the  $x$ -axis. The points where the vertical line touches on  $\lambda_j^s(\alpha)$  traces are the optimal arrival rates. Next, we present a case study with three charger rates; 6 kW, 24 kW, and 48 kW, unit time demand is assumed to be 6 kWh (so that  $\mu_1 = 1$ ,  $\mu_2 = 4$ , and  $\mu_3 = 8$ ) and unit PEV arrival rate is assumed to be 11.15. As shown in Figure 5, the optimal arrival rates can be calculated as  $\lambda_1^s = 7.1$ ,  $\lambda_2^s = 3.36$ , and  $\lambda_3^s = 0.69$ . Note that a higher arrival rate is assigned to fast charger which minimises the waiting time.

Next, we present the explicit solution for socially optimal arrival rates  $\lambda_j^s$ . As a first step, let us sort the  $\mu_j$  in a



**FIGURE 5** Socially optimal arrival rates using graphical method for three chargers (6 kW, 24 kW, and 48 kW) [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

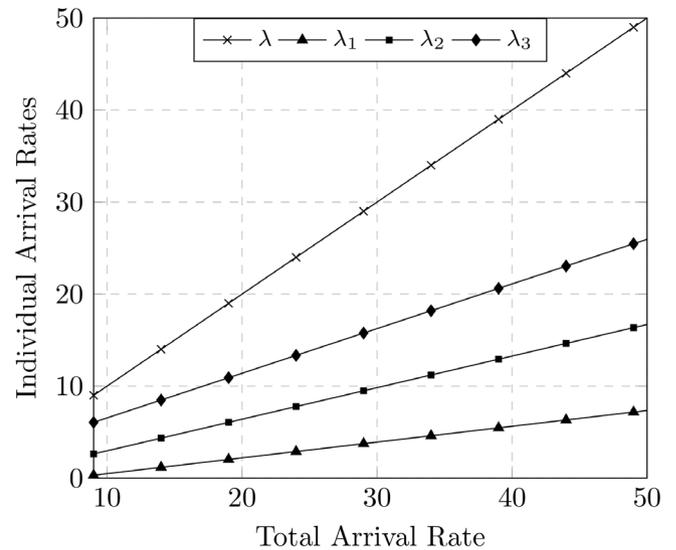
descending order  $\mu_1 \geq \mu_2 \geq \dots \geq \mu_n$ . Since  $\lambda^s(\alpha)$  is a continuous and strictly increasing function of the Lagrange multiplier  $\alpha$  (see Equation (17) and Figure 5) for  $\alpha \geq \mu_1^{-1}$ . Let  $\alpha(\lambda)$  denote the inverse of  $\lambda^s(\alpha)$ . Then,  $\alpha(\lambda)$  is solved separately for each interval, that is,  $\mu_1^{-1} \leq \alpha \leq \mu_2^{-1}$ ,  $\mu_2^{-1} \leq \alpha \leq \mu_3^{-1}$ , .... For instance, for the first interval  $\mu_1^{-1} \leq \alpha \leq \mu_2^{-1}$ , socially optimal arrival rates are

$$\begin{aligned} \lambda_1^s(\alpha) &= \mu_1 - \sqrt{\mu_1/\alpha} \\ \lambda_j^s(\alpha) &= 0, j = 2, \dots, n \end{aligned}$$

From the above equations, it can be seen that  $\lambda_1^s(\lambda) = \mu_1 - (\mu_1/\sqrt{\mu_1})(\mu_1 - \lambda) = \lambda$ . This approach can be expanded for each interval and the following explicit solution can be devised

$$\begin{aligned} \lambda_j^s(\alpha) &= \mu_j - \left( \frac{\sqrt{\mu_j}}{\sum_{i=1}^k \sqrt{\mu_i}} \right) \left( \sum_{i=1}^k \mu_i - \lambda \right), j = 1, \dots, k \\ \lambda_j^s(\alpha) &= 0, j = k + 1, \dots, n \end{aligned} \quad (18)$$

Next, we present a sample evaluation. Let us assume that the total arrival rate is  $\lambda = 12$  and there are three chargers as described above. Then from Equation (18), the solution would be  $\lambda_1 = 0.829$ ,  $\lambda_2 = 3.657$ , and  $\lambda_3 = 1 = 7.514$ . Similar to the previous case, more customer demand is allocated to the first charger with 48 kW rating. Moreover, in Figure 6 a wider range of calculations is presented.



**FIGURE 6** Socially optimal arrival rates using explicit solution for three chargers (6 kW, 24 kW, and 48 kW)

### 3.1 | Individual optimality

We proceed to calculate individually optimal arrival rates. As described in the previous section, allocating arrival rates to each charger  $j$  is equivalent to assigning the probability  $a_j = \lambda_j/\lambda$  of joining the charger  $j$ . For a new customer who chooses charger  $j$  with probability  $a_j = \lambda_j/\lambda$ , the expected waiting time is  $(\mu_j - \lambda_j)^{-1}$ . Similarly, the overall waiting time becomes  $\sum_{j=1}^n a_j (\mu_j - \lambda_j)^{-1}$ . Recall from Section 2.2.1 that individual optimality is achieved when customers have no incentive to deviate from the allocated arrival rates. The optimality conditions will hold if and only if the following conditions hold: (a) the objective function (waiting times) attain the same value for all allocation combinations, that is,  $(\mu_j - \lambda_j)^{-1} = (\mu_k - \lambda_k)^{-1}$  for  $\lambda_j > 0$  and  $\lambda_k > 0$ , and (b) the waiting time is less than the average service duration, that is,  $(\mu_j - \lambda_j)^{-1} \leq \mu_k^{-1}$ , if  $\lambda_j > 0$  and  $\lambda_k = 0$ . Individual optimality can also be written in the following format for  $j = 1, \dots, n$ ,

$$\begin{aligned} \frac{1}{\mu_j - \lambda_j} &= \alpha, \text{ if } \lambda_j > 0, \\ \frac{1}{\mu_j} &\geq \alpha, \text{ if } \lambda_j = 0 \end{aligned}$$

where  $\alpha$  is the Lagrange multiplier. To that end, individually optimal arrival rates can be written as  $\lambda_j^e(\alpha) = (\mu_j - 1/\alpha)^+$ , for  $j = 1, \dots, n$ . Next, we calculate individually optimal arrival rates and compare rates with socially optimal arrival rates discussed above. In Figure 7, optimal arrival rates are computed for the same three charger rates.

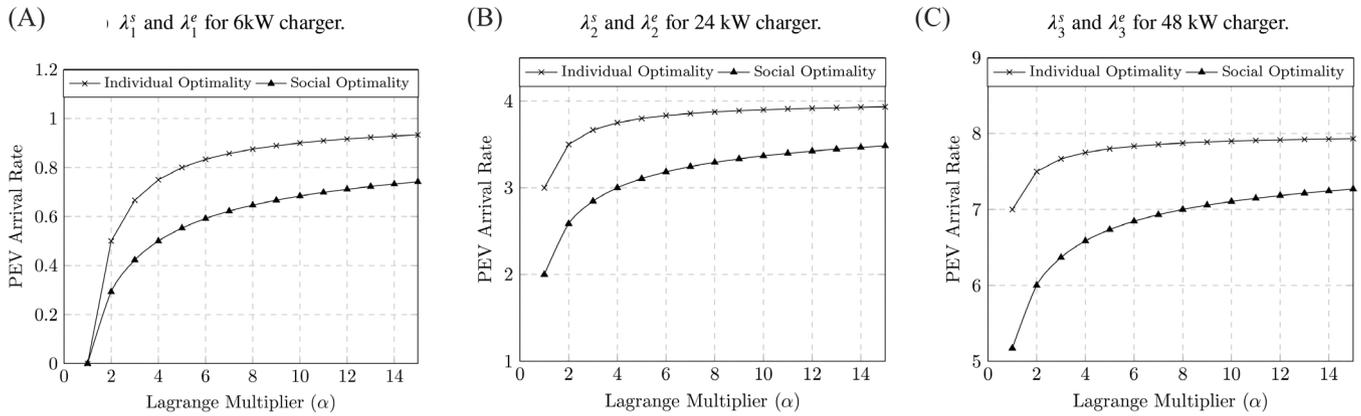


FIGURE 7 Comparison of social and individual optimality

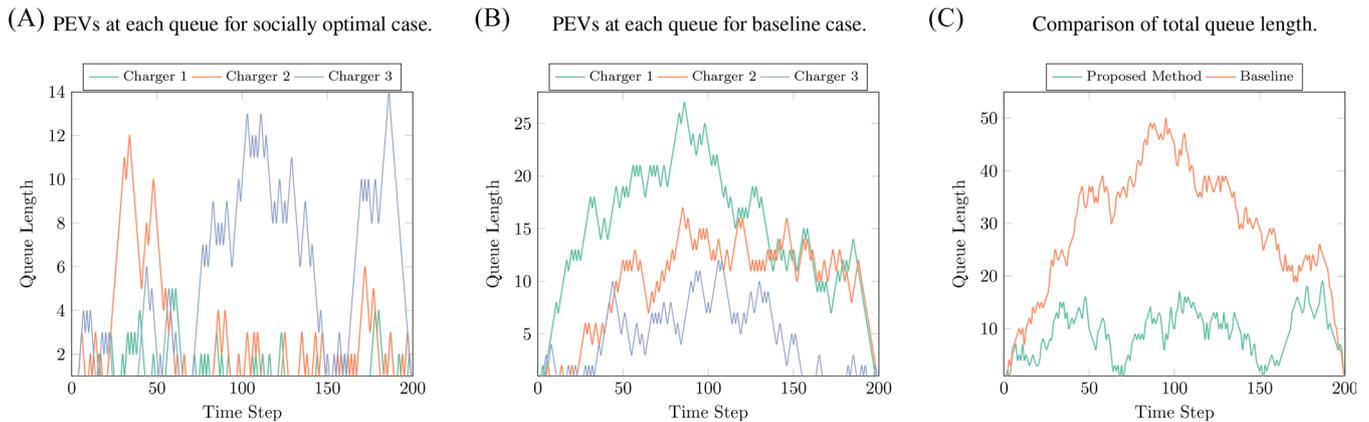


FIGURE 8 Discrete event simulation flow diagram [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

The results confirm the discussion in the previous section and the relationship between social and individual optimality given in Equation (14) is clearly depicted in this figure as for each evaluation social optimality is attained at a lower arrival rate than the individual optimality.

### 3.2 | Case study with discrete event simulation

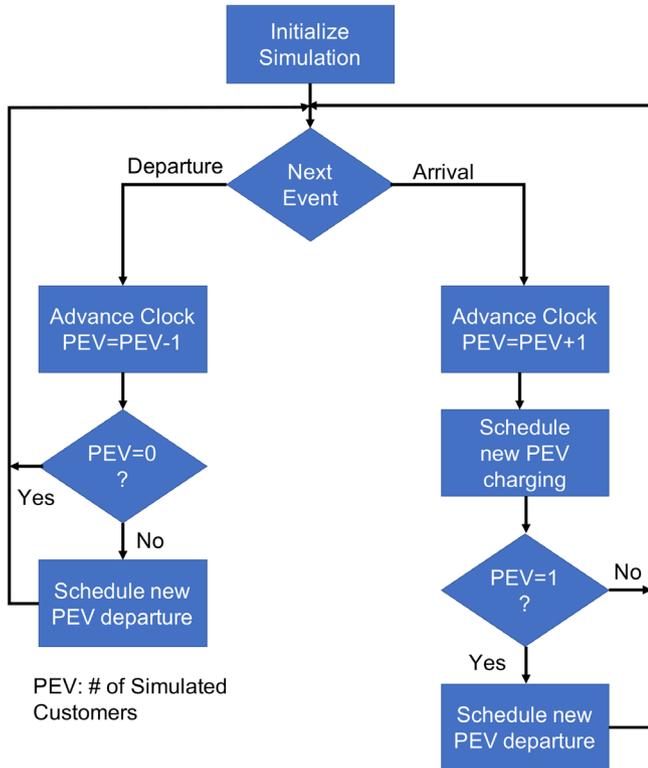
The proposed optimisation method is evaluated with a simulation-based case study with the following parameters. Similar to preceding examples, the parking lot has three charger types, 6 kW, 24 kW, and 48 kW with  $\mu_1 = 1$ ,  $\mu_2 = 4$ , and  $\mu_3 = 8$ . Total PEV arrival rate is assumed to be  $\lambda = 11.125$ . Considering the stability condition  $\mu_j > \lambda_j$ , the following arrival rates are assigned in the baseline scenario;  $\lambda_1^b = 0.98$ ,  $\lambda_2^b = 3.98$ , and  $\lambda_3^b = 6.165$ . For the optimal allocation case, optimisation problem given in Equation (15) is solved and the following rates are obtained for socially optimal case  $\lambda_1^s = 0.375$ ,  $\lambda_2^s = 3.375$ , and  $\lambda_3^s = 7.375$ . Each charger is simulated with discrete event

simulation (DES) developed in Matlab (flow diagram depicted in Figure 8).<sup>41</sup> Details of the DES are given as follows. The system state is described by the number of PEVs in the station. The system state changes with one of the two possible events: PEV arrival or departure. As shown in the flow diagram (Figure 8), an arrival increases the system state by 1 and a departure decrements it by 1. In the simulation, a module (subroutine) for each event is created for sequencing the event occurrences and executing the appropriate event at the right time. For each module (arrival and departure), the following actions are taken: internal clock is updated; state vector is updated (number of PEVs) according to the event; update a counter that keeps track of the occurrence of this event; schedule the time of the next event of the same type; and update the other system variables such as service durations and waiting times that are used to compute the performance of the system. Note that the service and arrival patterns follow Poisson distribution, hence, the next arrival or departure is generated by generating a random number from an exponential distribution. The simulation is repeated until 100 000 PEVs are served to capture

**TABLE 3** Simulation results for parking lot with three different chargers

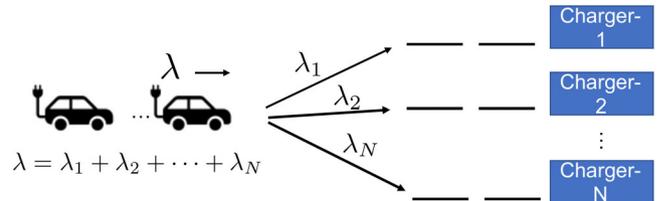
	Charger 1 (6 kW)		Charger 2 (24 kW)		Charger 3 (48 kW)		Overall	
	PM	BS	PM	BS	PM	BS	PM	BS
Waiting time (h)	1.59	6.94	1.23	1.59	0.6	0.5	<b>1.14</b>	<b>3.01</b>
Queue length (# of PEVs)	1.05	6.32	4.32	5.93	4.42	3.32	<b>3.26</b>	<b>5.19</b>

Note: Comparison of proposed method (PM) and baseline scenario (BS). Results show average values.

**FIGURE 9** Evaluation of proposed optimisation method [Colour figure can be viewed at wileyonlinelibrary.com]

steady-state probability distribution. Each simulation is repeated 30 times to capture randomness and results are presented in Table 3. For the proposed method socially optimal arrival rates are used (alternatively individual optimality could be adopted as well). An analysis of DES techniques is presented in Reference 41.

For each charger and the overall parking lot, average waiting times (hours) and queue lengths are presented. It can be seen that waiting times are reduced from 3.01 hours to 1.14 hours which is reasonable considering long parking durations. Similarly, waiting times for chargers 1 and 2 are considerable reduced, while the waiting time for charger 3 is slightly higher than baseline scenario because the socially optimal arrival rate is higher than the baseline scenario ( $\lambda_3^s > \lambda_3^b$ ). Moreover, Figure 9 presents the simulated behaviour of each charger and the number of PEVs waiting in the parking to be served.

**FIGURE 10** Addressing capacity scaling problem with Poisson superposition property [Colour figure can be viewed at wileyonlinelibrary.com]

Recall from the first section and reference<sup>20</sup> that more than 80% of the customers have more than 1 hour of laxity. Hence, reducing waiting times via optimal arrival rate assignments improves service quality for customers.

### 3.3 | Capacity scaling

Capacity planning frameworks discussed in the previous sections assumed single or multiple charger technologies and each station is equipped with one charger of each type. By assuming that the parking lots at commercial premises have large physical space, the proposed system model can be scaled-up by simply assuming that multiple queues at different floors or parts of the site can co-exist. Mathematically, the scaling problem can be addressed by the superposition property of Poisson process detail of which is given as follows. The superposition of the Poisson process states that the superposition (union) of independent Poisson point processes  $N_1, N_2, \dots, N_n$  with mean rates  $\lambda_1, \lambda_2, \dots, \lambda_N$  will also be a Poisson process with mean  $\sum_{i=1}^N \lambda_i$ . In other words, the union of multiple Poisson processes is another Poisson process.<sup>42</sup> As shown in Figure 10, PEV arrivals (with rate  $\lambda$ ) can be split into  $N$  different chargers and the total capacity calculation becomes solving the optimisation problems for  $N$  different chargers with rate  $\lambda_i, i = 1, \dots, N$ . For instance, in Figure 2 the optimal service rate for a charger was 14.48 kW. Now suppose that there are 5 chargers and the total arrival rate (per hour) is  $\lambda = 5$  and for the sake of simplicity it is assumed that each charger demand is equally shared, that is,  $\lambda_1 = \dots = \lambda_5 = 10$ . Then, the total system capacity would be  $5 \times 14.48 = 72.4$  kW.

## 4 | CONCLUSIONS

In this paper, we have presented optimal design framework for charging stations located at commercial premises. We considered the following two cases. The first one is parking lots located in workplaces or park and ride facilities. In such facilities, vehicles remain parked during most of the working hours and customer arrival and departure statistics are predictable. Hence, we proposed an optimisation framework to calculate station capacity with respect to customer demand and service waiting times. In the second problem, we assumed that parking lot is located at a public place such as shopping mall or hospital. In this case, parking durations were assumed to be shorter. In this case, we proposed an optimisation framework to calculate optimal arrival rates for a given station capacity. The stations were modelled as M/M/1 queues and linear cost functions were adopted for service and waiting costs. Using the convexity of the queueing model, closed-form solutions were devised using Lagrange multiplier techniques. Moreover, we discussed both social and individual optimality and solved case studies for both cases. Next, we expanded the second problem and assumed that there can be more one charger technology in a parking lot. We solved optimal arrival rates for each charger type. In the final part, a case study was presented to show how the optimal load allocation improves the station's performance in serving customers. A discrete event simulation was performed and it was shown that average service waiting time was reduced by 60% while the station waiting queue has reduced by 42%. Moreover, we have shown that the station capacity can be scaled linearly by using superposition property, hence, proposed calculation can be adopted to any station with an arbitrary number of chargers. In this paper, the optimal arrival rate calculation for the multiple charger case was devised for  $c_w = 1$  and  $c_r = \alpha$ . As a future step, we will find a general case solution for the optimisation problem (3). In addition, we will study non-linear cost functions for electricity and waiting times.

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### NOMENCLATURE

$\alpha$	Lagrange multiplier
$\lambda$	PEV arrival rate per unit time (PEVs/h)
$\lambda_e; \mu_e$	individually optimal arrival and departure rates
$\lambda^s; \mu^s$	socially optimal arrival and departure rates

$\mu$	PEV service rate per unit time (PEVs/h)
$B(\lambda)$	net benefit function
$C(\mu)$	station cost function
$c_p$	service cost rate (USD/kW)
$c_r$	average number of customers in the station
$c_w$	customer waiting cost (USD/h)
$p^s$	acceptance probability
$W(\mu)$	average number of customers in the station

### DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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