

# Topological algorithm to form power of compensating devices of distribution networks

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**Abstract.** Effectiveness of providing consumers with technical limits of voltage deviation relates with integrating digital technologies in automation voltage regulation control. Automatized compensating devices, intended control voltage, tunes without consideration of influence of other compensating devices installed for the same purpose. It leads to their energetical efficiency deterioration. Thus, there is a need to develop new algorithms needed power of compensating devices.

## 1 Introduction

Compensation of reactive power in distribution networks extends to all power consumers and is mandatory for all power supply organizations regardless of their departmental affiliation [1]. It is known that the transfer of reactive power from the 6- 35 kV network to the low-voltage network, as a rule, is economically unprofitable. Continuous control of compensating device modes is required in order to provide standard parameters of electric power quality, which is associated with great difficulties due to mutual influence and variety of their modes. The most practical and theoretical importance is the development of new methods for controlling the reactive power of compensating devices of distribution networks [2]. Voltages at different points of distribution networks vary and depend on network parameters and flow distribution mode. Voltage deviation from nominal values in distribution networks is limited by quantitative values of electromagnetic compatibility [3,4]. Stringent requirements for maintaining voltage within technically defined limits necessitate its automatic control. One of the main methods of voltage control in electrical networks is the use of compensating devices at lowering substations operating on the principle of voltage stabilization [5,6]. In order to increase the efficiency of compensating devices, it is necessary to determine the conditions of the adequacy of the control range of reactive power to ensure the desired voltage level on the substation low-voltage buses in the mode of greatest loads. This problem is solved in a traditional way individually for each load node based on the results of calculating the steady-state mode of electric networks of power systems. The solution of this problem is directly related to conditions of reactive power balance of the given unit. The installation of a compensating device at an arbitrary point in the network leads to the redistribution of reactive power over the distribution

network as a whole. Achieving the desired voltage level at the same time becomes a rather difficult task. Therefore, the development of a systematic method for determining the required reactive power of compensating devices is an urgent task in the conditions of the digitalization of voltage control in electrical networks.

## 2 Topological equations of node voltages

Topological models of node voltages allow you to define mode parameters based on natural parameters and node loads of the network [7]. In work [8] the method of calculation of system function of resistance with application of coefficients of distribution of setting currents determined on the basis of the topology of the electric network is developed. Developed algorithms are implemented in the form of software complex of search and determination of all possible and specific trees of a directed graph of the complex electrical network [9,10]. The distribution coefficients of the setting currents are formalized based on the topological expression [7]:

$$\underline{C}_{ij} = \frac{\sum F_{ij}}{\sum F} \quad (1)$$

where  $\sum F_{ij}$  - the algebraic sum of specific i-branch trees relative to the j-th node;

$\sum F$  - an arithmetic sum of all possible graph trees.

Node voltages are determined by topological expression obtained by analytical transformation of known equations of electric state of networks [13-16]:

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$$\dot{U} = \dot{U}_0 - \underline{C}^t \underline{Z}_\delta \underline{C} \hat{U}_\delta^{-1} \hat{S} \quad (2)$$

where  $\underline{C}$  - matrix of complex coefficients of assignment currents distribution;  $\underline{Z}_\delta$  - matrix of complex coefficients of assignment currents distribution;  $\dot{U}_0$  - base node voltage;  $\hat{U}_\delta$  - diagonal matrix of a conjugated complex of node voltages;  $\hat{S}$  - vector of a conjugated complex of nodal loads;  $t$  - sign of matrix transposition.

### 3 Modelling of compensating devices power

As is known, calculations of stationary modes of electric networks are accompanied by determination of the required capacities of compensating devices, which provide the desired voltage on the buses of secondary windings of transformers [2,5]. This made it possible to formulate the task of modelling the power of compensating devices from the position of synthesis of the permissible mode on the voltage of electric networks. Solution of this problem is achieved by conversion of topological equation (2) relative to power of compensating devices.

Suppose that the provision of the required voltage is achieved by switching on static capacitors on the buses of the low voltage winding of the transformer. Therefore, the desired voltage values on the downshift buses can be considered a predetermined value. Then, it is necessary to find the power of the compensating devices installed to provide the desired node voltage.

The analytical relationship between the power of the compensating devices and the mode and network parameters can be established based on the transformation of equation (2).

Consider a complex power grid where the desired node voltage value is provided by adjusting the power of the compensating devices. Then equation (2) is written as:

$$\dot{U}_{id} = \dot{U}_0 + \sum_{j=1}^n \sum_{k=1}^m C_{ik}^t Z_k C_{kj} \dot{U}_{jd}^{-1} (\dot{S}_j - iQK_j) \quad (3)$$

where  $\dot{U}_{id}$  - complex desired voltage value of the i-th node;  $QK_j$  - power of compensating device of j-th node.

Equation (3) with respect to the real and imaginary part is written as follows:

$$\begin{aligned} \omega_{\dot{U}_i}(\delta, QK) = & -U_{id} \cos \delta_i + U_0 + \\ & + \sum_{j=1}^n U_{\Delta ij} \cos(\delta_j - \varphi_j + \psi_{ij}) + \\ & + \sum_{j=1}^n Z_{ij} U_{jd}^{-1} QK_j \sin(\delta_j + \psi_{ij}), \end{aligned} \quad (4)$$

$$\begin{aligned} \omega_{\dot{U}_i}''(\delta, QK) = & -U_{id} \sin \delta_i + \\ & + \sum_{j=1}^n U_{\Delta ij} \sin(\delta_j - \varphi_j + \psi_{ij}) - \\ & - \sum_{j=1}^n Z_{ij} U_{jd}^{-1} QK_j \cos(\delta_j + \psi_{ij}), \end{aligned} \quad (5)$$

where

$$Z_{ij} = \sqrt{(\text{Re} \sum_{j=1}^m C_{ik}^t Z_j C_{jj})^2 + (\text{Im} \sum_{j=1}^m C_{ik}^t Z_j C_{jj})^2}$$

- mutual system function resistance module;

$$U_{\Delta ij} = Z_{ij} U_{jd}^{-1} \sqrt{P_j^2 + Q_j^2}$$

- module of the partial node voltage drop of the i-th node;

$U_{id}$  - desired voltage of i-th node ;

$P_j, Q_j$  - active and reactive power of i-th node ;

$U_i', U_i''$  - real and imaginary parts of complex voltage of i-th node ;

It can be seen from (4) and (5) that for a complex circuit of an electric network consisting of n independent nodes, a system of 2n equations with 2n unknowns is composed, which can be solved by the Newton method.

If we neglect the influence of the phases of the node voltages on the values of the driving currents, then the calculated expressions for the partial derivatives of the Jacobi matrix are simplified since the equations of the balance of the real and imaginary parts of the voltage of the i-th node are written in the form:

$$\begin{aligned} \partial \omega_{U_i'}(\delta, QK) = & -U_{i0c} \sin \delta_i + U_0 + \\ & + \sum_{j=1}^n U_{\Delta ij} \cos(-\varphi_j + \psi_{ij}) + \\ & + \sum_{j=1}^n Z_{ij} U_{jd}^{-1} QK_j \sin \psi_{ij} = 0, \end{aligned} \quad (6)$$

$$\begin{aligned} \partial \omega_{v_i}(\delta, Q) &= -U_{i\text{inc}} \sin \delta_i + \\ &+ \sum_{j=1}^n U_{\Delta ij} \sin(-\varphi_j + \psi_{ij}) - \\ &- \sum_{j=1}^n Z_{ij} U_{j\text{inc}}^{-1} QK_j \cos \psi_{ij} = 0. \end{aligned} \quad (7)$$

The following expressions are valid for Jacobi matrix elements:

$$\begin{aligned} \frac{\partial \omega_{v_i}(\delta, QK)}{\partial \delta} &= U_{id} \sin \delta_i; \\ \frac{\partial \omega_{v_i}(\delta, QK)}{\partial QK} &= \sum_{j=1}^n Z_{ij} U_{jd}^{-1} QK_j \sin \psi_{ij}; \\ \frac{\partial \omega_{v_i}(\delta, QK)}{\partial \delta} &= -U_{id} \cos \delta_i; \\ \frac{\partial \omega_{v_i}(\delta, QK)}{\partial QK} &= -\sum_{j=1}^n Z_{ij} U_{jd}^{-1} QK_j \cos \psi_{ij}. \end{aligned}$$

When calculating distribution networks, the voltage of the *i*-th node is described by the equation of the form:

$$\begin{aligned} \omega_{\dot{U}_i}(QK) &= -U_{id} + U_0 + \\ &+ \sum_{j=1}^n U_{\Delta ij} \cos(-\varphi_j + \psi_{ij}) + \\ &+ \sum_{j=1}^n Z_{ij} U_{jd}^{-1} QK_j \sin \psi_{ij} = 0 \end{aligned} \quad (8)$$

It can be seen from (8) that the compensating devices' powers are determined by solving a system of *n* linear equations.

Based on providing the desired node voltage at a known value prior to reactive power compensation, the system of equations for calculating power of the compensating device can be written as:

$$\omega_{\dot{U}_i}(QK) = -U_{i\text{inc}} + U_i + \sum_{j=1}^n Z_{ij} U_{j\text{inc}}^{-1} QK_j \sin \psi_{ij} = 0 \quad (9)$$

To calculate the simplest scheme, equation (9) is easily converted into a known expression:

$$QK = \frac{U_d - U}{X} U_d \quad (10)$$

where  $X = Z \sin \psi$  - the reactance of the network;

$U$  - unit voltage value before reactive power compensation;

$U_d$  - value of the desired node voltage after switching on the compensating device with power.

#### 4 Example of compensating device power calculation

As an example, the calculation of the required compensating device capacity for the section of the real 10 kV "Valikhanov" distribution network, shown in Fig. 1, is given.

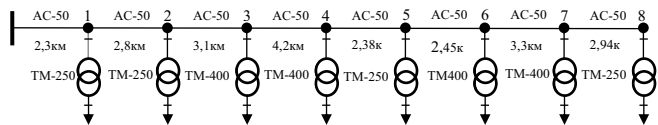


Fig. 1. 10 kV "Valikhanov" distribution network.

The settlement scheme is submitted in fig. 2.

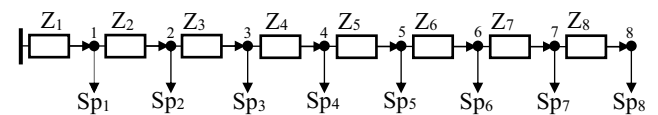


Fig. 2. Computational scheme.

The parameters required to perform the calculations are presented in matrix form:

$$Z_b = \begin{bmatrix} 1.368 + j0.902 \\ 1.666 + j1.098 \\ 1.845 + j1.215 \\ 2.499 + j1.646 \\ 1.416 + j0.933 \\ 1.458 + j0.96 \\ 1.963 + j1.294 \\ 1.749 + j1.152 \end{bmatrix} \text{ Ohm}$$

$$S = \begin{bmatrix} S_{p1} \\ S_{p2} \\ S_{p3} \\ S_{p4} \\ S_{p5} \\ S_{p6} \\ S_{p7} \\ S_{p8} \end{bmatrix} = \begin{bmatrix} 0.123 + j0.034 \\ 0.12 + j0.031 \\ 0.189 + j0.042 \\ 0.185 + j0.052 \\ 0.111 + j0.037 \\ 0.184 + j0.057 \\ 0.193 + j0.047 \\ 0.126 + j0.031 \end{bmatrix} \text{ MVA}$$

distribution coefficients of node currents

$$C=M^{-1}=\begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

Taking  $U_0 = 10.5$  kV, we determine the node voltages by formula (2):

$$U = U_0 - C^T Z_b C U S = \begin{bmatrix} 10.302 - j0.066 \\ 10.085 - j0.138 \\ 9.87 - j0.209 \\ 9.633 - j0.285 \\ 9.53 - j0.318 \\ 9.444 - j0.347 \\ 9.371 - j0.373 \\ 9.346 - j0.382 \end{bmatrix} \text{ kV}$$

It can be seen from the calculations that the node voltage deviations are within the permissible limits only for the first five nodes. In order to check the developed algorithms, we determine the values of the required CP capacities from the position of synthesis of the mode by voltage. Suppose the desired node voltage values are given by:  $U_{1d}=10.325$  kV,  $U_{2d}=10.136$  kV,  $U_{3d}=9.953$  kV,  $U_{4d}=9.758$  kV,  $U_{5d}=9.679$  kV,  $U_{6d}=9.618$  kV,  $U_{7d}=9.582$  kV,  $U_{8d}=9.58$  kV and it is necessary to find the power of the compensating devices providing the predetermined mode.

Having solved the system of equations (9), we determine the compensating devices powers, which are equal to:

$$Q_{CD} = \begin{bmatrix} 0.075 \\ 0.069 \\ 0.074 \\ -0.148 \\ 0.319 \\ 0.12 \\ 0.005 \\ -0.144 \end{bmatrix} \text{ MVAr}$$

Installation of the compensating devices with the capacities defined above shall provide the desired values

of the node voltages. In order to check the correctness of the CP power determination, we will calculate the voltage mode taking into account the reactive powers of the compensating devices according to formula (2).

The calculations confirm the efficiency of the topological algorithm in determining the necessary capacities of compensating devices to provide the desired voltages in the nodes of distribution networks.

## 5 Conclusion

1. A topological method for calculating the power of compensating devices from the position of synthesis of modes by voltage has been developed.
2. The developed algorithm can be used to determine the location and type of CS installation to provide the desired voltage level.

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