A NEW METHOD TO DETERMINE ELASTO-PLASTIC J -INTEGRAL FOR STEEL PIPES WITH LONGITUDINAL SEMI-ELLIPTICAL SURFACE CRACKS

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ABSTRACT

Steel pipes have been widely used in the water and petroleum industries where crack-like flaws have long been recognised as a major cause of their failures. This paper intends to develop a new method for determining the elasto-plastic J -integral of cracked pipes made of all types of steel. A solution to elasto-plastic J -integrals is derived for external longitudinal semi-elliptical surface cracks in pressurised steel pipes. A meshing technique is developed in the detailed three-dimensional finite element analysis that incorporates the mixed types of quadratic tetrahedron and hexahedron elements. The developed method is verified with detailed finite element results. It is found in the paper that the general trend of normalised fully plastic J -integral changes with the change of crack depths and aspect ratios, and the magnitude of normalised fully plastic J -integrals can be significantly affected by material properties and pipe dimensions. The developed method can provide a useful tool for the failure assessment of pressurized steel pipes, allowing for a wide range of pipe dimensions, crack geometries and material properties.

KEYWORDS

Elasto-plastic J ; thin-wall pipes; surface crack; 3D finite element; yielding plateau.

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1. INTRODUCTION

The surface cracks have been identified as the main cause of failures for pressurised steel pipes. To quantify the stress state in the vicinity of a crack front, a number of researchers, e.g., Anderson (2017), Li and Yang (2012), Laham (1999), and Raju and Newman (1982) etc., have developed various solutions to stress intensity factors (SIF) for the cracked pipes, based on linear elastic fracture mechanics (LEFM) (Fu et al. 2019 and Wang et al. 2019). As it is known, SIF solutions are only valid for linear elastic materials or ductile materials under small-scale yielding condition, where the deformation of the material is predominately linear elastic. For pipes made of ductile or elasto-plastic materials, such as steel pipes, however, large plastic deformation can occur around a crack before the fracture initiates. In this situation, the stresses around the crack-tip are relaxed, and as a result, the SIF solutions derived from LEFM become inapplicable in determining the real stress-state around the crack. To overcome this problem, the elasto-plastic fracture mechanics has been developed (Anderson 2017), in which J -integral, among others, is a key parameter.

A review of published literature (e.g., Surh et al. 2017; Cho et al. 2011; Kim et al. 2004; Lei 2004; Kumar et al. 1984,1981) shows that considerable research has been undertaken to determine the J -integral for cracked pipes made of ductile or elastoplastic materials. For example, the solutions to J -integral for through-wall cracks and continuous (full-length) part-through cracks in pipes were developed by the General Electric/Electric Power Research Institute (GE/EPRI) based on shell line-spring finite element (FE) models. The results of these solutions are well documented in EPRI-NP-1931 (Kumar et al. 1981) and EPRI-NP-3607 (Kumar et al. 1984). Other researchers, e.g., Park et al. (2019), Surh et al. (2017), Jeong et al. (2016) and Kim et al. (2001, 2002a, 2002b), further extended the existing solutions to J -integral for through-wall cracks in pipes under various loading conditions. As the crack-like defects

detected in real pipes are usually semi-circular or semi-elliptical (Bloom 1983), most studies (e.g., Liu et al. 2018, Cho et al. 2011 and Kim et al.(2004) investigated the J -integral for semi-elliptical cracks with focus on circumferential cracks.

Further review of the literature suggests that there is a lack of solutions to J -integral for the external longitudinal semi-elliptical surface cracks (LSSC) in steel pipes. The reason is that most solutions to J -integral were developed for the nuclear steel pipes (Østby et al. 2005), whereby circumferential cracks are considered more significant than the longitudinal ones in their risk assessment (Kim et al. 2003). The external LSSC, however, is one of the most commonly encountered defects in buried steel pipes used for water, oil and gas transport (Wang et al. 2019 and Makar et al. 2001), which is often initiated by corrosion pits (Koduru and Nessim 2017). As it is known a solution to elasto-plastic J -integral for circumferential cracks in pipes is not straightforward to that for longitudinal cracks. More sophisticated modelling and meshing techniques in 3D finite element analysis (FEA) is required due to the 3-D nature of LSSC.

Some researchers, e.g., Wang (2006), Lei (2004), Kim et al. (2002c) and Yagawa (1993) have derived solutions to the elasto-plastic J -integral for semi-elliptical surface cracks in plates under tension. They suggested that these solutions could be used for estimating the J -integral for cracked pipes. However, Kiefner et al. (1973) and Anderson and Sullivan (1966) have proven that application of the plate-based solutions to a curved structure subject to internal pressure, e.g., pressurised pipes, results in an underestimate of J -integral which is unconservative for structural design and assessment, leading to ultimate failures. The underestimate of J -integral is mainly caused by the neglect of the bulging resulting from the pressure acting on the curved walls (Kiefner et al. 1973), which can produce additional stress near the crack tip, leading to the increase of J -integral. Evidently there is a clear need to develop new solutions to J -integral for external LSSC in steel pipes.

The review also shows that most of current research (e.g., Park et al. 2019; Surh et al. 2017; Cho et al. 2011; Kim et al. 2004) used the reference stress (RS) method to estimate the elasto-plastic J integral for cracked structures. This is because the GE/EPRI method cannot determine the J -integral accurately when the materials do not follow stress-strain law of Ramberg-Osgood (R-O) (EDF Energy 2015). The stress-strain curve for those materials which follow the R-O model does not show a clear yielding plateau. Figure 1 shows the difference in stress-strain curves between R-O material without clear yielding and materials with clear yielding plateau, referred to as non-R-O materials in this paper. Since most structural steels, in particular steels used for pipes, exhibit a clear yielding plateau there is a compelling need to incorporate the yielding characteristics into the R-O model fitting.

Figure 1. Schematic of R-O and non-R-O materials

On the other hand, the difficulties in use of RS method for cracked pipes have been well recognised (Cho et al. 2011, Kamaya and Machida 2010, Staat and Vu 2007, Kim et al. 2003). In RS method, an appropriate (optimized) reference stress for a cracked structure needs to be found first based on its limit load. This often requires detailed limit load analysis based on the 3D FEA or experimental work, as well as the trial-and-error process (Cho et al. 2011). However, for a pressurized pipe subject to a semi-elliptical surface crack, available limit loads published in literature are not consistent and the majority of limit load models are inaccurate (Kamaya and Machida 2010, Staat and Vu 2007, Kim et al. 2003). In comparison, the GE/EPRI method is more practical and easier to use. As the inaccurate estimation of elasto-plastic J -integral by the general GE/EPRI method is mainly caused by the use of R-O mode, rather than by inherent deficiency of the method (Chattopadhyay 2006; Kim et al. 2001, 2002c), one feasible solution is to modify the original R-O model with parameter fitted from the test results to include both non-yielding materials (R-O materials) and materials with clear yielding plateau (non R-O

materials). With this modification the GE/EPRI method can be applied to all materials regardless of their yielding behaviour.

This paper aims to develop a method for estimating the elasto-plastic J -integral of cracked pipes made of all types of steel. Solutions to elasto-plastic J -integrals are derived for external longitudinal semi-elliptical surface cracks (LSSC) in pressurised steel pipes. The GE/EPRI method is modified to include both R-O and non-R-O materials in determining the elasto-plastic J -integral of cracked pipes. A meshing technique is developed in the detailed three-dimensional finite element analysis (3D FEA) that incorporates the mixed types of quadratic tetrahedron and hexahedron elements. The developed method is also verified for both R-O and non-R-O materials. Effects of a wide range of pipe dimensions, crack geometries and material properties on the change of J -integrals along the crack front are thoroughly investigated.

The merit of the developed method is that it effectively extends the application of the general GE/EPRI method to almost all types of steel which is the primarily construction material for all infrastructure. It also provides a useful tool for the failure assessment of pressurized steel pipes. The significance of this paper is that a 3D FEA and modified GE/EPRI method are combined to derive the solution to J -integral for external LSSC, which enables the consideration of a wide range of pipe dimensions, crack geometries and material properties.

2. DEVELOPMENT OF THE METHOD

As it is known an accurate J -integral for a pipe with LSSC can be obtained directly from 3D finite element analysis (FEA). However, it is often computation intensive and requires a high level of programming skills. Consequently, undertaking a detailed 3D FEA for any specific pipe and crack dimensions is unrealistic for practical engineering use. To overcome this, a practical method that can determine the J -integrals of cracked pipes by using a limited number of

solutions predetermined from FEA is to be developed here.

According to the GE/EPRI method (Kumar et al. 1981), a simple expression for the elasto-plastic J -integral for cracked structures made of R-O materials can be expressed as follows

$$J(a, P) = J^e(a_e, P) + J^p(a, P) \quad (1)$$

$$\text{with } J^p(a, P) = \alpha \varepsilon_o \sigma_o L h(g_i, n) \left(\frac{P}{P_o} \right)^{n+1} \quad (2)$$

where J^e is the elastic part of the J -integral, J^p is the fully plastic part of J -integral, a is the crack length, P is the applied load, a_e is the effective crack length that includes Irwin's small-scale yielding correction, L is an arbitrarily defined characteristic length and $g_i (i = 1, 2, \dots)$ represent geometric properties of the cracked structure. In Equation (2), $h(g_i, n)$ is defined as the normalised fully plastic J -integral, which can be expressed from Equations (1) and (2) as follows

$$h(g_i, n) = \frac{J(a, P) - J^e(a_e, P)}{\alpha \varepsilon_o \sigma_o L} \left(\frac{P}{P_o} \right)^{-n-1} \quad (3)$$

The above equations are theoretically valid only for Ramberg-Osgood materials, i.e., materials whose stress-strain relation follows the Ramberg-Osgood (R-O) model, which can be expressed as follows (Kumar et al. 1981)

$$\frac{\varepsilon}{\varepsilon_o} = \frac{\sigma}{\sigma_o} + \alpha \left(\frac{\sigma}{\sigma_o} \right)^n \quad (4)$$

where α and n are material constants, σ_o is normalising stress and $\varepsilon_o = \sigma_o/E$.

To extend the application of general GE/EPRI method, i.e., Equations (1) – (3) to non-R-O materials, instead of using Equation (4), a modified R-O model was proposed by (Chattopadhyay 2006) as follows

$$\frac{\varepsilon}{\varepsilon_o} = \alpha^{\frac{1}{n+1}} \left(\frac{\sigma}{\sigma_o} \right)^n \quad \text{for } \frac{\sigma}{\sigma_o} \leq 1.0 \quad (5a)$$

$$\varepsilon = \varepsilon_1 + \frac{\varepsilon_{1,2} - \varepsilon_1}{0.2} \left(\frac{\sigma}{\sigma_o - 1} \right) \quad \text{for } 1.0 < \frac{\sigma}{\sigma_o} < 1.2 \quad (5b)$$

$$\frac{\varepsilon}{\varepsilon_o} = \alpha \left(\frac{\sigma}{\sigma_o} \right)^n \quad \text{for } \frac{\sigma}{\sigma_o} \geq 1.2 \quad (5c)$$

138 where ε_1 is the strain obtained from the true stress-strain curve at yield strength σ_o , and $\varepsilon_{1,2}$
 139 is the strain determined at $\sigma = 1.2\sigma_o$. It can be seen that the parameters of original R-O model,
 140 i.e., Equation (4), are modified in the form of Equation (5) with different parameters fitted from
 141 test results. Equation (5) can be applied to both R-O materials and non-R-O materials.
 142 Accordingly, Equation (2) can be reformulated as follows

$$J^p(a, P) = \alpha^{\frac{1}{n+1}} \varepsilon_o \sigma_o Lh(g_i, n) \left(\frac{P}{P_o} \right)^{n+1} \quad \text{for } \frac{\sigma}{\sigma_o} \leq 1.0 \quad (6a)$$

$$J^p(a, P) = J^{p1} + \frac{J^{p1,2} - J^{p1}}{0.2} \left(\frac{P}{P_o} - 1 \right) \quad \text{for } 1.0 < \frac{\sigma}{\sigma_o} < 1.2 \quad (6b)$$

$$J^p(a, P) = \alpha \varepsilon_o \sigma_o Lh(g_i, n) \left(\frac{P}{P_o} \right)^{n+1} \quad \text{for } \frac{\sigma}{\sigma_o} \geq 1.2 \quad (6c)$$

143 where J^{p1} is the plastic part of J -integral at $\sigma = \sigma_o$ and $J^{p1,2}$ is the plastic part of J -integral
 144 at $\sigma = 1.2 \sigma_o$.

145 By applying Equation (6) to a pressurised pipe subject to external longitudinal semi-elliptical
 146 surface cracks (LSSC) as shown in Figure 2, the elasto-plastic J -integral can be determined
 147 through superposing the elastic and fully plastic portions according to Equation (1) as follows

$$\hat{J} = J^e + \alpha^{\frac{1}{n+1}} \varepsilon_o \sigma_o Lh\left(\frac{2\theta}{\pi}, \frac{a}{c}, \frac{a}{d}, n, \frac{d}{R_i}\right) \left(\frac{P}{P_o} \right)^{n+1} \quad \text{for } \frac{\sigma}{\sigma_o} \leq 1.0 \quad (7a)$$

$$\hat{J} = J^e + J^{p1} + \frac{J^{p1,2} - J^{p1}}{0.2} \left(\frac{P}{P_o} - 1 \right) \quad \text{for } 1.0 < \frac{\sigma}{\sigma_o} < 1.2 \quad (7b)$$

$$\hat{J} = J^e + \alpha \varepsilon_o \sigma_o L h \left(\frac{2\theta}{\pi}, \frac{a}{c}, \frac{a}{d}, n, \frac{d}{R_i} \right) \left(\frac{P}{P_o} \right)^{n+1} \text{ for } \frac{\sigma}{\sigma_o} \geq 1.2 \text{ or R-O materials} \quad (7c)$$

where $\hat{J}$ is the elasto-plastic J -integral, θ is the eccentric anomaly of the crack ellipse, c is the half length of crack, d is the pipe wall thickness, R_i is the inner radius of pipe and $P_o = d\sigma_o/R_i$. Equation (7) is the modified GE/EPRI method which can determine the elasto-plastic J -integral for both R-O and non-R-O steels.

Figure 2. Schematic for (a) a pipe with a semi-elliptical external surface crack; and (b) surface crack

3. THREE-DIMENSIONAL FINITE ELEMENT ANALYSIS

To apply the above method for predicting the elasto-plastic J -integral of a cracked pipe, the key is to derive a series of solutions to normalised fully plastic J -integrals for various geometrical and material properties. For external LSSC, the three-dimensional finite element method is apparently necessary to take into account the 3-D nature of the stress state in the vicinity of crack. Therefore, three-dimensional finite element models, based on Abaqus 6.14 software (Abaqus 2014), are developed in this study to evaluate the J -integrals.

3.1 Development of FE Models

In the developed FE models, a wide range of the relatively thick-wall pipes ($d/R_i=0.1$) to the thin-wall pipes ($d/R_i=0.025$) were considered to cover the dimensions of commonly used water, oil and gas pipes. For the geometries of the surface cracks, different aspect ratios (a/c from 0.2 to 0.8) and depths (a/d from 0.2 to 0.8) were considered. Also, different strain hardening exponents (n from 3 to 10) were used in the Ramberg-Osgood material model based on deformation theory of plasticity. A total of 108 cases were considered with the variables

summarized in Table 1. The length of the pipe under analysis was taken to be 20 times that of the corrosion pit length to reduce the boundary effects. Because of the symmetry, a quarter of the cylinder was modelled as illustrated in Figure 3. The pressure was directly applied to the inner surface of the pipe as a distributed surface load. The applied load increased proportionally and monotonically. The Poisson's ratio $\nu = 0.3$, Young's modulus $E = 207$ GPa, yield stress $\sigma_0 = 210$ MPa and material constant $\alpha = 1$. It should be noted that the $h\left(\frac{2\theta}{\pi}, \frac{a}{c}, \frac{a}{d}, n, \frac{d}{R_i}\right)$ as determined through Equation (3) is independent of the material parameters (E, ν, σ_0, α) since it is normalized with respect to materials.

Table 1. Variables considered in the parametric FE analyses

Figure 3. Finite element model of a thin-wall pipe with external semi-elliptical surface crack

The focused 15-node wedges elements (C3D15) (see Figure 3) were used at the crack front. The rest of the domain integral regions were meshed by 20-node hexahedron elements (C3D20R) with reduced integration points. Due to the complexity of the crack geometry, a meshing technique was developed such that the irregular region close to the crack region was meshed with quadratic tetrahedron elements. The tie constraints were applied to the interfaces between the irregular regions and domain integral regions.

The domain integral method was used to evaluate the total J -integral and the elastic J -integral. To consider the crack-tip strain singularity appropriately in the present elasto-plastic analysis, the crack-tip nodes were set free to displace (not tied) to include both $1/r$ and $1/\sqrt{r}$ singularity terms. But at the intersection of the free surface and the crack front, this crack-tip strain singularity does not exist in the power-law materials (Anderson 2017, Sih and Lee 1989). To reduce the effect of singularity change, a very fine mesh with 90-180 element segments was used at the front of cracks. This fine mesh can also ensure the accuracy of the J -integrals when

the material becomes highly non-linear. Besides, since the J -integrals evaluated at the first block of elements are normally inaccurate, the results from this first domain were neglected and the J -integrals from the outer four blocks were averaged to obtain the final result of J -integral. After the elastic J -integrals and the total J -integrals are obtained from FEA, the normalised fully plastic J -integrals (h) can be determined through Equation (3).

The normalised fully plastic J -integral h largely depends on applied loads if the load is small (Wang 2016). When the applied load increases, its dependence gradually decreases, eventually diminishing when the load becomes sufficiently large (Fu et al. 2017). Wang (2016) proposed a criterion for determining the fully plastic conditions of a cracked body; that is, the plastic J -integral component accounts for more than 95% of the total J -integral for all points along a crack front. This criterion was used in the present study.

3.2 Verification of Finite Element Models

Before conducting FEA for cracked pipes, it is essential to verify the accuracy of proposed FE models. For the SIFs determined based on LEFM and FE models, extensive verifications were previously conducted by Wang et al (2017) and Li and Yang (2012). Thus, in this paper, only the 3D FE method for determining the fully plastic J -integrals need to be verified. Two examples were used for verification.

The first one is an embedded penny-shaped crack in an infinite body (see Figures 4a). The length, width and height of the body with the embedded penny-shaped crack were taken to be 20 times that of the crack size (i.e., $H = 20a$) in the 3D FE model to reduce the boundary effect. The R-O material model was used with the strain hardening exponent of 3, 5 and 10. The cracked body was subject to far-field tensile stresses that are perpendicular to the crack surface. To determine the normalised fully plastic J -integral (h), Equation (4) was used with the inputs

(elastic J -integral, total J -integral) directly calculated by FE model. Also, ' P/P_o ' was replaced with ' σ_∞/σ_o ' and the characteristic length (L) was taken as the depth of crack (a) in Equation (4).

In relation to the crack in the infinite body, an analytical solution, which was developed by He and Hutchinson (1981) based on a perturbation technique as follows

$$h = \frac{6}{\pi} \left(1 + \frac{3}{n}\right)^{-0.5} \left(\frac{\sigma_1}{\sigma_\infty}\right)^2 \quad (8)$$

where σ_∞ is the effective stress, which equals σ_1 under the uniaxial tension loading and $|(1 - \lambda)\sigma_1|$ under the triaxial tension loading, and λ is the biaxiality ratio.

Figure 4. Schematic for (a) an embedded penny-shaped crack in an infinite body and a plate subject to semi-elliptical surface crack under far-field tensile stresses; and (b) one-quarter FE model of a cracked plate

Table 1 compares the normalised fully plastic J -integral (h) obtained from Equation (4) to that from Equation (8). As it can be seen, a good agreement between the results of the proposed 3D FEA and that of the He and Hutchinson's solution was obtained. The maximum difference is only 2.34%.

Table 2. Comparison of h for a penny-shaped crack against the proposed FE method

The second one is a plate subject to semi-elliptical surface crack (see Figures 4b). For the semi-elliptical surface crack in a plate, both the shallow ($a/d=0.2$) and deep ($a/d=0.8$) cracks were considered. The R-O material model was also used with three different strain hardening exponents ($n=3, 5$ and 10). The loading and boundary conditions were set to be the same as the first example. The characteristic length (L) was taken as the thickness of plate (d) in Equation (4). The results determined by the proposed FE models are compared with those previously

reported by Wang (2006). As Figures 5a and 5b show, there is also a good agreement with the maximum difference less than 3.18% between them for both shallow and deep cracks. These two examples indicate the accuracy of the proposed FE models in evaluating the J -integrals.

Figure 5. Comparison of h values from the present FE analyses and those of Wang (2006) with $n = 3, 5$ and 10 for: (a) $a/d = 0.2$; (b) $a/d = 0.8$

4. VERIFICATION OF MODIFIED GE/EPRI METHOD

After the FE models are verified, the normalised fully plastic J -integrals (h) for external LSSC in pressurized pipes can be derived with various crack geometries, the pipe dimensions and strain hardening exponents. The normalised fully plastic J -integrals were determined at different locations along the crack front (i.e., at every 10 degrees; $\theta = 10^\circ$). The characteristic length L in Equation (4) was taken as the length of the crack ($2c$). As the singularity change at the surface point can cause the simulated J -integral at surface point inaccurate (Sih and Lee 1989), the J -integral values at surface points were estimated by extrapolation from those next to the surface. The derived normalised fully plastic J -integrals are tabulated as shown in Tables 3-6. From these tables, the elasto-plastic J -integrals of a pressurized pipe subjected to external crack can be estimated by the modified GE/PERI method with any other dimensions and material properties. However, before the practical engineering application, it is necessary to verify the accuracy of the modified GE/PERI method. To do this, two groups of 3D FE analyses were performed for verification purpose.

Table 3. Normalised fully plastic J -integral h for cracked pipe with $a/c=0.2$

Table 4. Normalised fully plastic J -integral h for cracked pipe with $a/c=0.4$

Table 5. Normalised fully plastic J -integral h for cracked pipe with $a/c=0.6$

Table 6. Normalised fully plastic J -integral h for cracked pipe with $a/c=0.8$

The first group of FE analyses was conducted to verify the accuracy of proposed method in predicting the elasto-plastic J -integral for cracked pipes made of the R-O materials. Three materials were assumed, denoted as ‘Material A’, ‘Material B’, ‘Material C’, which have the following properties: for Material A: $\alpha = 1$, $n = 4$; for Material B: $\alpha = 3$, $n = 4$; for Material C: $\alpha = 3$, $n = 7$. The Young’s modulus, Poisson’s ratio and yield strength were assumed to be 207 GPa, 0.3 and 210 MPa, respectively. The geometries of crack and pipe were taken as $d/R_i=0.05$, $a/c=0.6$ and $a/d=0.5$. The loads were applied up to 1.6 times of P_o to ensure the results covering a wide range, i.e., from elastic to fully plastic conditions. The elasto-plastic J -integral of cracked pipe under different loads were extracted from FE models directly.

To predict the elasto-plastic J -integrals by use of Equation (7), the elastic part of J -integral for pipes with external LSSC was determined as follows (Fu et al. 2019)

$$J^e = \frac{\pi a (F \sigma^\infty)^2}{\bar{E} Q} \quad (9a)$$

with
$$Q = 1 + 1.47 \left(\frac{a}{c} \right)^{1.64} \quad (9b)$$

$$F = \left\{ h_1 + h_2 \left(\frac{a}{c} \right) + \left[h_3 + h_4 \left(\frac{a}{c} \right) + h_5 \left(\frac{a}{c} \right)^2 \right] \left(\frac{a}{d} \right)^2 + \left[h_6 + h_7 \left(\frac{a}{c} \right) + h_8 \left(\frac{a}{c} \right)^2 \right] \left(\frac{a}{d} \right)^4 \right\} \exp \left(h_9 \frac{d}{R_i} \right) \quad (9c)$$

where σ^∞ is the remote stress, and $\sigma^\infty = p R_i / d$ for a cracked pipe under internal pressure, $\bar{E} = E$ for plane stress problem and $\bar{E} = E / (1 - \nu^2)$ for plane strain problems, F is the influence coefficients function, and the coefficients $h_1, h_2, \dots, h_9$ are summarized in Table 7.

Table 7. Coefficients in Equation (9) for the deepest point of semi-elliptical surface crack

The plastic part of J -integral was calculated by first interpolating the normalised fully plastic

J -integrals for $n=4$ and $n=7$ from Table 5, and then substituting the values of α, n and P/P_0 etc. into Equation (7c). The comparison between the results of 3D FE analysis and that predicted from the proposed method is shown in Figure 6. It can be seen from Figure 6 that the estimated results have an overall good agreement with FE results. The comparison of results indicates that the developed method has a sufficient accuracy in estimating the elasto-plastic J -integrals for R-O materials.

Figure 6. Comparison of the elasto-plastic J -integrals estimated by Equation (7c) with the FE results based on R–O materials for $d/R_i=0.05$, $a/c=0.6$ and $a/d=0.5$.

The second group of analyses were conducted to verify the accuracy of proposed method in predicting the elasto-plastic J -integral of cracked pipes made of non-R–O materials. Detailed 3D FE analyses were performed based on the incremental plasticity material model. The stress-strain curves of two realistic steel materials, namely, G250 and G350 were used as the inputs for material model. The Elastic modulus of G250 and G350 are 203 GPa and 209 GPa, respectively. The Poisson's ratios of both materials were taken to be 0.3. The geometries of crack and pipe in the 3D FE models were taken to be the same as that of the first group of analyses. The loads were also applied up to 1.6 times of P_0 to ensure a fully plastic condition is reached.

The true stress-strain curves of G250 and G350 steels are presented in Figure 7. It can be seen from Figure 7 that there is yielding plateau in the stress-strain curves of G250 and G350 steels. To account for the characteristics of stress-strain curves, the Equation (5) was used with the fitted material parameters. For the purpose of comparison, the original R-O model (Equation 3) was also used. The fitted material parameters are presented in Table 8. The fitted curves are also shown in Figures 7a and 7b, for G250 and G350, respectively. It can be clearly seen that there is a distinct deviation near the region of yielding plateau between the curve determined by

Equation (3) and the stress-strain curve determined from the experiment. With use of the modified R-O model, i.e., Equation (5), this deviation was greatly reduced. Equation (7) was then used, based on the fitted material parameters, for predicting the elasto-plastic J -integrals of cracked pipe under different loads.

Table 8. R-O fitting results of G250 and G350 stress-strain curves (room temperature)

Figure 7. R-O fittings by use of Equation (3) and Equation (5) for steels with yielding plateau for: (a) G250; (b) G350

The comparison between the results predicted by the modified GE/EPRI method and that from the detailed 3D FE analyses is shown in Figure 8. Also, the results predicted by the general GE/EPRI method are also presented. From the results, it can be clearly seen that the elasto-plastic J -integrals of cracked pipes made of these two materials can be significantly underestimated with increasing the applied load if the original R-O fitting method was used. In comparison, this underestimation is greatly minimised by the modified GE/EPRI method, particularly after $P/P_o > 1.0$. This is mainly because that the fitted parameters of Equation (5) can characterise the stress-strain curve near the yielding plateau more accurately than Equation (3). It can be concluded that an overall good agreement between the results predicted by the modified GE/EPRI method and that from detailed 3D FE analyses has been obtained for both R-O materials and non R-O materials. This proves the effectiveness of the modified GE/EPRI method in predicting the elasto-plastic J -integral of cracked pipes made of different types of steel.

Figure 8. Comparison of elasto-plastic J -integrals determined by the developed method with the detailed FE results for $d/R_i=0.05$, $a/c=0.6$ and $a/d=0.5$: (a) G250; (b) G350

5. EFFECT OF AFFECTING FACTORS

The changes of normalised fully plastic J -integrals along the whole crack front are presented in Figures 9 to 10 for different pipe dimensions, crack geometries, material properties. First, the results of normalised fully plastic J -integrals with different crack aspect ratios ($a/c=0.2, 0.4, 0.6$ and 0.8) and various crack depth-to-wall ratios ($a/d=0.2, 0.5$ and 0.8) are presented in Figure 9. It can be found from Figure 9a that the distributions of normalised fully plastic J -integral are similar under different crack aspect ratios for shallow cracks ($a/d=0.2$). However, the general trends of normalised fully plastic J -integral are different for various crack aspect ratios when $a/d=0.5$ and 0.8 , as shown in Figure 9b and 9c. To be specific, the maximum values of normalised fully plastic J -integral are located at the deepest point of crack when $a/d=0.2$ while they are located between the surface point and the deepest point (approximately at $\theta = 30^\circ$) when $a/d=0.5$ and 0.8 . This is because when the ligament in the remaining wall decreases (i.e., crack depth increases), the constraint of crack back-face reduces, leading to the decrease of J -integral at the deepest point of crack. This also implies that a shallow crack will propagate downwards and become deeper when the J -integral exceed its critical value. It can be concluded that both the direction of crack propagation and the locations of critical point are different for cracks with different geometries.

Figure 9. Change of normalised fully plastic J -integral values along the whole crack front in cracked pipe with $n=5$, $d/R=0.1$ and different a/c for: (a) $a/d=0.2$; (b) $a/d=0.5$; (c) $a/d=0.8$

Second, the changes of normalised fully plastic J -integrals along the crack front are investigated under different pipe dimensions (d/R) and strain hardening exponents (n), as shown in Figure 10. It can be found that, in general, the pipe dimensions and strain hardening exponent do not significantly affect the trends of normalised fully plastic J -integrals. However, there is a difference of 7% to 12% in the normalised fully plastic J -integrals between the thick-wall pipes

and thin-wall pipes. Similar results are also obtained from other geometric parameters of cracks and thus not repeated here. This result implies that the J -integral of thin-wall pipes can be overestimated if the solutions derived from thick-wall pipes are used. Therefore, the solutions to J -integral derived from the present study will be useful in accurately predicting the elasto-plastic J -integrals for thin-wall pipes that are widely used in the current water or petroleum industries.

Figure 10. Change of normalised fully plastic J -integral values along the whole crack front in cracked pipe with different d/R and a/d for: (a) $n=3$; (b) $n=5$; (c) $n=10$

By comparing the Figures 10a, 10b and 10c, it can be seen that strain hardening exponent (n) of material has a significant effect on the normalised fully plastic J -integral. Across Figure 9 to Figure 10, strain hardening exponent has the largest effect on the magnitude of normalised fully plastic J -integral. It is also found from Figure 10c that, for a deep crack with a relatively large strain hardening exponent ($n=10$), the normalised fully plastic J -integral slightly fluctuates along the crack front. The reason of this phenomenon was initially investigated in this study by comparing the results with different densities of meshing (45 to 450 element segments) along the crack front. The comparison of results shows that this phenomenon is independent of the meshing density. Instead, this is because a material with a large strain hardening approaches to be perfect plastic. As a result, the crack front and the remaining pipe-wall ligament become easier to be locally deformed. This finding can further help understanding the failure mechanisms of external LSSC in pressurised steel pipes.

6. CONCLUSIONS

In this paper a method to determine the elasto-plastic J -integral of cracked pipes made of all types of steel has been developed and verified. A meshing technique has been developed in the

detailed three-dimensional finite element analysis (3D FEA) that incorporates the mixed types of quadratic tetrahedron and hexahedron elements. Solutions to elasto-plastic J -integrals have been derived for external longitudinal semi-elliptical surface cracks (LSSC) in pressurised steel pipes. It has been found that the distributions of normalised fully plastic J -integral change with different crack depth to wall ratio (a/d) and aspect ratios (a/c) due to the change of the crack back-face constraint, and that the crack extension directions and the positions of critical points are different for shallow and deep surface cracks. It has also been found that there is a significant difference in the derived solutions to J -integral between the thick-wall pipes and thin-wall pipes, and that the strain hardening exponent has the largest influence on the normalised fully plastic J -integral. It can be concluded that the developed method can help pipe engineers and managers accurately predict the likelihood of elasto-plastic fractures in steel pipes.

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Table 1 Variables considered in the parametric FE analyses

Variables	Value
a/c	0.2, 0.4, 0.6, 0.8
a/d	0.2, 0.5, 0.8
n	3, 5, 10
d/R_i	0.1, 0.05, 0.025

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533 **Table 2** Comparison of h for a penny shaped crack against the proposed FE method

n	Present FE results	He and Hutchinson (1981)	Difference (%)
3	1.383	1.350	2.34
5	1.537	1.510	1.75
10	1.670	1.675	-0.35

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537 **Table 3** Normalised fully plastic J -integral h for cracked pipe with $a/c=0.2$

θ	h	d/R_i	$a/d=0.2$			$a/d=0.5$			$a/d=0.8$		
			$n=3$	$n=5$	$n=10$	$n=3$	$n=5$	$n=10$	$n=3$	$n=5$	$n=10$
90		0.1	2.534	3.164	4.460	2.955	4.427	8.504	4.284	8.190	31.415
80			2.493	3.112	4.406	2.914	4.396	8.524	4.351	8.235	32.478
70			2.372	2.963	4.192	2.806	4.336	8.583	4.392	8.202	32.152
60			2.183	2.735	3.853	2.542	3.975	8.038	4.469	8.304	30.022
50			1.933	2.427	3.424	2.212	3.550	7.486	4.294	8.143	29.256
40			1.636	2.064	2.946	1.826	3.018	6.671	3.847	7.635	28.499
30			1.302	1.649	2.338	1.409	2.368	5.405	3.114	6.557	26.119
20			0.952	1.211	1.729	1.007	1.719	4.047	2.231	4.948	20.812
10			0.628	0.799	1.137	0.654	1.133	2.765	1.401	3.219	14.479
0			0.234	0.256	0.298	0.199	0.275	0.490	0.322	0.549	1.903
90		0.05	2.363	2.877	4.052	2.672	4.075	7.733	3.874	7.466	28.340
80			2.296	2.809	3.972	2.698	4.031	7.715	3.928	7.441	29.314
70			2.232	2.702	3.838	2.584	3.927	7.790	3.970	7.412	28.938
60			1.975	2.536	3.543	2.335	3.664	7.244	4.052	7.559	27.057
50			1.742	2.205	3.086	1.998	3.224	6.751	3.928	7.326	26.397
40			1.558	1.943	2.704	1.679	2.763	6.085	3.500	6.913	25.670
30			1.170	1.578	2.114	1.314	2.169	4.890	2.832	5.917	23.535
20			0.912	1.171	1.577	0.983	1.548	3.707	2.098	4.480	18.790
10			0.651	0.781	1.060	0.607	1.050	2.533	1.305	2.924	13.104
0			0.295	0.240	0.364	0.200	0.322	0.500	0.292	0.567	1.723
90		0.025	2.284	2.709	3.806	2.502	3.878	7.276	3.624	7.039	26.418
80			2.190	2.622	3.696	2.589	3.821	7.222	3.666	6.953	27.344
70			2.174	2.552	3.636	2.459	3.673	7.313	3.710	6.926	26.914
60			1.851	2.438	3.372	2.221	3.498	6.744	3.798	7.119	25.202
50			1.624	2.069	2.878	1.867	3.028	6.288	3.720	6.813	24.612
40			1.530	1.896	2.565	1.593	2.614	5.742	3.290	6.467	23.894
30			1.089	1.561	1.974	1.267	2.053	4.569	2.667	5.516	21.914
20			0.905	1.172	1.487	0.988	1.440	3.510	2.043	4.191	17.535
10			0.690	0.787	1.019	0.587	1.010	2.404	1.259	2.746	12.258
0			0.363	0.229	0.433	0.206	0.369	0.525	0.276	0.599	1.616

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541 **Table 4** Normalised fully plastic J -integral h for cracked pipe with $a/c=0.4$

θ	h d/R_i	$a/d=0.2$			$a/d=0.5$			$a/d=0.8$		
		$n=3$	$n=5$	$n=10$	$n=3$	$n=5$	$n=10$	$n=3$	$n=5$	$n=10$
90	0.1	3.758	4.534	6.037	2.828	3.880	6.518	2.679	4.126	9.356
80		3.703	4.500	5.997	2.814	3.860	6.451	2.704	4.133	9.210
70		3.574	4.347	5.799	2.769	3.832	6.455	2.799	4.231	9.384
60		3.352	4.100	5.420	2.679	3.762	6.469	2.926	4.355	9.226
50		3.068	3.778	5.015	2.525	3.609	6.314	3.063	4.655	9.849
40		2.719	3.371	4.481	2.304	3.354	6.018	3.086	4.852	10.552
30		2.332	2.908	3.886	2.021	2.981	5.426	2.922	4.733	10.683
20		1.932	2.427	3.231	1.707	2.531	4.670	2.605	4.276	9.816
10		1.556	1.940	2.527	1.386	2.007	3.624	2.149	3.459	7.861
0		0.585	1.273	1.545	0.463	0.568	0.836	0.939	1.260	2.343
90	0.05	3.406	4.125	5.533	2.573	3.554	5.937	2.463	3.787	8.479
80		3.399	4.136	5.494	2.563	3.498	5.817	2.508	3.762	8.295
70		3.228	4.014	5.308	2.503	3.533	5.888	2.540	3.830	8.523
60		3.030	3.785	4.903	2.415	3.467	5.915	2.719	3.968	8.406
50		2.807	3.476	4.599	2.352	3.306	5.704	2.848	4.200	8.931
40		2.500	3.044	4.087	2.152	3.106	5.505	2.785	4.386	9.583
30		2.178	2.673	3.597	1.862	2.744	4.938	2.716	4.276	9.689
20		1.787	2.203	2.936	1.589	2.335	4.264	2.374	3.855	8.859
10		1.465	1.747	2.279	1.340	1.887	3.289	1.956	3.180	7.131
0		0.538	1.227	1.402	0.442	0.562	0.804	0.905	1.217	2.133
90	0.025	3.190	3.884	5.244	2.419	3.368	5.591	2.341	3.592	7.940
80		3.230	3.932	5.203	2.418	3.277	5.421	2.411	3.544	7.721
70		3.017	3.832	5.025	2.337	3.371	5.549	2.381	3.584	8.007
60		2.832	3.615	4.585	2.248	3.307	5.593	2.611	3.744	7.916
50		2.655	3.307	4.357	2.271	3.132	5.330	2.742	3.921	8.372
40		2.376	2.842	3.857	2.083	2.980	5.207	2.594	4.099	9.001
30		2.106	2.539	3.441	1.775	2.613	4.644	2.614	3.996	9.088
20		1.711	2.065	2.759	1.527	2.230	4.026	2.241	3.588	8.259
10		1.423	1.626	2.122	1.335	1.832	3.089	1.840	3.024	6.688
0		0.517	1.225	1.320	0.439	0.572	0.795	0.901	1.215	2.011

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545 **Table 5** Normalised fully plastic J -integral h for cracked pipe with $a/c=0.6$

θ	d/R_i	$a/d=0.2$			$a/d=0.5$			$a/d=0.8$		
		$n=3$	$n=5$	$n=10$	$n=3$	$n=5$	$n=10$	$n=3$	$n=5$	$n=10$
90	0.1	4.388	5.305	7.071	2.613	3.469	5.539	2.116	3.154	6.548
80		4.350	5.234	6.879	2.609	3.466	5.577	2.163	3.200	6.511
70		4.245	5.115	6.700	2.609	3.506	5.712	2.233	3.276	6.626
60		4.075	4.924	6.463	2.588	3.524	5.820	2.349	3.340	6.362
50		3.850	4.666	6.147	2.534	3.489	5.786	2.511	3.565	6.663
40		3.583	4.353	5.755	2.436	3.383	5.690	2.651	3.886	7.489
30		3.280	3.971	5.238	2.299	3.203	5.347	2.689	3.993	7.873
20		2.971	3.559	4.647	2.128	2.944	4.942	2.635	3.951	7.868
10		2.627	3.053	3.883	1.898	2.523	4.044	2.460	3.607	6.952
0		1.303	1.393	1.561	0.762	0.897	1.223	0.903	1.090	1.670
90	0.05	4.010	4.779	6.385	2.437	3.188	5.075	1.910	2.881	5.990
80		4.015	4.805	6.226	2.355	3.155	5.052	1.946	2.908	5.937
70		3.841	4.612	6.068	2.362	3.199	5.149	2.063	2.992	6.003
60		3.677	4.428	5.897	2.359	3.181	5.251	2.156	3.057	5.801
50		3.469	4.243	5.545	2.367	3.185	5.214	2.323	3.221	6.031
40		3.258	3.939	5.272	2.286	3.070	5.167	2.413	3.559	6.779
30		2.988	3.638	4.760	2.101	2.935	4.900	2.433	3.606	7.135
20		2.735	3.226	4.206	1.977	2.674	4.495	2.400	3.566	7.132
10		2.438	2.839	3.546	1.736	2.294	3.680	2.312	3.255	6.297
0		1.179	1.307	1.492	0.728	0.844	1.169	0.833	0.987	1.504
90	0.025	3.788	4.445	5.961	2.355	3.031	4.808	1.778	2.725	5.666
80		3.833	4.567	5.824	2.197	2.966	4.728	1.812	2.733	5.600
70		3.597	4.292	5.681	2.210	3.014	4.798	1.974	2.823	5.618
60		3.433	4.118	5.567	2.222	2.971	4.896	2.046	2.894	5.473
50		3.230	3.985	5.168	2.292	3.005	4.853	2.225	3.003	5.646
40		3.066	3.687	5.001	2.216	2.883	4.850	2.273	3.367	6.343
30		2.815	3.448	4.470	1.985	2.785	4.643	2.275	3.369	6.687
20		2.605	3.021	3.933	1.899	2.516	4.229	2.256	3.328	6.682
10		2.338	2.734	3.353	1.640	2.160	3.467	2.249	3.033	5.899
0		1.106	1.272	1.475	0.721	0.819	1.157	0.797	0.923	1.401

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549 **Table 6** Normalised fully plastic J -integral h for cracked pipe with $a/c=0.8$

θ	h d/R_i	$a/d=0.2$			$a/d=0.5$			$a/d=0.8$		
		$n=3$	$n=5$	$n=10$	$n=3$	$n=5$	$n=10$	$n=3$	$n=5$	$n=10$
90	0.1	4.605	5.594	7.394	2.420	3.129	4.706	1.862	2.774	5.555
80		4.587	5.555	7.294	2.427	3.143	4.708	1.876	2.717	5.286
70		4.542	5.506	7.229	2.456	3.211	4.853	1.945	2.759	5.214
60		4.468	5.418	7.114	2.490	3.302	5.134	2.078	2.912	5.330
50		4.369	5.301	6.974	2.513	3.368	5.297	2.263	3.217	5.918
40		4.777	5.134	6.726	2.519	3.393	5.362	2.435	3.553	6.725
30		4.104	4.909	6.317	2.499	3.354	5.293	2.554	3.737	7.200
20		3.933	4.606	5.640	2.440	3.208	4.920	2.600	3.770	7.135
10		3.670	4.136	4.624	2.296	2.880	4.158	2.538	3.569	6.458
0		1.625	1.832	2.132	0.943	1.077	1.372	1.044	2.449	3.883
90	0.05	4.165	5.111	6.655	2.187	2.881	4.290	1.676	2.503	5.092
80		4.133	5.034	6.620	2.245	2.836	4.305	1.746	2.542	4.842
70		4.145	5.006	6.574	2.285	2.982	4.389	1.834	2.485	4.740
60		4.051	4.919	6.430	2.320	3.062	4.653	1.946	2.706	4.848
50		3.959	4.865	6.276	2.294	3.074	4.859	2.114	2.899	5.371
40		4.385	4.716	6.091	2.307	3.141	4.832	2.236	3.249	6.080
30		3.731	4.481	5.696	2.277	3.112	4.839	2.373	3.371	6.481
20		3.543	4.240	5.080	2.277	2.985	4.455	2.358	3.419	6.434
10		3.347	3.764	4.202	2.142	2.625	3.824	2.373	3.242	5.895
0		1.481	1.712	1.949	0.859	1.052	1.316	0.956	2.297	3.569
90	0.025	3.899	4.834	6.193	2.043	2.744	4.042	1.560	2.337	4.824
80		3.845	4.711	6.215	2.145	2.647	4.069	1.679	2.457	4.584
70		3.913	4.703	6.181	2.196	2.866	4.106	1.794	2.312	4.460
60		3.795	4.615	6.010	2.237	2.940	4.364	1.883	2.603	4.559
50		3.707	4.620	5.840	2.169	2.901	4.607	2.046	2.698	5.038
40		4.161	4.484	5.699	2.184	3.012	4.501	2.128	3.076	5.687
30		3.511	4.229	5.306	2.144	2.991	4.579	2.285	3.141	6.029
20		3.302	4.035	4.728	2.199	2.873	4.169	2.212	3.206	6.001
10		3.156	3.538	3.951	2.064	2.474	3.637	2.294	3.043	5.562
0		1.391	1.657	1.844	0.811	1.058	1.307	0.909	2.227	3.397

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Table 7 Coefficients in Equation (9) for the deepest point of crack

h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9
0.513	0.592	-0.204	3.966	-2.485	2.022	-6.519	3.552	-0.437

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Table 8 R-O fitting results of G250 and G350 stress-strain curves (room temperature)

	G250		G350	
	α	n	α	n
Equation (3)	5.365	5.096	7.757	6.387
Equation (5)	5.450	4.596	7.193	6.112

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Figure 1 Schematic of R-O and non-R-O materials

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(a)

(b)

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Figure 2 Schematic for (a) a pipe with a semi-elliptical external surface crack; and (b) surface crack

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Figure 3 Finite element model of a thin-wall pipe with external semi-elliptical surface crack

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(a)

(b)

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Figure 4 Schematic for (a) an embedded penny-shaped crack in an infinite body and a plate subject to semi-elliptical surface crack under far-field tensile stresses; and (b) one-quarter FE model of a cracked plate

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604 **Figure 5** Comparison of h values from the present FE analyses and those of Wang (2006)
605 with $n = 3, 5$ and 10 for: (a) $a/d = 0.2$; (b) $a/d = 0.8$
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Figure 6 Comparison of the elasto-plastic J -integrals estimated by Equation (7c) with the FE results based on R–O materials for $d/R_i=0.05$, $a/c=0.6$ and $a/d=0.5$.

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Figure 7 R-O fittings by use of Equation (4) and Equation (6) for steels with yielding plateau for: (a) G250; (b) G350.

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620 **Figure 8** Comparison of elasto-plastic J -integrals determined by the developed method
621 with the detailed FE results for $d/R_1=0.05$, $a/c=0.6$ and $a/d=0.5$: (a) G250; (b) G350

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625 **Figure 9** Change of normalised fully plastic J -integral values along the whole crack front
626 in cracked pipe with $n=5$, $d/R=0.1$ and different a/c for: (a) $a/d=0.2$; (b) $a/d=0.5$; (c) $a/d=0.8$

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628

(a)

(b)

(c)

Figure 10 Change of normalised fully plastic J -integral values along the whole crack front in cracked pipe with different d/R and a/d for: (a) $n=3$; (b) $n=5$; (c) $n=10$.