Event-triggered passivity of multi-weighted coupled delayed reaction-diffusion memristive neural networks with fixed and switching topologies

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Abstract

This paper solves the event-triggered passivity problem for multiple-weighted coupled delayed reaction-diffusion memristive neural networks (MWCDRD-MNNs) with fixed and switching topologies. On the one side, by designing appropriate event-triggered controllers, several passivity criteria for MWC-DRDMNNs with fixed topology are derived based on the Lyapunov functional method and some inequality techniques. Moreover, some adequate conditions for ensuring asymptotical stability of the event-triggered passive network are presented. On the other side, we take the switching topology in network model into consideration, and investigate the event-triggered passivity and passivity-based synchronization for MWCDRDMNNs with switching topology. Finally, two examples with numerical simulation results are provided to illustrate the effectiveness of the obtained theoretical results.

Keywords: Event-triggered control, Synchronization, Switching topology,

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Multi-weights, Reaction-diffusion terms, Passivity

1. Introduction

Recently, a lager number of scholars have paid extensive attention to complex networks (CNs) since they are ubiquitous in our daily life. Examples include food webs, communication networks, social networks, metabolic systems, etc. As a peculiar case of CNs, coupled neural networks (CNNs) have been put to use availably in various fields, for instance, harmonic oscillation generation, secure communication and chaos generators design [1, 2]. As we know, these applications are heavily relying on some dynamical behaviors of CNNs, e.g., synchronization. So far, some significant results on the synchronization of CNNs have been reported in recent years [3–7]. In [3], several adequate conditions were derived for randomly delayed CNNs to reach synchronization by employing the properties of random variables. Zhang and Gao [5] dealt with synchronization problem for delayed CNNs based on Lyapunov stability theory. Besides synchronization, passivity is also one of the most significant dynamical behaviors in CNNs because the internal stability of a complex system can be ensured by passive property in system theory. In the past few decades, the passivity analysis for CNNs has attracted increasing research interests because of their widespread applications in plenty of fields such as fuzzy control and sliding mode control [8]. Up to now, some significant results about the passivity of CNNs have been obtained [9–11]. In [9], Li and Cao investigated CNNs with time delay based on Lyapunov stability theory, and proposed several passivity criteria.

However, most of results in above-mentioned works [3–7, 9–11] ignore the reaction-diffusion phenomena (RDP). As we all know, the RDP is inevitable for many CNs such as cellular networks, neural networks, etc. [12, 13], especially when they are implemented through electric circuits in practical systems. Thus, it is necessary to investigate the coupled reaction-diffusion neural networks (CRDNNs). Recently, many meaningful results on the synchronization of CRDNNs have been obtained [14–18]. In [14], the authors established some synchronization conditions for CRDNNs. Some conditions were obtained for CRDNNs to make sure of the synchronization by designing suitable pinning controllers in [17]. Furthermore, some scholars have investigated the passivity for CRDNNs [19–22]. Several (pinning) passivity conditions were derived for CRDNNs in [20]. Huang et al. [21] addressed passivity problem for CRDNNs with nonlinear coupling, and some passivity criteria were presented. Nevertheless, in the above mentioned literatures [9–11, 19–22], the derived passivity results are based on the situation that the output has the same dimension as input. To our knowledge, the passivity of networks with different dimensional input and output only has been considered by a few scholars until now [23, 24]. In [23], the authors investigated the passivity for CRDNNs with the output and input in different dimensions, and some passivity criteria were established. Therefore, it is meaningful to further investigate the passivity of CRDNNs with different dimensional input and output.

It is well known that Chua first proposed the concept of memristor in 1970s. The memristor in neural networks can be exploited instead of resistor to better comprehend the neural processes of the human brain. Until now, many worthwhile and meaningful results on the research for memristive neural networks (MNNs) have been acquired [25–28]. In [26], the authors addressed the Lagrange stability of MNNs, and some conditions were derived based on nonsmooth analysis and control theory. Wu and Zeng [28] investigated the exponential stabilization for MNNs with time delays, and several exponential stabilization criteria were established on accordance of the Lyapunov stability theory. Nevertheless, only a few authors have investigated the synchronization of coupled memristive neural networks (CMNNs) [29–31]. Several sufficient conditions were acquired for CMNNs to make sure of the exponential synchronization in [30]. In [31], Wan and Cao dealt with the synchronization problem for CMNNs with supremums, and several synchronization criteria were obtained. It should be pointed out that there is no research results reported on passivity and passivity-based synchronization for coupled delayed reaction-diffusion memristive neural networks (CDRDMNNs).

To the best of our knowledge, many networks in the real-world should be modeled by multi-weighted complex networks (MWCNs), for instance, the World Web, transportation networks and social networks, in which the network nodes are coupled by multiple coupling forms. Over the past years, some results on the passivity and synchronization for MWCNs have been obtained [32–35]. In [33], several synchronization criteria for MWCNs were derived by designing appropriate pinning controllers based on Lyapunov functional method. Some synchronization conditions in a finite time were established for MWCNs in [34]. At present, only a few authors have investigated the passivity and synchronization problems for multi-weighted coupled neural networks (MWCNNs) [36–38]. Tang et al. [36] dealt with the passivity and exponential synchronization problems for MWCNNs by designing suitable impulsive controllers. Unfortunately, the passivity and passivity-based synchronization problems for multi-weighted CDRDMNNs (MWCDRDMNNs) have rarely been addressed.

As far as we know, many CNs couldn't realize passivity and synchronization by themselves in practical situation. Therefore, it is indispensable and significant for CNs to design suitable controllers that ensuring them achieve passivity and synchronization. Currently, the event-triggered control has become more and more popular. With the intensive research of CNs, event-triggered control strategy has been proven to be an effective method, which can overcome the shortcomings of the continuous control scheme and reduce the unnecessary transmission of the communication media in the process of information exchanging. Thus, some authors have investigated the synchronization of CNs under event-triggered control schemes [42–45]. In [43], the authors discussed synchronization problem for CNs with event-triggered control, and several event-triggered synchronization criteria were established. By designing suitable event-triggered controllers, some conditions were acquired to guarantee CNs achieve synchronization in [44]. However, only a few scholars have studied event-triggered synchronization of CNNs [46–49]. In [46], Huang et al. addressed synchronization-based passivity for partially CNNs with event-triggered communication by combining Lyapunov stability theory with some matrix inequality techniques. As far as we know, the problems of event-triggered passivity and passivity-based synchronization for MWCDRDMNNs with fixed and switching topologies have not yet been investigated.

According to the points discussed above, we investigate event-triggered passivity for MWCDRDMNNs with fixed and switching topologies. The primary contribution of the presented work lies on the following aspects.

(1) First, we propose two models that one is MWCDRDMNNs with fixed topology and the other is MWCDRDMNNs with switching topology based on event-triggered control. Compared with the CMNNs in [29–31], the presented network model in this paper is more complicated and general, which includes multi-weighted and reaction-diffusion terms (named as MWCDRDMNNs), and can reflect many real-world networks in a more accurate sense.

(2) Several event-triggered passivity criteria for MWCDRDMNNs with fixed topology are proposed. As we know, most of the existing works studied the traditional passivity of CNs [9–11, 19–24]. However, the obtained passivity results in our present work are based on the strategy of distributed triggering event, which can exploit the advantage of the discontinuous control scheme and reduce some unnecessary communication among neural networks when information exchanging.

(3) Some conditions are obtained for ensuring event-triggered passivity-based synchronization of the considered MWCDRDMNNs. Different from the traditional synchronization and event-triggered synchronization strategies in the previous works [3–7, 14–18, 46–49], we establish some passivity-based synchronization criteria under event-triggered condition by establishing the relationship between the obtained event-triggered output-strict passivity condition and asymptotical stability of the considered networks in this paper. Actually, these synchronization conditions are acquired based on the deducted passivity criteria, which are not derived

from the networks themselves.

(4) The event-triggered passivity are discussed for switched MWCDRDMNNs. In contrast to the event-triggered synchronization results of CNs in [42–49], we investigate the event-triggered passivity of MWCDRDMNNs with switching topology in this paper. Moreover, compared with the event-triggered conditions in [42–49], a distributed event-triggered conditions with switches is firstly designed in this paper. Therefore, the obtained event-triggered passivity results has more generalization and less conservatism in our present work.

The following shows the organization of this remaining paper. In Section 2, some mathematical notations, several related definitions of passivity and necessary lemmas are introduced. In Section 3, we firstly propose the network model of MWCDRDMNNs with fixed topology. Then, event-triggered passivity and passivity-based synchronization are studied for this kind of network. Section 4 is devoted to investigating event-triggered passivity and passivity-based synchronization for MWCDRDMNNs with switching topology. Two simulation examples are given in Section 5 to illustrate the validity of these obtained event-triggered passivity and synchronization results. Finally, we conclude our paper in Section 6 with summary of our work and some future works.

2. Preliminaries

2.1. Notations

Let $\mathcal{W} = (\nu, \Sigma, M)$ be a weighted connected digraph, in which $\nu = \{1, 2, \cdots, N\}$ denotes a set of nodes and $\Sigma \subseteq \nu \times \nu$ means a set of edges.

$(q,p) \in \Sigma$ is a directed edge from q to p. $\mathcal{N}_q = \{p \in \nu | (q,p) \in \Sigma\}$ represents a set of neighbors of node q and $p \neq q$. $M = (M_{qp})_{N\times N}$ with $M_{qq} = 0, M_{qp} > 0 \iff p \in \mathcal{N}_q$ is the connection adjacency matrix of the graph $\mathcal{W}$. For the matrix $K \in \mathbb{R}^{n\times n}$, the representation $K \leqslant 0$ $(K \geqslant 0, K < 0, K > 0)$ means K is symmetric and semi-negative (semi-positive, negative, positive) definite. $\otimes$ stands for the Kronecker product. $\lambda_M(\cdot)(\lambda_m(\cdot))$ is the largest (smallest) eigenvalue of the matrix. $\Phi = \{s = (s_1, s_2, \cdots, s_\rho)^T || s_\kappa| < \mu_\kappa, \kappa = 1, 2, \cdots, \rho\}$ signifies a bounded compact set with smooth boundary $\partial\Phi$. For the vector $\varepsilon(s,t) = (\varepsilon_1(s,t), \varepsilon_2(s,t), \cdots, \varepsilon_\psi(s,t))^T \in \mathbb{R}^\psi$, we have

$$||\varepsilon(\cdot,t)|| = \Big(\int_\Phi \sum_{j=1}^{\psi} \varepsilon_j^2(s,t)ds\Big)^{\frac{1}{2}}.$$

2.2. Definition and lemmas

Definition 2.1. (See [50]) If there is a nonnegative storage function $\mathcal{Q}$: $[0,+\infty) \longrightarrow [0,+\infty)$ satisfying

$$\int_{t_r}^{t_p} \Theta(y(s,t), u(s,t))dt \geqslant \mathcal{Q}(t_m) - \mathcal{Q}(t_r)$$

for any $t_r, t_p \in [0,+\infty)$ and $t_r \leqslant t_p$, where $\Theta(y,u)$ is supply rate, then the system with output $y(s,t) \in \mathbb{R}^{nN}$ and input $u(s,t) \in \mathbb{R}^{lN}$ is said to be dissipative. Especially, if a system is dissipative and

$$\Theta(y(s,t), u(s,t)) = \int_\Phi y^T(s,t)Pu(s,t)ds,$$

then the system is called to be passive, where $P \in \mathbb{R}^{nN\times lN}$ is a constant matrix. In addition, if a system is dissipative and

$$\Theta(y(s,t), u(s,t)) = \int_\Phi y^T(s,t)Pu(s,t)ds - \int_\Phi u^T(s,t)J_1u(s,t)ds$$

$$-\int_{\Phi} y^T(s,t)J_2 y(s,t)ds,$$

where $J_1 \in \mathbb{R}^{lN\times lN} \geqslant 0, J_2 \in \mathbb{R}^{nN\times nN} \geqslant 0, \lambda_m(J_1)+\lambda_m(J_2) > 0$ and $P \in \mathbb{R}^{nN\times lN}$, then the system is called to be strictly passive. Moreover, the system is said to be input-strictly passive if $J_1 > 0$ and output-strictly passive if $J_2 > 0$.

Lemma 2.1. (See [51]) Let Φ be a cube $|s_\kappa| < \mu_\kappa(\kappa = 1,2,\cdots,\rho)$ and real-valued function $z(s) \in C^1(\Phi)$ satisfy $z(s)|_{\partial\Phi} = 0$. Then

$$\int_{\Phi} z^2(s)ds \leqslant \mu_\kappa^2 \int_{\Phi} \Big(\frac{\partial z(s)}{\partial s_\kappa}\Big)^2 ds,$$

where $s = (s_1, s_2, \cdots, s_\rho)^T$.

Lemma 2.2. (See [52]) For $\forall \wp_1, \wp_2 \in \mathbb{R}^n$ and $G \in \mathbb{R}^{n\times n} > 0$, we have

$$2\wp_1^T \wp_2 \leqslant \wp_1^T G \wp_1 + \wp_2^T G^{-1} \wp_2.$$

3. Event-triggered passivity of MWCDRDMNNs with fixed topology

In this section, we discuss the event-triggered passivity problem for MWCDRDMNNs with fixed topology based on the designed event-triggered condition and Lyapunov functional method. First, the considered network model of MWCDRDMNNs with fixed topology is proposed in Subsection 3.1. Then, by designing a suitable even-triggered controller, several even-triggered passivity criteria are established in Subsection 3.2. After that, we study asymptotical stability of event-triggered passivity for the considered network, and obtain an event-triggered synchronization criterion based on the stability result in Subsection 3.3.

3.1. Network model

Consider the following MWCDRDMNNs model in this section:

$$\begin{cases} \frac{\partial \delta_q(s,t)}{\partial t} & = & \sum_{\kappa=1}^{\rho} L_\kappa \frac{\partial^2 \delta_q(s,t)}{\partial s_\kappa^2} - Z\delta_q(s,t) + E(\delta_q(s,t))f(\delta_q(s,t)) \\ & & +B(\delta_q(s,t))g(\overline{\delta_q(s,t)}) + O + Ku_q(s,t) + \eta_q(s,t) \\ & & +\sum_{r=1}^{\eta}\sum_{p=1}^{N}\xi_r C_{qp}^r \Gamma^r \delta_p(s,t) + v_q(s,t), \\ \delta_q(s,t) & = & \varrho_q(s,t), \qquad (s,t) \in \Phi \times [-\gamma, 0], \\ \delta_q(s,t) & = & 0, \qquad (s,t) \in \partial\Phi \times [-\gamma, +\infty), \end{cases} \tag{1}$$

where $q = 1, 2, \cdots, N, \delta_q(s,t) = (\delta_{q1}(s,t), \delta_{q2}(s,t), \cdots, \delta_{q\psi}(s,t))^T \in \mathbb{R}^\psi$ denotes the state vector of qth neuron; $L_\kappa = \text{diag}(\ell_{1\kappa}, \ell_{2\kappa}, \cdots, \ell_{\psi\kappa}) > 0; Z = \text{diag}(z_1, z_2, \cdots, z_\psi) > 0; \gamma_j(t)(j = 1, 2, \cdots, \psi)$ means the time varying delays, and $\gamma_j(t)$ satisfies $0 \leqslant \gamma_j(t) \leqslant \gamma_j, \gamma = \max_{j=1,2,\cdots,\psi}\{\gamma_j\}$ and $\dot{\gamma}_j(t) \leqslant \zeta_j < 1$; $f(\delta_q(s,t)) = (f_1(\delta_{q1}(s,t)), f_2(\delta_{q2}(s,t)), \cdots, f_\psi(\delta_{q\psi}(s,t)))^T, g(\overline{\delta_q(s,t)}) = (g_1(\delta_{q1} (s, t - \gamma_1(t))), g_2(\delta_{q2}(s, t - \gamma_2(t))), \cdots, g_\psi(\delta_{q\psi}(s, t - \gamma_\psi(t))))^T$ stand for the activation functions in neural network q; $O = (O_1, O_2, \cdots, O_\psi)^T \in \mathbb{R}^\psi$ represents the constant external input; $K \in \mathbb{R}^{\psi \times l}$ signifies a known matrix; $u_q(s,t) \in \mathbb{R}^l$ is the input vector of the neural network q; $\xi_r > 0$ denotes the overall coupling strength; $\Gamma^r \in \mathbb{R}^{\psi\times\psi} > 0$ stands for the inner coupling matrix; $C^r = (C_{qp}^r)_{N\times N}$ means the coupling configuration matrix, which satisfies $C_{qp}^r = C_{pq}^r > 0(q \neq p)$ if node q and node p are connected, or else $C_{qp}^r = 0$, and $C_{qq}^r = -\sum_{\substack{p=1 \\ p\neq q}}^{N} C_{qp}^r$; $\varrho_q(s,t)$ is bounded and continuous on $\Phi \times [-\gamma, 0]$; $\eta_q(s,t) \in \mathbb{R}^\psi, v_q(s,t) \in \mathbb{R}^\psi$ signify the controllers; $E(\delta_q(s,t)) = (e_{gh}(\delta_{qg}(s,t)))_{\psi\times\psi}, B(\delta_q(s,t)) = (b_{gh}(\delta_{qg}(s,t)))_{\psi\times\psi}$ stand for memristors synaptic connection weights, where $e_{gh}(\delta_{qg}(s,t))$ and $b_{gh}(\delta_{qg}(s,t))$ are defined by

$$e_{gh}(\delta_{qg}(s,t)) = \frac{\mathcal{E}_{gh}}{\mathcal{I}_g} \times \text{sign}_{gh},$$

$$b_{gh}(\delta_{qg}(s,t)) = \frac{\mathcal{B}_{gh}}{\mathcal{I}_g} \times \text{sign}_{gh},$$

$$\text{sign}_{gh} = \begin{cases} 1, & g \neq h, \\ -1, & g = h, \end{cases}$$

where $g, h = 1, 2, \cdots, \psi, \mathcal{E}_{gh}, \mathcal{B}_{gh}$ represent the memductances of memristors $\mathbf{E}_{gh}, \mathbf{B}_{gh}$, and $\mathbf{E}_{gh}$ means the memristor between $f_h(\delta_{qh}(s,t))$ and $\delta_{qg}(s,t)$, $\mathbf{B}_{gh}$ means the memristor between $g_h(\delta_{qh}(s, t-\gamma_h(t)))$ and $\delta_{qg}(s,t)$. Based on the voltage-current characteristic of memristor, the following memristive connection weights are described by:

$$e_{gh}(\delta_{qg}(s,t)) = \begin{cases} \hat{e}_{gh}, & |\delta_{qg}(s,t)| \leqslant \mathfrak{m}_{qg}, \\ \check{e}_{gh}, & |\delta_{qg}(s,t)| > \mathfrak{m}_{qg}, \end{cases}$$

$$b_{gh}(\delta_{qg}(s,t)) = \begin{cases} \hat{b}_{gh}, & |\delta_{qg}(s,t)| \leqslant \mathfrak{m}_{qg}, \\ \check{b}_{gh}, & |\delta_{qg}(s,t)| > \mathfrak{m}_{qg}, \end{cases}$$

where the switching jumps $\mathfrak{m}_{qg} > 0, \hat{e}_{gh}, \check{e}_{gh}, \hat{b}_{gh}, \check{b}_{gh}$ are constants and $g, h = 1, 2, \cdots, \psi$.

Throughout this paper, we define

$$\bar{e}_{gh} = \max\{|\check{e}_{gh}|, |\hat{e}_{gh}|\}, \bar{E} = \text{diag}(\sum_{h=1}^{\psi} \bar{e}_{1h}^2, \sum_{h=1}^{\psi} \bar{e}_{2h}^2, \cdots, \sum_{h=1}^{\psi} \bar{e}_{\psi h}^2),$$

$$\bar{b}_{gh} = \max\{|\check{b}_{gh}|, |\hat{b}_{gh}|\}, \bar{B} = \text{diag}(\sum_{h=1}^{\psi} \bar{b}_{1h}^2, \sum_{h=1}^{\psi} \bar{b}_{2h}^2, \cdots, \sum_{h=1}^{\psi} \bar{b}_{\psi h}^2),$$

$$\tilde{e}_{gh} = |\hat{e}_{gh} - \check{e}_{gh}|, \tilde{E} = (\tilde{e}_{gh})_{\psi\times\psi}, \tilde{b}_{gh} = |\hat{b}_{gh} - \check{b}_{gh}|, \tilde{B} = (\tilde{b}_{gh})_{\psi\times\psi}.$$

Assume that there exist some positive numbers $\varphi_j, \omega_j, \breve{\varphi}_j, \breve{\omega}_j$ such that

$$|f_j(\chi_1) - f_j(\chi_2)| \leqslant \varphi_j|\chi_1 - \chi_2|, \qquad |f_j(\chi)| \leqslant \breve{\varphi}_j,$$

$$|g_j(\chi_1) - g_j(\chi_2)| \leqslant \omega_j|\chi_1 - \chi_2|, \qquad |g_j(\chi)| \leqslant \breve{\omega}_j,$$

holds for all $\chi, \chi_1, \chi_2, \in \mathbb{R}, j = 1, 2, \cdots, \psi$.

Suppose that $\delta^*(s) = (\delta_1^*(s), \delta_2^*(s), \cdots, \delta_\psi^*(s))^T$ is an equilibrium solution of the network (1). Then

$$\sum_{\kappa=1}^{\rho} L_\kappa \frac{\partial^2 \delta^*(s)}{\partial s_\kappa^2} - Z\delta^*(s) + E(\delta^*(s))f(\delta^*(s)) + B(\delta^*(s))g(\delta^*(s)) + O = 0.$$

For network (1), we design the following state feedback controller $\eta_q(s,t)$ and event-triggered controller $v_q(s,t)$:

$$\begin{cases} \eta_q(s,t) &= -\mathrm{sign}(\delta_q(s,t) - \delta^*(s))(\tilde{E}\breve{\varphi} + \tilde{B}\breve{\omega}), \\ v_q(s,t) &= \sum_{r=1}^{\eta} \sum_{p\in\mathcal{N}_q} \beta_r M_{qp}^r(\delta_q(s,t) - \delta_p(s,t)), \end{cases}$$

where $\beta_r > 0, \breve{\varphi} = (\breve{\varphi}_1, \breve{\varphi}_2, \cdots, \breve{\varphi}_\psi)^T, \breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, \cdots, \breve{\omega}_\psi)^T$ and $\mathrm{sign}(\delta_q(s,t) - \delta^*(s)) = \mathrm{diag}(\mathrm{sign}(\delta_{q1}(s,t) - \delta_1^*(s)), \mathrm{sign}(\delta_{q2}(s,t) - \delta_2^*(s)), \cdots, \mathrm{sign}(\delta_{q\psi}(s,t) - \delta_\psi^*(s)))$.

Let $\{t_m^q\}_{m=1}^{\infty}$ signifies the sequence of increasing event-triggered time, where $t_m^q < t_{m+1}^q$ for all $q = 1, 2, \cdots, N$. According to the event-triggered strategy and sample data, $v_q(s,t)$ can be rewritten as follows:

$$v_q(s,t) = \sum_{r=1}^{\eta} \sum_{p\in\mathcal{N}_q} \beta_r M_{qp}^r(\delta_q(s, t_m^q) - \delta_p(s, t_m^p)), \tag{2}$$

where $M^r = (M_{qp}^r)_{N\times N}$ stands for the coupling configuration matrix, which satisfies $M_{qq}^r = 0, M_{qp}^r > 0$ when $p \in \mathcal{N}_q$, t_m^q means the event-triggered time instant of node q, $\delta_q(s, t_m^q)$ is the state of node q at t_m^q. Obviously, $t_m^q \in T$. As in [48, 49], the Zeno behavior can be naturally excluded as $t_{m+1}^q - t_m^q > 0$. From (2), the sampler samples the node state at a random sample instant.

Define the event-triggered measure error $\phi_q(s,t) = \delta_q(s,t_m^q) - \delta_q(s,t)$. For $t \in [t_m^q, t_{m+1}^q)$, we design the distributed triggering event as follows:

$$
\begin{aligned}
t_{m+1}^q &= \inf\Big\{t : t > t_m^q, \theta_q(s,t) > 0\Big\} \text{ and} \\
\theta_q(s,t) &= ||\phi_q(s,t)|| - \vartheta\Big|\Big|\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q} M_{qp}^r(\delta_q(s,t_m^q) - \delta_p(s,t_m^p))\Big|\Big|, \qquad (3)
\end{aligned}
$$

where $\vartheta > 0$.

For the error vector $\varepsilon_q(s,t) = (\varepsilon_{q1}(s,t), \varepsilon_{q2}(s,t), \cdots, \varepsilon_{q\psi}(s,t))^T = \delta_q(s,t) - \delta^*(s)$, we have

$$
\begin{aligned}
\frac{\partial \varepsilon_q(s,t)}{\partial t} =& \sum_{\kappa=1}^{\rho} L_\kappa \frac{\partial^2 \varepsilon_q(s,t)}{\partial s_\kappa^2} + E(\delta_q(s,t))\mathcal{F}(\varepsilon_q(s,t)) + B(\delta_q(s,t))\mathcal{G}(\overline{\varepsilon_q(s,t)}) \\
&+ [E(\delta_q(s,t)) - E(\delta^*(s))]f(\delta^*(s)) - Z\varepsilon_q(s,t) + [B(\delta_q(s,t)) \\
&- B(\delta^*(s))]g(\delta^*(s)) + \sum_{r=1}^{\eta}\sum_{p=1}^{N} \xi_r C_{qp}^r \Gamma^r \varepsilon_p(s,t) + Ku_q(s,t) \\
&+ \sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q} \beta_r M_{qp}^r(\varepsilon_q(s,t_m^q) - \varepsilon_p(s,t_m^p)) \\
&- \operatorname{sign}(\varepsilon_q(s,t))(\tilde{E}\breve{\varphi} + \tilde{B}\breve{\omega}), \qquad (4)
\end{aligned}
$$

where $t \in [t_m^q, t_{m+1}^q)$, $\mathcal{F}(\varepsilon_q(s,t)) = f(\delta_q(s,t)) - f(\delta^*(s))$, $\mathcal{G}(\overline{\varepsilon_q(s,t)}) = g(\overline{\delta_q(s,t)}) - g(\delta^*(s))$, $\overline{\varepsilon_q(s,t)} = (\varepsilon_{q1}(s,t-\gamma_1(t)), \varepsilon_{q2}(s,t-\gamma_2(t)), \cdots, \varepsilon_{q\psi}(s,t-\gamma_\psi(t)))^T$ and $q = 1,2,\cdots,N$.

As $\phi_q(s,t) = \delta_q(s,t_m^q) - \delta_q(s,t)$, we can derive from (4) that

$$
\begin{aligned}
\frac{\partial \varepsilon_q(s,t)}{\partial t} =& \sum_{\kappa=1}^{\rho} L_\kappa \frac{\partial^2 \varepsilon_q(s,t)}{\partial s_\kappa^2} + E(\delta_q(s,t))\mathcal{F}(\varepsilon_q(s,t)) + B(\delta_q(s,t))\mathcal{G}(\overline{\varepsilon_q(s,t)}) \\
&+ [E(\delta_q(s,t)) - E(\delta^*(s))]f(\delta^*(s)) - Z\varepsilon_q(s,t) + [B(\delta_q(s,t)) \\
&- B(\delta^*(s))]g(\delta^*(s)) + \sum_{r=1}^{\eta}\sum_{p=1}^{N} \xi_r C_{qp}^r \Gamma^r \varepsilon_p(s,t) + Ku_q(s,t)
\end{aligned}
$$

$$+\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}\beta_r M_{qp}^r(\varepsilon_q(s,t)-\varepsilon_p(s,t)+\phi_q(s,t)-\phi_p(s,t))$$
$$-\operatorname{sign}(\varepsilon_q(s,t))(\tilde{E}\breve{\varphi}+\tilde{B}\breve{\omega}). \tag{5}$$

According to event-triggered condition (3), we can obtain

$$\begin{aligned}
||\phi_q(s,t)|| \leqslant& \vartheta\Big|\Big|\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}M_{qp}^r(\delta_q(s,t_m^q)-\delta_p(s,t_m^p))\Big|\Big| \\
=&\vartheta\Big|\Big|\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}M_{qp}^r(\phi_q(s,t)+\varepsilon_q(s,t)-\phi_p(s,t)-\varepsilon_p(s,t))\Big|\Big| \\
\leqslant&\vartheta\Big[\Big|\Big|\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}M_{qp}^r(\varepsilon_q(s,t)-\varepsilon_p(s,t))\Big|\Big| \\
&+\Big|\Big|\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}M_{qp}^r(\phi_q(s,t)-\phi_p(s,t)\Big|\Big|\Big] \\
\leqslant&\vartheta\Big[\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}M_{qp}^r||\varepsilon_q(s,t)||+\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}M_{qp}^r||\varepsilon_p(s,t)|| \\
&+\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}M_{qp}^r||\phi_q(s,t)||+\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}M_{qp}^r||\phi_p(s,t)||\Big] \\
\leqslant&2\vartheta\sum_{r=1}^{\eta}\mathbf{m}_r(||\varepsilon(s,t)||+||\phi(s,t)||),
\end{aligned} \tag{6}$$

where $\mathbf{m}_r=\max\{\sum_{p\in\mathcal{N}_q}M_{qp}^r\}, r=1,2,\cdots,\eta, \varepsilon(s,t)=(\varepsilon_1^T(s,t),\varepsilon_2^T(s,t),\cdots,$ $\varepsilon_N^T(s,t))^T$ and $\phi(s,t)=(\phi_1^T(s,t),\phi_2^T(s,t),\cdots,\phi_N^T(s,t))^T$. Then,

$$||\phi(s,t)||\leqslant 2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r(||\varepsilon(s,t)||+||\phi(s,t)||).$$

Finally,

$$||\phi(s,t)||\leqslant\frac{2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r}{1-2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r}||\varepsilon(s,t)||, \tag{7}$$

where $0 < \vartheta < \frac{1}{2N\sum_{r=1}^{\eta}\mathbf{m}_r}$.

The output vector $y_q(s,t) \in \mathbb{R}^n$ for the network (5) is given as follows:

$$y_q(s,t) = Q_1\varepsilon_q(s,t) + Q_2u_q(s,t), \tag{8}$$

where $Q_1 \in \mathbb{R}^{n\times\psi}$ and $Q_2 \in \mathbb{R}^{n\times l}$.

Throughout this paper, we define

$$\begin{aligned}
&\Xi = \mathrm{diag}(\varphi_1^2, \varphi_2^2, \cdots, \varphi_\psi^2), \quad W = \mathrm{diag}(\omega_1^2, \omega_2^2, \cdots, \omega_\psi^2),\\
&u(s,t) = (u_1^T(s,t), u_2^T(s,t), \cdots, u_N^T(s,t))^T,\\
&y(s,t) = (y_1^T(s,t), y_2^T(s,t), \cdots, y_N^T(s,t))^T,\\
&H = \mathrm{diag}(\frac{1}{1-\zeta_1}, \frac{1}{1-\zeta_2}, \cdots, \frac{1}{1-\zeta_\psi}).
\end{aligned}$$

Remark 1. As is well known, Chua first proposed the concept of memristor in 1970s. Actually, the memristor in neural networks can be exploited instead of resistor to better comprehend the neural processes of the human brain. Until now, many worthwhile and meaningful results on the research for memristive neural networks (MNNs) have been acquired [25–28]. Nevertheless, only a few authors have investigated the synchronization of coupled memristive neural networks (CMNNs) [29–31]. It should be pointed out that there is no research results reported on the dynamical behaviors of coupled delayed reaction-diffusion memristive neural networks (CDRDMNNs). To the best of our knowledge , this paper is the first step toward to dealing with passivity and passivity-based synchronization problems of MWCDRDMNNs.

3.2. Event-triggered passivity

Theorem 3.1. *If there exist matrices* $F = \mathrm{diag}(\mathfrak{f}_1, \mathfrak{f}_2, \cdots, \mathfrak{f}_\psi) > 0, P \in \mathbb{R}^{nN\times lN}$ *and a constant* $0 < \vartheta < \frac{1}{2N\sum_{r=1}^{\eta}\mathbf{m}_r}$ *such that*

$$\begin{pmatrix} \mathcal{D}_1 & \Pi_1 \\ \Pi_1^T & \mathcal{H}_1 \end{pmatrix} \leqslant 0, \tag{9}$$

where $\mathcal{D}_1 = I_N \otimes [-\sum_{\kappa=1}^{\rho}\frac{2}{\mu_\kappa^2}FL_\kappa - 2FZ + WH + \Xi + F^2(\bar{E}+\bar{B}) + 2\sum_{r=1}^{\eta}\beta_r F^2 + 4N\sum_{r=1}^{\eta}\beta_r \mathbf{m}_r^2((\frac{2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r}{1-2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r})^2 + 1)I_\psi] + \sum_{r=1}^{\eta}\xi_r C^r \otimes (F\Gamma^r + \Gamma^r F), \Pi_1 = I_N \otimes (FK) - (I_N \otimes Q_1^T)P, \mathcal{H}_1 = -(I_N \otimes Q_2^T)P - P^T(I_N \otimes Q_2)$ *and* $\mathbf{m}_r = \max\{\sum_{p\in\mathcal{N}_q} M_{qp}^r\}, r = 1, 2, \cdots, \eta$*, then the system* (5) *realizes passivity under the event-triggered condition* (3)*.*

Proof. Construct the following Lyapunov functional for system (5):

$$\begin{aligned} V(t) =& \sum_{q=1}^{N}\sum_{j=1}^{\psi}\frac{\omega_j^2}{1-\zeta_j}\int_{t-\gamma_j(t)}^{t}\int_{\Phi}\varepsilon_{qj}^2(s,h)dsdh \\ &+ \sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)F\varepsilon_q(s,t)ds. \end{aligned} \tag{10}$$

Taking the upper right Dini derivative of $V(t)$ with respect to $t \in [t_m^q, t_{m+1}^q)$, we have

$$\begin{aligned} D^+V(t) =& 2\sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)F_q\frac{\partial\varepsilon_q(s,t)}{\partial t}ds + \sum_{q=1}^{N}\sum_{j=1}^{\psi}\frac{\omega_j^2}{1-\zeta_j}\int_{\Phi}\varepsilon_{qj}^2(s,t)ds \\ &- \sum_{q=1}^{N}\sum_{j=1}^{\psi}\frac{\omega_j^2(1-\dot{\gamma}_j(t))}{1-\zeta_j}\int_{\Phi}\varepsilon_{qj}^2(s,t-\gamma_j(t))ds \\ \leqslant& \sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)WH\varepsilon_q(s,t)ds + 2\sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)F\Big(\sum_{\kappa=1}^{\rho}L_\kappa\frac{\partial^2\varepsilon_q(s,t)}{\partial s_\kappa^2} \\ &+ E(\delta_q(s,t))\mathcal{F}(\varepsilon_q(s,t)) + B(\delta_q(s,t))\mathcal{G}(\overline{\varepsilon_q(s,t)}) + [E(\delta_q(s,t)) \end{aligned}$$

$$
\begin{aligned}
&- E(\delta^*(s))]f(\delta^*(s)) + [B(\delta_q(s,t)) - B(\delta^*(s))]g(\delta^*(s)) \\
&+ \sum_{r=1}^{\eta}\sum_{p=1}^{N}\xi_r C_{qp}^r \Gamma^r \varepsilon_p(s,t) + \sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}\beta_r M_{qp}^r(\varepsilon_q(s,t) \\
&- \varepsilon_p(s,t) + \phi_q(s,t) - \phi_p(s,t)) + Ku_q(s,t) \\
&- \text{sign}(\varepsilon_q(s,t))(\tilde{E}\breve{\varphi} + \tilde{B}\breve{\omega}) - Z\varepsilon_q(s,t)\Big)ds \\
&- \sum_{q=1}^{N}\int_{\Phi}\overline{\varepsilon_q(s,t)}^T W \overline{\varepsilon_q(s,t)} ds.
\end{aligned}
$$

From Green's formula and the Dirichlet boundary condition, one has

$$
\sum_{\kappa=1}^{\rho}\int_{\Phi}\varepsilon_{q\epsilon}(s,t)\frac{\partial^2\varepsilon_{q\epsilon}(s,t)}{\partial s_\kappa^2}ds = -\sum_{\kappa=1}^{\rho}\int_{\Phi}\Big(\frac{\partial\varepsilon_{q\epsilon}(s,t)}{\partial s_\kappa}\Big)^2 ds,
$$

where $\epsilon = 1, 2, \cdots, \psi, q = 1, 2, \cdots, N$. Then,

$$
\begin{aligned}
&\sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)F\sum_{\kappa=1}^{\rho}L_\kappa\frac{\partial^2\varepsilon_q(s,t)}{\partial s_\kappa^2}ds \\
=&\sum_{\kappa=1}^{\rho}\sum_{q=1}^{N}\sum_{\epsilon=1}^{\psi}\mathfrak{f}_\epsilon\ell_{\epsilon\kappa}\int_{\Phi}\varepsilon_{q\epsilon}(s,t)\frac{\partial^2\varepsilon_{q\epsilon}(s,t)}{\partial s_\kappa^2}ds \\
=&-\sum_{\kappa=1}^{\rho}\sum_{q=1}^{N}\sum_{\epsilon=1}^{\psi}\mathfrak{f}_\epsilon\ell_{\epsilon\kappa}\int_{\Phi}\Big(\frac{\partial\varepsilon_{q\epsilon}(s,t)}{\partial s_\kappa}\Big)^2 ds \\
=&-\sum_{\kappa=1}^{\rho}\sum_{q=1}^{N}\int_{\Phi}\Big(\frac{\partial\varepsilon_q(s,t)}{\partial s_\kappa}\Big)^T FL_\kappa\frac{\partial\varepsilon_q(s,t)}{\partial s_\kappa}ds.
\end{aligned}
$$

Then, we obtain

$$
\begin{aligned}
&2\sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)FL_\kappa\sum_{\kappa=1}^{\rho}\frac{\partial^2\varepsilon_q(s,t)}{\partial s_\kappa^2}ds \\
=&-\sum_{\kappa=1}^{\rho}\int_{\Phi}\Big(\frac{\partial\varepsilon(s,t)}{\partial s_\kappa}\Big)^T(I_N\otimes 2FL_\kappa)\frac{\partial\varepsilon(s,t)}{\partial s_\kappa}ds.
\end{aligned}
$$

Obviously, there is a real matrix $\Omega \in \mathbb{R}^{\psi N \times \psi N}$ satisfying

$$I_N \otimes 2FL_\kappa = \Omega^T \Omega.$$

Thus,

$$\Big(\frac{\partial \varepsilon(s,t)}{\partial s_\kappa}\Big)^T (I_N \otimes 2FL_\kappa) \frac{\partial \varepsilon(s,t)}{\partial s_\kappa} = \Big(\frac{\partial(\Omega \varepsilon(s,t))}{\partial s_\kappa}\Big)^T \frac{\partial(\Omega \varepsilon(s,t))}{\partial s_\kappa}.$$

Let $\pi(s,t) = \Omega\varepsilon(s,t)$, for $(s,t) \in \partial\Phi \times [-\gamma, +\infty)$. From the boundary condition of model (1), we get $\pi(s,t) = \Omega\varepsilon(s,t) = 0$. By Lemma 2.1, one has

$$\begin{aligned}
&\sum_{\kappa=1}^{\rho} \int_\Phi \Big(\frac{\partial \pi(s,t)}{\partial s_\kappa}\Big)^T \frac{\partial \pi(s,t)}{\partial s_\kappa} ds \\
\geqslant &\sum_{\kappa=1}^{\rho} \frac{1}{\mu_\kappa^2} \int_\Phi \pi^T(s,t)\pi(s,t) ds \\
= &\sum_{\kappa=1}^{\rho} \frac{2}{\mu_\kappa^2} \int_\Phi \varepsilon^T(s,t)(I_N \otimes FL_\kappa)\varepsilon(s,t) ds.
\end{aligned}$$

Therefore,

$$\begin{aligned}
&2\sum_{q=1}^{N} \int_\Phi \varepsilon_q^T(s,t) FL_\kappa \sum_{\kappa=1}^{\rho} \frac{\partial^2 \varepsilon_q(s,t)}{\partial s_\kappa^2} ds \\
\leqslant &-\sum_{\kappa=1}^{\rho} \frac{2}{\mu_\kappa^2} \int_\Phi \varepsilon^T(s,t)(I_N \otimes FL_\kappa)\varepsilon(s,t) ds. \qquad (11)
\end{aligned}$$

In addition, according to Lemma 2.2, we can derive that

$$\begin{aligned}
&2\sum_{q=1}^{N} \int_\Phi \varepsilon_q^T(s,t) FE\mathcal{F}(\varepsilon_q(s,t)) ds \\
=&2\sum_{q=1}^{N}\sum_{g=1}^{\psi}\sum_{h=1}^{\psi} \mathfrak{f}_g \int_\Phi \varepsilon_{qg}(s,t) e_{gh}(\delta_{qg}(s,t))(f_h(\delta_{qh}(s,t)) - f_h(\delta_h^*(s))) ds \\
\leqslant &2\sum_{q=1}^{N}\sum_{g=1}^{\psi}\sum_{h=1}^{\psi} \mathfrak{f}_g \bar{e}_{gh} \int_\Phi |\varepsilon_{qg}(s,t)||f_h(\delta_{qh}(s,t)) - f_h(\delta_h^*(s))| ds
\end{aligned}$$

$$\leqslant \sum_{q=1}^{N}\sum_{g=1}^{\psi}\sum_{h=1}^{\psi}\int_{\Phi}\mathfrak{f}_g^2\bar{e}_{gh}^2\varepsilon_{qg}^2(s,t)ds + \sum_{q=1}^{N}\sum_{h=1}^{\psi}\int_{\Phi}\varphi_h^2\varepsilon_{qh}^2(s,t)ds$$

$$\leqslant \sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)F^2\bar{E}\varepsilon_q(s,t)ds + \sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)\Xi\varepsilon_q(s,t)ds$$

$$= \int_{\Phi}\varepsilon^T(s,t)[I_N\otimes(F^2\bar{E}+\Xi)]\varepsilon(s,t)ds. \tag{12}$$

Similarly, we have

$$2\sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)FB(\delta_q(s,t))\mathcal{G}(\overline{\varepsilon_q(s,t)})ds$$

$$=2\sum_{q=1}^{N}\sum_{g=1}^{\psi}\sum_{h=1}^{\psi}\mathfrak{f}_g\int_{\Phi}\varepsilon_{qg}(s,t)b_{gh}(\delta_{qg}(s,t))(g_h(\delta_{qh}(s,t-\gamma_h(t)))-g_h(\delta_h^*(s)))ds$$

$$\leqslant 2\sum_{q=1}^{N}\sum_{g=1}^{\psi}\sum_{h=1}^{\psi}\mathfrak{f}_g\bar{b}_{gh}\int_{\Phi}|\varepsilon_{qg}(s,t)||g_h(\delta_{qh}(s,t-\gamma_h(t)))-g_h(\delta_h^*(s))|ds$$

$$\leqslant \sum_{q=1}^{N}\sum_{g=1}^{\psi}\sum_{h=1}^{\psi}\int_{\Phi}\mathfrak{f}_g^2\bar{b}_{gh}^2\varepsilon_{qg}^2(s,t)ds + \sum_{q=1}^{N}\sum_{h=1}^{\psi}\int_{\Phi}\omega_h^2\varepsilon_{qh}^2(s,t-\gamma_h(t))ds$$

$$\leqslant \sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)F^2\bar{B}\varepsilon_q(s,t)ds + \sum_{q=1}^{N}\int_{\Phi}\overline{\varepsilon_q(s,t)}^TW\overline{\varepsilon_q(s,t)}ds$$

$$= \sum_{q=1}^{N}\int_{\Phi}\overline{\varepsilon_q(s,t)}^TW\overline{\varepsilon_q(s,t)}ds + \int_{\Phi}\varepsilon^T(s,t)[I_N\otimes(F^2\bar{B})]\varepsilon(s,t)ds. \tag{13}$$

Furthermore,

$$2\sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)F[E(\delta_q(s,t))-E(\delta^*(s))]f(\delta^*(s))ds$$

$$=2\sum_{q=1}^{N}\sum_{g=1}^{\psi}\sum_{h=1}^{\psi}\mathfrak{f}_g\int_{\Phi}\varepsilon_{qg}(s,t)(e_{gh}(\delta_{qg}(s,t))-e_{gh}(\delta_g^*(s)))f_h(\delta_h^*(s))ds$$

$$\leqslant 2\sum_{q=1}^{N}\sum_{g=1}^{\psi}\sum_{h=1}^{\psi}\mathfrak{f}_g|\hat{e}_{gh}-\check{e}_{gh}|\breve{\varphi}_h\int_{\Phi}|\varepsilon_{qg}(s,t)|ds$$

$$\leqslant 2\sum_{q=1}^{N}\int_{\Phi}|\varepsilon_q(s,t)|F\tilde{E}\breve{\varphi}ds. \tag{14}$$

Similarly, one has

$$\begin{aligned}
&2\sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)F[B(\delta_q(s,t))-B(\delta^*(s))]g(\delta^*(s))ds\\
=&2\sum_{q=1}^{N}\sum_{g=1}^{\psi}\sum_{h=1}^{\psi}\mathfrak{f}_g\int_{\Phi}\varepsilon_{qg}(s,t)(b_{gh}(\delta_{qg}(s,t))-b_{gh}(\delta_g^*(s)))g_h(\delta_h^*(s))ds\\
\leqslant&2\sum_{q=1}^{N}\sum_{g=1}^{\psi}\sum_{h=1}^{\psi}\mathfrak{f}_g|\hat{b}_{gh}-\breve{b}_{gh}|\breve{\omega}_h\int_{\Phi}|\varepsilon_{qg}(s,t)|ds\\
\leqslant&2\sum_{q=1}^{N}\int_{\Phi}|\varepsilon_q(s,t)|F\tilde{B}\breve{\omega}ds.
\end{aligned} \tag{15}$$

Furthermore,

$$\begin{aligned}
&2\sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)F\sum_{r=1}^{\eta}\sum_{p=1}^{N}\xi_r C_{qp}^r\Gamma^r\varepsilon_p(s,t)ds\\
=&2\int_{\Phi}\varepsilon^T(s,t)F\sum_{r=1}^{\eta}\xi_r(C^r\otimes\Gamma^r)\varepsilon(s,t)ds\\
\leqslant&\int_{\Phi}\varepsilon^T(s,t)\sum_{r=1}^{\eta}\xi_r C^r\otimes(F\Gamma^r+\Gamma^r F)\varepsilon(s,t)ds.
\end{aligned} \tag{16}$$

In addition, we have

$$\begin{aligned}
&2\sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)F\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}\beta_r M_{qp}^r(\varepsilon_q(s,t)-\varepsilon_p(s,t))ds\\
\leqslant&\sum_{q=1}^{N}\sum_{r=1}^{\eta}\beta_r\int_{\Phi}\varepsilon_q^T(s,t)F^2\varepsilon_q(s,t)ds+\sum_{q=1}^{N}\sum_{r=1}^{\eta}\beta_r\Big|\Big|\sum_{p\in\mathcal{N}_q}M_{qp}^r(\varepsilon_q(s,t)\\
&-\varepsilon_p(s,t))\Big|\Big|^2\\
\leqslant&\sum_{q=1}^{N}\sum_{r=1}^{\eta}\beta_r\int_{\Phi}\varepsilon_q^T(s,t)F^2\varepsilon_q(s,t)ds+\sum_{q=1}^{N}\sum_{r=1}^{\eta}\beta_r\Big[\sum_{p\in\mathcal{N}_q}M_{qp}^r||\varepsilon_q(s,t)||
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{p\in\mathcal{N}_q} M^r_{qp}||\varepsilon_p(s,t))||\Big]^2\\
&\leqslant \sum_{r=1}^{\eta}\beta_r\int_\Phi \varepsilon^T(s,t)I_N\otimes F^2\varepsilon(s,t)ds+4\sum_{r=1}^{\eta}\beta_r\mathbf{m}_r^2N||\varepsilon(s,t)||^2\\
&=\sum_{r=1}^{\eta}\beta_r\int_\Phi \varepsilon^T(s,t)I_N\otimes(F^2+4N\mathbf{m}_r^2I_\psi)\varepsilon(s,t)ds. \qquad (17)
\end{aligned}
$$

From (7), one has

$$
\begin{aligned}
&2\sum_{q=1}^{N}\int_\Phi \varepsilon_q^T(s,t)F\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}\beta_r M^r_{qp}(\phi_q(s,t)-\phi_p(s,t))ds\\
&\leqslant \sum_{r=1}^{\eta}\beta_r\int_\Phi \varepsilon^T(s,t)I_N\otimes F^2\varepsilon(s,t)ds+\sum_{r=1}^{\eta}\beta_r\int_\Phi 4\mathbf{m}_r^2N\phi^T(s,t)\phi(s,t)ds\\
&\leqslant \sum_{r=1}^{\eta}\beta_r\int_\Phi \varepsilon^T(s,t)I_N\otimes\Big(F^2+4N\mathbf{m}_r^2\Big(\frac{2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r}{1-2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r}\Big)^2I_\psi\Big)\varepsilon(s,t)ds. \qquad (18)
\end{aligned}
$$

From (11)-(18), we can derive that

$$
\begin{aligned}
D^+V(t)\leqslant&\int_\Phi \varepsilon^T(s,t)\Big\{I_N\otimes\Big[-\sum_{\kappa=1}^{\rho}\frac{2}{\mu_\kappa^2}FL_\kappa-2FZ+F^2(\bar{E}+\bar{B})+2\sum_{r=1}^{\eta}\beta_rF^2\\
&+\Xi+WH+4N\sum_{r=1}^{\eta}\beta_r\mathbf{m}_r^2\Big(1+\Big(\frac{2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r}{1-2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r}\Big)^2\Big)I_\psi\Big]\\
&+\sum_{r=1}^{\eta}\xi_rC^r\otimes(F\Gamma^r+\Gamma^rF)\Big\}\varepsilon(s,t)ds\\
&+2\int_\Phi \varepsilon^T(s,t)(I_N\otimes FK)u(s,t)ds. \qquad (19)
\end{aligned}
$$

From (19), one gets

$$
\begin{aligned}
&D^+V(t)-2\int_\Phi y^T(s,t)Pu(s,t)ds\\
\leqslant&\int_\Phi u^T(s,t)(-(I_N\otimes Q_2^T)P-P^T(I_N\otimes Q_2))u(s,t)ds+\int_\Phi \varepsilon^T(s,t)\Big\{I_N\otimes[WH
\end{aligned}
$$

$$
+\Xi+F^2(\bar{E}+\bar{B})-\sum_{\kappa=1}^{\rho}\frac{2}{\mu_\kappa^2}FL_\kappa+4N\sum_{r=1}^{\eta}\beta_r\mathbf{m}_r^2\Big(\Big(\frac{2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r}{1-2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r}\Big)^2
$$

$$
+1\Big)I_\psi-2FZ+2\sum_{r=1}^{\eta}\beta_rF^2]+\sum_{r=1}^{\eta}\xi_rC^r\otimes(F\Gamma^r+\Gamma^rF)\Big\}\varepsilon(s,t)ds
$$

$$
+2\int_\Phi\varepsilon^T(s,t)(I_N\otimes(FK)-(I_N\otimes Q_1^T)P)u(s,t)ds
$$

$$
=\int_\Phi\varpi^T(s,t)\begin{pmatrix}\mathcal{D}_1 & \Pi_1\\ \Pi_1^T & \mathcal{H}_1\end{pmatrix}\varpi(s,t)ds,
$$

where $\varpi(s,t)=(\varepsilon^T(s,t),u^T(s,t))^T$. From (9), we have

$$
2\int_\Phi y^T(s,t)Pu(s,t)ds\geqslant D^+V(t).
$$

$$
\begin{aligned}
2\int_{t_r}^{t_p}\int_\Phi y^T(s,t)Pu(s,t)dsdt\geqslant&\int_{t_r}^{t_p}D^+V(t)dt\\
=&\int_{t_r}^{t_{k+1}}D^+V(t)dt+\int_{t_{k+1}}^{t_{k+2}}D^+V(t)dt\\
&+\cdots+\int_{t_{n-1}}^{t_n}D^+V(t)dt+\int_{t_n}^{t_p}D^+V(t)dt\\
=&V(t_{k+1})-V(t_r)+V(t_{k+2})-V(t_{k+1})\\
&+\cdots+V(t_n)-V(t_{n-1})+V(t_p)-V(t_n)\\
=&V(t_p)-V(t_r)
\end{aligned}
$$

for any $t_r,t_p\in[0,+\infty)$ and $t_p\geqslant t_r$, $t_{k+1},t_{k+2},\cdots,t_{n-1},t_n$ are event-triggered times between t_r and t_p if any, i.e., $t_k<t_r\leqslant t_{k+1},t_n<t_p\leqslant t_{n+1}$. In other words,

$$
\int_{t_r}^{t_p}\int_\Phi y^T(s,t)Pu(s,t)dsdt\geqslant\mathcal{Q}(t_p)-\mathcal{Q}(t_r),
$$

where $\mathcal{Q}(t)=\frac{V(t)}{2}$.

Similarly, we can derive the following results.

Theorem 3.2. *If there exist matrices* $F = \mathrm{diag}(\mathfrak{f}_1, \mathfrak{f}_2, \cdots, \mathfrak{f}_\psi) > 0, P \in \mathbb{R}^{nN\times lN}, 0 < J_1 \in \mathbb{R}^{lN\times lN}$ *and a constant* $0 < \vartheta < \frac{1}{2N\sum_{r=1}^{\eta}\mathbf{m}_r}$ *such that*

$$\begin{pmatrix} \mathcal{D}_1 & \Pi_1 \\ \Pi_1^T & \mathcal{H}_2 \end{pmatrix} \leqslant 0, \tag{20}$$

where $\mathcal{D}_1 = I_N \otimes [-\sum_{\kappa=1}^{\rho}\frac{2}{\mu_\kappa^2}FL_\kappa - 2FZ + WH + \Xi + F^2(\bar{E}+\bar{B}) + 2\sum_{r=1}^{\eta}\beta_r F^2 + 4N\sum_{r=1}^{\eta}\beta_r\mathbf{m}_r^2((\frac{2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r}{1-2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r})^2 + 1)I_\psi] + \sum_{r=1}^{\eta}\xi_r C^r \otimes (F\Gamma^r + \Gamma^r F), \Pi_1 = I_N \otimes (FK) - (I_N \otimes Q_1^T)P, \mathcal{H}_2 = J_1 - (I_N \otimes Q_2^T)P - P^T(I_N \otimes Q_2)$ *and* $\mathbf{m}_r = \max\{\sum_{p\in\mathcal{N}_q} M_{qp}^r\}, r = 1, 2, \cdots, \eta$, *then the system* (5) *realizes input-strict passivity under the event-triggered condition* (3).

Proof. According to (19), we have

$$\begin{aligned}
& D^+V(t) - 2\int_\Phi y^T(s,t)Pu(s,t)ds + \int_\Phi u^T(s,t)J_1u(s,t)ds \\
\leqslant & 2\int_\Phi \varepsilon^T(s,t)(I_N \otimes (FK) - (I_N \otimes Q_1^T)P)u(s,t)ds + \int_\Phi \varepsilon^T(s,t)\Big\{I_N \otimes [WH \\
& + \Xi + F^2(\bar{E}+\bar{B}) - \sum_{\kappa=1}^{\rho}\frac{2}{\mu_\kappa^2}FL_\kappa + 4N\sum_{r=1}^{\eta}\beta_r\mathbf{m}_r^2\Big(\Big(\frac{2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r}{1-2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r}\Big)^2 \\
& + 1\Big)I_\psi - 2FZ + 2\sum_{r=1}^{\eta}\beta_r F^2] + \sum_{r=1}^{\eta}\xi_r C^r \otimes (F\Gamma^r + \Gamma^r F)\Big\}\varepsilon(s,t)ds \\
& + \int_\Phi u^T(s,t)(J_1 - (I_N \otimes Q_2^T)P - P^T(I_N \otimes Q_2))u(s,t)ds \\
& = \int_\Phi \varpi^T(s,t)\begin{pmatrix} \mathcal{D}_1 & \Pi_1 \\ \Pi_1^T & \mathcal{H}_2 \end{pmatrix}\varpi(s,t)ds,
\end{aligned}$$

where $\mathcal{H}_2 = J_1 - (I_N \otimes Q_2^T)P - P^T(I_N \otimes Q_2)$. From (20), we can obtain

$$2\int_\Phi y^T(s,t)Pu(s,t)ds - \int_\Phi u^T(s,t)J_1u(s,t)ds \geqslant D^+V(t).$$

Then

$$
\begin{aligned}
2\int_{t_r}^{t_p}\int_{\Phi} y^T(s,t)Pu(s,t)dsdt \geqslant & \int_{t_r}^{t_p} D^+V(t)dt + \int_{t_r}^{t_p}\int_{\Phi} u^T(s,t)J_1u(s,t)dsdt \\
=& V(t_p) - V(t_r) + \int_{t_r}^{t_p}\int_{\Phi} u^T(s,t)J_1u(s,t)dsdt
\end{aligned}
$$

for any $t_r, t_p \in [0, +\infty)$ and $t_p \geqslant t_r$. In other words,

$$
\int_{t_r}^{t_p}\int_{\Phi}\Big(y^T(s,t)Pu(s,t) - u^T(s,t)\frac{J_1}{2}u(s,t)\Big)dsdt \geqslant \mathcal{Q}(t_p) - \mathcal{Q}(t_r),
$$

where $\mathcal{Q}(t) = \frac{V(t)}{2}$.

Theorem 3.3. *If there exist matrices* $F = \mathrm{diag}(\mathfrak{f}_1, \mathfrak{f}_2, \cdots, \mathfrak{f}_\psi) > 0, P \in \mathbb{R}^{nN\times lN}, 0 < J_2 \in \mathbb{R}^{nN\times nN}$ *and a constant* $0 < \vartheta < \frac{1}{2N\sum_{r=1}^{\eta}\mathbf{m}_r}$ *such that*

$$
\begin{pmatrix} \mathcal{D}_2 & \Pi_2 \\ \Pi_2^T & \mathcal{H}_3 \end{pmatrix} \leqslant 0, \tag{21}
$$

where $\mathcal{D}_2 = I_N\otimes[-\sum_{\kappa=1}^{\rho}\frac{2}{\mu_\kappa^2}FL_\kappa - 2FZ + WH + \Xi + F^2(\bar{E}+\bar{B}) + 2\sum_{r=1}^{\eta}\beta_rF^2 + 4N\sum_{r=1}^{\eta}\beta_r\mathbf{m}_r^2((\frac{2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r}{1-2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r})^2+1)I_\psi] + \sum_{r=1}^{\eta}\xi_rC^r\otimes(F\Gamma^r + \Gamma^rF) + (I_N\otimes Q_1^T)J_2(I_N\otimes Q_1), \Pi_2 = I_N\otimes(FK) + (I_N\otimes Q_1^T)J_2(I_N\otimes Q_2) - (I_N\otimes Q_1^T)P, \mathcal{H}_3 = (I_N\otimes Q_2^T)J_2(I_N\otimes Q_2) - (I_N\otimes Q_2^T)P - P^T(I_N\otimes Q_2)$ *and* $\mathbf{m}_r = \max\{\sum_{p\in\mathcal{N}_q}M_{qp}^r\}$, $r = 1, 2, \cdots, \eta$, *then the system* (5) *is output-strictly passive under the event-triggered condition* (3).

Proof. From (19), we can obtain

$$
\begin{aligned}
& D^+V(t) - 2\int_{\Phi} y^T(s,t)Pu(s,t)ds + \int_{\Phi} y^T(s,t)J_2y(s,t)ds \\
\leqslant & 2\int_{\Phi}\varepsilon^T(s,t)(I_N\otimes(FK) + (I_N\otimes Q_1^T)J_2(I_N\otimes Q_2) - (I_N\otimes Q_1^T)P)u(s,t)ds \\
& + \int_{\Phi}\varepsilon^T(s,t)\Big\{I_N\otimes[4N\sum_{r=1}^{\eta}\beta_r\mathbf{m}_r^2\Big(\Big(\frac{2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r}{1-2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r}\Big)^2+1\Big)I_\psi
\end{aligned}
$$

$$
\begin{aligned}
&+ \Xi + WH + F^2(\bar{E} + \bar{B}) - \sum_{\kappa=1}^{\rho} \frac{2}{\mu_\kappa^2} FL_\kappa + 2\sum_{r=1}^{\eta} \beta_r F^2 - 2FZ] \\
&+ (I_N \otimes Q_1^T) J_2 (I_N \otimes Q_1) + \sum_{r=1}^{\eta} \xi_r C^r \otimes (F\Gamma^r + \Gamma^r F) \Big\} \varepsilon(s,t) ds \\
&+ \int_\Phi u^T(s,t)((I_N \otimes Q_2^T) J_2 (I_N \otimes Q_2) - (I_N \otimes Q_2^T) P \\
&- P^T (I_N \otimes Q_2)) u(s,t) ds \\
&= \int_\Phi \varpi^T(s,t) \begin{pmatrix} \mathcal{D}_2 & \Pi_2 \\ \Pi_2^T & \mathcal{H}_3 \end{pmatrix} \varpi(s,t) ds,
\end{aligned}
$$

where $\mathcal{H}_3 = (I_N \otimes Q_2^T) J_2 (I_N \otimes Q_2) - (I_N \otimes Q_2^T) P - P^T (I_N \otimes Q_2)$. From (21), one has

$$
2 \int_\Phi y^T(s,t) P u(s,t) ds - \int_\Phi y^T(s,t) J_2 y(s,t) ds \geqslant D^+ V(t).
$$

Then

$$
\begin{aligned}
2 \int_{t_r}^{t_p} \int_\Phi y^T(s,t) P u(s,t) ds dt \geqslant & \int_{t_r}^{t_p} D^+ V(t) dt + \int_{t_r}^{t_p} \int_\Phi y^T(s,t) J_2 y(s,t) ds dt \\
= & V(t_p) - V(t_r) + \int_{t_r}^{t_p} \int_\Phi y^T(s,t) J_2 y(s,t) ds dt
\end{aligned}
$$

for any $t_r, t_p \in [0, +\infty)$ and $t_p \geqslant t_r$. In other words,

$$
\int_{t_r}^{t_p} \int_\Phi \Big(y^T(s,t) P u(s,t) - y^T(s,t) \frac{J_2}{2} y(s,t) \Big) ds dt \geqslant \mathcal{Q}(t_p) - \mathcal{Q}(t_r),
$$

where $\mathcal{Q}(t) = \frac{V(t)}{2}$.

Remark 2. As a matter of fact, a system's internal stability can be ensured by the passive property in system theory. Moreover, passivity has revealed comprehensive applications in a large number of domains. Thus, it is very necessary to conduct the research on the passivity of neural networks. In the past several years, many meaningful results about passivity of CNNs and CRDNNs have been derived [9–11, 19–22]. However, the derived passivity results on the aforementioned studies [9–11, 19–22] are based on the situation that the output has the same dimension as input. In addition, the networks discussed in these works coupled by single weight. Moreover, these networks did not include the memristive term. To our knowledge, the event-triggered passivity problem of MWCDRDMNNs with non-identical dimensional input and output has not been yet discussed, which motivates our research work in this paper. Due to the introduction of multi-weighted term and memristive term in our network model, the problem for achieving the event-triggered passivity of the considered network in our paper becomes more complicated, which cannot be dealt with by using the existing event-triggered control techniques for CNNs with single weights or without memristive term. For overcoming this difficulty, a novel event-triggered condition (3) is designed by utilizing the own characteristics of our network model, which is an improvement of some existing event-triggered conditions. Moreover, some new inequality techniques need to be employed in (6) and the proof of our theoretical results because of the memristive term, multi-weighted coupling term, and the new designed event-triggered condition.

3.3. Asymptotical stability of event-triggered passivity

Definition 3.1. *The network* (1) *under the event-triggered condition* (3) *is called to be synchronized if for all* $q = 1, 2, \cdots, N$,

$$\lim_{t\to+\infty} ||\delta_q(\cdot, t) - \delta^*(s)|| = 0$$

under the condition $u_q(s, t) = 0$.

Theorem 3.4. *Assume that $\mathcal{Q} : [0, +\infty) \longrightarrow [0, +\infty)$ is continuously differentiable satisfying*

$$\sigma_1(||\varepsilon(s,t)||) \leqslant \mathcal{Q}(t) \leqslant \sigma_2(||\varepsilon(s,t)||), \tag{22}$$

where $\sigma_1, \sigma_2 : [0, +\infty) \longrightarrow [0, +\infty)$ are continuous and strict increased functions, $\sigma_1(x) > 0, \sigma_2(x) > 0$ for $x > 0$ with $\sigma_1(0) = 0, \sigma_2(0) = 0$, then network (1) *realizes asymptotical stability if the system* (5) *is output-strictly passive under the event-triggered condition* (3) *with respect to $\mathcal{Q}(t)$ and $Q_1 \in \mathbb{R}^{n\times\psi}$ is a nonsingular matrix.*

Proof. If the system (5) achieves event-triggered output-strict passivity, then there exist matrices $P \in \mathbb{R}^{nN\times lN}$ and $0 < J_2 \in \mathbb{R}^{nN\times nN}$ such that

$$\mathcal{Q}(t+\theta) - \mathcal{Q}(t) \leqslant \int_t^{t+\theta} \int_\Phi \Big(y^T(s,t)Pu(s,t) - y^T(s,t)J_2 y(s,t)\Big)dsdt,$$

where $\theta > 0$ and $t \in [0, +\infty)$. Then, it is easy to derive that

$$\frac{\mathcal{Q}(t+\theta) - \mathcal{Q}(t)}{\theta} \leqslant \frac{1}{\theta}\int_t^{t+\theta} \int_\Phi \Big(y^T(s,t)Pu(s,t) - y^T(s,t)J_2 y(s,t)\Big)dsdt. \tag{23}$$

From (23), we take $\theta \to 0$, one has

$$\int_\Phi y^T(s,t)Pu(s,t)ds - \int_\Phi y^T(s,t)J_2 y(s,t)ds \geqslant D^+\mathcal{Q}(t).$$

Let $u(s,t) = 0$, we can derive

$$\begin{aligned} D^+\mathcal{Q}(t) &\leqslant -\int_\Phi \varepsilon^T(s,t)(I_N \otimes Q_1^T)J_2(I_N \otimes Q_1)\varepsilon(s,t)ds \\ &\leqslant -\lambda_m((I_N \otimes Q_1^T)J_2(I_N \otimes Q_1))||\varepsilon(s,t)||^2. \end{aligned} \tag{24}$$

According to (22) and (24), then the system (5) is asymptotically stable.

From Theorems 3.3 and 3.4, the following conclusion can be easily derived.

Corollary 3.1. *If there exist matrices* $F = \mathrm{diag}(\mathfrak{f}_1, \mathfrak{f}_2, \cdots, \mathfrak{f}_\psi) > 0, P \in \mathbb{R}^{nN\times lN}, 0 < J_2 \in \mathbb{R}^{nN\times nN}$ *and a constant* $0 < \vartheta < \frac{1}{2N\sum_{r=1}^{\eta}\mathbf{m}_r}$ *such that*

$$\begin{pmatrix} \mathcal{D}_2 & \Pi_2 \\ \Pi_2^T & \mathcal{H}_3 \end{pmatrix} \leqslant 0, \tag{25}$$

where $Q_1 \in \mathbb{R}^{n\times\psi}$ *is a nonsingular matrix,* $\mathcal{D}_2 = I_N \otimes [-\sum_{\kappa=1}^{\rho}\frac{2}{\mu_\kappa^2}FL_\kappa - 2FZ + WH + \Xi + F^2(\bar{E}+\bar{B}) + 2\sum_{r=1}^{\eta}\beta_r F^2 + 4N\sum_{r=1}^{\eta}\beta_r\mathbf{m}_r^2((\frac{2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r}{1-2\vartheta N\sum_{r=1}^{\eta}\mathbf{m}_r})^2 + 1)I_\psi] + \sum_{r=1}^{\eta}\xi_r C^r \otimes (F\Gamma^r + \Gamma^r F) + (I_N \otimes Q_1^T)J_2(I_N \otimes Q_1), \Pi_2 = I_N \otimes (FK) + (I_N \otimes Q_1^T)J_2(I_N \otimes Q_2) - (I_N \otimes Q_1^T)P, \mathcal{H}_3 = (I_N \otimes Q_2^T)J_2(I_N \otimes Q_2) - (I_N \otimes Q_2^T)P - P^T(I_N \otimes Q_2)$ *and* $\mathbf{m}_r = \max\{\sum_{p\in\mathcal{N}_q} M_{qp}^r\}, r = 1, 2, \cdots, \eta$, *then the network* (1) *is synchronized under the event-triggered condition* (3).

Remark 3. Strictly speaking, most of CNs in practical situations can not achieve synchronization and passivity by means of themselves. Hence, some effective control methods can be employed in this case in order to reach network's synchronization and passivity. Recently, event-triggered control, as a popular discrete control method, has been well adopted to realizing synchronization and passivity for CNs and CNNs [42–49] because it can cut down some redundant transmission of communication media when data information is exchanging. However, the event-triggered passivity and passivity-based synchronization of MWCDRDMNNs has not been discussed before, which prompts our research work in this paper. By mean of selecting an appropriate Lyapunov functional and designing a suitable event-triggered condition, several event-triggered passivity criteria are established in Theorems 3.1-3.3, and an asymptotical stability criterion and a passivity-based synchronization criterion under event-triggered condition are derived in Theorem 3.4 and Corollary 3.1 respectively.

4. Event-triggered passivity of MWCDRDMNNs with switching topology

In many real-world networks, the connection topology may change very quickly by switches. Therefore, we further address the event-triggered passivity of MWCDRDMNNs with switching topology in Section 4. First, we introduce the network model of MWCDRDMNNs with switching topology in Subsection 4.1. Then, by designing appropriate even-triggered condition, some sufficient conditions are derived for ensuring the considered network realize even-triggered passivity in Subsection 4.2. Furthermore, the event-triggered passivity-based synchronization criterion is acquired for MWCDRDMNNs with switching topology in Subsection 4.3.

4.1. Network model

In this section, we take the switching topology into account, the following MWCDRDMNNs is described by:

$$\begin{cases} \frac{\partial \delta_q(s,t)}{\partial t} &= \sum_{\kappa=1}^{\rho} L_\kappa \frac{\partial^2 \delta_q(s,t)}{\partial s_\kappa^2} - Z\delta_q(s,t) + E(\delta_q(s,t))f(\delta_q(s,t)) \\ & \quad +B(\delta_q(s,t))g(\overline{\delta_q(s,t)}) + O + Ku_q(s,t) + \eta_q(s,t) \\ & \quad + \sum_{r=1}^{\eta}\sum_{p=1}^{N} \xi_r C_{qp}^{r,\ell(t)}\Gamma^r \delta_p(s,t) + v_q(s,t), \\ \delta_q(s,t) &= \varrho_q(s,t), \qquad (s,t) \in \Phi \times [-\gamma, 0], \\ \delta_q(s,t) &= 0, \qquad (s,t) \in \partial\Phi \times [-\gamma, +\infty), \end{cases} \tag{26}$$

where $q = 1, 2, \cdots, N$, $\delta_q(s,t)$, $E(\delta_q(s,t))$, $f(\delta_q(s,t))$, $B(\delta_q(s,t))$, $g(\overline{\delta_q(s,t)})$, Z, $L_\kappa, O, K, u_q(s,t), \xi_r, \Gamma^r, \eta_q(s,t), \varrho_q(s,t)$ denote the same meanings as those in model (1); $v_q(s,t) = \sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q} \beta_r M_{qp}^{r,\ell(t)}(\delta_q(s,t_m^q) - \delta_p(s,t_m^p))$; $\ell(t) : [0,+\infty) \to \mathcal{V} = \{1, 2, \cdots, \upsilon\}$ stands for switching signal that is defined by a serial of switching sequence:

$$\mathbf{W} = \{(\varsigma_0, t_0), (\varsigma_1, t_1), \ldots, (\varsigma_\alpha, t_\alpha), \ldots | \varsigma_\alpha \in \mathcal{V}, \alpha \in \mathbb{N}\},$$

where the initial time is t_0, and ς_α represents the serial number of the activated subsystem at t_α; $C^{r,\Im} = (C^{r,\Im}_{qp})_{N\times N}$ for each $\Im \in \mathcal{V}$, $C^{r,\Im}$ satisfies $C^{r,\Im}_{qp} = C^{r,\Im}_{pq} > 0 (q \neq p)$ if node q and node p for the $\Im$th topology are connected, or else $C^{r,\Im}_{qp} = 0$, and $C^{r,\Im}_{qq} = -\sum_{\substack{p=1\\p\neq q}}^{N} C^{r,\Im}_{qp}$; $M^{r,\Im} = (M^{r,\Im}_{qp})_{N\times N}$, which satisfies $M^{r,\Im}_{qq} = 0, M^{r,\Im}_{qp} > 0$.

Suppose that $\delta^*(s) = (\delta^*_1(s), \delta^*_2(s), \cdots, \delta^*_\psi(s))^T$ is an equilibrium solution of the network (26). Then

$$\sum_{\kappa=1}^{\rho} L_\kappa \frac{\partial^2 \delta^*(s)}{\partial s_\kappa^2} - Z\delta^*(s) + E(\delta^*(s))f(\delta^*(s)) + B(\delta^*(s))g(\delta^*(s)) + O = 0.$$

Let $\phi_q(s,t) = \delta_q(s, t^q_m) - \delta_q(s,t)$. For $t \in [t^q_m, t^q_{m+1})$, we design the distributed triggering event as follows:

$$t^q_{m+1} = \inf\left\{t : t > t^q_m, \tilde{\theta}_q(s,t) > 0\right\} \text{ and}$$
$$\tilde{\theta}_q(s,t) = ||\phi_q(s,t)|| - \vartheta_\Im \Big|\Big| \sum_{r=1}^{\eta} \sum_{p\in\mathcal{N}_q} M^{r,\Im}_{qp} (\delta_q(s, t^q_m) - \delta_p(s, t^p_m)) \Big|\Big|, \qquad (27)$$

for all $\Im = 1, 2, \cdots, \upsilon$, and $\vartheta_\Im > 0$.

Let $\varepsilon_q(s,t) = \delta_q(s,t) - \delta^*(s)$. From the distributed triggering event (27), the error system of network (26) is stated by:

$$\begin{aligned}\frac{\partial \varepsilon_q(s,t)}{\partial t} =& \sum_{\kappa=1}^{\rho} L_\kappa \frac{\partial^2 \varepsilon_q(s,t)}{\partial s_\kappa^2} + E(\delta_q(s,t))\mathcal{F}(\varepsilon_q(s,t)) + B(\delta_q(s,t))\mathcal{G}(\overline{\varepsilon_q(s,t)}) \\ &+ [E(\delta_q(s,t)) - E(\delta^*(s))]f(\delta^*(s)) - Z\varepsilon_q(s,t) + [B(\delta_q(s,t)) \\ &- B(\delta^*(s))]g(\delta^*(s)) + \sum_{r=1}^{\eta}\sum_{p=1}^{N} \xi_r C^{r,\Im}_{qp} \Gamma^r \varepsilon_p(s,t) + Ku_q(s,t)\end{aligned}$$

$$
\begin{aligned}
&+\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}\beta_r M_{qp}^{r,\Im}(\varepsilon_q(s,t)-\varepsilon_p(s,t)+\phi_q(s,t)-\phi_p(s,t))\\
&-\operatorname{sign}(\varepsilon_q(s,t))(\tilde{E}\breve{\varphi}+\tilde{B}\breve{\omega}). \qquad (28)
\end{aligned}
$$

where $t\in[t_m^q,t_{m+1}^q), q=1,2,\cdots,N, \mathcal{F}(e_q(m,t))$ and $\mathcal{G}(\overline{e_q(m,t)})$ have the same meanings as those in system (5).

From event-triggered condition (27), one has

$$
\begin{aligned}
||\phi_q(s,t)||\leqslant&\vartheta_\Im\Big|\Big|\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}M_{qp}^{r,\Im}(\delta_q(s,t_m^q)-\delta_p(s,t_m^p))\Big|\Big|\\
\leqslant&\vartheta_\Im\Big[\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}M_{qp}^{r,\Im}||\varepsilon_q(s,t)||+\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}M_{qp}^{r,\Im}||\varepsilon_p(s,t)||\\
&+\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}M_{qp}^{r,\Im}||\phi_q(s,t)||+\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}M_{qp}^{r,\Im}||\phi_p(s,t)||\Big]\\
\leqslant&2\vartheta_\Im\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}(||\varepsilon(s,t)||+||\phi(s,t)||),
\end{aligned}
$$

where $\mathbf{m}_{r,\Im}=\max\{\sum_{p\in\mathcal{N}_q}M_{qp}^{r,\Im}\}, r=1,2,\cdots,\eta,\Im=1,2,\cdots,\upsilon$. Then,

$$
||\phi(s,t)||\leqslant 2\vartheta_\Im N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}(||\varepsilon(s,t)||+||\phi(s,t)||).
$$

Finally,

$$
||\phi(s,t)||\leqslant\frac{2\vartheta_\Im N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}}{1-2\vartheta_\Im N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}}||\varepsilon(s,t)||, \qquad (29)
$$

where $0<\vartheta_\Im<\frac{1}{2N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}}$.

Similarly, the output vector $y_q(s,t)$ for system (28) can be defined as (8).

4.2. Event-triggered passivity

Theorem 4.1. *If there exist matrices* $F = \mathrm{diag}(\mathfrak{f}_1, \mathfrak{f}_2, \cdots, \mathfrak{f}_\psi) > 0, P \in \mathbb{R}^{nN\times lN}$ *and a constant* $0 < \vartheta_\Im < \frac{1}{2N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}}$ *such that*

$$\begin{pmatrix} \mathcal{D}_3^{\Im} & \Pi_1 \\ \Pi_1^T & \mathcal{H}_1 \end{pmatrix} \leqslant 0, \tag{30}$$

where $\mathcal{D}_3^{\Im} = I_N\otimes[-\sum_{\kappa=1}^{\rho}\frac{2}{\mu_\kappa^2}FL_\kappa - 2FZ + WH + \Xi + F^2(\bar{E}+\bar{B}) + 2\sum_{r=1}^{\eta}\beta_r F^2 + 4N\sum_{r=1}^{\eta}\beta_r\mathbf{m}_{r,\Im}^2((\frac{2\vartheta_\Im N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}}{1-2\vartheta_\Im N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}})^2+1)I_\psi] + \sum_{r=1}^{\eta}\xi_r C^{r,\Im}\otimes(F\Gamma^r + \Gamma^r F), \Pi_1 = I_N\otimes(FK) - (I_N\otimes Q_1^T)P, \mathcal{H}_1 = -(I_N\otimes Q_2^T)P - P^T(I_N\otimes Q_2)$ *and* $\mathbf{m}_{r,\Im} = \max\{\sum_{p\in\mathcal{N}_q}M_{qp}^{r,\Im}\}, r = 1, 2, \cdots, \eta, \Im = 1, 2, \cdots, \upsilon,$ *then the system* (28) *realizes passivity under the event-triggered condition* (27).

Proof. Choose the same Lyapunov functional as (10) for system (28). Then, we have

$$\begin{aligned}
D^+V(t) \leqslant& \sum_{q=1}^{N}\int_\Phi \varepsilon_q^T(s,t)WH\varepsilon_q(s,t)ds + 2\sum_{q=1}^{N}\int_\Phi \varepsilon_q^T(s,t)F\Big(\sum_{\kappa=1}^{\rho}L_\kappa\frac{\partial^2\varepsilon_q(s,t)}{\partial s_\kappa^2} \\
&+ E(\delta_q(s,t))\mathcal{F}(\varepsilon_q(s,t)) + B(\delta_q(s,t))\mathcal{G}(\overline{\varepsilon_q(s,t)}) + [E(\delta_q(s,t)) \\
&- E(\delta^*(s))]f(\delta^*(s)) + [B(\delta_q(s,t)) - B(\delta^*(s))]g(\delta^*(s)) \\
&+ \sum_{r=1}^{\eta}\sum_{p=1}^{N}\xi_r C_{qp}^{r,\Im}\Gamma^r\varepsilon_p(s,t) + \sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}\beta_r M_{qp}^{r,\Im}(\varepsilon_q(s,t) \\
&- \varepsilon_p(s,t) + \phi_q(s,t) - \phi_p(s,t)) + Ku_q(s,t) \\
&- \mathrm{sign}(\varepsilon_q(s,t))(\tilde{E}\breve{\varphi} + \tilde{B}\breve{\omega}) - Z\varepsilon_q(s,t)\Big)ds \\
&- \sum_{q=1}^{N}\int_\Phi \overline{\varepsilon_q(s,t)}^T W\overline{\varepsilon_q(s,t)}ds.
\end{aligned}$$

Obviously,

$$
\begin{aligned}
&2\sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)F\sum_{r=1}^{\eta}\sum_{p=1}^{N}\xi_r C_{qp}^{r,\Im}\Gamma^r\varepsilon_p(s,t)ds\\
\leqslant&\int_{\Phi}\varepsilon^T(s,t)\sum_{r=1}^{\eta}\xi_r C^{r,\Im}\otimes(F\Gamma^r+\Gamma^r F)\varepsilon(s,t)ds. \qquad (31)
\end{aligned}
$$

Similar as the deduction of (17), we have

$$
\begin{aligned}
&2\sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)F\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}\beta_r M_{qp}^{r,\Im}(\varepsilon_q(s,t)-\varepsilon_p(s,t))ds\\
\leqslant&\sum_{q=1}^{N}\sum_{r=1}^{\eta}\beta_r\int_{\Phi}\varepsilon_q^T(s,t)F^2\varepsilon_q(s,t)ds+\sum_{q=1}^{N}\sum_{r=1}^{\eta}\beta_r\Big|\Big|\sum_{p\in\mathcal{N}_q}M_{qp}^{r,\Im}(\varepsilon_q(s,t)\\
&-\varepsilon_p(s,t))\Big|\Big|^2\\
\leqslant&\sum_{q=1}^{N}\sum_{r=1}^{\eta}\beta_r\int_{\Phi}\varepsilon_q^T(s,t)F^2\varepsilon_q(s,t)ds+\sum_{q=1}^{N}\sum_{r=1}^{\eta}\beta_r\Big[\sum_{p\in\mathcal{N}_q}M_{qp}^{r,\Im}||\varepsilon_q(s,t)||\\
&+\sum_{p\in\mathcal{N}_q}M_{qp}^{r,\Im}||\varepsilon_p(s,t))||\Big]^2\\
\leqslant&\sum_{r=1}^{\eta}\beta_r\int_{\Phi}\varepsilon^T(s,t)I_N\otimes F^2\varepsilon(s,t)ds+4\sum_{r=1}^{\eta}\beta_r\mathbf{m}_{r,\Im}^2N||\varepsilon(s,t)||^2\\
=&\sum_{r=1}^{\eta}\beta_r\int_{\Phi}\varepsilon^T(s,t)I_N\otimes(F^2+4N\mathbf{m}_{r,\Im}^2I_\psi)\varepsilon(s,t)ds. \qquad (32)
\end{aligned}
$$

From (29), one has

$$
\begin{aligned}
&2\sum_{q=1}^{N}\int_{\Phi}\varepsilon_q^T(s,t)F\sum_{r=1}^{\eta}\sum_{p\in\mathcal{N}_q}\beta_r M_{qp}^{r,\Im}(\phi_q(s,t)-\phi_p(s,t))ds\\
\leqslant&\sum_{r=1}^{\eta}\beta_r\int_{\Phi}\varepsilon^T(s,t)I_N\otimes F^2\varepsilon(s,t)ds+\sum_{r=1}^{\eta}\beta_r\int_{\Phi}4\mathbf{m}_{r,\Im}^2N\phi^T(s,t)\phi(s,t)ds\\
\leqslant&\sum_{r=1}^{\eta}\beta_r\int_{\Phi}\varepsilon^T(s,t)I_N\otimes\Big(F^2+4N\mathbf{m}_{r,\Im}^2\Big(\frac{2\vartheta_\Im N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}}{1-2\vartheta_\Im N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}}\Big)^2I_\psi\Big)\varepsilon(s,t)ds. \qquad (33)
\end{aligned}
$$

In terms of (11)-(15) and (31)-(33), one gets

$$
\begin{aligned}
D^+V(t) \leqslant & \int_\Phi \varepsilon^T(s,t)\Big\{ I_N \otimes \Big[-\sum_{\kappa=1}^{\rho} \frac{2}{\mu_\kappa^2} F L_\kappa - 2FZ + F^2(\bar{E}+\bar{B}) + 2\sum_{r=1}^{\eta} \beta_r F^2 \\
& + \Xi + WH + 4N \sum_{r=1}^{\eta} \beta_r \mathbf{m}_{r,\Im}^2 \Big(1 + \Big(\frac{2\vartheta_\Im N \sum_{r=1}^{\eta} \mathbf{m}_{r,\Im}}{1 - 2\vartheta_\Im N \sum_{r=1}^{\eta} \mathbf{m}_{r,\Im}}\Big)^2\Big) I_\psi \Big] \\
& + \sum_{r=1}^{\eta} \xi_r C^{r,\Im} \otimes (F\Gamma^r + \Gamma^r F)\Big\} \varepsilon(s,t) ds \\
& + 2\int_\Phi \varepsilon^T(s,t)(I_N \otimes FK) u(s,t) ds. \qquad (34)
\end{aligned}
$$

From (34), one gets

$$
\begin{aligned}
& D^+V(t) - 2\int_\Phi y^T(s,t) P u(s,t) ds \\
\leqslant & \int_\Phi u^T(s,t)(-(I_N \otimes Q_2^T)P - P^T(I_N \otimes Q_2)) u(s,t) ds + \int_\Phi \varepsilon^T(s,t)\Big\{ I_N \otimes [WH \\
& + \Xi + F^2(\bar{E}+\bar{B}) - \sum_{\kappa=1}^{\rho} \frac{2}{\mu_\kappa^2} F L_\kappa + 4N \sum_{r=1}^{\eta} \beta_r \mathbf{m}_{r,\Im}^2 \Big(\Big(\frac{2\vartheta_\Im N \sum_{r=1}^{\eta} \mathbf{m}_{r,\Im}}{1 - 2\vartheta_\Im N \sum_{r=1}^{\eta} \mathbf{m}_{r,\Im}}\Big)^2 \\
& + 1\Big) I_\psi - 2FZ + 2\sum_{r=1}^{\eta} \beta_r F^2] + \sum_{r=1}^{\eta} \xi_r C^{r,\Im} \otimes (F\Gamma^r + \Gamma^r F)\Big\} \varepsilon(s,t) ds \\
& + 2\int_\Phi \varepsilon^T(s,t)(I_N \otimes (FK) - (I_N \otimes Q_1^T)P) u(s,t) ds \\
& = \int_\Phi \varpi^T(s,t) \begin{pmatrix} \mathcal{D}_3^\Im & \Pi_1 \\ \Pi_1^T & \mathcal{H}_1 \end{pmatrix} \varpi(s,t) ds.
\end{aligned}
$$

From (30), we have

$$
2\int_\Phi y^T(s,t) P u(s,t) ds \geqslant D^+V(t).
$$

Then

$$
2\int_{t_r}^{t_p} \int_\Phi y^T(s,t) P u(s,t) ds dt \geqslant \int_{t_r}^{t_p} D^+V(t) dt = V(t_p) - V(t_r)
$$

for any $t_r, t_p \in [0, +\infty)$ and $t_p \geqslant t_r$. In other words,

$$\int_{t_r}^{t_p} \int_{\Phi} y^T(s,t) P u(s,t) ds dt \geqslant \mathcal{Q}(t_p) - \mathcal{Q}(t_r),$$

where $\mathcal{Q}(t) = \frac{V(t)}{2}$.

Similarly, we can derive the following results.

Theorem 4.2. *If there exist matrices $F = \mathrm{diag}(\mathfrak{f}_1, \mathfrak{f}_2, \cdots, \mathfrak{f}_\psi) > 0, P \in \mathbb{R}^{nN \times lN}, 0 < J_1 \in \mathbb{R}^{lN \times lN}$ and a constant $0 < \vartheta_{\Im} < \frac{1}{2N \sum_{r=1}^{\eta} \mathbf{m}_{r,\Im}}$ such that*

$$\begin{pmatrix} \mathcal{D}_3^{\Im} & \Pi_1 \\ \Pi_1^T & \mathcal{H}_2 \end{pmatrix} \leqslant 0, \tag{35}$$

where $\mathcal{D}_3^{\Im} = I_N \otimes [-\sum_{\kappa=1}^{\rho} \frac{2}{\mu_\kappa^2} F L_\kappa - 2FZ + WH + \Xi + F^2(\bar{E} + \bar{B}) + 2\sum_{r=1}^{\eta} \beta_r F^2 + 4N \sum_{r=1}^{\eta} \beta_r \mathbf{m}_{r,\Im}^2 ((\frac{2\vartheta_{\Im} N \sum_{r=1}^{\eta} \mathbf{m}_{r,\Im}}{1 - 2\vartheta_{\Im} N \sum_{r=1}^{\eta} \mathbf{m}_{r,\Im}})^2 + 1) I_\psi] + \sum_{r=1}^{\eta} \xi_r C^{r,\Im} \otimes (F\Gamma^r + \Gamma^r F), \Pi_1 = I_N \otimes (FK) - (I_N \otimes Q_1^T) P, \mathcal{H}_2 = J_1 - (I_N \otimes Q_2^T) P - P^T (I_N \otimes Q_2)$ and $\mathbf{m}_{r,\Im} = \max\{\sum_{p \in \mathcal{N}_q} M_{qp}^{r,\Im}\}, r = 1, 2, \cdots, \eta, \Im = 1, 2, \cdots, \upsilon$, then the system (28) *realizes input-strict passivity under the event-triggered condition* (27)*.*

Proof. From (34), one gets

$$\begin{aligned}
&D^+ V(t) - 2\int_{\Phi} y^T(s,t) P u(s,t) ds + \int_{\Phi} u^T(s,t) J_1 u(s,t) ds \\
\leqslant & 2\int_{\Phi} \varepsilon^T(s,t) (I_N \otimes (FK) - (I_N \otimes Q_1^T) P) u(s,t) ds + \int_{\Phi} \varepsilon^T(s,t) \Big\{ I_N \otimes [WH + \Xi \\
&+ F^2(\bar{E} + \bar{B}) - \sum_{\kappa=1}^{\rho} \frac{2}{\mu_\kappa^2} F L_\kappa + 4N \sum_{r=1}^{\eta} \beta_r \mathbf{m}_{r,\Im}^2 \Big(\Big(\frac{2\vartheta_{\Im} N \sum_{r=1}^{\eta} \mathbf{m}_{r,\Im}}{1 - 2\vartheta_{\Im} N \sum_{r=1}^{\eta} \mathbf{m}_{r,\Im}} \Big)^2 \\
&+ 1 \Big) I_\psi - 2FZ + 2\sum_{r=1}^{\eta} \beta_r F^2] + \sum_{r=1}^{\eta} \xi_r C^{r,\Im} \otimes (F\Gamma^r + \Gamma^r F) \Big\} \varepsilon(s,t) ds \\
&+ \int_{\Phi} u^T(s,t) (J_1 - (I_N \otimes Q_2^T) P - P^T (I_N \otimes Q_2)) u(s,t) ds
\end{aligned}$$

$$= \int_{\Phi} \varpi^T(s,t) \begin{pmatrix} \mathcal{D}_3^{\Im} & \Pi_1 \\ \Pi_1^T & \mathcal{H}_2 \end{pmatrix} \varpi(s,t)ds.$$

From (35), we can obtain

$$2\int_{\Phi} y^T(s,t)Pu(s,t)ds - \int_{\Phi} u^T(s,t)J_1u(s,t)ds \geqslant D^+V(t).$$

Then

$$\begin{aligned} 2\int_{t_r}^{t_p}\int_{\Phi} y^T(s,t)Pu(s,t)dsdt \geqslant & \int_{t_r}^{t_p} D^+V(t)dt + \int_{t_r}^{t_p}\int_{\Phi} u^T(s,t)J_1u(s,t)dsdt \\ =& V(t_p) - V(t_r) + \int_{t_r}^{t_p}\int_{\Phi} u^T(s,t)J_1u(s,t)dsdt \end{aligned}$$

for any $t_r, t_p \in [0, +\infty)$ and $t_p \geqslant t_r$. In other words,

$$\int_{t_r}^{t_p}\int_{\Phi} \Big(y^T(s,t)Pu(s,t) - u^T(s,t)\frac{J_1}{2}u(s,t)\Big)dsdt \geqslant \mathcal{Q}(t_p) - \mathcal{Q}(t_r),$$

where $\mathcal{Q}(t) = \frac{V(t)}{2}$.

Theorem 4.3. *If there exist matrices* $F = \mathrm{diag}(\mathfrak{f}_1, \mathfrak{f}_2, \cdots, \mathfrak{f}_\psi) > 0, P \in \mathbb{R}^{nN\times lN}, 0 < J_2 \in \mathbb{R}^{nN\times nN}$ *and a constant* $0 < \vartheta_{\Im} < \frac{1}{2N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}}$ *such that*

$$\begin{pmatrix} \mathcal{D}_4^{\Im} & \Pi_2 \\ \Pi_2^T & \mathcal{H}_3 \end{pmatrix} \leqslant 0, \tag{36}$$

where $\mathcal{D}_4^{\Im} = I_N \otimes [-\sum_{\kappa=1}^{\rho}\frac{2}{\mu_\kappa^2}FL_\kappa - 2FZ + WH + \Xi + F^2(\bar{E}+\bar{B}) + 2\sum_{r=1}^{\eta}\beta_r F^2 + 4N\sum_{r=1}^{\eta}\beta_r\mathbf{m}_{r,\Im}^2((\frac{2\vartheta_{\Im}N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}}{1-2\vartheta_{\Im}N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}})^2 + 1)I_\psi] + \sum_{r=1}^{\eta}\xi_r C^{r,\Im} \otimes (F\Gamma^r + \Gamma^r F) + (I_N \otimes Q_1^T)J_2(I_N \otimes Q_1), \Pi_2 = I_N \otimes (FK) + (I_N \otimes Q_1^T)J_2(I_N \otimes Q_2) - (I_N \otimes Q_1^T)P, \mathcal{H}_3 = (I_N \otimes Q_2^T)J_2(I_N \otimes Q_2) - (I_N \otimes Q_2^T)P - P^T(I_N \otimes Q_2)$ *and* $\mathbf{m}_{r,\Im} = \max\{\sum_{p\in\mathcal{N}_q} M_{qp}^{r,\Im}\}, r = 1, 2, \cdots, \eta, \Im = 1, 2, \cdots, \upsilon$*, then the system* (28) *is output-strictly passive under the event-triggered condition* (27).

Proof. From (34), we can obtain

$$\begin{aligned}
&D^{+}V(t)-2\int_{\Phi}y^{T}(s,t)Pu(s,t)ds+\int_{\Phi}y^{T}(s,t)J_{2}y(s,t)ds\\
\leqslant&2\int_{\Phi}\varepsilon^{T}(s,t)(I_{N}\otimes(FK)+(I_{N}\otimes Q_{1}^{T})J_{2}(I_{N}\otimes Q_{2})-(I_{N}\otimes Q_{1}^{T})P)u(s,t)ds\\
&+\int_{\Phi}\varepsilon^{T}(s,t)\Big\{I_{N}\otimes[4N\sum_{r=1}^{\eta}\beta_{r}\mathbf{m}_{r,\Im}^{2}\Big(\Big(\frac{2\vartheta_{\Im}N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}}{1-2\vartheta_{\Im}N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}}\Big)^{2}+1\Big)I_{\psi}\\
&+\Xi+WH+F^{2}(\bar{E}+\bar{B})-\sum_{\kappa=1}^{\rho}\frac{2}{\mu_{\kappa}^{2}}FL_{\kappa}+2\sum_{r=1}^{\eta}\beta_{r}F^{2}-2FZ]\\
&+(I_{N}\otimes Q_{1}^{T})J_{2}(I_{N}\otimes Q_{1})+\sum_{r=1}^{\eta}\xi_{r}C^{r,\Im}\otimes(F\Gamma^{r}+\Gamma^{r}F)\Big\}\varepsilon(s,t)ds\\
&+\int_{\Phi}u^{T}(s,t)((I_{N}\otimes Q_{2}^{T})J_{2}(I_{N}\otimes Q_{2})-(I_{N}\otimes Q_{2}^{T})P\\
&-P^{T}(I_{N}\otimes Q_{2}))u(s,t)ds\\
&=\int_{\Phi}\varpi^{T}(s,t)\begin{pmatrix}\mathcal{D}_{4}^{\Im} & \Pi_{2}\\ \Pi_{2}^{T} & \mathcal{H}_{3}\end{pmatrix}\varpi(s,t)ds.
\end{aligned}$$

According to (36), one has

$$2\int_{\Phi}y^{T}(s,t)Pu(s,t)ds-\int_{\Phi}y^{T}(s,t)J_{2}y(s,t)ds\geqslant D^{+}V(t).$$

Then

$$\begin{aligned}
2\int_{t_{r}}^{t_{p}}\int_{\Phi}y^{T}(s,t)Pu(s,t)dsdt\geqslant&\int_{t_{r}}^{t_{p}}D^{+}V(t)dt+\int_{t_{r}}^{t_{p}}\int_{\Phi}y^{T}(s,t)J_{2}y(s,t)dsdt\\
=&V(t_{p})-V(t_{r})+\int_{t_{r}}^{t_{p}}\int_{\Phi}y^{T}(s,t)J_{2}y(s,t)dsdt
\end{aligned}$$

for any $t_r, t_p \in [0, +\infty)$ and $t_p \geqslant t_r$. In other words,

$$\int_{t_{r}}^{t_{p}}\int_{\Phi}\Big(y^{T}(s,t)Pu(s,t)-y^{T}(s,t)\frac{J_{2}}{2}y(s,t)\Big)dsdt\geqslant\mathcal{Q}(t_{p})-\mathcal{Q}(t_{r}),$$

where $\mathcal{Q}(t)=\frac{V(t)}{2}$.

4.3. Asymptotical stability of event-triggered passivity

Theorem 4.4. *Assume that $\mathcal{Q} : [0, +\infty) \longrightarrow [0, +\infty)$ is continuously differentiable satisfying*

$$\sigma_1(||\varepsilon(s,t)||) \leqslant \mathcal{Q}(t) \leqslant \sigma_2(||\varepsilon(s,t)||), \tag{37}$$

where $\sigma_1, \sigma_2 : [0, +\infty) \longrightarrow [0, +\infty)$ are continuous and strict increased functions, $\sigma_1(x) > 0, \sigma_2(x) > 0$ for $x > 0$ with $\sigma_1(0) = 0, \sigma_2(0) = 0$, then network (26) *realizes asymptotical stability if the system* (28) *is output-strictly passive under the event-triggered condition* (27) *with respect to $\mathcal{Q}(t)$ and $Q_1 \in \mathbb{R}^{n\times\psi}$ is a nonsingular matrix.*

Proof. If the system (28) achieves event-triggered output-strict passivity, then there exist matrices $P \in \mathbb{R}^{nN\times lN}$ and $0 < J_2 \in \mathbb{R}^{nN\times nN}$ such that

$$\mathcal{Q}(t+\theta) - \mathcal{Q}(t) \leqslant \int_t^{t+\theta} \int_\Phi \Big(y^T(s,t)Pu(s,t) - y^T(s,t)J_2 y(s,t)\Big) ds dt,$$

where $\theta > 0$ and $t \in [0, +\infty)$. Then, it is easy to derive that

$$\frac{\mathcal{Q}(t+\theta) - \mathcal{Q}(t)}{\theta} \leqslant \frac{1}{\theta}\int_t^{t+\theta} \int_\Phi \Big(y^T(s,t)Pu(s,t) - y^T(s,t)J_2 y(s,t)\Big) ds dt. \tag{38}$$

From (38), we take $\theta \to 0$, one has

$$\int_\Phi y^T(s,t)Pu(s,t)ds - \int_\Phi y^T(s,t)J_2 y(s,t)ds \geqslant D^+\mathcal{Q}(t).$$

Let $u(s,t) = 0$, we can derive

$$\begin{aligned} D^+\mathcal{Q}(t) &\leqslant -\int_\Phi \varepsilon^T(s,t)(I_N \otimes Q_1^T)J_2(I_N \otimes Q_1)\varepsilon(s,t)ds \\ &\leqslant -\lambda_m((I_N \otimes Q_1^T)J_2(I_N \otimes Q_1))||\varepsilon(s,t)||^2. \end{aligned} \tag{39}$$

From (37) and (39), then the system (28) is asymptotically stable.

According to Theorems 4.3 and 4.4, the following conclusion can be easily derived.

Corollary 4.1. *If there exist matrices* $F = \mathrm{diag}(\mathfrak{f}_1, \mathfrak{f}_2, \cdots, \mathfrak{f}_\psi) > 0, P \in \mathbb{R}^{nN\times lN}, 0 < J_2 \in \mathbb{R}^{nN\times nN}$ *and a constant* $0 < \vartheta_\Im < \frac{1}{2N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}}$ *such that*

$$\begin{pmatrix} \mathcal{D}_4^\Im & \Pi_2 \\ \Pi_2^T & \mathcal{H}_3 \end{pmatrix} \leqslant 0, \tag{40}$$

where $Q_1 \in \mathbb{R}^{n\times\psi}$ *is a nonsingular matrix,* $\mathcal{D}_4^\Im = I_N \otimes [-\sum_{\kappa=1}^{\rho}\frac{2}{\mu_\kappa^2}FL_\kappa - 2FZ + WH + \Xi + F^2(\bar{E}+\bar{B}) + 2\sum_{r=1}^{\eta}\beta_r F^2 + 4N\sum_{r=1}^{\eta}\beta_r \mathbf{m}_{r,\Im}^2((\frac{2\vartheta_\Im N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}}{1-2\vartheta_\Im N\sum_{r=1}^{\eta}\mathbf{m}_{r,\Im}})^2 + 1)I_\psi] + \sum_{r=1}^{\eta}\xi_r C^{r,\Im}\otimes(F\Gamma^r + \Gamma^r F) + (I_N\otimes Q_1^T)J_2(I_N\otimes Q_1), \Pi_2 = I_N\otimes(FK) + (I_N\otimes Q_1^T)J_2(I_N\otimes Q_2) - (I_N\otimes Q_1^T)P, \mathcal{H}_3 = (I_N\otimes Q_2^T)J_2(I_N\otimes Q_2) - (I_N\otimes Q_2^T)P - P^T(I_N\otimes Q_2)$ *and* $\mathbf{m}^{r,\Im} = \max\{\sum_{p\in\mathcal{N}_q} M_{qp}^{r,\Im}\}, r = 1, 2, \cdots, \eta,$ *then the network* (26) *is synchronized under the event-triggered condition* (27).

Remark 4. Note that the coupling configuration matrices C_{qp}^r, M_{qp}^r in the considered MWCDRDMNNs (1) are certain, which means the topology of network (1) are entirely fixed. Nevertheless, it is utterly general that the connection topology in real networks changes very quickly by switches because of stochastic disturbances and link failure. Consequently, it is necessary to discuss the dynamical properties of MWCDRDMNNs with switching topology. In this section, we derive some event-triggered passivity and passivity-based synchronization conditions for MWCDRDMNNs with switching topology in Theorems 4.1-4.4 and Corollary 4.1, respectively.

5. Numerical examples

In this section, two examples are given to illustrate the obtained theoretical results in Sections 3 and 4. More precisely, the event-triggered passivity and synchronization results for MWCDRDMNNs with fixed topology in Theorem 3.1 and Corollary 3.1 are demonstrated in Case 1 and Case 2 of Example 5.1 respectively. Similarly, the results for the network of MWCDRDMNNs with switching topology is validated in Example 5.2. In addition, some simulation figures are also provided for better understanding the effectiveness of our results.

Example 5.1. Considering MWCDRDMNNs with fixed topology under event-triggered control as follows:

$$\begin{aligned}\frac{\partial \delta_q(s,t)}{\partial t} =& L\frac{\partial^2 \delta_q(s,t)}{\partial s^2} - Z\delta_q(s,t) + E(\delta_q(s,t))f(\delta_q(s,t)) + O + Ku_q(s,t)\\ &+ B(\delta_q(s,t))g(\overline{\delta_q(s,t)}) + \eta_q(s,t) + \sum_{r=1}^{\eta}\sum_{p=1}^{5}\xi_r C_{qp}^r \Gamma^r \delta_p(s,t)\\ &+ v_q(s,t), \end{aligned} \tag{41}$$

where $q = 1,2,3,4,5, \psi = 3, \Phi = \{s|-0.5 < s < 0.5\}, f_\iota(\chi) = \frac{|\chi+1|-|\chi-1|}{8}, g_\iota(\chi) = \frac{|\chi+1|-|\chi-1|}{4}, \iota = 1,2,3; L = \mathrm{diag}(1.2, 1.6, 1.5), Z = \mathrm{diag}(6,7,8); \gamma_j(t) = 1 - \frac{1}{j+4}e^{-t}, \zeta_j = \frac{1}{j+4}, j = 1,2,3; O = (0,0,0)^T, \Gamma^1 = \mathrm{diag}(1.2, 0.8, 1.8), \Gamma^2 = \mathrm{diag}(0.9, 0.6, 1.5); \xi_1 = 0.08, \xi_2 = 0.4, v_q(s,t) = \sum_{r=1}^{2}\beta_r\sum_{p\in\mathcal{N}_q} M_{qp}^r(\delta_q(s,t_m^q) - \delta_p(s,t_m^p)); \eta_q(s,t) = -\mathrm{sign}(\delta_q(s,t))(\tilde{E}\breve{\varphi} + \tilde{B}\breve{\omega}), \beta_1 = 0.05, \beta_2 = 0.2; u_q(s,t) = (1.2q\sqrt{t}\cos(\pi s), 1.5q\sqrt{t}\cos(\pi s))^T$. The following matrices $C^r = (C_{qp}^r)_{5\times5}, M^r = (M_{qp}^r)_{5\times5}, E(\delta_q(s,t)), B(\delta_q(s,t)), K, Q_1, Q_2$ are chosen respectively:

$$e_{11}(\delta_{q1}(s,t)) = \begin{cases} -0.24, & |\delta_{q1}(s,t)| \leqslant 2,\\ 0.28, & |\delta_{q1}(s,t)| > 2,\end{cases}$$

$$e_{12}(\delta_{q1}(s,t)) = \begin{cases} -0.18, & |\delta_{q1}(s,t)| \leqslant 2, \\ 0.35, & |\delta_{q1}(s,t)| > 2, \end{cases}$$

$$e_{13}(\delta_{q1}(s,t)) = \begin{cases} -0.32, & |\delta_{q1}(s,t)| \leqslant 2, \\ 0.36, & |\delta_{q1}(s,t)| > 2, \end{cases}$$

$$e_{21}(\delta_{q2}(s,t)) = \begin{cases} 0.38, & |\delta_{q2}(s,t)| \leqslant 2, \\ -0.56, & |\delta_{q2}(s,t)| > 2, \end{cases}$$

$$e_{22}(\delta_{q2}(s,t)) = \begin{cases} -0.24, & |\delta_{q2}(s,t)| \leqslant 2, \\ 0.32, & |\delta_{q2}(s,t)| > 2, \end{cases}$$

$$e_{23}(\delta_{q2}(s,t)) = \begin{cases} 0.43, & |\delta_{q2}(s,t)| \leqslant 2, \\ -0.52, & |\delta_{q2}(s,t)| > 2, \end{cases}$$

$$e_{31}(\delta_{q3}(s,t)) = \begin{cases} 0.32, & |\delta_{q3}(s,t)| \leqslant 2, \\ -0.45, & |\delta_{q3}(s,t)| > 2, \end{cases}$$

$$e_{32}(\delta_{q3}(s,t)) = \begin{cases} 0.27, & |\delta_{q3}(s,t)| \leqslant 2, \\ -0.53, & |\delta_{q3}(s,t)| > 2, \end{cases}$$

$$e_{33}(\delta_{q3}(s,t)) = \begin{cases} -0.18, & |\delta_{q3}(s,t)| \leqslant 2, \\ -0.36, & |\delta_{q3}(s,t)| > 2, \end{cases}$$

$$b_{11}(\delta_{q1}(s,t)) = \begin{cases} 0.42, & |\delta_{q1}(s,t)| \leqslant 2, \\ -0.45, & |\delta_{q1}(s,t)| > 2, \end{cases}$$

$$b_{12}(\delta_{q1}(s,t)) = \begin{cases} -0.32, & |\delta_{q1}(s,t)| \leqslant 2, \\ 0.26, & |\delta_{q1}(s,t)| > 2, \end{cases}$$

$$b_{13}(\delta_{q1}(s,t)) = \begin{cases} -0.46, & |\delta_{q1}(s,t)| \leqslant 2, \\ 0.29, & |\delta_{q1}(s,t)| > 2, \end{cases}$$

$$b_{21}(\delta_{q2}(s,t)) = \begin{cases} -0.23, & |\delta_{q2}(s,t)| \leqslant 2, \\ -0.36, & |\delta_{q2}(s,t)| > 2, \end{cases}$$

$$b_{22}(\delta_{q2}(s,t)) = \begin{cases} 0.32, & |\delta_{q2}(s,t)| \leqslant 2, \\ -0.47, & |\delta_{q2}(s,t)| > 2, \end{cases}$$

$$b_{23}(\delta_{q2}(s,t)) = \begin{cases} -0.18, & |\delta_{q2}(s,t)| \leqslant 2, \\ 0.35, & |\delta_{q2}(s,t)| > 2, \end{cases}$$

$$b_{31}(\delta_{q3}(s,t)) = \begin{cases} 0.34, & |\delta_{q3}(s,t)| \leqslant 2, \\ -0.42, & |\delta_{q3}(s,t)| > 2, \end{cases}$$

$$b_{32}(\delta_{q3}(s,t)) = \begin{cases} 0.36, & |\delta_{q3}(s,t)| \leqslant 2, \\ -0.25, & |\delta_{q3}(s,t)| > 2, \end{cases}$$

$$b_{33}(\delta_{q3}(s,t)) = \begin{cases} -0.31, & |\delta_{q3}(s,t)| \leqslant 2, \\ 0.29, & |\delta_{q3}(s,t)| > 2, \end{cases}$$

$$Q_1 = \begin{pmatrix} 0.4 & 0.5 & 0.2 \\ 0.2 & 0.8 & 0.4 \\ 0.3 & 0.4 & 0.2 \end{pmatrix}, Q_2 = \begin{pmatrix} 0.6 & 0.3 \\ 0.3 & 0.4 \\ 0.5 & 0.3 \end{pmatrix},$$

$$C^1 = \begin{pmatrix} -0.5 & 0.1 & 0 & 0.3 & 0.1 \\ 0.1 & -0.6 & 0.1 & 0.2 & 0.2 \\ 0 & 0.1 & -0.3 & 0 & 0.2 \\ 0.3 & 0.2 & 0 & -0.7 & 0.2 \\ 0.1 & 0.2 & 0.2 & 0.2 & -0.7 \end{pmatrix},$$

$$C^2 = \begin{pmatrix} -0.7 & 0.2 & 0 & 0.2 & 0.3 \\ 0.2 & -0.6 & 0.2 & 0.1 & 0.1 \\ 0 & 0.2 & -0.6 & 0 & 0.4 \\ 0.2 & 0.1 & 0 & -0.5 & 0.2 \\ 0.3 & 0.1 & 0.4 & 0.2 & -1 \end{pmatrix},$$

$$M^1 = \begin{pmatrix} 0 & 0.02 & 0.03 & 0.02 & 0.04 \\ 0.1 & 0 & 0.4 & 0.2 & 0.3 \\ 0.02 & 0.02 & 0 & 0.05 & 0.02 \\ 0.2 & 0.5 & 0.3 & 0 & 0.3 \\ 0.03 & 0.02 & 0.03 & 0.06 & 0 \end{pmatrix},$$

$$M^2 = \begin{pmatrix} 0 & 0.04 & 0.01 & 0.02 & 0.03 \\ 0.02 & 0 & 0.02 & 0.02 & 0.3 \\ 0.03 & 0.02 & 0 & 0.05 & 0.02 \\ 0.06 & 0.05 & 0.01 & 0 & 0.03 \\ 0.03 & 0.02 & 0.03 & 0.02 & 0 \end{pmatrix},$$

$$K = \begin{pmatrix} 0.2 & 0.3 \\ 0.3 & 0.2 \\ 0.4 & 0.6 \end{pmatrix}.$$

Hence,

$$\tilde{E} = \begin{pmatrix} 0.52 & 0.53 & 0.68 \\ 0.94 & 0.56 & 0.95 \\ 0.77 & 0.8 & 0.18 \end{pmatrix}, \tilde{B} = \begin{pmatrix} 0.87 & 0.58 & 0.75 \\ 0.13 & 0.79 & 0.53 \\ 0.76 & 0.61 & 0.6 \end{pmatrix}.$$

Case 1: Obviously, the equilibrium solution of the isolated nodes of network (41) is $\delta^*(s) = (0, 0, 0)^T, \varphi_j = \breve{\varphi}_j = 0.25, \omega_j = \breve{\omega}_j = 0.5$. Take $\vartheta = 0.036$

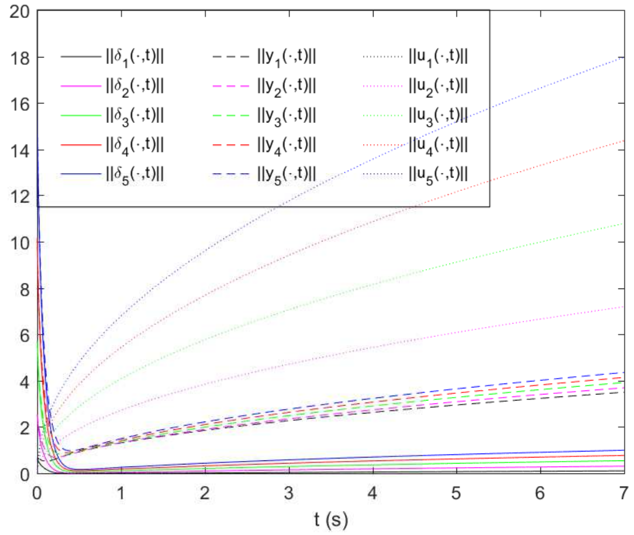

Figure 1: The norms of the state vectors $\delta_q(s,t)$, output vectors $y_q(s,t)$ and output vectors $u_q(s,t)$ in network (41), where $q = 1, 2, \cdots, 5$.

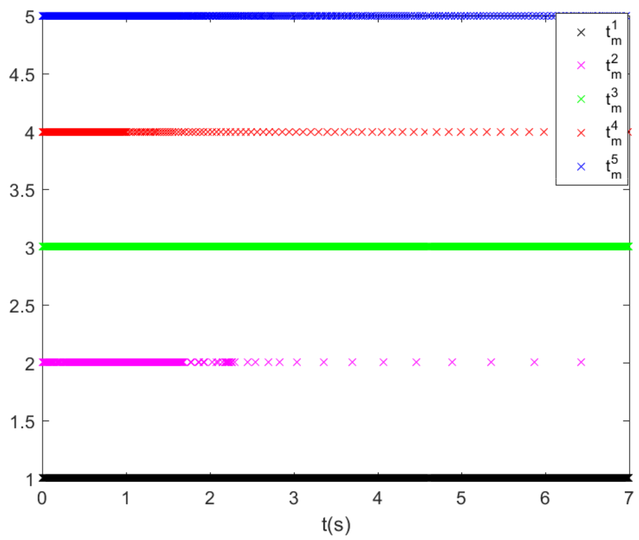

Figure 2: The event-triggered time instants t_m^q in network (41), where $q = 1, 2, \cdots, 5$.

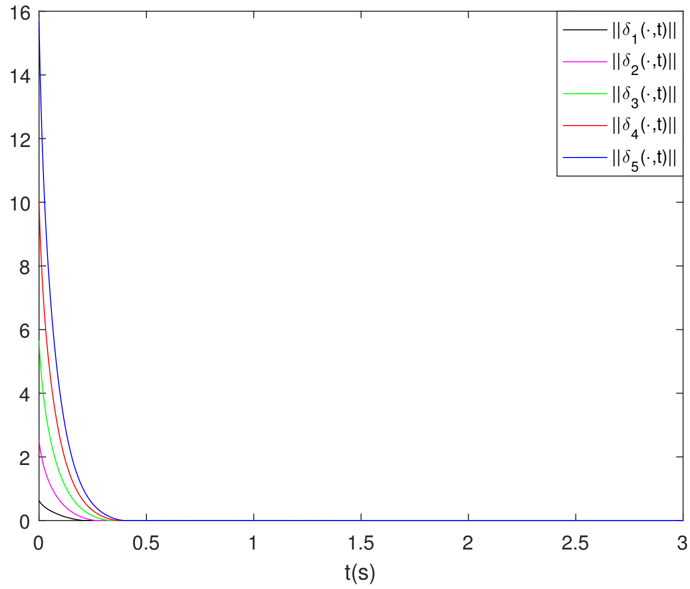

Figure 3: The norms of the state vectors $\delta_q(s,t)$ in network (41),where $q = 1, 2, \cdots, 5$.

in the event-triggered condition (3), we can easily compute the following matrices F, P satisfying (9):

$$F = \begin{pmatrix} 0.7268 & 0 & 0 \\ 0 & 0.5585 & 0 \\ 0 & 0 & 0.5366 \end{pmatrix}, P = I_5 \otimes \begin{pmatrix} 4.8964 & -38.4809 \\ -4.9820 & 0.9163 \\ 3.2910 & 46.5522 \end{pmatrix}.$$

Base on Theorem 3.1, the network (41) realizes event-triggered passivity. Figure 1 shows the norms of the state vectors, output vectors and output vectors when the network (41) is passive. In addition, the event-triggered time instants $t_m^q (q = 1, 2, \cdots, 5)$ are displayed in Figure 2.

Case 2: It is easy to know that $|Q_1| = 0.008$. Therefore, Q_1 is a nonsingular matrix. In addition, $\varphi_j, \breve{\varphi}_j, \omega_j, \breve{\omega}_j, \delta^*(s)$ and ϑ denote the same meanings as in *Case 1*. By employing the MATLAB, the following matrices F, P, J_2

satisfying (25) can be computed:

$$F=\begin{pmatrix} 0.7623 & 0 & 0 \\ 0 & 0.3427 & 0 \\ 0 & 0 & 0.5780 \end{pmatrix}, P = I_5 \otimes \begin{pmatrix} 17.7958 & -42.8839 \\ 0.1994 & 5.4934 \\ -12.8364 & 50.2961 \end{pmatrix},$$

$$J_2 = I_5 \otimes \begin{pmatrix} 8.6238 & -125.1243 & 88.8230 \\ -125.1243 & -26.3285 & 172.8574 \\ 88.8230 & 172.8574 & -234.2138 \end{pmatrix}.$$

From Corollary 3.1, the network (41) achieves event-triggered synchronization. Figure 3 displays network synchronization's simulation results.

Example 5.2. The following MWCDRDMNNs with switching topology under event-triggered control is considered:

$$\begin{aligned}\frac{\partial \delta_q(s,t)}{\partial t} =& L\frac{\partial^2 \delta_q(s,t)}{\partial s^2} - Z\delta_q(s,t) + E(\delta_q(s,t))f(\delta_q(s,t)) + O + Ku_q(s,t) \\ &+ B(\delta_q(s,t))g(\overline{\delta_q(s,t)}) + \eta_q(s,t) + \sum_{r=1}^{\eta}\sum_{p=1}^{6} \xi_r C_{qp}^{r,\Im}\Gamma^r \delta_p(s,t) \\ &+ v_q(s,t), \end{aligned} \tag{42}$$

where $q = 1,2,3,4,5,6, \psi = 3, \Im = 1,2, \Phi = \{s| -0.5 < s < 0.5\}, f_\iota(\chi) = \frac{|\chi+1|-|\chi-1|}{4}, g_\iota(\chi) = \frac{|\chi+1|-|\chi-1|}{8}, \iota = 1,2,3; L = \mathrm{diag}(0.6, 1.2, 2), Z = \mathrm{diag}(4,2,6);$ $\gamma_j(t) = 1-\frac{1}{j+3}e^{-t}, \zeta_j = \frac{1}{j+3}, j = 1,2,3; O = (0,0,0)^T, \Gamma^1 = \mathrm{diag}(0.6, 0.9, 1.4), \Gamma^2$ $= \mathrm{diag}(0.7, 0.5, 1.2); \xi_1 = 3.56, \xi_2 = 2.9, v_q(s,t) = \sum_{r=1}^{2}\beta_r \sum_{p\in\mathcal{N}_q} M_{qp}^{r,\Im}(\delta_q(s,t_m^q)$ $-\delta_p(s,t_m^p)); \eta_q(s,t) = -\mathrm{sign}(\delta_q(s,t))(\tilde{E}\breve{\varphi}+\tilde{B}\breve{\omega}), \beta_1 = 0.12, \beta_2 = 1.8; u_q(s,t) =$ $(0.8q\sqrt{t}\cos(\pi s), 2q\sqrt{t}\cos(\pi s))^T$. The following matrices $Q_1, Q_2, E(\delta_q(s,t)), K,$ $B(\delta_q(s,t))$ are chosen respectively:

$$e_{11}(\delta_{q1}(s,t)) = \begin{cases} -0.18, & |\delta_{q1}(s,t)| \leqslant 3.5, \\ 0.35, & |\delta_{q1}(s,t)| > 3.5, \end{cases}$$

$$e_{12}(\delta_{q1}(s,t)) = \begin{cases} -0.25, & |\delta_{q1}(s,t)| \leqslant 3.5, \\ 0.46, & |\delta_{q1}(s,t)| > 3.5, \end{cases}$$

$$e_{13}(\delta_{q1}(s,t)) = \begin{cases} 0.32, & |\delta_{q1}(s,t)| \leqslant 3.5, \\ -0.23, & |\delta_{q1}(s,t)| > 3.5, \end{cases}$$

$$e_{21}(\delta_{q2}(s,t)) = \begin{cases} -0.29, & |\delta_{q2}(s,t)| \leqslant 3.5, \\ 0.46, & |\delta_{q2}(s,t)| > 3.5, \end{cases}$$

$$e_{22}(\delta_{q2}(s,t)) = \begin{cases} -0.34, & |\delta_{q2}(s,t)| \leqslant 3.5, \\ 0.26, & |\delta_{q2}(s,t)| > 3.5, \end{cases}$$

$$e_{23}(\delta_{q2}(s,t)) = \begin{cases} -0.42, & |\delta_{q2}(s,t)| \leqslant 3.5, \\ 0.38, & |\delta_{q2}(s,t)| > 3.5, \end{cases}$$

$$e_{31}(\delta_{q3}(s,t)) = \begin{cases} 0.15, & |\delta_{q3}(s,t)| \leqslant 3.5, \\ -0.38, & |\delta_{q3}(s,t)| > 3.5, \end{cases}$$

$$e_{32}(\delta_{q3}(s,t)) = \begin{cases} 0.27, & |\delta_{q3}(s,t)| \leqslant 3.5, \\ -0.47, & |\delta_{q3}(s,t)| > 3.5, \end{cases}$$

$$e_{33}(\delta_{q3}(s,t)) = \begin{cases} 0.32, & |\delta_{q3}(s,t)| \leqslant 3.5, \\ -0.46, & |\delta_{q3}(s,t)| > 3.5, \end{cases}$$

$$b_{11}(\delta_{q1}(s,t)) = \begin{cases} 0.51, & |\delta_{q1}(s,t)| \leqslant 3.5, \\ -0.25, & |\delta_{q1}(s,t)| > 3.5, \end{cases}$$

$$b_{12}(\delta_{q1}(s,t)) = \begin{cases} -0.46, & |\delta_{q1}(s,t)| \leqslant 3.5, \\ 0.34, & |\delta_{q1}(s,t)| > 3.5, \end{cases}$$

$$b_{13}(\delta_{q1}(s,t)) = \begin{cases} -0.35, & |\delta_{q1}(s,t)| \leqslant 3.5, \\ 0.42, & |\delta_{q1}(s,t)| > 3.5, \end{cases}$$

$$b_{21}(\delta_{q2}(s,t)) = \begin{cases} 0.37, & |\delta_{q2}(s,t)| \leqslant 3.5, \\ -0.41, & |\delta_{q2}(s,t)| > 3.5, \end{cases}$$

$$b_{22}(\delta_{q2}(s,t)) = \begin{cases} 0.27, & |\delta_{q2}(s,t)| \leqslant 3.5, \\ -0.53, & |\delta_{q2}(s,t)| > 3.5, \end{cases}$$

$$b_{23}(\delta_{q2}(s,t)) = \begin{cases} 0.54, & |\delta_{q2}(s,t)| \leqslant 3.5, \\ -0.28, & |\delta_{q2}(s,t)| > 3.5, \end{cases}$$

$$b_{31}(\delta_{q3}(s,t)) = \begin{cases} -0.21, & |\delta_{q3}(s,t)| \leqslant 3.5, \\ 0.45, & |\delta_{q3}(s,t)| > 3.5, \end{cases}$$

$$b_{32}(\delta_{q3}(s,t)) = \begin{cases} 0.62, & |\delta_{q3}(s,t)| \leqslant 3.5, \\ -0.45, & |\delta_{q3}(s,t)| > 3.5, \end{cases}$$

$$b_{33}(\delta_{q3}(s,t)) = \begin{cases} 0.38, & |\delta_{q3}(s,t)| \leqslant 3.5, \\ -0.21, & |\delta_{q3}(s,t)| > 3.5, \end{cases}$$

$$Q_1 = \begin{pmatrix} 0.5 & 0.7 & 0.9 \\ 0.4 & 0.6 & 0.3 \\ 0.1 & 0.2 & 0.4 \end{pmatrix}, Q_2 = \begin{pmatrix} 0.5 & 0.2 \\ 0.8 & 0.6 \\ 0.3 & 0.6 \end{pmatrix}, \; K = \begin{pmatrix} 0.4 & 0.1 \\ 0.2 & 0.7 \\ 0.5 & 0.1 \end{pmatrix}.$$

Hence,

$$\tilde{E} = \begin{pmatrix} 0.53 & 0.71 & 0.55 \\ 0.75 & 0.6 & 0.8 \\ 0.53 & 0.74 & 0.78 \end{pmatrix}, \tilde{B} = \begin{pmatrix} 0.76 & 0.8 & 0.77 \\ 0.78 & 0.8 & 0.82 \\ 0.66 & 1.07 & 0.59 \end{pmatrix}.$$

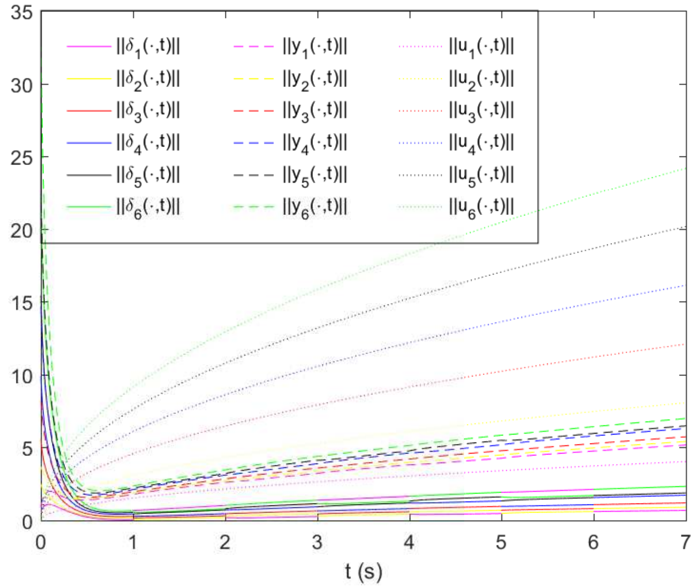

Figure 4: The norms of the state vectors $\delta_q(s,t)$, output vectors $y_q(s,t)$ and output vectors $u_q(s,t)$ in network (42), where $q = 1, 2, \cdots, 6$.

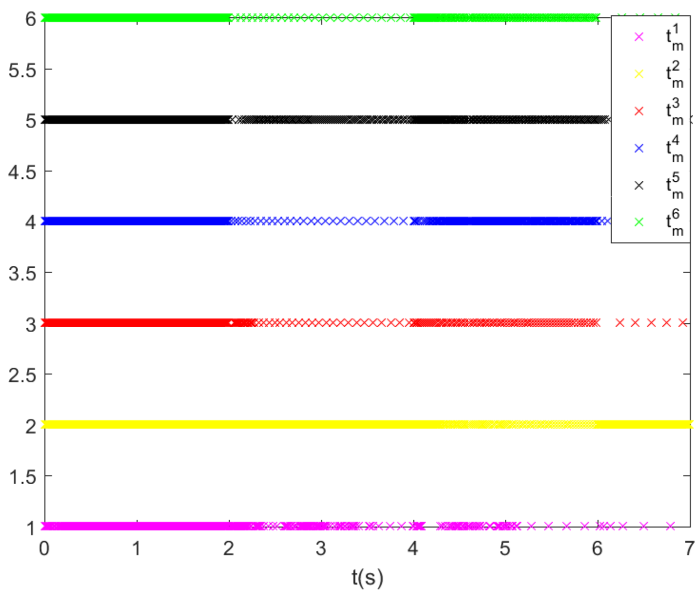

Figure 5: The event-triggered time instants t_m^q in network (42), where $q = 1, 2, \cdots, 6$.

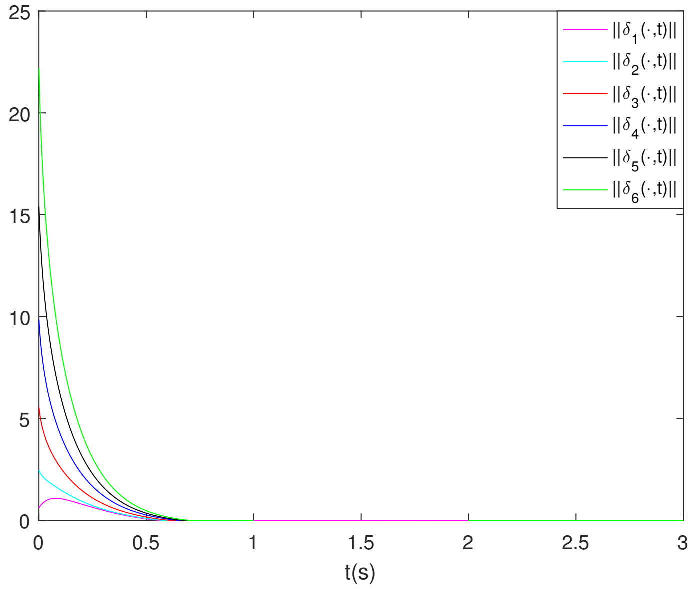

Figure 6: The norms of the state vectors $\delta_q(s,t)$ in network (42), where $q = 1, 2, \cdots, 6$.

Case 1: Obviously, the equilibrium solution of the isolated nodes of network (42) is $\delta^*(s) = (0,0,0)^T, \varphi_j = \breve{\varphi}_j = 0.5$ and $\omega_j = \breve{\omega}_j = 0.25$. $C^{1,1}$ and $C^{1,2}$($C^{2,1}$ and $C^{2,2}$; $M^{1,1}$ and $M^{1,2}$; $M^{2,1}$ and $M^{2,2}$) denote two possible topologies which are switched as $C^{1,1} \to C^{1,2} \to C^{1,1} \to C^{1,2} \cdots$, and each topology is active for 1s, the $C^{2,1}, C^{2,2}, M^{1,1}, M^{1,2}, M^{2,1}, M^{2,2}$ are switched similarly. The coupling matrices $C^{1,1} = (C^{1,1}_{qp})_{6\times6}, C^{1,2} = (C^{1,2}_{qp})_{6\times6}, C^{2,1} = (C^{2,1}_{qp})_{6\times6}, C^{2,2} = (C^{2,2}_{qp})_{6\times6}, M^{1,1} = (M^{1,1}_{qp})_{6\times6}, M^{1,2} = (M^{1,2}_{qp})_{6\times6}, M^{2,1} =$

$(M_{qp}^{2,1})_{6\times6}, M^{2,2} = (M_{qp}^{2,2})_{6\times6}$ are chosen as follows:

$$C^{1,1} = \begin{pmatrix} -0.7 & 0.2 & 0.3 & 0.1 & 0 & 0.1 \\ 0.2 & -0.6 & 0.2 & 0 & 0.2 & 0 \\ 0.3 & 0.2 & -0.8 & 0 & 0.1 & 0.2 \\ 0.1 & 0 & 0 & -0.2 & 0.1 & 0 \\ 0 & 0.2 & 0.1 & 0.1 & -0.6 & 0.2 \\ 0.1 & 0 & 0.2 & 0 & 0.2 & -0.5 \end{pmatrix},$$

$$C^{1,2} = \begin{pmatrix} -0.6 & 0 & 0.3 & 0 & 0.2 & 0.1 \\ 0 & -0.4 & 0.2 & 0 & 0.2 & 0 \\ 0.3 & 0.2 & -0.7 & 0 & 0 & 0.2 \\ 0 & 0 & 0 & -0.1 & 0.1 & 0 \\ 0.2 & 0.2 & 0 & 0.1 & -0.6 & 0.1 \\ 0.1 & 0 & 0.2 & 0 & 0.1 & -0.4 \end{pmatrix},$$

$$C^{2,1} = \begin{pmatrix} -0.6 & 0.1 & 0.3 & 0.1 & 0 & 0.1 \\ 0.1 & -0.5 & 0.2 & 0 & 0.2 & 0 \\ 0.3 & 0.2 & -0.7 & 0 & 0.1 & 0.1 \\ 0.1 & 0 & 0 & -0.3 & 0.2 & 0 \\ 0 & 0.2 & 0.1 & 0.2 & -0.7 & 0.2 \\ 0.1 & 0 & 0.1 & 0 & 0.2 & -0.4 \end{pmatrix},$$

$$C^{2,2} = \begin{pmatrix} -0.5 & 0 & 0.1 & 0 & 0.2 & 0.2 \\ 0 & -0.4 & 0.2 & 0 & 0.2 & 0 \\ 0.1 & 0.2 & -0.4 & 0 & 0 & 0.1 \\ 0 & 0 & 0 & -0.2 & 0.2 & 0 \\ 0.2 & 0.2 & 0 & 0.2 & -0.8 & 0.2 \\ 0.2 & 0 & 0.1 & 0 & 0.2 & -0.5 \end{pmatrix},$$

$$M^{1,1} = \begin{pmatrix} 0 & 0.02 & 0.02 & 0.03 & 0.4 & 0.05 \\ 0.03 & 0 & 0.05 & 0.02 & 0.03 & 0.04 \\ 0.01 & 0.6 & 0 & 0.1 & 0.02 & 0.3 \\ 0.003 & 0.07 & 0.02 & 0 & 0.003 & 0.04 \\ 0.04 & 0.02 & 0.03 & 0.004 & 0 & 0.001 \\ 0.01 & 0.02 & 0.03 & 0.02 & 0.05 & 0 \end{pmatrix},$$

$$M^{1,2} = \begin{pmatrix} 0 & 0.001 & 0.003 & 0.002 & 0.04 & 0.005 \\ 0.006 & 0 & 0.004 & 0.002 & 0.003 & 0.004 \\ 0.001 & 0.06 & 0 & 0.002 & 0.06 & 0.03 \\ 0.005 & 0.008 & 0.02 & 0 & 0.003 & 0.004 \\ 0.003 & 0.002 & 0.005 & 0.003 & 0 & 0.001 \\ 0.02 & 0.05 & 0.003 & 0.002 & 0.004 & 0 \end{pmatrix},$$

$$M^{2,1} = \begin{pmatrix} 0 & 0.06 & 0.2 & 0.05 & 0.011 & 0.04 \\ 0.02 & 0 & 0.004 & 0.01 & 0.05 & 0.002 \\ 0.01 & 0.06 & 0 & 0.021 & 0.02 & 0.025 \\ 0.001 & 0.09 & 0.005 & 0 & 0.008 & 0.001 \\ 0.01 & 0.03 & 0.05 & 0.006 & 0 & 0.003 \\ 0.025 & 0.01 & 0.02 & 0.04 & 0.005 & 0 \end{pmatrix},$$

$$M^{2,2} = \begin{pmatrix} 0 & 0.03 & 0.06 & 0.02 & 0.001 & 0.003 \\ 0.08 & 0 & 0.005 & 0.004 & 0.007 & 0.002 \\ 0.003 & 0.05 & 0 & 0.006 & 0.002 & 0.05 \\ 0.07 & 0.09 & 0.005 & 0 & 0.001 & 0.002 \\ 0.008 & 0.004 & 0.03 & 0.06 & 0 & 0.02 \\ 0.002 & 0.06 & 0.001 & 0.005 & 0.009 & 0 \end{pmatrix}.$$

Take $\vartheta_1 = 0.02, \vartheta_2 = 0.2$ in the event-triggered condition (27), we can easily compute the following matrix F, P satisfying (30):

$$F = \begin{pmatrix} 2.5553 & 0 & 0 \\ 0 & 2.7259 & 0 \\ 0 & 0 & 5.8855 \end{pmatrix}, P = I_6 \otimes \begin{pmatrix} -165.9814 & -290.3731 \\ 160.5884 & 226.1919 \\ 172.6068 & 478.4970 \end{pmatrix}.$$

According to Theorem 4.1, the network (42) achieves event-triggered passivity. Figure 4 shows the norms of the state vectors, output vectors and output vectors when the network (42) is passive. Moreover, the event-triggered time instants $t_m^q (q = 1, 2, \cdots, 6)$ are displayed in Figure 5.

Case 2: It is easy to know that $|Q_1| = 0.017$. Therefore, Q_1 is a nonsingular matrix. In addition, $\varphi_j, \breve{\varphi}_j, \omega_j, \breve{\omega}_j, \delta^*(s)$ and ϑ_1, ϑ_2 denote the same meanings as in *Case 1*. By making use of the MATLAB Toolbox, the following matrices F, P, J_2 satisfying (40) can be computed:

$$F = \begin{pmatrix} 2.5741 & 0 & 0 \\ 0 & 2.5097 & 0 \\ 0 & 0 & 2.4259 \end{pmatrix}, P = I_6 \otimes \begin{pmatrix} -11.3600 & 5.1747 \\ 39.0468 & 22.3267 \\ -48.1544 & -36.0722 \end{pmatrix},$$

$$J_2 = I_6 \otimes \begin{pmatrix} -71.1673 & -9.0216 & 136.7154 \\ -9.0216 & -66.5516 & 86.7348 \\ 136.7154 & 86.7348 & -416.6542 \end{pmatrix}.$$

From Corollary 4.1, the network (42) achieves event-triggered synchronization. Figure 6 displays the simulation results.

6. Conclusion

In this paper, the event-triggered passivity problem for MWCDRDMNNs with fixed and switching topologies has been investigated through designing suitable event-triggered controllers. Firstly, several event-triggered passivity and passivity-based synchronization conditions for MWCDRDMNNs with fixed topology have been established by making use of Lyapunov stability theory. Secondly, event-triggered passivity problem for MWCDRDMNNs with switching topology have also been addressed, and a synchronization criterion has been derived based on the obtained stability and output-strict passivity conditions. Finally, two numerical examples have been given to show the validity of these acquired results. In the future, it would be very interesting to study event-triggered synchronization and $\mathcal{H}_\infty$ synchronization problems of MWCDRDMNNs. In addition, note that the considered topology structure in this paper is undirected. Recently, there are some novel results about fixed or time-varying unbalanced directed topologies [53, 54], which motivates us to further apply the proposed event-triggered control strategy in this paper into the MWCDRDMNNs under directed topology or other directed networks. That would be another research topic in our future works.

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Declarations of interest

None.

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