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PERMIT MARKETS WITH POLITICAL AND MARKET DISTORTIOINS

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Permit markets with political and market distortions

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Abstract

This article investigates the cost effectiveness of cap-and-trade markets in the presence of both political and market distortions. We create a model where dominant firms have the ability to rent seek for a share of pollution permits as well as influence the market equilibrium with their choice of permit exchange because of market power. We derive the subgame-perfect equilibrium and show the interaction of these two distortions has consequences for the resulting allocative efficiency of the market. We find that if the dominant rent-seeking firms are all permit buyers (or a composition of buyers and sellers) then allocative efficiency is improved relative to the case without rent seeking; by contrast, if the dominant rent-seeking firms are all permit sellers then allocative efficiency reduces.

Key words: Pollution market, Market power, rent seeking.

JEL classification: D43, D72, Q58.

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1 Introduction

Cap-and-trade markets have been justified as a cost-effective form of pollution control. The varied scope of these markets can be observed within the European Union and United States, among others.[1] Although permit markets have performed relatively well, the implementation of these schemes has raised a number of concerns. First, concern exists over the potential market power of participating firms that may result in allocative inefficiency (e.g., Hintermann, 2017). Participating firms are normally sourced from a small number of concentrated industries (such as the energy sector), which may increase the likelihood of market-power effects. Second, it has been well documented that investments in rent-seeking efforts have been used to alter the distribution of initial permit allocations (e.g., Joskow and Schmalensee, 1998; Ellerman et al., 2007). Evidence exists for rent seeking over pollution permits within the US Acid Rain Program (Joskow and Schmalensee, 1998) as well as the European Union Emissions Trading Scheme (Ellerman et al., 2007). Thus, within contemporary cap-and-trade markets, there exists the potential for both political and market distortions. Yet it is *a priori* unclear how these distortions interact and the consequences for the cost effectiveness of pollution control.[2]

In this article we investigate the interactions between political and market distortions within a cap-and-trade market. To achieve this, we create a model of a cap-and-trade market with large dominant players and a competitive fringe of individually insignificant players. The dominant players have the ability to: (i) rent seek for their initial allocation of permits; and (ii) choose their level of permit exchange. We allow the contestability of the initial allocation of pollution permits to vary from being non-contestable to fully contestable via a rent-seeking process. This allows us to consider how alternative initial allocation mechanisms operate when there is a difference in sensitivity towards rent seeking or, indeed, to consider alternative regulatory systems where the degree of rent seeking varies. We focus on the direct comparison between two environments: when permits are either contestable or non-contestable. We show the resulting allocative efficiency in a contestable environment depends on the composition of buyers and sellers in the market: in the presence of active rent seeking, if the dominant firms are all permit buyers (or a composition of buyers and sellers) then allocative efficiency *increases* above what is observed in a non-contestable environment. Whereas allocative efficiency decreases relative

[1] While notable successes exists such as the US Acid Rain Program, Regional Greenhouse Gas initiative (RGGI), and the European Emissions Trading Scheme (EU-ETS), many schemes have been less successful (i.e., not implemented), such as the Australian carbon pricing scheme.

[2] Although it is well known—from the the theory of second best (Lipsey and Lancaster, 1956)—that the addition of another distortion may dampen the effects of a pre-existing distortion, the aim in this paper is different as we focus on identifying the interactive mechanism between both distortions. Only then can we detail how overall allocative efficiency is altered in the presence of both political and market distortions.

to the non-contestable environment when the firms are all sellers.

The examination of market power within cap-and-trade markets is well developed. The main branch of literature extends the work of Hahn (1984) that focuses on a dominant firm with a competitive fringe of price-taking firms (e.g., Misiolek and Elder, 1989; von der Fehr, 1993; Westskog, 1996; Sartzetakis, 1997; Liski and Montero, 2011; Hintermann, 2011, 2016, 2017; D'Amato et al., 2017).[3] Hahn (1984) shows the dominant firm will select their permit holdings to either decrease (increase) the permit price if they are a permit buyer (seller). Consequently, the existence of market power results in increased costs of pollution control. Although the literature on market power is well established, it does not extend the analysis to incorporate rent-seeking activities. Yet, in a market with a concentrated set of dominant firms, it is plausible that rent seeking would be more prevalent than in a perfectly competitive market. In this article, we advance the literature on market power within cap-and-trade markets by allowing the dominant firms to additionally rent seek for the initial permit allocation. Our analysis shows that the allocative inefficiency normally associated with market power is significantly altered by the presence of political distortions.

The rent-seeking literature focusing on cap-and-trade markets is also well established (Dijkstra, 1998; Malueg and Yates, 2006a; Lai, 2007, 2008; Hanley and MacKenzie, 2010; MacKenzie and Ohndorf, 2012; MacKenzie, 2017).[4] The majority of this literature focuses on rent seeking for initial permit endowments in a competitive permit market.[5] Usually rent seeking is modeled as a Tullock (1980) contest where firms invest effort in order to obtain permit endowments, the outcome being determined by their relative effort; thus, it is feasible to analyze the equilibrium rent-seeking outcomes under alternative permit market settings. The issue of market power within the permit market has, however, so far been ignored in these models. Yet to fully understand the influence of rent seeking on the permit market it is important to understand how the interaction between market-power distortions and rent seeking affect the operation of the market. The main aim of this article is to provide the first comprehensive analysis of a cap-and-trade market with dual distortionary effects. We analyze the subgame-perfect equilibrium

[3]A second branch of literature allows for all firms in the market to exert market power (Malueg and Yates, 2009; Wirl, 2009; Lange, 2012; Haita, 2014; Dickson and MacKenzie, 2018). Again, it is shown that market power generates allocative inefficiency within the market.

[4]Our focus here is on rent seeking associated with the distribution of permits and, therefore, the impact it has on the working and efficiency of the market. Other distinct literature exists on lobbying over environmental policy targets as well as the choice of policy instrument (see, for example, Buchanan and Tullock, 1975; Aidt, 1998, 2010). For a comprehensive review of the literature see Oates and Portney (2003).

[5]Hanley and MacKenzie (2010) extend their basic perfectly competitive model to include one dominant firm. Although they do obtain a rent-seeking equilibrium, they abstract from the interactions between rent seeking and market exchange and, therefore, they do not consider the full (subgame-perfect) analysis of a cap-and-trade market. As a consequence no conclusions can be drawn about how the interaction of political and market distortions can affect the overall cost effectiveness of cap-and-trade schemes.

of a cap-and-trade market with two distortionary effects and identify conditions where overall allocative efficiency is either improved or reduced by investment in rent-seeking activity.

We therefore provide a model that bridges the gap between the rent-seeking and market-power literatures. We follow the market-power literature by developing a competitive-fringe framework with two dominant firms. Our model has two stages. In the first stage, the dominant firms invest in rent-seeking effort in order to obtain a share of the initial allocation of permits. Rent-seeking efforts are sunk costs, such as lobbying and persuasive activities, that alter the distribution of the initial permit allocation. We develop this process as a strategic contest (e.g., Tullock, 1980; Malueg and Yates, 2006b; Long, 2013; Dickson et al., 2018), where the equilibrium share of permits is determined by a firm's rent-seeking efforts relative to total outlays. We allow firms' initial permit allocations to be determined by a contestable component and a non-contestable component thus allowing a continuum between these two extremes which can capture the realistic institutional setups of initial allocation processes. While it is clear that firms can rent seek for their permits, there can also be exogenous restrictions on the initial allocation of permits. We introduce a parameter α that captures the degree of contestability. This could represent the varying degrees of rent-seeking culture, such as how responsive bureaucrats are to the rent-seeking process. Further, it could represent specific rules from legislation that allow for the earmarking of permits or specific allocation mechanisms like auctioning or grandfathering.[6]

Our major innovation is thus the investigation of permit markets in the presence of both market and political distortions. This may provide insight to policymakers regarding the contestability of allocated permits and the likely consequences for the operation of the permit market. The article is organized as follows. In Section 2 the model is outlined. Section 3 characterizes the equilibrium of the game. Section 4 details the results that link rent-seeking activity and market power to changes in allocative efficiency. Section 5 concludes. Proofs of the theoretical results are contained in the appendix.

[6]Note that while it is unlikely that firms will invest rent-seeking effort to alter the distribution of permits in an auction, effort may be used to capture the auction revenue (MacKenzie and Ohndorf, 2012; MacKenzie, 2017). For an analysis of auction and grandfathering aspects related to market power in the permit market see Álvarez and André (2015). Further, the consequences of market power within a multi-unit auction process usually take the form of lower clearing prices due to firms' demand reduction (Khezr and MacKenzie, 2018a,b).

2 The model

2.1 Preliminaries

Consider a pollution permit market with $N+2$ regulated firms. Denote two dominant firms by $i, j \in \{1,2\}$ and a competitive fringe of N firms denoted by index f.[7] Denote a dominant firm i's pollution abatement by $a^i \equiv e - [\omega^i + x^i]$, where e is the level of unconstrained emissions (common to both firms), ω^i is the initial allocation of permits and x^i represents permits transacted in the market; $x^i > 0$ for purchases and $x^i < 0$ for sales. The cost of abatement is $C^i(a)$ with $C^{i\prime}(\cdot) > 0$, $C^{i\prime\prime}(\cdot) > 0$, and $C^{i\prime\prime\prime}(\cdot) = 0$. We use analogous notation for the fringe firms and impose the same assumptions on their cost functions. For the two dominant firms we assume that $C^1(0) = C^2(0)$ and that $C^{1\prime\prime}(a) \geq C^{2\prime\prime}(a)\, \forall a$, which combined implies that firm 1 has a weakly higher marginal abatement cost than firm 2 for all levels of abatement.

Our framework consists of two stages. In Stage 1, the dominant firms invest in rent-seeking effort in order to alter their initial permit allocation from the regulator. In Stage 2, the initial permit allocations become common knowledge and firms subsequently engage in permit exchange in the presence of a competitive fringe of small firms. The solution concept we use in this dynamic game of complete information is subgame-perfect Nash equilibrium.

2.2 Stage 1: Rent seeking over pollution permits

In Stage 1 each dominant firm chooses a level of rent-seeking effort $k^i \geq 0$ in order to influence their initial permit allocation. We follow the rent-seeking literature and assume the cost of rent seeking is $v^i(k)$ with $v^{i\prime}(\cdot) > 0$, $v^{i\prime\prime}(\cdot) = 0$, and $v^{i\prime}(0) = 0$. Let Ω denote the number of permits available for allocation among the two dominant firms. To allow for a wide variety of institutional settings, we allow the dominant firms' initial permit allocation to be based on both contestable and non-contestable components. To achieve this, denote by $\alpha \in [0,1]$ the share of Ω that can be contested via rent-seeking. Formally, then, firm i's initial permit allocation $\omega^i(k^i, k^j)$ is given by

$$\omega^i(k^i, k^j) = [1-\alpha]\frac{\Omega}{2} + \alpha\frac{k^i}{k^i + k^j}\Omega \quad \text{for } i = 1,2; \quad i \neq j. \tag{1}$$

Note that $\omega^i(k^i, k^j) + \omega^j(k^i, k^j) = \Omega$ and since each fringe firm exogenously receives ω^f, the total emissions cap is $N\omega^f + \Omega$.

From (1) the first term illustrates the non-contestable component of pollution permits al-

[7] Note that our focus on two dominant firms is for simplicity of exposition, but the analysis can be logically extended to incorporate several large firms.

located to firm i and the second term illustrates the permits obtained via rent seeking. The contestable component follows the convention within the rent-seeking literature that uses a Tullock contest to model the influence of political pressure (Dickson et al., 2018). As can be observed from (1), firm i's share of contestable permits is based on their rent-seeking effort relative to total outlays of effort. Note that variations of α can represent many important institutional applications. First it can represent the political and regulatory culture of how rent seeking affects regulatory outcomes. Second, it could represent alternative initial allocation mechanisms with varying degrees of potential contestability. For example, under auctioned permits it is less likely that rent seeking will influence the distribution of permits which instead will be based on the auction rules (so α may be rather small) whereas rent seeking may be more severe under alternative mechanisms, such as a grandfathered (free allocation) approach (see, for example, MacKenzie and Ohndorf, 2012).

2.3 Stage 2: Permit exchange

In Stage 2, once initial permit allocations become common knowledge, the dominant firms exchange permits within a competitive-fringe model. Market clearing requires

$$x^i + x^j + Nx^f = 0.$$

If p is the price of permits, each firm in the competitive fringe will seek to minimize the cost of abatement to solve:

$$\min_{x^f} C^f(e - [\omega^f + x^f]) + px^f$$

and therefore equilibrium demand from the fringe will satisfy:

$$\tilde{x}^f(p) = \{x^f : C^{f\prime}(e - [\omega^f + x^f]) = p\}. \tag{2}$$

As such, market clearing requires the price to be set such that:

$$x^i + x^j + N\tilde{x}^f(p) = 0$$

and we write

$$\tilde{p}(x^i + x^j) = \tilde{x}^{f-1}\left(-\frac{x^i + x^j}{N}\right).$$

It follows that

$$\tilde{p}'(x^i + x^j) = -\frac{1}{N}\frac{1}{\tilde{x}^{f\prime}} = \frac{C^{f\prime\prime}(e - [\omega^f + \tilde{x}^f(\tilde{p}(x^i + x^j))])}{N} > 0,$$

which shows that the equilibrium permit price is increasing in the dominant firms' permit purchases.[8]

Each of the two dominant firms cares about their overall cost of emissions, which includes their abatement cost taking into account the effect of their actions on the permit price and the cost of rent seeking. For firm i this cost takes the form:

$$C^i(e - [\omega^i(k^i, k^j) + x^i]) + x^i\tilde{p}(x^i + x^j) + v^i(k^i) \text{ for } i = 1, 2, i \neq j. \tag{3}$$

3 Equilibrium Analysis

We now start the equilibrium analysis of the game using backward induction to identify the subgame-perfect equilibrium. In particular, we first derive the Nash equilibrium of the permit market exchange given the choice of rent-seeking efforts from Stage 1, and then turn attention to the rent-seeking equilibrium.

3.1 Stage 2: permit market choices

In a Nash equilibrium of the Stage 2 game, each firm can be seen as choosing their permit allocation to minimize the cost detailed in (3). The solution to this, which yields each firm's reaction function, is given by

$$\tilde{x}^i(x^j; \omega^i(\alpha; k^i, k^j)) = \left\{x^i : -C^{i\prime}(e - [\omega^i(\alpha; k^i, k^j) + x^i]) + \tilde{p}(x^i + x^j) + x^i\tilde{p}'(x^i + x^j) = 0\right\}, \tag{4}$$

subject to the second-order condition being satisfied. Let us define $\tilde{l}^i(x^i, x^j; \omega^i(\alpha; k^i, k^j))$ as the left-hand side of the first-order condition:

$$\tilde{l}^i(x^i, x^j; \omega^i(\alpha; k^i, k^j)) \equiv -C^{i\prime}(e - [\omega^i(\alpha; k^i, k^j) + x^i]) + \tilde{p}(x^i + x^j) + x^i\tilde{p}'(x^i + x^j). \tag{5}$$

[8] Note that since $C^{f\prime\prime\prime} = 0$, $\tilde{p}'' = 0$, so inverse demand is linear.

Before we engage in a discussion of the nature of the equilibrium, let us make the preliminary observations that[9]

$$\begin{aligned}\tilde{l}^i_{x^i} &= C^{i\prime\prime} + 2\tilde{p}' > 0,\\ \tilde{l}^i_{x^j} &= \tilde{p}' > 0,\\ \tilde{l}^i_{k^i} &= \tilde{l}^i_{\omega^i}\omega^i_{k^i} = C^{i\prime\prime}\omega^i_{k^i} > 0,\\ \tilde{l}^i_{k^j} &= \tilde{l}^i_{\omega^i}\omega^i_{k^j} = C^{i\prime\prime}\omega^i_{k^j} < 0,\end{aligned}$$

where $\omega^i_{k^i} = \frac{k^j}{[k^i+k^j]^2}\alpha\Omega$ and $\omega^i_{k^j} = -\frac{k^i}{[k^i+k^j]^2}\alpha\Omega$. Noting $\tilde{l}^i_{x^i} > 0$ allows us to conclude that the second-order condition is indeed satisfied so the reaction function is identified by (4).

To understand how the dominant firms interact, we use implicit differentiation of (4) to deduce the slope of the reaction function:

$$\tilde{x}^i_{x^j} = -\frac{\tilde{l}^i_{x^j}}{\tilde{l}^i_{x^i}} = -\frac{\tilde{p}'}{C^{i\prime\prime} + 2\tilde{p}'} \in (-1/2, 0), \tag{6}$$

so this is a game of strategic substitutes with downward-sloping reaction functions. Note that when $x^i = 0$, $\tilde{l}^i(0, x^j; \omega^i(\alpha; k^i, k^j)) = -C^{i\prime}(e^i - \omega^i(\alpha; k^i, k_j)) + \tilde{p}(x^j)$. Recalling that $\tilde{l}^i_{x^i} > 0$, if this is greater than zero then $\tilde{x}^i$ will be negative (i will be a seller of permits) for this given x^j; while if it is less than zero then $\tilde{x}^i$ will be positive (i will be a buyer of permits). In terms of firm i's reaction function, the point at which $\tilde{l}^i(0, x^j; \omega^i(\alpha; k^i, k^j)) = 0$ determines where firm i's reaction function crosses the horizontal axis, i.e., the x^j such that $C^{i\prime}(e^i - [\omega^i(\alpha; k^i, k_j) + x^i]) = \tilde{p}(x^j)$. Given its negative slope, for any x^j smaller than this, firm i will be a buyer of permits, while for any x^j larger, firm i will be a seller. Figure 1 illustrates one possible example where firm i is a net permit seller in the equilibrium whereas firm j is net buyer of permits.

We now derive the unique Nash equilibrium of permit exchange and provide comparative statics of how the equilibrium changes relative to endowments and rent-seeking effort chosen in Stage 1.

Proposition 1. *Suppose $\alpha > 0$, then for any $k^1, k^2 > 0$ there exists a unique Nash equilibrium in the permit market that we denote by $\{x^{1*}(k^1, k^2), x^{2*}(k^1, k^2)\}$ with the property that $\frac{\mathrm{d}x^{i*}}{\mathrm{d}\omega^i} < 0$ and consequently*

$$x^{i*}_{k^i} < 0, x^{i*}_{k^j} > 0 \textit{ for } i = 1, 2, i \neq j.$$

Proposition 1 shows that as a firm's permit endowment increases they decrease their pur-

[9] By convention, for a function of many variables we use subscripts to denote the derivative with respect to the variable highlighted; for functions of single variables we use $'$s to indicate derivatives.

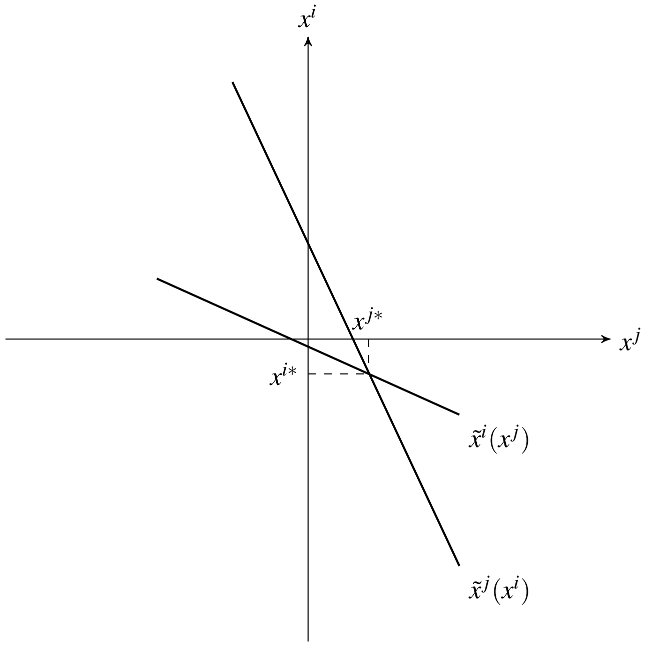

Figure 1: Reaction functions where, in equilibrium, firm *i* is a permit seller and firm *j* is a permit buyer.

chase of permits, or sell more permits. As such, as a firm's rent seeking increases this—through a channel of obtaining more permits as $\omega^i_{k^i} > 0$—results in a decrease in permit purchase or increase in permits sold. By contrast, increased rent-seeking activity from a rival results in an increase in permit purchases or reduction in permits sold, as their rival's actions imply a reduction in the allocation of permits to the firm in question.

It is interesting to note that when a firm engages in more rent seeking in the first stage, which *ceteris paribus* gives them more permits, there is an offsetting reduction in permit market transactions in the second stage. However, overall, an increase in rent seeking will result in an increase in the number of permits a firm holds in equilibrium, since

$$\underbrace{\omega^i_{k^i}}_{>0} + \underbrace{x^{i*}_{k^i}}_{<0} = \frac{\frac{k^j}{[k^i+k^j]^2}\alpha\Omega}{[C^{i\prime\prime}+2\tilde{p}'][C^{j\prime\prime}+2\tilde{p}']}[\tilde{p}'C^{j\prime\prime}+4[\tilde{p}']^2] > 0. \tag{7}$$

Thus it is clear that more rent seeking by a firm leads to less pollution abatement by that firm.

Note that in the permit-market equilibrium, we could either have both firms on one side of the market, or firms on different sides of the market. From the first-order condition presented in (4) observe that if $C^{i\prime} > \tilde{p}^*$ in equilibrium then it must be the case that $x^{i*} > 0$ and if $C^{i\prime} < \tilde{p}^*$ in equilibrium then it must be the case that $x^{i*} < 0$. Thus, depending on the relationship between the equilibrium levels of $C^{i\prime}, C^{j\prime}$, and $\tilde{p}^*$, we could have both firms on the same side of the market (if both firms equilibrium marginal abatement costs are larger, or smaller, than the equilibrium permit price), or firms on opposite sides of the market, in which case our assumption that firm 1 always has a larger abatement cost than firm 2 will imply it is firm 1 that will be the buyer of permits while firm 2 will be the seller of permits.

3.2 Stage 1: rent-seeking choices

In this subsection, we analyze the equilibrium rent-seeking choices of the dominant firms, accounting for the second-stage permit-market equilibrium. Each firm can be seen as choosing their rent-seeking effort to minimize the overall cost of emissions that include abatement costs, the net cost of permit purchases, and rent-seeking costs. Firm i's problem is therefore to

$$\min_{k^i \geq 0} C^i(e - [\omega^i(k^i, k^j) + x^{i*}(k^i, k^j)]) + x^{i*}(k^i, k^j)\tilde{p}(x^{i*}(k^i, k^j) + x^{j*}(k^i, k^j)) + v^i(k^i).$$

For ease of notation, define the cost of emissions as

$$G^i(k^i, k^j) = C^i(e - [\omega^i(k^i, k^j) + x^{i*}(k^i, k^j)]) + x^{i*}(k^i, k^j)\tilde{p}(x^{i*}(k^i, k^j) + x^{j*}(k^i, k^j)).$$

As such, firm i's reaction function will take the form

$$\hat{k}^i(k^j) = \{k^i : l^i(k^i, k^j) \equiv G^i_{k^i}(k^i, k^j) + v^{i\prime}(k^i) = 0\}.$$

The effect of increasing rent-seeking effort on the cost of emissions is given by

$$G^i_{k^i} = \frac{\partial G^i}{\partial k^i} + \frac{\partial G^i}{\partial x^{i*}} x^{i*}_{k^i} + \frac{\partial G^i}{\partial x^{j*}} x^{j*}_{k^i}.$$

From the first-order optimality condition in Stage 2 (see (4)), $\frac{\partial G^i}{\partial x^{i*}} = 0$. For tractability, we restrict our attention to settings where, while each firm anticipates the effect of its rent-seeking effort on its own permit market activity—it takes the permit market activity of the other firm as given.[10] Thus noting that $\frac{\partial G^i}{\partial k^i} = -C^{i\prime}(e^i - [\omega^i(k^i, k^j) + x^{i*}])\omega^i_{k^i}$, the expression for firm i's reaction function simplifies to

$$\hat{k}^i(k^j) = \left\{k^i : l^i(k^i, k^j) \equiv -C^{i\prime}\left(e^i - \left[[1-\alpha]\frac{\Omega}{2} + \frac{k^i}{k^i + k^j}\alpha\Omega + x^{i*}(k^i, k^j)\right]\right)\frac{k^j}{[k^i + k^j]^2}\alpha\Omega \right.$$
$$\left. + v^{i\prime}(k^i) = 0\right\}. \quad (8)$$

[10]This appeals to the conventional assumption of zero conjectural variation in Nash equilibrium—each player makes their choice assuming the action of the other player is fixed—which is adapted so that the actions we are assuming are fixed occur at a later stage (i.e., the firm anticipates this action, but doesn't anticipate it changing, so $x^{j*}_{k^i} = 0$).

Rather than work with reaction functions directly—whose properties are difficult to determine—we instead take an aggregative games approach and consider the behavior of each individual consistent with an equilibrium in which the aggregate rent-seeking effort, $K \equiv k^i + k^j$, takes a particular value (the aggregative games approach is detailed in the proof of Proposition 2 within the appendix). We work with expressions that consider an individual's share of the aggregate rent-seeking effort, $\sigma^i \equiv k^i/K$, consistent with equilibrium (as opposed to the level) because identifying the equilibrium becomes straightforward as consistency of the aggregate rent-seeking effort requires individuals to have shares with this aggregate effort that sum to 1. As such, the properties of the aggregation of share functions are instructive as to the existence and uniqueness of equilibrium.

We utilize expression (8) in the following proposition that demonstrates that there is a unique equilibrium in rent-seeking choices in which both firms are active in rent seeking.

Proposition 2. *For any $\alpha > 0$, there is a unique subgame-perfect Nash equilibrium with first-stage rent-seeking efforts $k^{1*}, k^{2*} > 0$.*

To proceed with the analysis, let us impose some structure on the nature of firms. In particular, suppose that the marginal cost of rent seeking ($v^{i\prime}$) is the same for each firm (as is standard within the rent-seeking literature[11]), and that firm 1—who has the higher marginal abatement cost for a given level of abatement—always values the outcome of rent-seeking more than firm 2 (as we would intuitively expect). A sufficient condition for this latter supposition is $C^{1\prime}(0) > C^{2\prime}(e)$, in which case firm 1 always has a higher marginal abatement cost than firm 2 regardless of the level of abatement of each firm. We instead assume a much weaker condition as follows:

Assumption 1. *For combinations of σ^1, σ^2 and K where the first-order conditions are satisfied,*

$$C^{1\prime}(e - [[1-\alpha]\frac{\Omega}{2} + \sigma^1\alpha\Omega + x^{1*}(\sigma^1 K, [1-\sigma^1]K)]) > \\ C^{2\prime}(e - [[1-\alpha]\frac{\Omega}{2} + \sigma^2\alpha\Omega + x^{2*}(\sigma^2 K, [1-\sigma^2]K)]).$$

With this structure on the nature of firms, we can then derive the composition of rent seeking between the dominant firms.

Proposition 3. *Suppose $v^{1\prime}(\cdot) = v^{2\prime}(\cdot)$ and Assumption 1 is satisfied. Then in the unique subgame-perfect Nash equilibrium, $k^{1*} > k^{2*}$.*

[11]Identical marginal costs of rent seeking can be interpreted as each firm expending a monetary cost to influence the initial endowment of pollution permits (Congleton et al., 2008). Note that firms continue to have heterogeneous values of the outcome of the rent-seeking process.

Proposition 3 is consistent with the rent-seeking literature in that if firm 1 values the outcome of rent seeking more then they also have a higher level of rent-seeking effort in equilibrium. Having established the composition of rent-seeking effort between the dominant firms, we now turn to investigate how the degree of permit contestability influences the cost effectiveness of the permit market.

4 The effect of permit contestability on allocative efficiency

In the previous section, we derived both the permit market and rent-seeking equilibrium of the game. In this section, our focus is on how permit contestability can influence the allocative efficiency of a permit market with pre-existing market-power distortions. To begin, let us start by examining the case where dominant firms are symmetric.

Proposition 4. *If dominant firms are symmetric then the permit market efficiency is independent of permit contestability.*

Proposition 4 shows that if the dominant firms are symmetric then the contestability does not alter the allocative efficiency in the market. When dominant firms are symmetric then their rent-seeking activity is identical, which results in an equal sharing of the initial permit allocation. Thus firm i's initial permit allocation is given by

$$\omega^i(k^i, k^j) = [1-\alpha]\frac{\Omega}{2} + \alpha\frac{\Omega}{2} = \frac{\Omega}{2} \quad \text{for } i = 1, 2; \quad i \neq j.$$

and is independent of α, the contestability of initial permit allocation. We now turn to the case where the dominant firms are asymmetric.

Proposition 5. *Allocative inefficiency, as measured by $[|x^{1*}| + |x^{2*}|]\tilde{p}'$, is lower in the presence of active rent seeking if both firms are buyers of permits on the market, or firm 1 is a buyer and firm 2 is a seller, but is higher if both firms are sellers on the market.*

Proposition 5 shows that how rent seeking and market power interact varies depending on the composition of the dominant firms. If all dominant firms are net permit buyers then the existence of contestable permits—and thus the existence of rent seeking over the distribution of permits—will increase the allocative efficiency of the market relative to case where permits are non-contestable. Thus contestability can improve the cost effectiveness of a permit market with market-power distortions. When the dominant firms are buyers, they must have relatively high abatement costs. With the ability to rent seek, firm 1 has more influence on their initial endowment. Intuitively the rent-seeking process allows the relatively high cost firm 1 to move towards an initial endowment that is their more preferred, which results in less distortionary behavior on the market as it purchases less permits. This offsets an increase in permit purchase from firm 2 that now obtains less initial permit endowment when permits are distributed using the rent-seeking process. Note that these market outcomes continue to hold even when firm 2 is a net seller: the dominance of firm 1's reduction in permit purchase fully offsets any distortion created by firm 2's increase in permits sold.

If all dominant firms are permit sellers in the market then Proposition 5 shows permit contestability has an adverse effect on permit market allocative efficiency. Now, if firms acquire the ability to rent seek then an increased initial endowment for the highest cost firm 1 will result in the ability of that firm to use its dominant position to sell more permits (i.e., x^{1*} becomes more negative) offsetting any reduction in permits sold by firm 2 as a net seller of permits.

For policymakers, then, it is important to realize that in a realistic environment—in which there exists rent seeking and market power—the degree of cost effectiveness in pollution control relies on the market composition of dominant players. Markets that experience dominant net permit sellers may therefore be less cost effective than if the dominant firms were in more diverse positions within the market.

5 Concluding remarks

The purpose of this article is to investigate how the interaction of political and market distortions affect the cost effectiveness of cap-and-trade markets. We develop a two-stage permit market model where dominant firms, first, have the ability to invest in rent-seeking effort to obtain an initial allocation of permits and then, second, choose their level of permit exchange. We derive the unique subgame-perfect equilibrium of the game.

In the permit market, for a given initial allocation of permits, a unique equilibrium exists where equilibrium permit holdings are decreasing (purchasing less; selling more) in the level of the initial allocation. Further, a dominant firm's equilibrium permit holdings decrease in the firms level of rent seeking. We show that if a dominant firm invests in rent seeking they increase their permit endowment but there is also an offsetting reduction in permit transactions. We show the net effect is positive: rent seeking will result in less individual pollution abatement. In the rent-seeking stage, we derive the unique equilibrium accounting for the second-stage permit market exchange.

Our main focus is whether permit contestability—how vulnerable the initial allocation of permits is to rent seeking—affects the allocative efficiency of the market. In a market with pre-existing market-power issues, we compare how allocative efficiency is altered when permits become contestable. We show how the cost effectiveness of the market is altered depends on the permit exchange activity of the dominant firms. In particular, we show that if dominant firms are permit buyers in equilibrium (or a mixture of buyer and seller) then allocative efficiency can actually improve relative to the conventional market-power efficiency problems. Whereas if all dominant firms are sellers then allocative efficiency is reduced.

The main aim of this article is to analyze how rent-seeking activity over the initial distribution of permits affects the cost effectiveness of a permit market under the presence of market power. Consequently, we have abstracted from the social cost of rent seeking. To include this note that the conventional methodology assumes that, in terms of social welfare, there are both losses associated with higher pollution control costs as well as the social cost of rent seeking (e.g., MacKenzie, 2017). As we have observed, rent-seeking activity can reduce the total pollution control costs from a situation with market power. To showcase the consequences for social welfare note that the key determinant is the social planner's (ad hoc) weights on pollution costs and rent seeking costs. Clearly, more weight on pollution abatement costs (and less on rent seeking) may result in improvements of social welfare and vice versa. As such, only a normative approach can be taken with limited insight. In contrast, our positive analysis shows that the cost effectiveness of a permit market—with both political and market distortions—is determined by the composition of market participants.

Appendix

Proof of Proposition 1. Since reaction functions have a slope whose absolute value is less than 1 they will intersect once and only once, which identifies the unique Nash equilibrium that we denote by $\{x^{1*}(k^1,k^2), x^{2*}(k^1,k^2)\}$: for any k^i, k^j there is a unique Nash equilibrium in the second stage subgame. Implicit differentiation of the first-order condition (4) allows us to understand the effect of a change in the permit endowment on the reaction function:

$$\tilde{x}^i_{\omega^i} = -\frac{\tilde{l}^i_{\omega^i}}{\tilde{l}^i_{x^i}} = -\frac{C^{i\prime\prime}}{C^{i\prime\prime} + 2\tilde{p}'} < 0. \tag{9}$$

So if firm i is a buyer of permits it buys fewer with a larger endowment, and if it is a seller a larger endowment means it will sell more. Considering how equilibrium permit transactions change, we note that this can be decomposed into the direct and strategic effects:

$$\begin{aligned} \frac{\mathrm{d}x^{i*}}{\mathrm{d}\omega^i} &= \tilde{x}^i_{\omega^i} + \tilde{x}^i_{x^j}\tilde{x}^j_{x^i}\tilde{x}^i_{\omega^i} \\ &= \tilde{x}^i_{\omega^i}[1 + \tilde{x}^i_{x^j}\tilde{x}^j_{x^i}]. \end{aligned}$$

Since this is a game of strategic substitutes, $\tilde{x}^i_{x^j}\tilde{x}^j_{x^i} > 0$, which allows us to conclude that $\frac{\mathrm{d}x^{i*}}{\mathrm{d}\omega^i} < 0$.

Turning now to understand the effect of a change in k^i on the permit-market equilibrium, we know that

$$x^{i*}_{k^i} = \tilde{x}^i_{\omega^i}\omega^i_{k^i} + \tilde{x}^i_{x^j}\tilde{x}^j_{\omega^j}\omega^j_{k^i}.$$

We can obtain

$$\begin{aligned} \omega^i_{k^i} &= \frac{k^j}{[k^i + k^j]^2}\alpha\Omega, \\ \tilde{x}^i_{x^j} &= -\frac{\tilde{l}^i_{x^j}}{\tilde{l}^i_{x^i}} = -\frac{\tilde{p}'}{C^{i\prime\prime} + 2\tilde{p}'}, \text{ and} \\ \omega^j_{k^i} &= -\frac{k^j}{[k^i + k^j]^2}\alpha\Omega. \end{aligned}$$

As such, it follows that

$$x^{i*}_{k^i} = -\frac{\frac{k^j}{[k^i+k^j]^2}\alpha\Omega}{C^{i\prime\prime} + 2\tilde{p}'}[C^{i\prime\prime} + \frac{C^{j\prime\prime}\tilde{p}'}{C^{j\prime\prime} + 2\tilde{p}'}] < 0.$$

Similarly, for firm j:

$$x_{k^i}^{j*} = \tilde{x}_{\omega^j}^{j}\omega_{k^i}^{j} + \tilde{x}_{x^i}^{j}\tilde{x}_{\omega^i}^{i}\omega_{k^i}^{i},$$

which is given by:

$$x_{k^i}^{j*} = \frac{\frac{k^j}{[k^i+k^j]^2}\alpha\Omega}{C^{j\prime\prime}+2\tilde{p}^{\prime}}[C^{j\prime\prime} + \frac{C^{i\prime\prime}\tilde{p}^{\prime}}{C^{i\prime\prime}+2\tilde{p}^{\prime}}] > 0.$$

□

Proof or Proposition 2. Recall from (8) the expression for firm i's reaction function. First, we replace k^j with $K - k^i$ in this expression, where $K \equiv k^i + k^j$ is the aggregate rent-seeking effort. This would give firm i's rent-seeking effort consistent with an equilibrium in which the aggregate rent-seeking effort is K. But rather than working with levels, we want to work with shares of the aggregate effort, and so we take an additional step by replacing k^i with $\sigma^i K$ where $\sigma^i \equiv k^i/K$ represents firm i's share of the aggregate rent-seeking effort. Firm i's share function consequently takes the form:

$$s^i(K) = \Big\{\sigma^i : l^i(\sigma^i K, [1-\sigma^i]K) \equiv$$
$$- C^{i\prime}\left(e^i - \left[[1-\alpha]\frac{\Omega}{2} + \sigma^i\alpha\Omega + x^{i*}(\sigma^i K, [1-\sigma^i]K)\right]\right)[1-\sigma^i]\frac{\alpha\Omega}{K} + v^{i\prime}(\sigma^i K) = 0\Big\}.$$

We want to understand the properties of these share functions, and we begin by understanding how share functions vary with K. Using the implicit function theorem,

$$s_K^i = -\frac{\frac{\mathrm{d}l^i}{\mathrm{d}K}}{\frac{\mathrm{d}l^i}{\mathrm{d}\sigma^i}}.$$

Before we deduce what this expression is, we need some preliminary observations. First, recall that since $v^{i\prime\prime} = 0$ the marginal cost of rent seeking in the above expression is constant.

Next, note that since we can write equilibrium permit market actions as $x^{i*}(\sigma^i K, [1-\sigma^i]K)$,

$$\frac{\mathrm{d}x^{i*}}{\mathrm{d}\sigma^i} = K[x_{k^i}^{i*} - x_{k^j}^{i*}] \text{ and}$$
$$\frac{\mathrm{d}x^{i*}}{\mathrm{d}K} = \sigma^i x_{k^i}^{i*} + [1-\sigma^i]x_{k^j}^{i*}.$$

Recalling from the proof of Proposition 1 that

$$x_{k^i}^{i*} = -[1-\sigma^i]\frac{\alpha\Omega}{K}\frac{C^{i\prime\prime} + \frac{C^{j\prime\prime}\tilde{p}'}{C^{j\prime\prime}+2\tilde{p}'}}{C^{i\prime\prime}+2\tilde{p}'} \text{ and}$$
$$x_{k^j}^{i*} = \sigma^i\frac{\alpha\Omega}{K}\frac{C^{i\prime\prime} + \frac{C^{j\prime\prime}\tilde{p}'}{C^{j\prime\prime}+2\tilde{p}'}}{C^{i\prime\prime}+2\tilde{p}'}$$

allows us to deduce, following simplification, that

$$\frac{\mathrm{d}x^{i*}}{\mathrm{d}\sigma^i} = -\alpha\Omega\frac{C^{i\prime\prime} + \frac{C^{j\prime\prime}\tilde{p}'}{C^{j\prime\prime}+2\tilde{p}'}}{C^{i\prime\prime}+2\tilde{p}'} \text{ and}$$
$$\frac{\mathrm{d}x^{i*}}{\mathrm{d}K} = 0.$$

We can now deduce that

$$\frac{\mathrm{d}l^i}{\mathrm{d}\sigma^i} = \frac{\alpha\Omega}{K}\left[[1-\sigma^i]C^{i\prime\prime}[\alpha\Omega + \frac{\mathrm{d}x^{i*}}{\mathrm{d}\sigma^i}] + C^{i\prime}\right],$$

and in this expression

$$\alpha\Omega + \frac{\mathrm{d}x^{i*}}{\mathrm{d}\sigma^i} = \alpha\Omega\left[1 - \frac{C^{i\prime\prime} + \frac{C^{j\prime\prime}\tilde{p}'}{C^{j\prime\prime}+2\tilde{p}'}}{C^{i\prime\prime}+2\tilde{p}'}\right]$$
$$= \frac{\alpha\Omega\tilde{p}'}{[C^{i\prime\prime} = 2\tilde{p}'][C^{j\prime\prime}+2\tilde{p}']}[C^{j\prime\prime}+4\tilde{p}'] > 0,$$

following some simplification. Note also that this implies the first-order condition is both necessary and sufficient in identifying optimal actions.

In addition,

$$\frac{\mathrm{d}l^i}{\mathrm{d}K} = [1-\sigma^i]\frac{\alpha\Omega}{K}C^{i\prime\prime}\frac{\mathrm{d}x^{i*}}{\mathrm{d}K} + C^{i\prime}[1-\sigma^i]\frac{\alpha\Omega}{K^2}$$
$$= C^{i\prime}[1-\sigma^i]\frac{\alpha\Omega}{K^2} > 0$$

as $\frac{\mathrm{d}x^{i*}}{\mathrm{d}K} = 0$ from above. As such, this allows us to conclude that $s_K^i < 0$, so individual share functions are strictly decreasing in the aggregate rent-seeking effort.

When $C^{i\prime}(e^i - [[1-\alpha]\frac{\Omega}{2} + \sigma^i\alpha\Omega + x^{i*}(\sigma^iK, [1-\sigma^i]K)])[1-\sigma^i]\frac{\alpha\Omega}{K}$ and $v^{i\prime}(\sigma^iK)$ are plotted as functions of σ^i, the intersection of these for a given K identifies the value of the share function for that K. Recall that $v^{i\prime}(\sigma^iK)$ is of course constant in σ^i, and our deductions above imply the first function is decreasing in both σ^i and K. The function $C^{i\prime}(e^i - [[1-\alpha]\frac{\Omega}{2} +$

$\sigma^i\alpha\Omega + x^{i*}(\sigma^i K, [1-\sigma^i]K)])[1-\sigma^i]\frac{\alpha\Omega}{K}$ takes the value of zero when $\sigma^i = 1$, and the value $C^{i\prime}(e^i - [[1-\alpha]\frac{\Omega}{2} + x^{i*}(0,K)])\frac{\alpha\Omega}{K}$ when $\sigma^i = 0$. If K is such that this exceeds $v^{i\prime}(\sigma^i K)$ then there will be a positive solution and the share function will be positive. But if K is such that this is less than $v^{i\prime}(\sigma^i K)$ the share function will take the value of zero. Let us define $\bar{K}^i$ as the value of K such that $C^{i\prime}(e^i - [[1-\alpha]\frac{\Omega}{2} + x^{i*}(0,K)])\frac{\alpha\Omega}{K} = v^{i\prime}(0)$, then the fact that $C^{i\prime}(e^i - [[1-\alpha]\frac{\Omega}{2} + \sigma^i\alpha\Omega + x^{i*}(\sigma^i K, [1-\sigma^i]K)])[1-\sigma^i]\frac{\alpha\Omega}{K}$ is decreasing in K implies that the share function will be positive for all $K < \bar{K}^i$ and zero for all $K \geq \bar{K}^i$. In addition, note that as $K \to 0$ the function $C^{i\prime}(e^i - [[1-\alpha]\frac{\Omega}{2} + \sigma^i\alpha\Omega + x^{i*}(\sigma^i K, [1-\sigma^i]K)])[1-\sigma^i]\frac{\alpha\Omega}{K}$ increases without bound for all σ^i, and we therefore deduce that $s^i(K) \to 1$ as $K \to 0$. To summarize: each firm's share function takes the value of 1 when K is arbitrarily close to zero, it is strictly decreasing in $K > 0$ and is equal to zero for all $K > \bar{K}^i$.

We now seek to identify a Nash equilibrium. This requires the sum of individual rent-seeking efforts to be equal to the aggregate rent-seeking effort, or, dividing both sides of this equation by the aggregate rent-seeking effort, for the sum of share functions to be equal to 1.

Again, we sum the two share functions up and find the value of K where the sum of share functions is equal to 1. Since the sum of share functions exceeds 1 when K is arbitrarily small, is less than one when $K = \min_i\{\bar{K}^i\}$ (since one firm's share function is zero and the other's is less than one by definition), and is strictly decreasing in K, the intermediate value theorem implies there is a unique $K^* \in (0, \min_i\{\bar{K}^i\})$ at which $s^i(K^*) + s^j(K^*) = 1$, and therefore a unique Nash equilibrium in which the shares of the two players are $s^{i*} = s^i(K^*)$ and $s^{j*} = s^j(K^*)$ and their rent-seeking efforts are $k^{i*} = K^* s^{i*} > 0$ and $k^{j*} = K^* s^{j*} > 0$. □

Proof of Proposition 3. By Assumption 1,

$$C^{1\prime}\left(e^1 - \left[[1-\alpha]\frac{\Omega}{2} + \sigma\alpha\Omega + x^{1*}(\sigma K, [1-\sigma]K)\right]\right) > C^{2\prime}\left(e^2 - \left[[1-\alpha]\frac{\Omega}{2} + \sigma\alpha\Omega + x^{2*}(\sigma K, [1-\sigma]K)\right]\right)$$

and since $v^{1\prime}(\cdot) = v^{2\prime}(\cdot)$ this implies that firm 1's share function, as identified by the level of σ where $C^{1\prime}(e^1 - [[1-\alpha]\frac{\Omega}{2} + \sigma\alpha\Omega + x^{1*}(\sigma K, [1-\sigma]K)])[1-\sigma]\frac{\alpha\Omega}{K} = v^{i\prime}(\sigma K)$ will always be larger than that of firm 2 which is identified by the analogous condition. As such, for any K where they are both defined, $s^1(K) > s^2(K)$, which therefore implies $k^{1*} = K^* s^1(K^*) > K^* s^2(K^*) = k^{2*}$. □

Proof of Proposition 4. If $C^{i\prime\prime} = C^{j\prime\prime}$ then $k^{i*} = k^{j*}$ and consequently $\omega^i(k^{i*}, k^{j*}) = [1-\alpha]\frac{\Omega}{2} +$

$\alpha \frac{k^{i*}}{k^{i*}+k^{j*}}\Omega = \frac{\Omega}{2}$ for $i = 1, 2$. As such, the equilibrium in the permit market x^{i*}, x^{j*} doesn't depend on α, so inefficiency doesn't depend on α. If α increases, which increases the contestability of permits in the mixed sharing rule, the equilibrium still awards each firm with $1/2$ of $\alpha\Omega$, hence each individual ω^i remains constant. □

Proof of Proposition 5. We want to compare the case of no contestability with the case where permits are contestable through rent seeking prior to permit exchange. With $\alpha = 0$, it follows that $\omega^i = \frac{\Omega}{2}$. With $\alpha > 0$, we need to consider the contest equilibrium. As shown in Proposition 3, under Assumption 1 $k^{1*} > k^{2*}$ and therefore,

$$\omega^1(k^{1*}, k^{2*}) = [1-\alpha]\frac{\Omega}{2} + \frac{k^{1*}}{k^{1*}+k^{2*}}\alpha\Omega > \frac{\Omega}{2},$$
$$\omega^2(k^{1*}, k^{2*}) = [1-\alpha]\frac{\Omega}{2} + \frac{k^{2*}}{k^{1*}+k^{2*}}\alpha\Omega < \frac{\Omega}{2}.$$

Note that[12] $\omega^1(k^{1*}, k^{2*}) - \frac{\Omega}{2} = \frac{\Omega}{2} - \omega^2(k^{1*}, k^{2*})$, so when comparing endowments with $\alpha > 0$ with those when $\alpha = 0$ we can write $\Delta\omega^1 = -\Delta\omega^2$. Given our earlier results on how x^{i*} changes with ω^i in Proposition 1, we can therefore directly conclude, abusing notation slightly, that:

$$x^{1*}\Big|_{\alpha>0} < x^{1*}\Big|_{\alpha=0}$$
$$x^{2*}\big|_{\alpha>0} > x^{2*}\big|_{\alpha=0}.$$

Now, what we are interested in is the effect of rent seeking on the allocative efficiency of the market, which is given by $[|x^{1*}| + |x^{2*}|]\tilde{p}'$.[13] Since $\tilde{p}'$ is a constant we focus on $[|x^{1*}| + |x^{2*}|]$, There are three cases to consider:

1. Firm 1 is a buyer of permits and firm 2 is a seller of permits[14];
2. Both firms are buyers of permits;
3. Both firms are sellers of permits.

[12]To see this, note that since $\frac{k^2}{k^1+k^2} = 1 - \frac{k^1}{k^1+k^2}$ we can write $\omega^2 = [1-\alpha]\frac{\Omega}{2} + \left[1 - \frac{k^1}{k^1+k^2}\right]\alpha\Omega$. As such, $\frac{\Omega}{2} - \omega^2(k^{1*}, k^{2*}) = -\alpha\frac{\Omega}{2} + \frac{k^{1*}}{k^{1*}+k^{2*}}\alpha\Omega$, which is precisely $\omega^1(k^{1*}, k^{2*}) - \frac{\Omega}{2}$.

[13]Allocative efficiency following trade in the permit market is given by the sum of the differences between the equilibrium marginal abatement cost and the price. From the first-order condition (4), for each dominant firm this is given by $x^{i*}\tilde{p}'$, and for the fringe firms it is zero, hence our focus on the stated expression.

[14]The reverse cannot be true under Assumption 1. Suppose, by contrast, that $x^{1*} < 0 < x^{2*}$. Then $\tilde{p}^* + x^{1*}\tilde{p}^{*\prime} < \tilde{p}^* + x^{2*}\tilde{p}^{*\prime}$ but then the stage 2 first-order conditions imply $C^{1\prime}(e^1 - [\omega^{1*} + x^{1*}]) < C^{2\prime}(e^2 - [\omega^{2*} + x^{2*}])$, but this is in direct contradiction to Assumption 1.

Case 1. If firm 1 is a buyer then $x^{1*} > 0$ and so decreases with $\alpha > 0$; If firm 2 is a seller then $x^{2*} < 0$ and so gets less negative when $\alpha > 0$. As such $x^{1*} + |x^{2*}|\Big|_{\alpha>0} < x^{1*} + |x^{2*}|\Big|_{\alpha=0}$ so allocative inefficiency decreases in the presence of rent seeking.

Case 2. If firm 1 is a buyer $x^{1*} > 0$ and so decreases when $\alpha > 0$; and if firm 2 is a buyer $x^{2*} > 0$ also increases when $\alpha > 0$. The overall effect, assuming small changes, is given by:

$$dx^{1*} + dx^{2*} = x^{1*}_{\omega^1} d\omega^1 + x^{2*}_{\omega^2} d\omega^2.$$

As noted previously $d\omega^1 = -d\omega^2$, and therefore

$$dx^{1*} + dx^{2*} = [x^{1*}_{\omega^1} - x^{2*}_{\omega^2}]d\omega^1 \text{ where } d\omega^1 > 0.$$

Recall from the proof of Proposition 1 that $\frac{\mathrm{d}x^{i*}}{\mathrm{d}\omega^i} = \tilde{x}^i_{\omega^i}[1 + \tilde{x}^i_{x^j}\tilde{x}^j_{x^i}]$. As such,

$$dx^{1*} + dx^{2*} = [\tilde{x}^1_{\omega^1} - \tilde{x}^2_{\omega^2}][1 + \tilde{x}^i_{x^j}\tilde{x}^j_{x^i}]d\omega^1.$$

Recall further that

$$x^{i*}_{\omega^i} = -\frac{C^{i\prime\prime}}{C^{i\prime\prime} + 2\tilde{p}^{\prime}},$$

then the term in the first square brackets is

$$\left[\frac{C^{1\prime\prime}}{C^{1\prime\prime} + 2\tilde{p}'} - \frac{C^{2\prime\prime}}{C^{2\prime\prime} + 2\tilde{p}'}\right] < 0$$

implying $dx^{1*} + dx^{2*} < 0$. Again allocative inefficiency decreases in the presence of rent seeking.

Case 3. If firm 1 and 2 are both sellers of permits then $x^{1*} < 0$ and $x^{2*} < 0$, so x^{1*} gets more negative and x^{2*} gets less negative. The overall effect, again assuming small changes is

$$\begin{aligned}
\left|dx^{1*}\right| + \left|dx^{2*}\right| &= \left|x^{1*}_{\omega^1}\right| d\omega^1 + \left|x^{2*}_{\omega^2}\right| d\omega^2 \\
&= \left[\left|x^{1*}_{\omega^1}\right| - \left|x^{2*}_{\omega^2}\right|\right] d\omega^1 \\
&= \left[\frac{C^{1\prime\prime}}{C^{1\prime\prime} + 2\tilde{p}'} - \frac{C^{2\prime\prime}}{C^{2\prime\prime} + 2\tilde{p}'}\right] d\omega^1 > 0
\end{aligned}$$

In this case, allocative inefficiency *increases* in the presence of rent seeking. □

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