Parameter Design Oriented Analysis of the Current Control Stability of the Weak-Grid-Tied VSC

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Abstract—This paper studies the dynamic behaviors of weak-grid-tied VSCs with simplified transfer functions, which provides an accurate stability analysis and useful indications for tuning system parameters. A reduced-order multi-input multi-output (MIMO) transfer function that contains four single-input single-output (SISO) transfer functions for the weak-grid-tied VSC is first presented. It is found that the four SISO transfer functions share the same equivalent open-loop transfer function, i.e., the same stability conclusion. The Bode plots of the equivalent open-loop transfer function show that the inner current loop behaves as a band-pass filter whose maximum gain is approximately at the frequency of the PLL’s bandwidth. By stability criterion, the harmonic amplification and instability occur when its maximum gain exceeds 0dB caused by high PLL’s bandwidth, large grid impedance or high active power. It is also found that the target system is less stable when it works as an inverter than as a rectifier, due to the risk of the local positive feedback in the inverter mode. An effective criterion is further proposed to guide the selection of a proper PLL’s bandwidth to ensure the stability of the VSC system. Simulation results validate the correctness of the analysis and the efficacy of the criterion.

Index Terms—Weak-grid-tied VSCs, MIMO transfer function, system parameter tuning, PLL’s bandwidth, current control stability.

I. INTRODUCTION

The increasing penetration of renewable energies and high voltage direct current transmission (HVDC) promotes the wide application of power electronics devices in the power system [1], [2]. Among various kinds of power electronics devices, the voltage-source converter (VSC) is the most commonly used. For example, almost all Photovoltaics (PVs) and direct-drive permanent magnet synchronous generators (PMSGs) for wind turbine applications are connected to the grid through VSCs [2].

Especially in China, the VSC-HVDC, whose two terminals are VSCs, has been widely used to interconnect AC grids and integrate large-scale renewable energy. That power electronics converters replacing traditional synchronous generators can decrease the strength of the grid, even resulting in weak grids [3], [4]. For instance, after the back-to-back VSC-HVDC project (±420kV/1250MW) between Chongqing and Hubei province in China is put into operation, the minimum short circuit ratio (SCR) in Chongqing side can be decreased to 1.9 [5]. Weak grids can severely worsen the stability of the VSC system as well as bring great challenges to the control of the VSC [6]. The most commonly installed VSC converter is the two- and three-level used in wind and PV applications [7], [8], while the MMC-HVDC (modular multilevel converter based HVDC) converter is widely used in high voltage direct current transmission applications.

The stability analysis of weak-grid-tied VSCs has drawn much attention from both academic researchers and industrial engineers. Several models have been proposed for analyzing the small-signal stability of this system, including the impedance-based methods [9]–[14], the complex-torque-based method [15]–[17] and the eigenvalue analysis [3], [4], [18], [19]. It is reported that the impacts of the inner current loop and the outer loop on the small-signal stability of the VSC can be investigated separately, due to different time scales [17]. The outer loop of the VSC provides d and q current references for the inner loop, where one component is related to the AC active power control or DC voltage control and the other is related to the AC voltage or reactive power control [3], [4]. Following standard cascaded control tuning methodologies, the outer loops are designed much slower than the inner loop [20]. Thus, the current control stability is a precondition of the system stability for the design of the outer loops. In other words, the inner current loop’s small-signal stability is a necessary condition for the small-signal stability of the overall system.

The phase-locked loop (PLL) plays an important role in determining the inner current loop’s stability of the weak-grid-tied VSC according to [13], [15], [16]. From the viewpoint of impedance characteristics, the grid-tied inverter’s impedance with the current control and the PLL was investigated in [13]. And it shows that a higher PLL bandwidth yields a wider frequency range of negative resistance behavior, which will deteriorate the small-signal stability. From the viewpoint of damping, eigenvalue analysis and complex torque analysis of the weak-grid-tied inverter with the current control and the PLL were carried out during deep voltage sag in [15], [16]. And it shows that the increase of PLL’s bandwidth can decrease the total damping and deteriorate the small-signal stability. These methods are well applied for analyzing the stability considering the PLL and the current control, but are limited for providing more insights into the PLL’s impacts on the stability from the viewpoint of classical control theory.

The transfer function is one classical method to investigate the stability of the weak-grid-tied voltage source converter.
grid and the electrical components of the VSC. The converter controller is composed by a current control and a PLL. The PLL provides the synchronization with the grid, the inner loop controls the active and reactive current through the converter line reactor. And \( u_s \), \( u_e \), \( u_c \) are the voltage phasors of the grid, the PCC (point of common coupling) and the VSC, \( i_c \) is the phaser of the current injecting to the grid. One can refer to the Appendix for the mathematical descriptions for the physical part and control part of the target system.

A. Model Linearization

The mapping relations of one small perturbation in the two different frames shown in Fig. 2 can be described by

\[
\begin{bmatrix}
\Delta f_d \\
\Delta f_q
\end{bmatrix} = T_{dqcf2dq0} \begin{bmatrix}
\Delta f_d^{cf} \\
\Delta f_q^{cf}
\end{bmatrix}, \quad T_{dq2dqcf0} \begin{bmatrix}
\Delta f_d \\
\Delta f_q
\end{bmatrix} = T_{dqcf2dq0} \begin{bmatrix}
\Delta f_d^{cf} \\
\Delta f_q^{cf}
\end{bmatrix}
\]

where \( f \) denotes \( u_s \), \( u_e \), \( u_c \) or \( i_c \), the superscript "cf" denotes that the variable is in the control system’s dq frame, \( \theta_{PLL} \) is the initial phase of the PCC voltage observed by PLL, the \( \omega_1 \) in Fig. 2 is 2\( \pi \)50rad/s, and the matrix of \( T_{dqcf2dq0} \) and \( T_{dq2dqcf0} \) are as follows,

\[
T_{dqcf2dq0} = \begin{bmatrix}
1 & 0 & -f_{e0}^{cf} \\
0 & 1 & f_{d0}^{cf}
\end{bmatrix}, \quad T_{dq2dqcf0} = \begin{bmatrix}
1 & 0 & f_{e0} \\
0 & 1 & -f_{d0}
\end{bmatrix}
\]

Fig. 3(a) shows the linearized model of each component of the inner current loop of the target system. Specifically, the linearized representation of the plant can be written as

\[
\begin{align*}
&\{sL_q\Delta i_{cd} = -R_q\Delta i_{cd} + \omega_1 L_q\Delta i_{cq} + \Delta u_{sd} - \Delta u_{qd}, \\
&\{sL_q\Delta i_{cq} = -\omega_1 L_q\Delta i_{cd} - R_q\Delta i_{cq} + \Delta u_{sq} - \Delta u_{eq}, \\
&\{sL_{eq}\Delta i_{cd} = -R_{eq}\Delta i_{cd} + R_{eq}\Delta i_{cq} + \Delta u_{cd} - \Delta u_{ed}, \\
&\{sL_{eq}\Delta i_{cq} = -R_{eq}\Delta i_{cd} + R_{eq}\Delta i_{cq} + \Delta u_{eq} - \Delta u_{qeq}.
\end{align*}
\]

where \( s \) is the Laplace variable, and the subscripts d and q indicate d- and q-axis components of a variable in the grid’s dq frame; \( i_c \) is the current across the PCC; \( u_s \) is the PCC voltage; \( u_e \) is the voltage of the equivalent grid voltage source; \( u_c \) is the output voltage of the VSC; \( R_q \) and \( L_q \) are the equivalent resistance and inductance of the AC grid, respectively; \( R_{eq} \) and \( L_{eq} \) are the equivalent resistance and inductance between the VSC and the PCC, respectively.

The linearized representation of the PLL can be written as \( \Delta \theta_{PLL} = G_{PLL} \Delta u_{eq} \), where \( G_{PLL} \) is the transfer function of the PLL and can be represented as

\[
G_{PLL} = \frac{(2\xi \omega_{PLL} s + \omega_{PLL}^2)^2}{s^2 + 2\xi \omega_{PLL} s + \omega_{PLL}^2},
\]

where \( \xi \) is the damping ratio, \( \omega_{PLL} \) is the bandwidth of the PLL, and the subscript 0 denotes steady-state values. Combing \( \Delta u_{ed} = 0, \Delta u_{eq} = 0 \) and (3), the final linear representation of the PLL can be described as

\[
\Delta \theta_{PLL} = G_{PLL} \left[ \omega_1 L_q \Delta i_{cd} + (sL_q + R_q) \Delta i_{cq} \right].
\]

By substituting the linear representations of the current control and the PLL into that of the plant in (4), the small-signal current outputs of \( \Delta i_{cd}, \Delta i_{cq} \) can be presented as the
functions of other small-signals including $\Delta i_{cd}$, $\Delta u_{aq}$, $\Delta i_{cq}$, $\Delta i_{aq}$, $\Delta i_{eq}$, $\Delta q_{a}$ and $\Delta i_{dc}$, which is explicitly shown in (7). Equation (7) is too complicated and further proper simplifications should be carried out for effective analysis.

B. Impact of the Time Delays and Measurement Filters

To study the impact of the time delays and the measurement filters, the magnitude responses with different time delays and measurement filters are compared, as shown in Fig. 4. The other parameters are presented in Table I. It can be seen that the time delays and the measurement filters mainly play a role in the high-frequency range $\omega>1500$ rad/s (200Hz), which is much higher than the cut-off frequency of the inner current loop (about 300 rad/s) and the bandwidth of the PLL (around 10 Hz). The high frequency stability issues also exist in the weak-grid-tied VSC, however it is out of the scope of this paper which focuses on the medium-frequency stability.

C. Impact of the Outer Loop

Fig. 5 shows the magnitude response of the outer loop of the active power control, where the cut-off frequency of outer loop is around 60 rad/s. Fig. 4 shows that the cut-off frequency of the inner loop is around 300 rad/s. It can be seen that the outer loop is much slower than the current loop. Thus, it is reasonable to simply the outer loop when analyzing the stability for the medium frequency ranges.

D. Reduced-order MIMO Transfer Function

To obtain the reduced-order as well as valid MIMO transfer function of the inner loop, the following reasonable assumptions are adopted.

1) The time delay usually plays a role in high frequency stability issues [23], but this paper focuses on the medium frequency stability. For the sake of simplifying, the delays of the control system, voltage modulation and the voltage measurement filter are neglected, thus $G_d=1$, $G_1=1$.

2) The perturbation of the voltage on the DC side is small enough to be neglected, i.e., $\Delta v_{dc}=0$.

### TABLE I

**PARAMETERS OF THE TEST CASE IN MODEL VALIDATION**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated/Base power</td>
<td>MW</td>
<td>1500</td>
</tr>
<tr>
<td>Rated/Base voltage</td>
<td>kV</td>
<td>525</td>
</tr>
<tr>
<td>Rated frequency</td>
<td>Hz</td>
<td>50</td>
</tr>
<tr>
<td>SCR</td>
<td>1</td>
<td>1.1</td>
</tr>
<tr>
<td>$R_L$, $L_d$</td>
<td>$\Omega$, mH</td>
<td>1.67, 531.7</td>
</tr>
<tr>
<td>$R_{eq}$, $L_{eq}$</td>
<td>$\Omega$, mH</td>
<td>0.58, 184.8</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1</td>
<td>0.707</td>
</tr>
<tr>
<td>$\omega_{cl}$, $k_P$, $T_{1, cl}$</td>
<td>Hz, rad/(V.s), s</td>
<td>16, $3.32 \times 10^{-4}$, 0.0141</td>
</tr>
<tr>
<td>$\omega_{cl}$, $k_P$, $T_{1, cl}$</td>
<td>Hz, V/A, s</td>
<td>125, 145.16, 0.3183</td>
</tr>
<tr>
<td>$\omega_{cl}$, $k_P$, $T_{dc}$, $T_{dc}$</td>
<td>Hz, A/W, A(Ws)</td>
<td>6, $3.56 \times 10^{-7}$, 2.80 $\times 10^{-4}$</td>
</tr>
<tr>
<td>$\omega_{cl}$, $k_P$, $T_{dc}$, $T_{dc}$</td>
<td>Hz, A(W, A(Ws))</td>
<td>6, $3.56 \times 10^{-7}$, 2.80 $\times 10^{-4}$</td>
</tr>
<tr>
<td>$P_{ref}$, $Q_{ref}$</td>
<td>p.u.</td>
<td>1, 0.8</td>
</tr>
</tbody>
</table>
By substituting $G_d=1$, $G_f=1$, $\Delta v_{dc}=0$ and (6) into (7), we get a simplified MIMO linearized mathematical representation of (7) as shown in (8), where $Z_{ic}=sL_{eq}+R_{eq}+G_{CL}$, and $G_{CL}$ is the PI controller in the current control. Based on (8), the reduced-order MIMO transfer function with $\Delta i_{cd}$, $\Delta i_{cq}$ as inputs and $\Delta i_{cd}$, $\Delta i_{cq}$ as outputs, can be obtained as

$$
\begin{bmatrix}
\Delta i_{cd} \\
\Delta i_{cq}
\end{bmatrix} =
\begin{bmatrix}
G_{CL} & 0 \\
0 & Z_{ic}
\end{bmatrix}
\begin{bmatrix}
\Delta i_{cd} \\
\Delta i_{cq}
\end{bmatrix}
+ \begin{bmatrix}
\Delta u_{cd} \\
\Delta u_{cq}
\end{bmatrix}

+ \begin{bmatrix}
\Delta i_{cd} \\
\Delta i_{cq}
\end{bmatrix}

+ \begin{bmatrix}
\Delta v_{dc}
\end{bmatrix}

(9)
$$

Fig. 3(b) shows a simplified block diagram of the inner current loop according to (9). And the $A$, $B$, $C$ and $D$ in (9) can be represented as

$$
A = \frac{G_{CL}}{Z_{ic}}, \quad B = \frac{-G_{dq}}{Z_{ic}}, \quad C = \frac{G_{CL}}{Z_{ic}}, \quad D = \frac{1}{Z_{ic}}
$$

where $G_{dd}$, $G_{dq}$, $G_{d}$ and $G_{q}$ can be written as

$$
G_{dd} = \frac{G_{CL}G_{pi}(\omega L_{eq}Z_{ic})}{Z_{ic}}, \quad G_{dq} = \frac{G_{CL}G_{pi}(sL_{eq}+R_{eq})}{Z_{ic}}, \quad G_{d} = \frac{G_{CL}G_{pi}(sL_{eq}+R_{eq})}{Z_{ic}}, \quad G_{q} = \frac{-G_{CL}G_{pi}(sL_{eq}+R_{eq})}{Z_{ic}}
$$

And according to (26), $G_{CL}/Z_{ic}$ in (10) and (11) is written as

$$
G_{CL} = \frac{sL_{eq}+R_{eq}+\omega CL}{Z_{ic}} = \frac{1}{s/\omega CL + 1}
$$

which is a first-order lag with a time constant of $1/\omega CL$, and $\omega CL$ is the bandwidth of the inner loop.

E. Equivalent Open Loop Transfer Function

The MIMO transfer function (9) is stable if and only if all elements in the transfer function matrix are stable. As shown in (10), each $A$, $B$, $C$ and $D$ has two multiplying terms: the first term on the right hand side of each equation of (10) is $G_{CL}/Z_{ic}$, which is a common term; the second term on the right hand side of each equation of (10) has the same denominator $(1+G_{dd}-G_{qq})$. From (12), $G_{CL}/Z_{ic}$ is table as it has a negative root $-\omega CL$. Thus, the stability of the current loop depends on the second terms of $A$, $B$, $C$ and $D$.

By linear control theory, the second terms of $A$, $B$, $C$, and $D$ are stable if and only if all the poles of each second term are on the left half plane (LHP). Also, as the denominators of $G_{dd}$, $G_{dq}$, $G_{d}$ and $G_{q}$ are the same according to (11), the second terms of $A$, $B$, $C$, and $D$ have the same characteristic equation $1+G_{dd}-G_{qq}=0$. In other words, they have the same poles.

From the above analysis, we draw the conclusion that the small-signal stability of the inner loop of the VSC can be transformed to computing the roots of the characteristic equation $1+G_{dd}-G_{qq}=0$. Furthermore, the complex root computation can be replaced by the analysis on the Bode plots of $G_{dd}-G_{qq}$, which is quite mature in classical control theory. Therefore, we define $G_0 = G_{dd}-G_{qq}$ as the equivalent open-loop transfer function that can be expressed as

$$
G_0 = \frac{G_{CL}G_{pi}(\omega L_{eq}Z_{ic})}{Z_{ic}} - (sL_{eq}+R_{eq})\frac{G_{pi}(sL_{eq}+R_{eq})}{Z_{ic}}
$$

When $i_{cd0} \neq 0$ and $R_{eq}=0$, (13) can be rewritten as

$$
G_0 = \frac{G_{CL}G_{pi}(sL_{eq}+R_{eq})}{Z_{ic}} \left(s + \frac{-\omega L_{eq}Z_{ic}}{i_{cd0}}\right) (i_{cd0} \neq 0)
$$

Then, in the condition that $i_{cd0}=0$ (unit power factor), by substituting (5) and (12) into (14), the equivalent open loop transfer function $G_0$ in the form of the multiplication of factors can be obtained as

$$
G_0 = \frac{-L_{eq}i_{cd0}s}{u_{d0}(1+s/\omega CL)} \left(1 + \frac{s(2\omega L_{eq}Z_{ic})}{\omega CL} + \frac{s^2}{\omega^2 CL}\right)
$$

Equation (15) has five factors including one proportional factor, one differential factor, one first-order lead factor, one first-order lag factor and one second-order lag factor.

III. PLL’s High-pass Filter Effect in Weak-grid-tied VSCs and Stability Analysis

In this section, the PLL’s effect as a high-pass filter in the weak-grid-tied VSC is revealed and the current control stability of the weak-grid-tied VSC is investigated based on the equivalent open-loop transfer function obtained.

The characteristics of each factor in \( G_0 \) (\( i_{c0d} = 0, i_{c0q} > 0 \))

<table>
<thead>
<tr>
<th>No.</th>
<th>Factor</th>
<th>Corner frequency (rad/s)</th>
<th>Gain slope (dB/dec)</th>
<th>Phase (deg)</th>
<th>Cumulative Gain Slope (dB/dec)</th>
<th>Cumulative Phase (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(-L_g i_{c0d}/i_{u0d})</td>
<td>(\cdots)</td>
<td>(\cdots)</td>
<td>(-180)</td>
<td>(-180)</td>
<td>(-180)</td>
</tr>
<tr>
<td>2</td>
<td>(x)</td>
<td>(-20)</td>
<td>(90)</td>
<td>(+20)</td>
<td>(-90)</td>
<td>(-180)</td>
</tr>
<tr>
<td>3</td>
<td>(1 + 2x\xi/\omega_{pil})</td>
<td>(\omega_{pil}/(2\xi))</td>
<td>(+20)</td>
<td>(90)</td>
<td>(+40)</td>
<td>(0)</td>
</tr>
<tr>
<td>4</td>
<td>((1 + 2\xi\omega_{pil} + x^2/\omega_{pil}^2)^{-1})</td>
<td>(\omega_{pil})</td>
<td>(-40)</td>
<td>(-180)</td>
<td>(0)</td>
<td>(-180)</td>
</tr>
<tr>
<td>5</td>
<td>((1 + x/\omega_{cL})^{-1})</td>
<td>(\omega_{cL})</td>
<td>(-20)</td>
<td>(-90)</td>
<td>(-20)</td>
<td>(-270)</td>
</tr>
</tbody>
</table>

Fig. 6. (a) The PLL’s high-pass filter effect in weak-grid condition. (b) The current control’s low-pass filter effect.

### A. PLL’s High-pass Filter Effect in Weak-grid-tied VSCs

The corner frequency, the gain slope and the phase of each factor at its corner frequency in (15) are listed in Table II and sorted by their corner frequency when the VSC works as an inverter that injects active power to the AC power grid with \( i_{c0d} > 0 \). And the cumulative gain slope and phase at each corner frequency are also obtained. Based on Table II, the Bode plots of the equivalent open loop transfer function as an inverter that injects active power to the AC power grid with \( i_{c0d} > 0 \). And the cumulative gain slope and phase at each corner frequency are also obtained. Based on Table II, the Bode plots of the equivalent open loop transfer function can be drawn by hand. According to (15) and Table II, four factors’ corner frequencies are below the frequency of the PLL’s bandwidth and they are listed as follows.

1. \(-L_g i_{c0d}/i_{u0d}\) is the proportional factor, in which the equivalent grid inductance \( L_g \) indicates the AC system strength, and the active power current \( i_{c0d} \) represents the active power injected to the grid. This proportional factor contributes a value added to the amplitude-frequency curve of \( G_0 \) for the entire frequency range \((0, \infty)\).

2. The differential factor \( x \) contributes a 20dB/dec gain slope for the entire frequency range \((0, \infty)\).

3. The first-order lead factor \((1 + 2\xi\omega_{pil})\) comes from the numerator of \( G_{pil} \) and contributes a 20dB/dec gain slope in the frequency range \((\omega_{pil}/(2\xi), \infty)\). Thus, the cumulative gain slope becomes 40dB/dec at \( \omega = \omega_{pil}/(2\xi) \).

4. The second-order lag factor \((1 + 2\xi\omega_{pil} + x^2/\omega_{pil}^2)^{-1}\) comes from the denominator of \( G_{pil} \) and contributes a -40dB/dec gain slope in the frequency range \((\omega_{pil}, \infty)\). Thus, the cumulative gain slope becomes 0dB/dec at \( \omega = \omega_{pil} \), which means that the largest value of \( G_0 \)’s gain appears at \( \omega = \omega_{pil} \) and a larger \( \omega_{pil} \) results in a larger gain at \( \omega = \omega_{cL} \). Obviously, a larger \( L_g \) or \( i_{c0d} \) also means a larger gain at the frequency \( \omega = \omega_{pil} \).

Fig. 6(a) shows the Bode plots of \( G_0 \) considering the PLL, the grid impedance, and the active power. It shows that the PLL can behave as a high-pass filter in the weak-grid-tied VSC when \(-\text{Ln} \left| G_0 \left( j\omega_{pil} \right) \right| < -3\text{dB}\), which will let the high frequency harmonics pass.

### B. Total Band-pass Filter Effect and Stability Analysis

The first-order lag factor \((1 + s/\omega_{cL})^{-1}\) stems from the current control and contributes a -20dB/dec gain slope in the frequency range \((\omega_{cL}, \infty)\). Thus, the current control behaves as a low-pass filter as shown in Fig. 6(b). And the cumulative gain slope becomes -20dB/dec in the frequency range \((\omega_{cL}, \infty)\) due to the current control. Therefore, the equivalent open-loop transfer function of the weak-grid-tied VSC can behave as a band-pass filter as shown in Fig. 7. From the Bode plots in Fig. 7, we can see that the amplitude-frequency curve begins to decline after the frequency reaches the current control’s bandwidth.

The stability criterion based on the Bode plots is

\[ k_g = -\text{Ln} \left| G_0 \left( j\omega_{pil} \right) \right| > 0, \]

where \( k_g \) is the phase-crossover frequency, and \( k_g \) is the gain margin at \( \omega_g \). By the stability criterion (16), the inner current loop is stable if and only if \( G_0 \)’s gain at the phase-crossover frequency is smaller than 0dB. According to Table II, \( G_0 \)’s phase-crossover frequency approximately equals to the PLL’s bandwidth \( \omega_{pil} \), as the cumulative phase equals to \(180^\circ \) at \( \omega_{pil} \).
Thus, the impact of each factor on the current control stability can be transformed to investigating how each factor in $G_0$ impacts the gain at the phase-crossover frequency $\omega = \omega_{pll}$. The detailed analysis on the impacts of each factor as well as their corresponding physical meanings on the small-signal stability of the current control are listed as follows.

1) Fig. 8(a)-Fig. 8(c) show that, a larger $L_g$, $i_{cd0}(i_{cd0} > 0)$, or PLL’s bandwidth $\omega_{pll}$ can all result in a larger value of $G_0$’s gain at the phase-crossover frequency $\omega_{pll}$, thus decreasing the gain margin. Because the largest value of $G_0$’s gain appears at the phase-crossover frequency $\omega = \omega_{pll}$ as shown in Fig. 7. And the gain margin is defined as the amplitude difference between the 0dB line and the amplitude-frequency line of $G_0$ at $\omega_{pll}$, i.e., $0dB - 20\log |G_0(\omega = \omega_{pll})|dB.$

2) Fig. 8(d) presents that the current control doesn’t have impact on the maximum value of the magnitude response, i.e., the current control stability of the VSC system. One important function of the current control is to suppress the high-frequency harmonics. As a general rule-of-thumb, its bandwidth is chosen to be 0.1 to 0.2 times of the power converter switching frequency [4]. As the PLL is usually designed to be much slower than the current control to avoid control resonance and provide enough time for current damping in engineering practice. Thus, $\omega_{PLL} < \omega_{CL}$ is satisfied. Therefore, the inner loop current control’s bandwidth doesn’t change $20\log |G_0(\omega = \omega_{PLL})|dB$, and doesn’t impact the current control stability.

3) The current control always has a positive stability margin when the VSC works as a rectifier ($i_{cd0} < 0$). Because the phase of the proportional factor is $0^\circ$ and the Bode phase-frequency curve of $G_0$ doesn’t have any intersection points with the line of $-180^\circ$ when $i_{cd0} < 0$. Thus, the phase of $G_0$ is always in the range of $(-90^\circ, 90^\circ)$ and there is no phase-crossover frequency on $G_0$’s Bode plots when $i_{cd0} < 0$. However, the phase of the proportional factor is $-180^\circ$ when $i_{cd0} > 0$ and there is a phase-crossover frequency on $G_0$’s Bode plots. Once the gain margin is negative, there is a risk of the local positive feedback in the inverter mode.

C. Physical Understandings of the PLL’s High-pass Filter Effect

Fig. 9 presents the small-signal block diagram of the current control of the weak-grid-tied VSC according to equation (8). It shows that there are four feedback loops in the weak-grid-tied VSC, where feedback loops 1 and 2 are the d and q current feedback control, feedback loops 3 and 4 are introduced by the voltage drop on the grid impedance through the PLL. In strong-grid conditions, $L_g \approx 0$, $R_g \approx 0$, feedback loop 3 and 4 disappear. However, in weak-grid condition, $L_g \approx 0$ doesn’t satisfy, and feedback loop 3 and 4 exist.

In weak-grid condition, the d-axis and q-axis current controls are tightly coupled because of the large grid impedance, see loop 3 and 4. In loop 4, the grid inductance can amplify the disturbance, the current control suppresses the disturbance, the PLL’s denominator suppresses and PLL’s numerator amplifies the disturbance. If the disturbance suppression effect is smaller than the disturbance amplification effect, the harmonics in the q-axis current disturbance can be amplified by the feedback loop 4. And the PLL’s high-pass dynamics emerges. The large grid inductance is the main factor that can lead to the PLL’s high-pass dynamics. Theoretically, a larger grid inductance ($L_g$) in feedback loop 4 can move the gain-frequency curve in Fig. 8(c) upward till above 0dB, which results in the high-pass filter effect of the PLL.

D. Impact of the Reactive Current

In Section III-B, it is assumed that $i_{cd0} = 0$ (unit power factor). When $i_{cd0} \neq 0$, by substituting (5) and (12) into (14), the equivalent open loop transfer function $G_0$ in the form of the multiplication of factors can be obtained as

$$G_0 = \frac{\omega_0 L_g i_{cd0}}{u_{cd0}(1 + s/\omega_{CL})} \frac{1 + s(\omega_{PLL})}{1 + s(2\xi/\omega_{PLL}) + s^2/\omega_{PLL}^2}. \quad (17)$$

Equation (17) also has five factors including one proportional factor, two first-order lead factor, one first-order lag factor and one second-order lag factor.

The Bode plots of $G_0$ when $i_{cd0} \neq 0$ can also be drawn according to (17). Differing from (15), the proportional factor in (15) is replaced by a first-order lead factor in (17) with a corner frequency of $(-\omega_{1}(i_{cd0})/i_{cd0})$. The value of $(-\omega_{1}(i_{cd0})/i_{cd0})$ depends on $i_{cd0}$ and $i_{cd0}$, and has three cases of below $\omega_{PLL}(2\xi)$, above $\omega_{PLL}$, or in the interval of $[\omega_{PLL}/(2\xi), \omega_{PLL}]$. For the three cases, Table III, Table IV and Table V present the corner frequency, the gain slope and the phase of the factors in $G_0$ at its corner frequency, and is sorted by their corner frequency. And the cumulative gain slope and phase at each corner frequency are also presented. The Bode plots of $G_0$ is presented in Fig. 10 when $(-\omega_{1}(i_{cd0})/i_{cd0})$ is below $\omega_{PLL}/(2\xi)$, above $\omega_{PLL}$.

When $(-\omega_{1}(i_{cd0})/i_{cd0}) > \omega_{PLL}/(2\xi)$ and $[\omega_{PLL}/(2\xi) < (-\omega_{1}(i_{cd0})/i_{cd0}) < \omega_{PLL}]$, Table III and Table IV show that the maximum gain of $G_0$ is also approximately at the frequency of the PLL’s bandwidth. When $(-\omega_{1}(i_{cd0})/i_{cd0}) > \omega_{PLL}$. Table V shows that the maximum gain of $G_0$ is in the interval of $[\omega_{PLL}/(2\xi), \omega_{PLL}]$, which is near PLL’s bandwidth. Thus, the Bode plots of $G_0$ presented in Fig. 10 when $i_{cd0} \neq 0$ have a similar form as Fig. 7. Impacts of different factors on the
Fig. 10. The Bode plots of \( G_0 \): (a) when \( |-(\omega_1+i\omega_0)/(\omega_0+q)| < \omega_{pil}/(2\xi) \); (b) when \( |-(\omega_1+i\omega_0)/(\omega_0+q)| > \omega_{pil} \); \( \omega_{pil} > 0, \omega_0 < 0 \).

### TABLE III

The Factors in \( G_0 \) when \( |-(\omega_1+i\omega_0)/(\omega_0+q)| < \omega_{pil}/(2\xi) \), \( \omega_{pil} > 0, \omega_0 > 0 \)

<table>
<thead>
<tr>
<th>No.</th>
<th>Factor</th>
<th>Corner Frequency (rad/s)</th>
<th>Gain Slope (dB/dec)</th>
<th>Phase Slope (deg)</th>
<th>Cumulative Gain Slope (dB)</th>
<th>Cumulative Phase Slope (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \omega_{pil} )</td>
<td>( \omega_{pil}/(2\xi) )</td>
<td>0</td>
<td>-180</td>
<td>0</td>
<td>-180</td>
</tr>
<tr>
<td>2</td>
<td>( 1 + si_{CQ} )</td>
<td>( -20 )</td>
<td>90</td>
<td>+20</td>
<td>0</td>
<td>-180</td>
</tr>
<tr>
<td>3</td>
<td>( 1 + 2i_{CQ}/(\omega_{pil}/2) )</td>
<td>( \omega_{pil}/(2\xi) )</td>
<td>+20</td>
<td>90</td>
<td>+40</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>( 1 + 2i_{CQ}/(\omega_{pil})+s^2/\omega_{cl}^2 )</td>
<td>( \omega_{pil} )</td>
<td>-40</td>
<td>-180</td>
<td>0</td>
<td>-180</td>
</tr>
<tr>
<td>5</td>
<td>( 1 + s/\omega_{CL} )</td>
<td>( \omega_{CL} )</td>
<td>-20</td>
<td>-90</td>
<td>-20</td>
<td>-270</td>
</tr>
</tbody>
</table>

The time domain simulation results of the cases with \( \omega_{pil}=50Hz \) and \( \omega_{pil}=80Hz \) are shown in Fig. 11(b) and Fig. 11(c). It shows that oscillations occur when \( \omega_{pil}=80Hz \) and the oscillation frequency is about 95Hz.

### TABLE IV

The Factors in \( G_0 \) when \( |-(\omega_1+i\omega_0)/(\omega_0+q)| < \omega_{pil}/(2\xi) \), \( \omega_{pil} < 0, \omega_0 > 0 \)

<table>
<thead>
<tr>
<th>No.</th>
<th>Factor</th>
<th>Corner Frequency (rad/s)</th>
<th>Gain Slope (dB/dec)</th>
<th>Phase Slope (deg)</th>
<th>Cumulative Gain Slope (dB)</th>
<th>Cumulative Phase Slope (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \omega_{pil} )</td>
<td>( \omega_{pil}/(2\xi) )</td>
<td>0</td>
<td>-180</td>
<td>0</td>
<td>-180</td>
</tr>
<tr>
<td>2</td>
<td>( 1 + 2i_{CQ}/(\omega_{pil}/2) )</td>
<td>( \omega_{pil}/(2\xi) )</td>
<td>+20</td>
<td>90</td>
<td>+90</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>( 1 + 2i_{CQ}/(\omega_{pil})+s^2/\omega_{cl}^2 )</td>
<td>( \omega_{pil} )</td>
<td>-40</td>
<td>-180</td>
<td>0</td>
<td>-180</td>
</tr>
<tr>
<td>4</td>
<td>( 1 + s/\omega_{CL} )</td>
<td>( \omega_{CL} )</td>
<td>-20</td>
<td>-90</td>
<td>-20</td>
<td>-270</td>
</tr>
</tbody>
</table>

### TABLE V

The Factors in \( G_0 \) when \( |-(\omega_1+i\omega_0)/(\omega_0+q)| > \omega_{pil} \), \( \omega_{pil} < 0, \omega_0 > 0 \)

<table>
<thead>
<tr>
<th>No.</th>
<th>Factor</th>
<th>Corner Frequency (rad/s)</th>
<th>Gain Slope (dB/dec)</th>
<th>Phase Slope (deg)</th>
<th>Cumulative Gain Slope (dB)</th>
<th>Cumulative Phase Slope (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \omega_{pil} )</td>
<td>( \omega_{pil}/(2\xi) )</td>
<td>0</td>
<td>-180</td>
<td>0</td>
<td>-180</td>
</tr>
<tr>
<td>2</td>
<td>( 1 + 2i_{CQ}/(\omega_{pil}/2) )</td>
<td>( \omega_{pil}/(2\xi) )</td>
<td>+20</td>
<td>90</td>
<td>+90</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>( 1 + 2i_{CQ}/(\omega_{pil})+s^2/\omega_{cl}^2 )</td>
<td>( \omega_{pil} )</td>
<td>-40</td>
<td>-180</td>
<td>0</td>
<td>-180</td>
</tr>
<tr>
<td>4</td>
<td>( 1 + s/\omega_{CL} )</td>
<td>( \omega_{CL} )</td>
<td>-20</td>
<td>-90</td>
<td>-20</td>
<td>-270</td>
</tr>
</tbody>
</table>

### IV. Simulation Results

To validate the proposed method and the impacts of different impacting factors on the stability, three cases with the varying PLL bandwidth, current control bandwidth and the active power and are studied. The d-axis and q-axis current step from the initial value to a higher value at \( t=0.8s \). The weak system defined in the paper is a weak grid with SCR=1.1. The SCR is defined as the ratio of the short circuit capacity over the rated power.

#### A. Impact of the Bandwidth of the PLL

From the analysis in Section III-B, we can see that the increase of the bandwidth of the PLL enlarges the value of \( G_0 \)’s gain at the phase-crossover frequency \( \omega=\omega_{pil} \), which results in \( G_0 \)’s effect as a band-pass filter and deteriorates the current control stability.

In this section, four cases with the PLL’s bandwidth being 6Hz, 16Hz, 50Hz and 80Hz are chosen for the simulation to verify the above conclusion. The other parameters are the same with those in Table I. The Bode plots of the four cases are presented in Fig. 11(a). It can be seen that the gain at the phase-crossover frequency increases when the PLL has a larger bandwidth. The value of the gain is larger than 0dB at its phase-crossover frequency of 600rad/s(95.5Hz) when \( \omega_{pil}=80Hz \), which means that the current control is unstable.

The time domain simulation results of the cases with \( \omega_{pil}=50Hz \) and \( \omega_{pil}=80Hz \) are shown in Fig. 11(b) and Fig. 11(c). It shows that oscillations occur when \( \omega_{pil}=80Hz \) and the oscillation frequency is about 95Hz.

#### B. Impact of the Active Power

From the analysis in Section III-B, we can see that when the active power \( P_e \) increases, the gain at the phase-crossover frequency \( \omega_{pil} \) will increase and be closer to the stability boundary of 0dB. In this section, we do simulations to verify this conclusion that large \( P_e \) worsens the small-signal stability of the current control.

Four cases with active power \( P_e \) being 0.5p.u., 0.4p.u. (inverter), -0.4p.u., and -0.5p.u. (rectifier) are chosen. In these cases, \( \omega_{pil}=80Hz \), and the other parameters are the same with those in Table I. The Bode plots of the four cases are presented in Fig. 12(a). It can be seen that the amplitude-frequency curve moves upward when the active power \( P_e \) increases. The value of \( G_0 \)’s gain at the phase-crossover frequency exceeds 0dB when \( P_e \) equals to 0.5p.u., indicating that the current control is unstable. Note that, when \( P_e = -0.4p.u. \) and \( -0.5p.u. \), the VSC operates as a rectifier whose small-signal stability is excellent.

Fig. 12(b) and Fig. 12(c) show the time domain simulation results in the cases of \( P_e=0.5p.u. \) and \( -0.5p.u. \). It can be seen that the current control is unstable when \( P_e=0.5p.u. \), with oscillations observed.

#### C. Impact of the Bandwidth of the Current Control

As demonstrated in Section III-B, \( \omega_{pil} = \omega_{cl} \). Thus, the bandwidth of the current control doesn’t influence the value of the gain of \( G_0 \) at the phase-crossover frequency \( \omega=\omega_{pil} \), i.e., it doesn’t influence the current control stability. In this subsection, we do simulations to verify this conclusion.

Four cases with the current control’s bandwidth being 125Hz, 250Hz, 375Hz and 500Hz are selected. The other parameters in the test cases are the same with those in Table I. Fig. 13(a) shows the Bode plots of the four cases. It can be seen that \( G_0 \)’s gain at the phase-crossover frequency remains almost unchanged as the current control’s bandwidth increases.

The time domain simulation results of the cases with \( \omega_{cl}=125Hz \) and \( \omega_{cl}=500Hz \) are presented in Fig. 13(b) and Fig. 13(c). It can be seen that both cases are stable, and that the d-axis current response is faster than when \( \omega_{cl}=500Hz \) than that when \( \omega_{cl}=125Hz \).
D. Impact of the Reactive Power

As demonstrated in Section III-D, the absolute value of the reactive power $Q_i$ increases, the gain at the phase-crossover frequency $\omega_{pill}$ will increase. In this subsection, we do simulations to verify this conclusion. Two cases with the reactive power being $-0.62\text{p.u.}$ and $-0.31\text{p.u.}$ are selected. The other parameters in the test cases are the same with those in Table I. Fig. 14(a) shows the Bode plots of the two cases. It can be seen that $G_o$’s gain at the phase-crossover frequency increases as the absolute value of the reactive power increases. The time domain simulation results of the cases with $Q_i=-0.62\text{p.u.}$ and $Q_i=-0.31\text{p.u.}$ are presented in Fig. 14(b) and Fig. 14(c). It can be seen that both cases are stable, as the $G_o$’s gain increases when $Q_i=-0.62\text{p.u.}$ but is still below 0dB.

V. A CRITERION FOR THE BANDWIDTH DESIGN OF THE PLL

In engineering practice, some system parameters are given and can’t be adjusted readily or economically, like $L_g$ and $i_{cd0}$. Up to now, to select a proper bandwidth for the PLL is one of the most feasible ways to ensure the stability of the VSC system’s current control.

A. The Criterion For the Bandwidth Design of the PLL

In this section, we propose a criterion to select proper bandwidth for the PLL based on the transfer function derived.

\[
|G_0|_{\omega_{pill}} = \frac{-L_g i_{cd0} s \left[ 1 + s(2\xi/\omega_{pill}) \right]}{u_{d0} (1 + s/\omega_{CL}) \left[ 1 + s(2\xi/\omega_{pill}) + s^2/\omega_{pill}^2 \right]} < 1.
\]  

From (18), $\omega_{pill}$ should satisfy the following constraint,

\[
\frac{\omega_{pill}}{\sqrt{\left(\omega_{pill}/\omega_{CL}\right)^2 + 1}} < \frac{u_{d0} (2\xi)}{L_g i_{cd0} \sqrt{2(\xi)^2 + 1}}.
\]  

which demonstrates that the PLL’s bandwidth has an upper limit when the other system parameters are given. And the upper limit is inversely proportional to the grid’s equivalent inductance $L_g$ and the injected active current $i_{cd0}$. Taking the test case whose parameters are shown in Table I as an example, by substituting $u_{d0}$, $i_{cd0}$, $L_g$ and $\xi$ into (19), we can get $\omega_{pill}$, which ensures the small-signal stability of the current control of the test cases in this paper.

In order to ensure the small-signal stability of the current control of the test cases in this paper, the bandwidth of the PLL should not exceed 75Hz, otherwise, the current control will be unstable. The simulation results in Fig. 11(b) show that the current control is small-signal stable when $\omega_{pill}=50$Hz.
This verifies the analysis in Section III that a smaller PLL’s bandwidth is 80 Hz and the oscillations can be observed while unstable when $\omega_{\text{PLL}}=80$ Hz, which matches well with this criterion.

B. Simulation Results with the Two-level Converter Considering the Outer Loops

The simulation results of the cases considering the outer loop are presented in Fig. 15 (the PLL’s bandwidth is 70 Hz) and Fig. 16 (the PLL’s bandwidth is 80 Hz). The parameters of the outer loop in Table I are used. In the two cases, the real power is increased to 0.8 p.u. and the reactive power is increased to 0.5 p.u. The system is stable when the PLL’s bandwidth is 70 Hz. However, it loses stability when the PLL’s bandwidth is 80 Hz and the oscillations can be observed in the real/reactive power and the d-axis and q-axis current. This verifies the analysis in Section III that a smaller PLL’s bandwidth is helpful to improve the current control stability.

The two-level converter is analyzed in this paper as integrating the wind farms and PVs to the weak grid is really a big challenge. The MMCs are usually connected to the high voltage system which might also be a weak grid, however it is not normal. Also, if the MMC internal loops of the balancing and circulating energy loops are properly tuned, a two level and an MMC have similar performance [24]. Then, the weak grid-tied MMC can be studied with our methodology.

VI. CONCLUSION

In this paper, the PLL’s effect as a high-pass filter and its impact on the current control stability of the weak-grid-tied VSC is revealed based on simplified transfer functions. By neglecting time delay components and the high-frequency voltage filters, a reduced-order MIMO transfer function and its equivalent open-loop transfer function is obtained. The analysis based on the equivalent open-loop transfer function shows that: The PLL can behave as a high-pass filter in weak-grid conditions due to the large grid impedance, whereas the current control behaves as a low-pass filter, making the inner loop of the weak-grid-tied VSC a band-pass filter; A higher PLL’s bandwidth will increase the open-loop transfer function’s maximum gain and the risk of harmonic amplification or system instability; The VSC system is less stable when it works as an inverter than as a rectifier, due to the risk of the local positive feedback in inverter mode; The current controller has slight impacts on the stability of the inner loop when its bandwidth is 0.1 to 0.2 times of the switching frequency of the power electronic switches. Based on the equivalent open-loop transfer function, an effective criterion that provides straightforward guidance for selecting a proper bandwidth of the PLL to secure the gain stability margin of the inner loop is further proposed and validated. Future work will discuss the small-signal stability of the outer loop and the design of the...
Similarly, by applying KVL across the equivalent impedance $R_{eq} + j\omega_1 L_{eq}$ between the VSC and the PCC, we can get
\[
\begin{bmatrix}
    s_i_{cd} \\
    s_i_{cq}
\end{bmatrix}
= \begin{bmatrix}
    -\frac{R_{eq}}{L_{eq}} & \omega_1 \\
    -\omega_1 & -\frac{R_{eq}}{L_{eq}}
\end{bmatrix}
\begin{bmatrix}
    i_{cd} \\
    i_{cq}
\end{bmatrix}
+ \frac{1}{L_{eq}} \begin{bmatrix}
    u_{cd} - u_{sd} \\
    u_{cq} - u_{sq}
\end{bmatrix},
\]
(21)

B. Modelling of the VSC Control System

As shown in Fig. 2, the PLL synchronizes the control system’s frame to the grid’s frame with $\omega_{r}$ orientation. The mapping relation of one variable in the two different frames is
\[
F_{dq}^{cl} = T_{dq2dqcf} F_{dq},
\]
where $F$ denotes $u_{s}$, $u_{c}$, $u_{g}$ or $i_{c}$, the superscript "cf" denotes that the variable is in the control system’s dq frame, and $T_{dq2dqcf}$ is written as
\[
T_{dq2dqcf} = \begin{bmatrix}
    \cos \theta_{PLL} & \sin \theta_{PLL} \\
    -\sin \theta_{PLL} & \cos \theta_{PLL}
\end{bmatrix},
\]
where $\theta_{PLL}$ is the initial phase of the PCC voltage observed by PLL. In the following, we explain each part of the control system in Fig. 1 in detail.

1) PLL: The representation of the PLL shown in Fig. 1 is as follows,
\[
\theta = \left( G_{PI-PLL} u_{cl}^{cf} + \omega_{r} \right) \frac{1}{s},
\]
(23)
where $G_{PI-PLL}$ is the transfer function of the PI controller in the PLL, and can be written as
\[
G_{PI-PLL} = k_{p-PLL} \left( 1 + \frac{1}{sT_{i-PLL}} \right),
\]
(24)
where $k_{p-PLL}$ and $T_{i-PLL}$ are the proportional coefficient and the integral time constant of $G_{PI-PLL}$, and can be obtained by $k_{p-PLL} = 2 \xi \omega_{PLL}/u_{sd0}$, $T_{i-PLL} = 2 \xi / \omega_{PLL}$. And $\omega_{PLL} = G_{PLL}(s)\omega_{cl}$.

2) Current Control: The current control can be described as
\[
\begin{bmatrix}
    u_{cd}^{cf} \\
    u_{cq}^{cf}
\end{bmatrix}
= \begin{bmatrix}
    G_{f} u_{cl}^{cf} + G_{CL}(\omega_{cl}^{cf} - \omega_{cl}^{cf}) - \omega_{1} L_{eq} \omega_{cl}^{cf} \\
    G_{f} u_{cl}^{cf} + G_{CL}(\omega_{cl}^{cf} - \omega_{cl}^{cf}) + \omega_{1} L_{eq} \omega_{cl}^{cf}
\end{bmatrix},
\]
(25)
where $G_{f}$ is the voltage filter, and $G_{CL}$ is the PI controller in the inner loop. $G_{f}$ and $G_{CL}$ can be expressed as
\[
\begin{bmatrix}
    G_{f} = \frac{\omega_{fr}}{s + \omega_{fr}} = \frac{1}{1 + sT_{f}} \\
    G_{CL} = k_{p-CL} \left( 1 + \frac{1}{sT_{i-CL}} \right)
\end{bmatrix},
\]
(26)
where $\omega_{fr}$ is the bandwidth of the voltage filter, $T_{f}$ is the time constant of the voltage filter, $k_{p-CL}$ and $T_{i-CL}$ are the proportional coefficient and the integral time constant of $G_{CL}$, respectively, and they can be obtained by $k_{p-CL} = \omega_{CL} L_{eq}$, $k_{i-CL} = \omega_{CL} R_{eq} = k_{p-CL}/T_{i-CL}$, where $\omega_{CL}$ is the bandwidth of the inner loop.

The time delay caused by the signal processing and transmission in the VSC’s control system can be modelled as a lumped time delay from $u_{cl}^{cf}$ to $u_{cl}^{cf}$, and can be described by
\[
\begin{bmatrix}
    u_{cd}^{cf} \\
    u_{cq}^{cf}
\end{bmatrix}
= \begin{bmatrix}
    G_{d} u_{cd}^{cf} \\
    G_{d} u_{cq}^{cf}
\end{bmatrix},
\]
(27)
where $G_{d}$ is the time delay function and can be described as
\[
G_{d} = \frac{1}{1 + sT_{d}},
\]
(28)
where $T_d$ is the delayed time of the PWM, the measurement and the control communication. The voltage signal $u_{c}^{*}$ is in the dq frame of the control system and needs to be transformed to the dq frame of the grid by

$$
\begin{bmatrix}
    u_{cq}^* \\
    u_{cd}^*
\end{bmatrix} = \mathbf{T}_{dq2dc}^{-1} \begin{bmatrix}
    u_{c}^{*} \\
    u_{c}^{*}
\end{bmatrix}.
$$

(29)

The VSC generates the voltage $u_{c}$ by modulating the DC voltage through the on-off actions of the switches in the VSC, which is described by

$$
u_{cd} = \frac{u_{cd}^{*}}{V_{dc}}, v_{cq} = \frac{u_{cq}^{*}}{V_{dc}},
$$

(30)

where $V_{dc}$ is the DC voltage, and $V_{dc0}$ is its steady-state value.

3) Outer Loop Control: The outer loop can be described by

$$
\hat{e}_{cd}^s = G_{PC}(P_{s} - P_{s}^{*}), \hat{e}_{cq}^s = G_{AC}(U_{s}^{*} - U_{c}^{*}).
$$

(31)

where $G_{PC}$ and $G_{AC}$ can be described by

$$
G_{PC} = k_{PC} \left( 1 + \frac{1}{sT_{1,PC}} \right), G_{QC} = -k_{QC} \left( 1 + \frac{1}{sT_{1,QC}} \right).
$$

(32)

where $k_{PC}$ and $T_{1,PC}$ are the proportional coefficient and integral time constant of $G_{PC}$ respectively, and $k_{QC}$ and $T_{1,QC}$ are the proportional coefficient and integral time constant of $G_{QC}$ respectively. They can be obtained by

$$
T_{1,PC} = \frac{1}{\omega_{OC,PC} k_{PC} T_{1,PC}}, k_{PC} = \omega_{OC,PC} T_{1,PC}/(1.5 u_{sm}), k_{QC} = -\omega_{OC,QC} T_{1,QC}/(1.5 u_{sm}),
$$

where $u_{sm}$ is the magnitude of the grid’s phase voltage.

REFERENCES


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