An efficient approach for computing analytical non-parametric fragility curves

Domenico Altieri¹, Edoardo Patelli^{1*}

¹ *Institute for Risk and Uncertainty, University of Liverpool, Peach Street, Chadwick Building, Liverpool L69 7ZF, UK,
D.Altieri@liverpool.ac.uk, Edoardo.Patelli@liverpool.ac.uk*

Abstract

Fragility curves are used in earthquake engineering for assessing the seismic vulnerability of structures or systems. Direct estimations of fragility curves by means of simulation-based approaches lead generally to relevant computational costs, especially when the failure region is characterized by small probabilities of occurrence. Simplified hypotheses are therefore introduced in the common practice to approximate the dependency between the structural response and the associated seismic intensity level.

The study proposes a non-parametric methodology to estimate analytical fragility curves without specific assumptions on their final shape. The approach starts by identifying all the subsets characterized by the same values of the chosen seismic intensity measure parameter. Then, the failure region is mapped by means of a classification algorithm coupled with a polynomial kernel. Finally, the conditional failure probability is computed by associating the samples generated in each subset to the corresponding classification score. A stochastic earthquake model is employed to define the seismic dataset and the uncertainty associated with the ground motion records.

Two case studies are analyzed in which the non-parametric methodology is compared against three popular parametric approaches and a reference solution. The proposed approach shows an overall higher accuracy and efficiency, especially in case of rare failure domains.

Keywords: Seismic fragility curves, Conditional failure probability, Reliability analysis, Performance based earthquake engineering

1. Introduction

Seismic fragility curves describe how the failure probability of structural systems or components evolves for different given levels of a seismic intensity measure parameter (IM) adopted to characterize the ground motion record (e.g. peak ground acceleration, spectral acceleration at the fundamental frequency). They are gaining popularity especially in the last years due to their importance in the framework of a performance based earthquake engineering ($PBEE$) approach [1, 2]. For example, by combining the fragility curve and the seismic hazard curve it is possible to compute the mean annual rate of the failure state [3, 4].

The dataset of the structural response under different seismic actions, needed to compute fragility curves, can be obtained from both empirical data (observed failures) (e.g. [5, 6]) and analytical model evaluations (e.g. [7, 8]). In the first case data are collected by recording the damage state of a set of structures along with the intensity seismic levels, that can be estimated from a ground motion instrument or in absence of that from a ground motion map [9, 10] or a proper attenuation law.

In case of analytical fragilities curves (i.e. computed on the basis of nonlinear dynamic analyses), different procedures have been proposed to efficiently perform nonlinear dynamic structural analyses for increasing seismic intensity measures. Classic analytical parametric methodologies are based on the assumption that fragility curves can be defined by means of a lognormal cumulative distribution function so that two parameters, median $\hat{\mu}$ and standard deviation $\hat{\beta}$, are sufficient to derive conditional seismic reliability measures for any given failure criterion [11, 12, 13]. The statistical parameters $\hat{\mu}$ and $\hat{\beta}$ refer to the dataset of IM s corresponding to the failure state, since the randomness of the seismic input can lead to different dynamic response even having the same IM . The estimation of $\hat{\mu}$ and $\hat{\beta}$ is commonly obtained by means of three different approaches: method of moments, maximum likelihood method or by adopting a regression demand model (mostly linear) in the log-scale between the IM s and the structural response [14, 15]. One of the most common parametric approach performs a sequence of dynamic analysis by repeatedly scaling the ground motion until the failure state is reached (Incremental Dynamic Analysis - IDA) [11]. However, IDA may require significant computational efforts and the final fragility curve may be affected by the scaling stage since the ground motion frequency content may be not representative of the corresponding intensity level [16, 17]. In this

regard, [18] proposes an hybrid procedure to reduce the number of analysis required by a classic *IDA* by previously estimating the seismic intensity level corresponding to the failure state by means of the so-called Cloud Analysis [19, 13] that provides a simple linear relation between the un-scaled seismic inputs and the corresponding Engineering Demand Parameter (*EDP*) values. Alternatively, the so-called Multiple Stripes Analysis (*MSA*) allows estimating the median and the dispersion of a sequence of 'stripes' of structural responses for increasing constant *IMs*, until the collapse is reached along the whole stripe [20, 21].

In general, parametric fragility models provide a smooth fragility curve easy to estimate. However they can lead to a lack of fit with respect the real data distribution due to the restriction on the final shape [22]. While parametric methods have been widely addressed due their robustness and simplicity, this is not the case for non-parametric procedures that result mainly based on the use of kernel smoothing methods (*KSMs*) [22, 23, 24] or the copula function [25]. These numerical approaches turn out to be extremely flexible and efficient to compute the joint probability $P(EDP, IM)$ or the conditional probability $P(EDP|IM = im_i)$ for the fragility curve estimation. However, *KSMs* can suffer of over-fitting or show bias near the boundaries of the analyzed domain, while a copula function cannot be always properly fitted and the selection of the optimal copula remains somehow murky [26]. The approach introduced in this article does not work directly on the joint probability $P(EDP, IM)$ and does not require an ambiguous tuning stage.

More specifically, this study proposes a methodology able to derive seismic fragility curves without any assumption on the final conditional cumulative density function, through a failure region mapping process that employs a Multinomial Logistic Regression coupled with a polynomial kernel. The absence of a predefined final shape reduces the risk of lack of fitting, while the use of probabilistic scores instead of a classical indicator function makes the approach more flexible and applicable even in case of noise along the failure region.

The paper is organized as follows. Section 2 provides a brief description of three parametric approaches employed to validate and test the proposed procedure. In section 3 the proposed methodology is described into details along with the failure region mapping process. In section 4 the problem raised by the seismic dataset selection is presented and two alternatives discussed. In section 5 two different case studies are presented and finally, all the results are reported and the conclusions drawn.

2. Fragility curves and parametric models

Analytical fragility curves allow selecting the IM levels for the seismic dataset definition along with the number of dynamic analysis required for each IM value [12]. This aspect contributes to define methods increasingly able to perform an efficient dataset selection by reducing the number of analyses performed.

Generally, by assuming that the IM s causing the failure are lognormal distributed then the final fragility curve can be derived from just 2 parameters, the median capacity $\hat{\mu}$ (i.e. the IM level associated to a probability of failure of 0.5) and the dispersion measure $\hat{\beta}$. The fragility curve can therefore be expressed as [11, 12, 13]:

$$P(Failure|IM_i) = \Phi\left(\frac{\log IM_i/\hat{\mu}}{\hat{\beta}}\right) \quad (1)$$

where Φ represents the standard normal cumulative density function.

A brief overview about three of the most established parametric procedures based on the lognormal assumption is provided in this section.

2.1. Incremental Dynamic Analysis

Incremental dynamic analysis computes the structural response for a sequence of scaled ground motion records until the failure state is reached [11]. Thus, for any given failure criterion, a dataset of seismic intensity measures corresponding to the failure can be obtained through a linear interpolation and subsequently assumed as lognormal distributed. The fragility curve parameters $\hat{\mu}$ and $\hat{\beta}$ are obtained by fitting the interpolated IM_i through the method of moments [27]:

$$\log \hat{\mu} = \frac{1}{n} \sum_{i=1}^n \log IM_i \quad (2)$$

$$\hat{\beta} = \sqrt{\frac{\sum_{i=1}^N (\log IM_i - \log \hat{\mu})^2}{N - 1}} \quad (3)$$

where N is the number of ground motion records considered in the analysis. For the scaling of the initial seismic dataset, instead of simply using a constant incremental step until the failure occurs, a more efficient algorithm is employed in this study allowing to control at the same time the tolerance resulting from the final linear interpolation [11].

104 2.2. Multiple Stripes Analysis

105 Similarly to the *IDA*, the Multiple Stripe Analysis (*MSA*) [20, 21] performs
 106 a sequence of dynamic analyses for different seismic datasets, each one corre-
 107 sponding to a fixed *IM* level. The final 'stripes' are therefore characterized
 108 by a constant *IM* level *vs* multiple structural responses. The method consid-
 109 ers increasing *IM* levels until the failure is reached for the whole intermediate
 110 seismic dataset. The final fractions of analyses causing failure can be fitted
 111 using the maximum likelihood method. In particular, the logarithmic median
 112 $\hat{\mu}$ and standard deviation $\hat{\beta}$ are computed as following:

$$\{\hat{\mu}, \hat{\beta}\} = \max_{\mu, \beta} \prod_{i=1}^{N_{IM_i}} \binom{n_i}{f_i} \Phi\left(\frac{\log IM_i/\mu}{\beta}\right)^{f_i} \left(1 - \Phi\left(\frac{\log IM_i/\mu}{\beta}\right)\right)^{n_i-f_i} \quad (4)$$

113 where, considering N_{IM_i} number of intermediate *IM* levels, n_i and f_i rep-
 114 resent the corresponding number of collected ground motion records and
 115 system failures respectively.

116 To reduce the computational effort the procedure can be stopped before the
 117 whole dataset causes the failure, affecting the final accuracy, or alternatively
 118 the median demand for a given *IM* and the associated dispersion can be
 119 computed from the percentiles of the stripes, as described in [13].

120 2.3. Cloud Analysis

121 The method employs a simple conditional *IM*-based approach by defining a
 122 linear probabilistic model in the logarithmic space after computing the *EDP*
 123 values for a set of ground motion records characterized by different *IM* levels
 124 [19, 13, 28]. The probabilistic model is defined as following:

$$\log \hat{\mu}_{Y|IM} = \log a + b \cdot \log IM \quad (5)$$

125

$$\hat{\beta}_{\log Y|IM} = \sqrt{\frac{\sum_{i=1}^N (\log Y_i - \log \hat{\mu}_{Y|IM})^2}{N - 2}} \quad (6)$$

126 where *Y* represents the critical demand to capacity ratio and *a* and *b* are
 127 the two linear regression coefficients in the logarithmic scale (similarly a
 128 power-law curve can be adopted to fit the data in the original scale). $\hat{\mu}_{Y|IM}$
 129 represents the median for *Y* given a seismic intensity value *IM*, while $\hat{\beta}_{\log Y|IM}$

130 is the logarithmic standard deviation for Y conditioned to the IM . Finally,
 131 the system fragility curve can be obtained as:

$$P(\log Y > 0|IM) = \Phi\left(\frac{\log \hat{\mu}_{Y|IM}}{\hat{\beta}_{\log Y|IM}}\right) \quad (7)$$

132 3. Proposed non-parametric approximation

133 The proposed approach aims at reducing the computational effort by pro-
 134 viding a non-parametric approximated solution to compute conditional reli-
 135 ability measures. For the sake of clarity, small letters are used to indicate
 136 parameters while capital letters refer to random variables or subsets of the
 137 sample space.

138 Given a failure event F and a sequence of subsets $B_i = \{\vec{M}, \vec{R}|im_i =$
 139 *constant* $\}$ in the 2-dimensional sample space [moment magnitude - M , epi-
 140 central distance - R], the failure probability for a given B_i can be expressed
 141 following the Kolmogorov definition as:

$$P(F|B_i) = \frac{P(F \cap B_i)}{P(B_i)} \quad (8)$$

143 where $P(F \cap B_i)$ represents the joint probability of the events F and B_i .
 144 Moreover, for any failure event F , the associated probability of occurrence
 145 can be computed as:

$$P(F) = P_f = \int_F \Phi(\vec{M}, \vec{R}) dx = \int_{[M,R]} \Phi(\vec{M}, \vec{R}) \cdot \mathbb{I} dx \quad (9)$$

147 where $\mathbb{I}$ is an indicator function equal to 1 when the system fails and 0
 148 otherwise, while $\Phi(\cdot)$ represents the joint probability density function in the
 149 standard normal space [29]. The mapping from the physical space to the
 150 standard normal space can be easily obtained by assuming that the cumula-
 151 tive density functions of the random variables (M and R) remain the same
 152 after the transformation in standard normal distributions [30].

153 The multi-dimensional integration over the failure domain F is often impos-
 154 sible to perform in a closed form [31]. Hence a simulation-based approach,
 155 that approximates the exact solution, is required.

156 The joint probability $P(F \cap B_i)$, according to Equation (9), can be computed
 157 as:

$$P(F \cap B_i) = \int_{F \cap B_i} \Phi(\vec{M}, \vec{R}) dx = \int_{B_i} \Phi(\vec{M}, \vec{R}) \cdot \mathbb{I} dx \quad (10)$$

158 while the probability $P(B_i)$ in Equation (8) can be computed through the
 159 integral of $\Phi(\cdot)$ over B_i . Equation (8) can be now rewritten as:

$$P(F|B_i) = \frac{P(F \cap B_i)}{P(B_i)} = \frac{\int_{B_i} \Phi(\vec{M}, \vec{R}) \cdot \mathbb{I} dx}{\int_{B_i} \Phi(\vec{M}, \vec{R}) dx} \quad (11)$$

160 An approximated solution of $\int_{B_i} \Phi(\vec{M}, \vec{R}) dx$ can be easily reached without
 161 additional computational efforts by means of a simulation-based approach,
 162 namely by sampling in the standard normal space along each event B_i . In
 163 general, to compute the failure probability conditioned to the value assumed
 164 by a specific input, a sequence of samples must be generated in all the subsets
 165 of the sample space characterized by a constant value for the input of interest.
 166 An efficient solution of Equation (11) still needs an additional procedure to
 167 reduce the number of model realizations related to the indicator function $\mathbb{I}$
 168 and thus, to the failure region identification.

169 3.1. Failure region mapping

170 A Multinomial Logistic Regression (*MLR*) [32] is adopted to predict the
 171 probability that a specific structural system reaches the failure region or
 172 not for a given *IM*. The chosen classification algorithm allows the definition
 173 of intermediate classes in case conditional failure probabilities with respect
 174 other final performance levels are required (i.e. multiple failure regions). The
 175 indicator function $\mathbb{I}$ in Equation (9) is therefore replaced by the probabilistic
 176 score w_i that expresses the confidence level of each prevision Y_i . In particular,
 177 the probability that Y_i belongs to the class K can be calculated as:

$$P(Y_i = K) = w_i = \frac{e^{\beta_K \cdot [\vec{M}_i, \vec{R}_i]}}{1 + \sum_{k=1}^K e^{\beta_k \cdot [\vec{M}_i, \vec{R}_i]}} \quad (12)$$

178 where β_K represents the vector of regression coefficients associated with the
 179 class K . Two classes are defined for each failure criterion (Failure or Not-
 180 Failure) and, if required, multi-dimensional damage measures [33] can be
 181 adopted by increasing the number of attributes describing each observation
 182 i . In case of synthetic ground motion records (stochastic earthquake model),
 183 the initial seismic dataset for the *MLR* training can be obtained by means
 184 of a Design of Experiments (*DoE*) simulation based on surrogate probability
 185 density functions (*PDF*) of M and R , in order to easily reach the failure

186 region getting, at the same time, a higher grade of coverage of the sample
 187 space (Table 4). In particular, deterministic samples are generated from uni-
 188 form distributions with upper and lower boundaries (U , L) chosen according
 189 to the real PDF s, so that $F(L)$ and $1 - F(U)$ can be considered negligible,
 190 with $F(\cdot)$ representing the associated CDF .

191 In absence of a stochastic earthquake model the training of the MLR has to
 192 be performed over a dataset of real ground motion records consistent with
 193 the probabilistic density functions adopted for M and R and able to cover
 194 as much as possible the sample space of interest $[M, R]$. Further details are
 195 provided in the next section.

196 Finally, despite the limitations of the MLR algorithm being a linear classifier
 197 based on a separating hyperplane, more complex and general failure region
 198 shapes can be still mapped by adopting alternative kernels [34]. In this study
 199 a second grade polynomial kernel is employed (this choice will be motivated
 200 afterwards).

201 3.2. Subsets definition

202 An estimation of the solution of Equation (10) can now be obtained by
 203 sampling from the surrogate PDF s along each predefined subset B_i and by
 204 evaluating the probabilistic score w_i (see Equation 12) associated with each
 205 realization:

$$P(F \cap B_i) \approx \sum_{B_i} \Phi(\vec{M}, \vec{R}) \cdot w_i \quad (13)$$

Figure 1: a) Seismic regression model with a sequence of subsets B_i , b) Subsets B_i in the standard normal space

Figure 2: Flowchart describing the proposed approach

206 In case of seismic fragility curves each subset B_i is characterized by a con-
 207 stant IM value. A seismic prediction model $g(M, R)$ is therefore required
 208 to identify the coordinates in terms of magnitude and epicentral distance of
 209 each $B_i \subset [M, R]$. A polynomial regression model is trained over a synthetic
 210 ground motion dataset of 5000 seismic samples and adopted as attenuation
 211 law. Figure (1a) reports the regression model and a sequence of subsets B_i
 212 corresponding to discrete values of the spectral acceleration Sa (assumed as
 213 IM). Finally, after mapping each subset B_i in the standard normal space
 214 (Figure 1b), the sought solution can be obtained by combining Equations
 215 (13) and (11):

$$P(F|B_i) = \frac{P(F \cap B_i)}{P(B_i)} \approx \frac{\sum_{B_i} \Phi(\vec{M}, \vec{R}) \cdot w_i}{\sum_{B_i} \Phi(\vec{M}, \vec{R})} \quad (14)$$

216 The samples generated along B_i , for a simulation-based estimation of $\Phi(\cdot)$,
 217 are affected by a regression error ε assumed to be normal distributed with
 218 mean zero and standard deviation provided by the polynomial regression
 219 $g(M, R)$. In Figure (2) a flowchart describes step-by-step the algorithm pro-
 220 posed for the estimation of the conditional failure probability corresponding
 221 to different B_i . It can be summarized as follows:

- 222 1 Definition of the subset $B_i = \{\vec{M}, \vec{R} | im_i = constant\}$ by means of a
 223 proper ground motion prediction model $g(M, R)$.
- 224 2 Definition of surrogate uniform distributions for M and R .
- 225 3 Design of Experiment simulation to generate the MLR training dataset
 226 by exploring the sample space $[M, R]$. The output variable is repre-
 227 sented by the structural response.
- 228 4 Training of the Multinomial Logistic Regression coupled with a poly-
 229 nomial kernel.
- 230 5 Sampling along each event B_i to compute $\Phi(\cdot)$ and w_i .
- 231 6 For each subset B_i compute the corresponding conditional failure probail-
 232 ity by means of Equation 14.

233 4. Seismic demand

234 Seismic reliability analyses are based on ground motion records consistent
 235 with the local seismic hazard. Despite the seismic record selection strategy
 236 is not the focus of this study, two possible approaches for the seismic demand
 237 definition are described below, both compatible with the proposed method-
 238 ology allowing a probabilistic description of the magnitude M , the epicentral
 239 distance R and the associated seismic intensity measure IM .

240 4.1. Stochastic earthquake model

241 The first alternative involves the adoption of a stochastic earthquake model
 242 to simulate the seismic 'record-to-record variability', as developed by Atkin-
 243 son and Silva for the California region (Appendix A). This option, adopted
 244 in the current study, allows to generate an extensive seismic dataset that in
 245 turn, allows to compute an '*exact*' seismic fragility curve. This curve can be
 246 used as target to assess the accuracy of the proposed approach in comparison
 247 to the parametric methodologies previously described in Section 2.

248 The use of synthetic time histories allows the generation of a large seismic
 249 dataset needed for the definition of a consistent regression model $g(M, R)$,
 250 subsequently employed to correctly identify all the events $B_i = \{\vec{M}, \vec{R} | im_i =$
 251 *constant* $\}$ (Figure 1). Moreover, a stochastic seismic model can be coupled
 252 with a *DoE* simulation for the generation of the seismic dataset for the *MLR*
 253 calibration, by improving the global efficiency of the method.

254 It is noted that the probability density functions adopted to define the mag-
 255 nitude and epicentral distance distribution do not affect the final shape of
 256 the fragility curves. The same is not true for the adopted regression model
 257 that, if poorly calibrated, can reduce the accuracy of the results due to the
 258 altered coordinates of all the subsets B_i .

259 In Appendix A it can be noted how the random generation of the ground
 260 motion time histories depends on only two seismological random variables,
 261 namely the moment magnitude and the epicentral distance, in addition to
 262 the initial white noise. Therefore, the proposed methodology is built on a
 263 two dimensional samples space, by assuming as constant all the remain pa-
 264 rameters required by the adopted seismic model. This requires to recalibrate
 265 the model in case of different sites of interest. However, the proposed ap-
 266 proach can be even generalized by adopting a multidimensional attenuation
 267 law able to account for additional sources of uncertainty.

268 4.2. Real ground motion records

269 Three of the basic steps that compose a classic Probabilistic Seismic Hazard
270 Analysis (*PSHA*) [35, 36] are necessary to couple the proposed approach with
271 real ground motion records.

- 272 1 The definition of a probability density function describing the occur-
273 rence of the earthquake magnitudes M .
- 274 2 A probabilistic description of the earthquake epicentral distances R
275 based on a previous identification of the earthquake source type (e.g.
276 line source, area source).
- 277 3 The ground motion intensity measure IM has to be predicted starting
278 from M and R . This can be done by means of existing ground motion
279 prediction models that are generally based and calibrated on statistical
280 regressions over a large dataset of observed seismic intensity measures.

281 All the subsets B_i can then be defined on the basis of the adopted attenuation
282 law, while the probability density functions of M and R allow computing $\Phi(\cdot)$
283 in the standard normal space. The dataset of real time histories can finally be
284 used for the *MLR* training. It is worth pointing out, however, that the use of
285 a real dataset will reduce the efficiency of the approach since advanced sam-
286 pling techniques can not be adopted for the definition of the training dataset.

287

288 5. Case studies

289 Two different structural systems are analyzed to test and validate the pro-
290 posed approach. Both models do not require an excessive computational
291 effort in order to derive the target seismic fragility curve through a Latin
292 Hypercube simulation. The opensource Matlab toolbox OpenCossan [37, 38]
293 is used for the probabilistic analysis.

294 5.1. Impact model

295 A dissipative Kelvin-Voight impact model [39], improved by Komodromos in
296 2007 [40], is added to a single-degree-of-freedom system (Figure 3) in order to
297 analyze the collisions between the superstructure and the lateral walls [41].
298 The magnitude of the impact force during the seismic event is computed as

Figure 3: SDOF impact model

follows:

$$F_{imp}(t + \Delta t) = k_{imp} \cdot \delta(t) + c_{imp} \cdot \dot{\delta}(t) \text{ when } F_{imp}(t) > 0 \quad (15)$$

$$F_{imp}(t + \Delta t) = 0 \text{ when } F_{imp}(t) \leq 0 \quad (16)$$

where $\delta(t)$ and $\dot{\delta}(t)$ represents the interpenetration depth and the relative velocity, respectively, computed in Matlab by means of an explicit Runge-Kutta method [42] and a time step of 10^{-5} seconds. k_{imp} is the impact stiffness while c_{imp} the impact damping coefficient [40]. Further details on the model characterization can be found in [41]. Table 1 reports the structural parameters adopted for the dynamic analysis, in particular a reduced Δ is employed in order to obtain a probability of having impacts almost equal to 1.

m [ton]	k_{imp}/k	T [s]	Δ [m]
100	10	1	0.001

Table 1: Adopted structural configuration for the SDOF impact model

5.2. Viscous Dampers

In the second case study the seismic reliability of a steel moment-resisting frame equipped with non linear viscous dampers is analyzed (Figure 4). The

314 structure has been already employed in several studies for retrofit applica-
 315 tions (e.g. [43, 44, 45]) and represents typical low-rise buildings of the Los
 316 Angeles, California region [43]. The dynamic analysis of the 2D model shown
 317 in Figure (4) is performed by the opensource FEM software OpenSees [46].
 318 The vibration periods associated to the first 3 modes are 0.99 s, 0.33 s and
 319 0.18 s respectively, while the total mass corresponding to the two lower levels
 320 is equal to 975 tons and it reaches 1040 tons at the top floor.
 321 Columns and beams are both wide flanges with section dimensions shown in
 322 Figure 4. A P-Delta column is employed within the model to account for the
 323 nonlinear geometrical effects due to the action of relevant vertical loads and
 324 a corotational transformation is employed to capture the large displacement-
 325 small strain effects. Finally, distributed plasticity is used to model nonlinear
 326 elements and fiber sections. Additional details on the FEM model and the
 327 structural properties can be found in [47, 43].
 328

Figure 4: Steel moment-resisting frame equipped with non linear viscous dampers

329 The damping force can be modeled by means of the following constitutive
 330 law:

$$F_d(\dot{u}) = C_d \cdot |\dot{u}|^\alpha \cdot \text{sign}(\dot{u}) \quad (17)$$

331 where C_d represents the damping constant, $\dot{u}$ is the relative velocity between
 332 the damper extremities and finally the exponent α controls the nonlinearity
 333 of the damper response. Seismic fragility curves are derived for the non linear
 334 case with $\alpha = 0.3$.

335 5.3. Results

336 The probabilistic analysis is carried out by considering three increasing struc-
 337 tural reliability levels associated with different dynamic performance re-
 338 quired. Table 2 reports the adopted thresholds for each case study, in terms
 339 of inter-storey drift (IDR) and impact force (IF). It is worth noting that the
 340 performance levels required to the steel moment resisting frame might ap-
 341 pear too low, however the presence of non linear viscous damper significantly
 342 increase the dissipative capacity of the original structure and an $IDR = 0.6\%$
 already generates a sufficiently rare failure region.

Impact model	Level 1	Level 2	Level 3
IF	200 kN	400 kN	600 kN
Viscous dampers	Level 1	Level 2	Level 3
IDR	0.1%	0.3%	0.6%

Table 2: Performance levels analyzed for each case study

343

344 5.3.1. Parametric approaches

345 A seismic dataset of 90 time histories is generated through a LHS by sampling
 346 M and R (Figure 9b,d) from the original distributions (Appendix A) and
 347 then employed to perform both the CA and IDA . The dataset defined for
 348 the proposed method (Figure 5), obtained by a 9^2 full factorial DoE , cannot
 349 be used for the IDA since the surrogate PDF_s would strongly affect the
 350 final distribution of the IM_s corresponding to the failure state, leading to
 351 inaccurate results.

352 A ground motion dataset based on 30 incremental values of Sa is defined
 353 for the MSA . In particular, 30 accelerograms have been selected for each
 354 considered IM level, reaching a total of 900 ground motion time histories.
 355 All the intermediate datasets of 30 samples are obtained through one single
 356 Subset Simulation analysis (SS) [48], able to efficiently direct the samples
 357 generation towards higher IM levels by mean of Markov chain Monte Carlo.
 358 It is worth highlighting that the number of stripes can be further optimized
 359 to reduce the seismic dataset required by the method. Indeed, the final
 360 efficiency and accuracy are also dependent on the amplitude of each stripe,
 361 as shown in Eads et al. [49]. However, to reach a priori a global optimal
 362 calibration of the MSA , its is required a more in depth knowledge on the
 363 system performances that, most of the times, is impossible to obtain.

364 In order to assess the performances of the proposed method in terms of
 365 both accuracy and efficiency, the analyzed parametric approaches have to be
 366 performed with a similar number of ground motion time histories to get final
 367 comparable results. However, the *MSA*, being based on multiple statistical
 368 analysis corresponding to increasing *IM* levels, requires a higher number of
 369 seismic samples comparing to the other approaches.
 370 Moreover, similarly to the proposed approach that is performed always on the
 371 same initial seismic dataset (regardless the specific case study or the required
 372 performance level), the datasets for the parametric approaches are purposely
 373 not calibrated according to the different analyzed case. Finally, it worths
 374 noting that the proposed approach is the one performed with the lowest
 375 number of seismic samples since it can be coupled with a *DoE* simulation to
 376 efficiently cover the sample space in case of artificial time histories.

Figure 5: M and R samples employed for the *CA* and the *IDA*.

377 For the *CA*, a probabilistic demand model is defined by means of a second
 378 grade polynomial regression that turned out to be more accurate than a
 379 simple linear model (Figure 6).

Figure 6: Probabilistic demand model employed by the Cloud Analysis for a) the steel frame equipped with viscous dampers and b) the SDOF impact model. Final structural threshold based on the performance level 2.

Figure 7: IDA curves for a) the steel frame equipped with viscous dampers and b) the SDOF impact model. Final structural threshold based on the performance level 2.

Figure 8: Structural samples for each IM value considered in the MSA for a) the steel frame equipped with viscous dampers and b) the SDOF impact model. Final structural threshold based on the performance level 2. The median of each stripe is reported together with the 84th and 16th percentiles

380 The *IDA* is performed by adopting a more efficient algorithm [11] able to
 381 adaptively scale the initial ground motion by respecting a tolerance of 5%
 382 on the final interpolated *IM* value corresponding to the failure state. Figure
 383 (7) shows the 90 *IDA* curves for the two case studies, assuming Level 2 as
 384 final threshold.

385 Figure (8) reports the structural responses associated with each *IM* value
 386 considered in the *MSA*, the median response with the 84th and 16th per-
 387 centiles are shown as well. Table 3 summarizes the employed seismic datasets
 388 together with the associated sampling technique.

Methodology	Seismic dataset dimension	Sampling Technique
<i>Method Proposed</i>	81	<i>DoE</i>
<i>CA</i>	90	<i>LHS</i>
<i>IDA</i>	90	<i>LHS</i>
<i>MSA</i>	900 (30x30)	<i>SS</i>

Table 3: Seismic datasets

389 5.3.2. Failure region mapping

390 The failure region mapping, required to compute the probabilistic scores w_i ,
 391 is performed by adopting a *MLR* coupled with a second grade polynomial

kernel. In particular, the *kernel method* allows to overcome the limitations of a linear classification algorithm by adding to the sample space different new features that depend on the original variables (in this case M and R) through a specific function or kernel. This simple numerical expedient moves the problem into a higher dimensional space where a linearly non-separable dataset (in the original space) is transformed in a dataset linearly separable by a hyperplane.

Figure 9: Multinomial logistic regression with failed and safe structural samples for a) the impact model and c) the viscous dampers. Sub-figures b) and d) show the efficiency of the adopted quadratic polynomial kernel (Level 2).

The optimal grade of the polynomial kernel depends on the shape of the failure region, for the analyzed case studies a second grade polynomial is

401 an acceptable compromise between the classification accuracy and the com-
 402 putational costs. Moreover, a higher polynomial grade associated with a
 403 reduced training dataset may increase the probability of having overfitting.
 404 For complex failure region shapes more performing kernels can be employed
 405 (e.g. Radial basis function kernel). Figures (9a) and (9c) show the trained
 406 logistic regression for two specific cases with the associated training dataset
 407 (81 structural responses) obtained through a 9^2 Full Factorial *DoE* simula-
 408 tion. The surrogate *PDFs*, employed for the training dataset definition, are
 409 reported in Table 4. From Figures (9b) and (9d) it can be observed how the
 410 *MLR* is able to perform a non-linear classification thanks to the adopted ker-
 411 nel. The employed multiclass classification algorithm allows to get multiple
 412 fragility curves for different structural thresholds after one single training
 413 stage. Though Figure (9b) shows an almost clean transition from the safe to
 414 the failure region, this depends on the number of levels adopted for the *DoE*
 415 simulation. In fact, the noise associated to the transition appears more visi-
 416 ble by increasing the number of samples within the sample space (Figure 10).
 417 Moreover, it should be noted that an additional level of uncertainty comes
 418 from the *Sa* associated to each couple (M, R) , that means we can experience
 419 failure or not-failure even with the same M and R , especially around the
 420 transition zone.

421 The *MLR* training is affected by the dimension of the initial seismic dataset
 422 and the same applies to the failure region shape. Figure (9d) is therefore
 423 a direct consequence of the reduced number of time histories employed by
 424 the proposed method. The adopted sparse grid of seismic samples (Figure
 425 10a) leads to a net separation between safe and failure samples and this is
 426 reflected in the mapping process. The method, however, is able to moder-
 427 ate the effects of this approximation by computing the probabilistic scores
 428 w_i over the whole subset B_i (Equation 14), and so, by overestimating and
 429 underestimating at the same time the failure probability around the edges of
 430 the failure region.

Figure 10: 9^2 a) and 20^2 b) Full Factorial DoE for the viscous dampers model (Level 2).

Quantity	Surrogate PDF	L	U
M	Uniform	5.5	8
R	Uniform	0 km	75 km

Table 4: Surrogate uniform distributions of M and r

431 5.3.3. Fragility curves

432 The seismic fragility curves obtained by means of parametric approaches
433 are compared with the results provided by the proposed procedure (Figures
434 12 and 13). Moreover, a target solution is computed with 20000 structural
435 dynamic analyses based on as many synthetic ground motion records gener-
436 ated by sampling M and R through a *LHS*. In particular, a discrete number
437 of finite S_a intervals (delimited by the vertical lines in Figure 11) have been
438 identified and for each of them the conditional failure probability is computed
439 as the ratio between the number of samples above the structural threshold of
440 interest (red lines in Figure 11) and the total number of sample in the same
441 S_a interval. The width of the S_a intervals is assumed constant and it has
442 been chosen by increasing as much as possible the final number of considered
443 intervals, ensuring at the same time a sufficient number of samples falling in
444 each of them.

Figure 11: LHS samples for the target fragility curve estimation with the three performance levels of interest and the finite intervals of S_a (viscous dampers).

The target conditional failure probability is estimated for a finite number of intervals of the seismic intensity measure (S_a). For each considered interval j , the estimated target value is reported together with the standard deviation of the seismic samples in j and the associated confidence bounds computed as the standard deviation of a Bernoulli distribution (failure/safe). In addition to Figures 12 and 13, a comparison in terms of Mean Squared Error (MSE) computed with respect to the target curve is provided in Figure 14.

Analyzing the results, the following are the most relevant aspects to highlight:

1. The proposed methodology returns seismic fragility curves with the best overall behavior if compared with the *LHS* target solution and the parametric approaches.
2. For the viscous dampers case study, the parametric methodologies tend to perform poorly in case of high structural thresholds, that are exceeded for seismic intensity measures characterized by a low probability of occurrence (Figure 12c). This trend is less clear for the impact model, since the performance levels considered are less demanding, generating larger failure regions (Figure 9b).
3. Thus, for failure regions reached only by rare seismic intensity levels, the Cloud Analysis needs to increase the employed dataset for the demand model definition while the Incremental Dynamic Analysis tends

to lose accuracy due to the scaling process.

Figure 12: Comparison between the parametric approaches, the proposed method and the target solution for the steel moment resisting frame. The result are reported for a) Level 1, b) Level 2 and c) Level 3 of the final structural threshold.

Figure 13: Comparison between the parametric approaches, the proposed method and the target solution for the SDOF impact model. The result are reported for a) Level 1, b) Level 2 and c) Level 3 of the final structural threshold.

(d)

(e)

Figure 14: Mean Squared Error between the target fragility curve and the different analyzed approaches for a) the viscous dampers case study and b) the impact model.

466 The Multiple Stripe Analysis can exploit an initial dataset 10 times
 467 larger than the other parametric approaches, obtaining more stable
 468 performances in almost every analyzed case.
 469 The proposed approach appears to be more reliable and efficient since
 470 all the considered parametric methodologies share an increase in terms

of samples required in case of rare failure events (e.g. Figure 9d).

4. Among the analyzed parametric procedures, the Cloud Analysis shows a higher accuracy for the impact model case study and it results slightly more accurate than the proposed method in 2 cases out of the 6 analyzed. The Incremental Dynamic Analysis shows problems for high-performance levels and this may be caused by the initial seismic dataset that needs to be heavily scaled in order to reach the failure region [17]. Finally, the Multiple Stripe Analysis thanks to the extended initial seismic dataset returns stable performances, providing the best fragility curve in only 1 case.

It worths noting that the analyzed parametric approaches have shown reliable performances in many other studies (e.g. [21, 50, 51, 52]). The structural models analyzed in the present work result more demanding and tricky due to the high nonlinearity of the response domain, but this is expected due the difficulty to efficiently populate the region of the sample space corresponding to high S_a values and multiple system failures [16]. In general, the accuracy of parametric fragility curves can therefore be more sensitive to the structural typology, in addition to the adopted *EDP*, *IM* and initial seismic dataset [11].

6. Conclusions

An efficient non-parametric methodology to compute seismic fragility curves has been proposed. A stochastic earthquake model is employed to generate the ground motion dataset. The failure region mapping process based on the use of surrogate *PDFs* and a Multinomial Logistic Regression allows decreasing the number of samples required. The method is able to manage noise along the failure region boundaries thanks to the adoption of probabilistic scores w_i as indicator function, while more complex failure regions can still be mapped by the adopted kernel. The non-parametric approach does not assume any predefined shape for the final fragility curve, preventing a possible lack of fitting and additional drawbacks characterizing the analyzed parametric methodologies.

The proposed approach returns the most accurate results for almost every performance level and case study considered. This appears even more evident in case of failure regions characterized by a low probability of occurrence where parametric approaches require an increase of computational cost

507 and/or initial seismic dataset dimensions.

508

509

510 **Acknowledgements**

511 The authors would like to acknowledge the gracious support of this work
512 through the EPSRC and ESRC Centre for Doctoral Training on Quantifi-
513 cation and Management of Risk and Uncertainty in Complex Systems and
514 Environments Grant number (EP/L015927/1).

515

Appendix A: Stochastic seismic model

The model is completely defined by the ground motion's radiation spectrum $A(f; M, R)$ and its time envelope function $e(t; M, R)$, both depending on the magnitude M and the epicentral distance R and defined as:

$$A(f; M, R) = A_0(f) \cdot V(f) \cdot 1/r \cdot \exp[-\gamma(f) \cdot R] \cdot \exp(-\pi f k) \quad (18)$$

$$e(t; M, R) = c_3 \cdot t^{c_1} \cdot \exp(-c_2 \cdot t) \cdot U(t) \quad (19)$$

In the Equation (18), A_0 is the 'equivalent point-source spectrum', $V(f)$ describes the amplifications through the crustal velocity gradient, $1/r$ is the geometric spreading factor, $\gamma(f)$ represents the anelastic attenuation and finally the term $\exp(-\pi f k)$ simulates the near surface attenuation of high-frequency amplitudes. In the Equation (19), $U(t)$ is a unit-step function, while the expressions of the parameters c_1 , c_2 and c_3 along with all the details to completely derive $A(f; M, R)$ and consequently the final ground motion sample are described in [53].

The uncertainty associated to M is modeled by adopting the following probability density function, obtained from a truncated Gutenberg-Richter (G-R) relationship (e.g. [54]) in the interval $[M_{min}, M_{max}] = [5.5, 8]$:

$$p(M) = \frac{b \cdot \exp(-b \cdot M)}{\exp(-b \cdot M_{min}) - \exp(-b \cdot M_{max})} \quad (20)$$

Finally, the uncertainty in the earthquake location is taken into account by assuming $\log(R)$ normally distributed with mean $\log(20)$ and standard deviation 0.5 (e.g. [55]).

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