Endogenous timing in a mixed duopoly

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Abstract

This paper addresses the issue of endogenizing the equilibrium solution when a private – domestic or foreign – firm competes in the quantities with a public, welfare maximizing firm.

Theoretical literature on mixed oligopolies, in fact, provides results and policy implications that crucially rely on the notion of equilibrium assumed, either sequential or simultaneous.

In the framework of the endogenous timing model of Hamilton and Slutsky (1990), we show that simultaneous play never emerges as the equilibrium of mixed duopoly games. We provide sufficient conditions for the emergence of public and/or private leadership equilibria.

These results are in sharp contrast with those obtained in private duopoly games in which simultaneous play is the general result. We show that the key difference lies in the fact that the objective of a welfare maximizing firm is generally increasing in the rival's output, while the contrary holds for private firms. We develop a comprehensive analysis of a mixed duopoly considering both the cases of domestic and international competition, and the possible strategic complementarity and substitutability. From a methodological viewpoint we make large use of the basic results of the theory of supermodular games in order to avoid extraneous assumptions such as concavity, existence and uniqueness of the equilibria.

Keywords: Mixed markets, endogenous timing, Cournot, Stackelberg

JEL codes: C72, D43, H42, L13

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1 Introduction

There are several important markets in which public firms compete with a small number of private ones. Examples include the banking industry in several countries (postal banks in UK, Germany, Italy, New Zealand), as well as TV broadcasting in most of the developed countries. Moreover, health care services and insurance, transportation, energy, overnight delivery (also in the US) are usually provided by state-owned enterprises in competition with private firms. In industrial organization the term *mixed oligopolies* has been used to describe such imperfectly competitive markets.

Theoretical literature began to devote closer attention to mixed oligopolies in the eighties of last century when a privatization wave started in several industrialized countries and soon extended to transition and developing economies. The aim of these works was to characterize the mixed oligopoly equilibria and to study the welfare effect of privatization by adapting the standard oligopoly models to the presence of welfare maximizing firms. Results and policy prescriptions turned out to crucially rely on the type of competition *assumed* (in quantity competition, for example, Cournot or Stackelberg games). In de Fraja and Delbono (1989) it is shown that, if a Stackelberg game with public leadership is played, privatization cannot improve welfare absent efficiency gains; on the contrary, under Cournot competition, this may occur.

In another paper Beato and Mas-Colell (1984) show that welfare may be higher when the public firm is the follower than when it is the leader in a Stackelberg game. In this way they provided an argument against the standard view of the so-called Second-Best literature (see for example Rees, 1984; Bös, 1986) that claimed the suboptimality of the marginal-cost pricing rule.[1]

During the last decade greater attention has been devoted to international mixed oligopolies, in which a domestic public firm competes with foreign private firms.[2] Again, results and policy prescriptions over privatization depend on the type of competition assumed, either simultaneous-move (Cournot) or sequential (Stackelberg) game.

More recently, a new trend has emerged, based on the idea that the order of play should result from the players' timing decisions. In many economic situations, in fact, it is often more reasonable to assume that firms choose not only what action to take, but also when to take it.[3]

The aim of the present work is to contribute to this literature identifying the endogenous timing equilibria of a mixed duopoly quantity game where a private – domestic or foreign – firm competes with a public, welfare maximizing firm. We use the model of endogenous timing with observable delay developed by Hamilton and Slutsky (1990). In their model, a preplay stage is added to the duopoly game, in which players simultaneously decide whether to move early or late in the basic duopoly game. At the end of this stage, timing decisions are revealed and then the basic game is played according to the announcements: if both choose the same timing, simultaneous game is then played; if timing decisions are different, sequential play under perfect information – with the order of moves as decided by the players – occurs. As a consequence, in the subgame perfect equilibrium (SPE) of the extended game, the relevant equilibrium solution for the basic game *endogenously* emerges.

Our main finding is that Cournot competition never arises as the equilibrium of the mixed duopoly game when both firms are active on the market.

[1] See de Fraja and Delbono (1990) for a survey of these models.

[2] See for example the works of Fjell and Pal (1996), Pal and White (1998), Fjell and Heywood (2002).

[3] See Pal (1998), Matsumura (2003), Cornes and Sepahvand (2003) and Sepahvand (2004) for endogenous timing games in mixed oligopolies.

We provide sufficient conditions for the emergence of public and/or private leadership Stackelberg equilibria in a comprehensive characterization of the mixed duopoly quantity game. We consider the cases in which the private competitor is either domestic or foreign, and the possible strategic complementarity and substitutability.

A domestic mixed duopoly differs from an international one since private firm's profits are not included in the public firm's objective function in the latter case.

We show that when quantities are strategic substitute for both firms (i.e. best-response correspondences are downward sloping) the Stackelberg equilibria with both private and public leadership arise as the SPE of the endogenous timing game.

A sufficient condition to have downward sloping best-response correspondence for the private firm is the log-concavity of the (inverse) demand function. For the public firm, the best-response correspondence is always downward sloping in a domestic duopoly, while a necessary and sufficient condition for this to occur in an international duopoly is the convexity of the demand function. Note that these conditions hold true regardless of the cost functions of the two firms.

Our results have to be contrasted with those obtained in the private duopoly framework. Amir and Grilo (1999) show that Cournot solution always emerge as the SPE when quantities are strategic substitute for both firms, giving theoretical support to the general preference for Cournot over Stackelberg games in the private oligopoly setting.

The main difference lies in the fact that the public firm's objective is generally increasing in private firm's output, while the contrary is true for private firms.[4]

As a consequence, under strategic substitutability for both firms, the public firm's output when Stackelberg leader is smaller than the one in a simultaneous-move game. In fact, in order to induce an increase in the rival's output, the public leader produces less that in any Cournot equilibrium. So, the private firm prefers to be follower than Cournot player and public leadership Pareto dominates any Cournot equilibrium. The same is true for private leadership: in fact, the private firm produces more when Stackelberg leader than under Cournot competition and the public firm prefers to be follower than simultaneous player.

When the reaction correspondence of the private firm is increasing while strategic substitutability holds for the public firm, private leadership is the unique SPE of the endogenous timing game in both domestic and international duopoly.[5] In this case Pareto dominance of private leadership over Cournot competition is again verified, while public leadership dominance is no longer true. In fact, in order to have a larger private production with respect to simultaneous-move game, the public leader produces more than under Cournot competition and the private firm's profits are reduced. Then, the private firm always prefers to move early and the best timing response of the public firm is to move late.

In an international mixed duopoly a third case may occur. If the demand function is concave, the best-response correspondence of the public firm is increasing while the private firm's one is decreasing. As a result, the Pareto dominance of the public leadership over Cournot competition holds, while the private leadership solution does not Pareto dominate the simultaneous equilibria. The latter is due to the fact that the private firm reduces its production when it is Stackelberg leader with respect to the Cournot solutions. Then, the public firm always prefers to move early in order to avoid private leadership, and public leadership emerges as the unique SPE of the endogenous timing game.

[4] as we will see, the public firm's objective is always increasing in the rival's output in an international mixed duopoly, while in a domestic duopoly this is true when the price is larger than the rival's marginal cost.

[5] A sufficient condition is log-convexity of demand function and zero production costs for the private firm.

From a methodological point of view, we make large use of the basic results of the theory of supermodular games.[6] These are also called games with strategic complementarities and are characterized by the monotonicity of the best responses to rival's action. The advantages of using this approach are that no concavity assumptions are needed and nevertheless existence of pure-strategy Cournot equilibria is guaranteed. Moreover we can deal with multiplicity of equilibria since they can be preference-ranked for both private and public firms.

The present paper is organized as follows. Section 2 describes the model, the solution concept for the endogenous timing, and the basic concepts of the supermodularity approach. Section 3 provides the results for a domestic mixed duopoly, while Section 4 contains the analysis of an international mixed duopoly. Section 5 briefly summarizes the conclusions. In Appendix A some intermediate results are provided, while all the proofs of Lemmas and Theorems are collected in Appendix B.

2 The model

In this paper we consider a market in which one private, profit maximizing firm competes with a public firm that maximizes domestic welfare. These firms, labeled respectively 1 and 0, compete in quantities and their products are perfect substitutes. The market inverse demand function is $P(\cdot)$, and the two cost functions are $C_1(q_1)$ and $C_0(q_0)$. The two firms' profit functions are then

$$\begin{aligned} \Pi_1(q_0, q_1) &= q_1 P(q_0 + q_1) - C_1(q_1) \\ \Pi_0(q_0, q_1) &= q_0 P(q_0 + q_1) - C_0(q_0) \end{aligned}$$

and consumer surplus is

$$CS(q_0, q_1) = \int_0^{q_0+q_1} P(z)\, dz - (q_0 + q_1) P(q_0 + q_1).$$

Since the public firm's objective function is social welfare, it depends on whether the private firm is domestic or foreign. We shall analyze the two cases separately. If the private firm is domestic, then the public firm maximizes

$$W^d(q_0, q_1) \triangleq CS(q_0, q_1) + \Pi_0(q_0, q_1) + \Pi_1(q_0, q_1) = \int_0^{q_0+q_1} P(z)\, dz - C_0(q_0) - C_1(q_1). \quad (1)$$

If the private firm is foreign, the public firm's objective is

$$W^f(q_0, q_1) \triangleq CS(q_0, q_1) + \Pi_0(q_0, q_1) = \int_0^{q_0+q_1} P(z)\, dz - q_1 P(q_0 + q_1) - C_0(q_0). \quad (2)$$

We define a mixed-duopoly Cournot equilibrium as a pair of (nonnegative) quantities $\left(q_0^C, q_1^C\right)$ such that

$$W^i\left(q_0^C, q_1^C\right) \geq W^i\left(q_0, q_1^C\right) \qquad \text{and} \qquad \Pi_1\left(q_0^C, q_1^C\right) \geq \Pi_1\left(q_0^C, q_1\right) \qquad \forall\ q_0, q_1 \geq 0; i = d, f.$$

Thus, in any mixed-duopoly Cournot equilibrium, each firm optimally replies to the action of the rival and so $\left(q_0^C, q_1^C\right)$ must lie on the best-response correspondences of both firms. The latter are defined in the standard way:

$$r_0(q_1) \triangleq \arg\max_{q_0 \geq 0} W^i(q_0, q_1) \qquad i = d, f \quad (3)$$

$$r_1(q_0) \triangleq \arg\max_{q_1 \geq 0} \Pi_1(q_0, q_1). \quad (4)$$

[6] This theory was initiated by Topkis (1978, 1979) and further developed by Vives (1990), Milgrom and Roberts (1990), and Milgrom and Shannon (1994). We base our analysis on the main results of Novshek (1985) and Amir (1996a) on the private Cournot duopoly.

We denote the set of mixed-duopoly Cournot equilibria by C.

A Stackelberg equilibrium of this game corresponds to the subgame perfect equilibrium (SPE) of a two stage game of perfect information in which the second mover (follower) chooses an action after having observed the action of the first mover (leader). A strategy for the leader is to pick a quantity $q_i \geq 0$ and a strategy for the follower is $\rho_j(q_i)$, where $\rho_j(\cdot)$ is a mapping from the domain of q_i to the domain of q_j. Assuming that firm 0 is the leader, a Stackelberg equilibrium of this mixed duopoly game with public leadership is then a pair $\left(q_0^l, \rho_1^*(\cdot)\right)$ such that

$$W^i\left(q_0^l, \rho_1^*\left(q_0^l\right)\right) \geq W^i\left(q_0, \rho_1^*(q_0)\right) \qquad \text{and} \qquad \Pi_1\left(q_0^l, \rho_1^*\left(q_0^l\right)\right) \geq \Pi_1\left(q_0^l, q_1\right)$$
$$\forall\ q_0, q_1 \geq 0; i = d, f$$

while a Stackelberg equilibrium with private leadership is a pair $\left(\rho_0^*(\cdot), q_1^l\right)$ such that

$$\Pi_1\left(\rho_0^*\left(q_1^l\right), q_1^l\right) \geq \Pi_1\left(\rho_0^*(q_1), q_1\right) \qquad \text{and} \qquad W^i\left(\rho_0^*\left(q_1^l\right), q_1^l\right) \geq W^i\left(q_0, q_1^l\right)$$
$$\forall\ q_0, q_1 \geq 0; i = d, f.$$

In other words, a Stackelberg equilibrium imposes that: (i) the strategy of the second mover is a single-valued selection from its best-response correspondence; and (ii) the first mover chooses an action that maximizes its objective function given the anticipation of the rival's reaction.

Then, a Stackelberg equilibrium with public leadership is a pair $\left(q_0^l, q_1^f\right)$ with $q_1^f \in r_1\left(q_0^l\right)$ and $q_0^l \in \arg\max_{q_0 \geq 0} W^i\left(q_0, r_1(q_0)\right)$, while a Stackelberg equilibrium with private leadership is a pair $\left(q_0^f, q_1^l\right)$ with $q_0^f \in r_0\left(q_1^l\right)$ and $q_1^l \in \arg\max_{q_1 \geq 0} \Pi_1\left(r_0(q_1), q_1\right)$. We denote the set of Stackelberg equilibria with public and private leadership by S_0 and S_1 respectively. Moreover, all the points in S_i must yield the same payoff to the leader (firm i).

The aim of this paper is to identify the appropriate sequencing of moves in a mixed duopoly game. We investigate how the choice between simultaneous (Cournot) and sequential (Stackelberg) games and the assignment of leader and follower roles in the latter case endogenously arise. To this end, we adopt the simple model of extended game with observable delay due to Hamilton and Slutsky (1990).

In this model, a preplay stage is added to the duopoly game. In this stage players simultaneously decide whether to move early or late in the basic duopoly game. At the end of this stage, timing decisions are revealed and the basic game is then played according to these announcements: if both players choose the same timing, simultaneous (Cournot) game is played; while, if timing decisions are different, sequential play under perfect information – with the order of moves as decided by the players – occurs. As a consequence, the subgame perfect equilibrium of the extended game *endogenously* determines the relevant equilibrium concept for the basic game. In figure 1 the game tree of the extended game with observable delay of Hamilton and Slutsky (1990) is depicted.

The following Proposition summarizes the results of the Hamilton and Slutsky (1990) model adapting it to the mixed duopoly setting. It characterizes the SPE of the extended game, that is a pair of timing announcements and the equilibrium quantities in the basic game. For the time being, we have to assume that leadership payoffs are strictly larger than any mixed Nash payoff and that all equilibria exist. In next Sections we define conditions for existence of equilibria in the two cases of domestic and foreign private firm, and show that in fact leader's payoffs are always larger than Nash payoff when both firms produces positive quantities.[7]

[7]The latter is always true when the reaction correspondence are continuous functions, but since in this model we allow for discontinuities, we have to prove it.

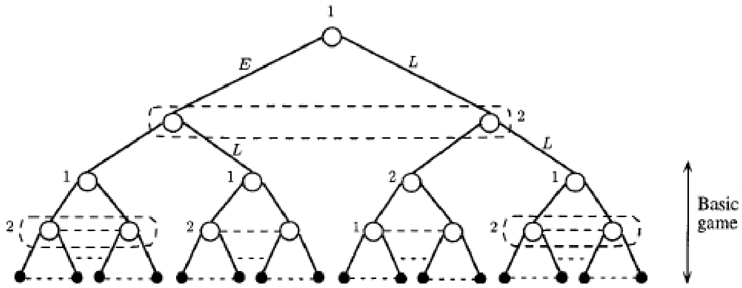

Figure 1: The extensive form of the Hamilton and Slutsky (1990)'s model extended game with observable delay.

Proposition 1 *Consider a mixed-duopoly quantity game in which Cournot and Stackelberg equilibria with both order of moves exist. Assume that both firms strictly prefers the leader payoff to any simultaneous play playoff. Then:*

(i) *if each firm's Stackelberg follower payoff is smaller than its least preferred simultaneous play payoff, then both firms decide to move early and Cournot equilibrium arises in any SPE of the extended game;*

(ii) *if each firm's Stackelberg follower payoff is strictly larger than its most preferred simultaneous play payoff, then Cournot equilibrium never arises in any SPE of the extended game and sequential play with either orders of move is played;*

(iii) *if firm i's Stackelberg follower payoff is strictly larger than its most preferred simultaneous play payoff and if firm j's Stackelberg follower payoff is strictly smaller than its least preferred Cournot payoff, then firm j moves early, firm i moves late and j-leadership arises in any SPE of the extended game.*

Proof. The proof of this Proposition follows easily from Theorems II, III and IV in Hamilton and Slutsky (1990). ■

Throughout the paper we assume the following.

Assumption 1 *The inverse demand function $P(\cdot)$ is twice continuously differentiable and positive (in the relevant range) with $P'(\cdot) < 0$ and $lim_{x\to\infty} P(x) = 0$.*

Assumption 2 *The cost functions $C_0(\cdot)$ and $C_1(\cdot)$ are strictly increasing and twice continuously differentiable with $C_i(0) = 0 \quad \forall i = 0, 1$.*

Under these assumptions both firms' action sets are compact since the firms never produce quantities larger than some upper-bound. This is due to the fact that price is strictly decreasing to 0 and marginal costs are strictly positive. As a consequence, there exists a k_i such that the outputs (k_i, ∞) are strictly dominated strategies for firm $i = 0, 1$.

In this work we invoke the results of the theory of supermodular games and so we need conditions on the primitives such that the players' best responses are monotonic in rivals' actions.

Topkis (1978) shows that $F(x, y) : \mathbb{R}_+^2 \to \mathbb{R}$ has the *Increasing Difference Property* or *IDP* (*Decreasing Difference Property* or *DDP*) if $\forall x' \geq x, y' \geq y$

$$F(x', y') - F(x, y') \geq (\leq) F(x', y) - F(x, y)$$

Furthermore, $F(x,y)$ has the *Single-Crossing Property* or *SCP* (*dual SCP*) if $\forall x' \geq x, y' \geq y$

$$F(x',y') - F(x,y') \geq (\leq)\, 0 \Longleftarrow F(x',y) - F(x,y) \geq (\leq)\, 0$$

Both IDP and SCP imply that $x^*(y) \triangleq \arg\max_{x \geq 0} F(x,y)$ has a minimal selection $\underline{x}(.)$, and a maximal selection $\overline{x}(.)$ that are non decreasing in y.[8] On the other hand, DDP and dual SCP imply that $x^*(y)$ has $\underline{x}(.)$ and $\overline{x}(.)$ that are non increasing in y.

In the framework of the present paper, under Assumptions 1 and 2, we make use of stronger results. Amir (1996b, Theorem 3) and Topkis (1998, p. 79) show that if $\frac{\partial F(x,y)}{\partial x}$ is strictly increasing (strictly decreasing) in y, then every selection of x^* is strictly increasing (strictly decreasing) in y.

3 Endogenous timing in mixed duopoly with a domestic private firm

In this section we consider the case in which the private firm is domestic, so that a social welfare maximizing public firm includes private profits in its own objective, as defined in equation (1). We prove that Cournot equilibrium never arises in any SPE of the endogenous timing game as long as it does not coincide with both Stackelberg equilibria. The latter may occur only on the boundary, when a monopolist solution arises.

Our results crucially relies on the effect of rival's action on the objective of the firms. While the private firm's profits are strictly decreasing in the public firm's quantity, welfare is nonmonotonic in the output of the private firm. Nonetheless, as long as price is higher than the private firm's marginal cost, welfare is increasing in the private firm's output. Indeed:

$$\frac{\partial \Pi_1(q_0,q_1)}{\partial q_0} = P'(q_0+q_1)\, q_1 < 0 \qquad \forall q_0, q_1 \geq 0 \tag{5}$$

$$\frac{\partial W^d(q_0,q_1)}{\partial q_1} = P(q_0+q_1) - C_1'(q_1) \tag{6}$$

In the following Lemmas we state sufficient conditions for monotonicity of best responses when the private firm is domestic.

Lemma 2 *Under Assumptions 1 and 2, any selection of the public firm's best-response correspondence $r_0(q_1)$ is strictly decreasing for every $q_1 \geq 0$ such that $r_0(q_1) > 0$.*

Lemma 3 *In addition to Assumptions 1 and 2:*

(a) Assume that either

$$P'(x) + xP''(x) < 0 \qquad or \qquad P(x)\,P''(x) - P'^2(x) \leq 0 \qquad \forall x \geq 0$$

hold. Then every selection of the private firm's best-response correspondence $r_1(q_0)$ is strictly decreasing for every $q_0 \geq 0$ such that $r_1(q_0) > 0$;

(b) Assume that $C_1(q_1) = 0\ \forall q_1$ and $P(\cdot)$ is strongly log-convex everywhere, i.e.

$$P(x)\,P''(x) - P'^2(x) > 0.$$

Then every selection of the best-response correspondence $r_1(q_0)$ is strictly increasing.

[8] Note that IDP is a cardinal property of the function F, while SCP is an ordinal property.

In what follows we first characterize the case in which the best-response correspondences are decreasing for both firms.

Under the assumptions of point (a) in Lemma 3, the mixed-duopoly quantity game is a supermodular game and the set C of pure strategy mixed-duopoly Cournot equilibria is nonempty.[9] Under the same assumptions, Stackelberg equilibria with either public or private leadership exist and the set S_0 and S_1 are nonempty.[10]

The following Theorem characterizes the SPE of the extended game when public and private quantities are strategic substitutes for both firms.

Theorem 4 *In addition to the assumptions of point (a) in Lemma 3, assume that the public firm's cost function is convex in the relevant range. Then, Stackelberg equilibria with either order of moves emerge in any SPE of the endogenous timing game. Mixed-duopoly Cournot equilibrium never occurs in any SPE of the game except when it coincides with both Stackelberg equilibria. The latter may occur only on the boundary, when a monopolist solution arises.*

The intuition for this result is the following.

If the public firm is the leader, it produces a lower quantity than in any mixed-duopoly Cournot equilibrium (point (ii) in Lemma 12). The reason is that in any Cournot equilibrium the public firm does not take into account the strategic (negative) effects of its action on private firm's production. So, when this effect is taken into account, the public leader reduces its quantity, and the private firm produces more than in any Cournot equilibrium. As a result, private profits are larger under public leadership than under simultaneous competition.

When the private firm acts as leader, it increases its quantity with respect to any mixed-duopoly Cournot equilibrium (point (i) in Lemma 12). In fact, it internalizes the strategic (negative) effects of its action on the rival's production and can increase profits by increasing its own output. The public firm prefers to be the follower than to play simultaneously because the variation in welfare has the same sign and larger magnitude of the variation in private firm's profits, as far as we compare two points on the public firm's reaction correspondence.

The next Theorem analyzes the case in which the private firm's reaction correspondence is increasing in the rival's output. Here we cannot invoke the results of the theory of supermodular games, since the two reaction correspondences slope in opposite directions, implying that it is not possible to re-order action sets in such a way that both reaction correspondences slope upward. As a consequence, to guarantee existence of Cournot equilibria, we need to revert to the standard approach for establishing existence of pure-strategy Cournot equilibrium, which requires strict quasi-concavity of each firm's objective function in own output to ensure single-valuedness and continuity of the firms' reaction functions. This is done via the sufficient conditions stated in the following Lemma.

Lemma 5 *Under the assumptions of point (b) in Lemma 3, the following sufficient conditions hold:*

(a) The private firm's profit function is quasi-concave in own output if $1/P(\cdot)$ *is a convex function, i.e. if* $P(\cdot)P''(\cdot) - 2(P')^2(\cdot) < 0$

[9] Following Vives (1990), reversing the natural order of the public firm's action set, the game becomes supermodular with effective strategy set $[0, k_0] \times [0, k_1]$. By Tarski's fixed point theorem the set C is not empty.

[10] Note that the effective action spaces are compact and the payoff functions are jointly continuous, so that the follower's best response correspondence has a closed graph. Then, as proved by Hellwig and Leininger (1987), Stackelberg equilibria exist.

(b) The public firm's objective function is strictly concave in own output if $P'(q_0+q_1) - C_0''(q_0) < 0$.

Proof. The proof of part (a) is in Amir (2003, Appendix), while the proof of part (b) is straightforward. ■

Theorem 6 *Under the conditions of Lemma 5,* $r_0(.)$ *and* $r_1(.)$ *are single-valued continuous functions strictly decreasing and increasing, respectively. Under interiority of the (unique) Cournot equilibrium, private leadership always emerges in the SPE of the extended game.*

In this case, differently from the one analyzed in Theorem 4, the strategic effect of public firm's action on private firm's production is positive. So, taking into account this effect, the public leader produces more than in the Cournot equilibrium. As a consequence, the private firm's profits are smaller under public leadership than in the Cournot equilibrium, and playing as follower is not an equilibrium for the private firm. The private leadership equilibrium always arises in the SPE given that the public firm prefers to be the follower for the very same reason as in Theorem 4.

4 Endogenous timing in mixed duopoly with a foreign private firm

When the private firm is foreign, its profits are not included in the objective function of the public firm, as defined in equation (2). Domestic welfare is strictly increasing in the rival's output under Assumptions 1 and 2 since

$$\frac{\partial W^f(q_0,q_1)}{\partial q_1} = -q_1 P'(q_0+q_1) > 0 \qquad \forall q_0, q_1 \geq 0. \tag{7}$$

This result simplifies the analysis with respect to the case of a domestic private firm. Nonetheless, the central result of the previous Section is confirmed: simultaneous play never occurs in any SPE of the endogenous timing game.

The next Lemma states sufficient conditions for monotonicity of the public firm's best-response correspondence, while Lemma 3 still holds for the private firm.

Lemma 7 *Under the standard assumptions, any selection of the public firm's best-response correspondence* $r_0(q_1)$ *is strictly decreasing (strictly increasing) as long as* $P''(x) > (<) 0 \ \forall x \geq 0$.

Note that it never occurs that both firms have nondecreasing best-response correspondences. In fact, by point (b) of Lemma 3, $P''(\cdot) > 0$ is a necessary condition to have a strictly increasing private firm's best response. But $P''(\cdot) > 0$ implies that the public firm's best response is downward sloping.

In what follows, we first characterize the case in which both best-response correspondences are strictly decreasing in the interior. As in the previous Section, this is a sufficient condition for the game to be supermodular and to have the existence results for both mixed-duopoly Cournot and Stackelberg equilibria. The following Theorem characterizes the SPE of the extended game with strategic substitutability for both firms.

Theorem 8 *Under the assumptions of point (a) of Lemma 3 and if $P''(\cdot) > 0$ $\forall q_0, q_1 \geq 0$, then Stackelberg equilibria with either order of moves arise in any SPE of the endogenous timing game. Mixed-duopoly Cournot equilibrium never emerges in any SPE of the game except when it coincides with both Stackelberg equilibria. The latter may occur only on the boundary, when a monopolist solution arises.*

The next Theorem analyzes the case in which the private firm's best-response correspondence is increasing in the rival's output. As in the previous Section, the game is not supermodular. We need again to assume quasi-concavity of welfare in public firm's quantity.

Theorem 9 *Under the assumptions of point (b) of Lemma 3, the private firm's reaction correspondence slopes upward and the reaction correspondence of the public firm slopes downward. With the assumptions of Lemma 5 in addition, welfare is concave in public firm's output. Assume also an interior equilibrium. Then, private leadership always emerge in the SPE of the extended game.*

Proof. See the proof of Theorem 6 ∎

To see that welfare is concave in own output, note that

$$\frac{\partial^2 W}{\partial q_0^2} = (P' - C_0'') - q_1 P'' < 0$$

since it is the sum of two negative terms (here, $P'' > 0$ by log-convexity).

When the private firm is foreign, the public firm may have a reaction correspondence that slopes upward, as proved by point (b) of Lemma 7. As in the previous case, the game is not supermodular and then further assumptions are needed to ensure existence. In the following Theorem we show that public leadership is the unique equilibrium of the extended game.

Theorem 10 *Under the assumption of point (b) in Lemma 7, the public firm best-response correspondence is increasing. Assume that the objective functions of both firms are quasi-concave in their own action and that an interior Cournot equilibrium exists. Then public leadership always emerge in the SPE of the endogenous timing game.*

5 Conclusion

Our main result is that Cournot equilibria never arise in the endogenous timing equilibrium of the mixed duopoly game as far as both firms produce positive quantities.

When quantities are strategic substitute for both firms the two Stackelberg equilibria arise in the SPE of the endogenous timing game.

When this is not the case, and one firm has upward sloping best response, then there is a unique equilibrium with this firm being the leader in the Stackelberg game.

Using the natural and simple endogenous time scheme of Hamilton and Slutsky (1990), our results provide a justification to the use of the Stackelberg concept of equilibrium in mixed duopolies.

If we couple this claim with the results in the private duopoly framework provided by Amir and Grilo (1999), where the Cournot solution is the general outcome, we gain new insights on the effect of privatization. The change in the objective of the former public firm is not the sole effect of privatization. In fact, when it is more reasonable to assume that firms choose not only what action to take, but also when to take them, also the timing of the game changes. Since the Stackelberg equilibria dominates Cournot solutions in terms of welfare, any positive effect of privatization derived under the assumption of a given timing of the game is weakened.

The striking difference between the result obtained in the mixed duopoly and in the private duopoly is entirely explained by the fact that the objective function of a public firm is generally increasing in the rival's output, while the opposite is true for a private firm.

Appendix A

This Appendix contains some intermediate results that are necessary to the proof of the main theorems of the paper.

Lemma 11 *Under the assumptions of point (a) in Lemma 3, the set C includes a point $\left(\underline{q}_0, \overline{q}_1\right)$ where the private firm produces its largest output and the public firm produces its smallest output in the set C. A sufficient condition for $\left(\underline{q}_0, \overline{q}_1\right)$ to be the Pareto dominant mixed-duopoly Cournot equilibrium is that the public firm's cost function is convex in the relevant range.*

Proof. Reversing the natural order of the public firm action set, the set C has a largest Cournot equilibrium in the new order, which is clearly $\left(\underline{q}_0, \overline{q}_1\right)$.[11] We show that this is the most preferred Cournot equilibrium by both private and public firms. Consider any other $(\widehat{q}_0, \widehat{q}_1) \in C$ different from $\left(\underline{q}_0, \overline{q}_1\right)$. By the extremal nature of $\left(\underline{q}_0, \overline{q}_1\right)$ and by Lemma 2 and point (a) of Lemma 3 it follows that $\underline{q}_0 < \widehat{q}_0$ and $\overline{q}_1 > \widehat{q}_1$.

(i) For the private firm the following relation holds:

$$\Pi_1\left(\underline{q}_0, \overline{q}_1\right) \geq \Pi_1\left(\underline{q}_0, \widehat{q}_1\right) > \Pi_1\left(\widehat{q}_0, \widehat{q}_1\right) \tag{8}$$

where the first inequality derives from $\overline{q}_1$ being best response to $\underline{q}_0$, and the second inequality is due to private firm's objective being strictly decreasing in q_0.

(ii) For the public firm, given that both $(\widehat{q}_0, \widehat{q}_1)$ and $\left(\underline{q}_0, \overline{q}_1\right)$ lay on $GR\ r_0\left(q_1\right)$, it follows that[12]

$$W^d\left(\underline{q}_0, \overline{q}_1\right) - W^d\left(\widehat{q}_0, \widehat{q}_1\right) = \int_{\widehat{q}_1}^{\overline{q}_1} \frac{dW^d\left(r_0\left(q_1\right), q_1\right)}{dq_1} dq_1 = \int_{\widehat{q}_1}^{\overline{q}_1} \frac{\partial W^d\left(r_0\left(q_1\right), q_1\right)}{\partial q_1} dq_1 \tag{9}$$

where the latter equality derives from the application of the *envelope theorem.* Therefore, it is possible to show that the following relation holds:

$$\frac{\partial W^d\left(r_0\left(q_1\right), q_1\right)}{\partial q_1} = P\left(r_0\left(q_1\right) + q_1\right) - C_1'\left(q_1\right) >$$
$$P\left(r_0\left(q_1\right) + q_1\right) - C_1'\left(q_1\right) + \left(1 + r_0'\left(q_1\right)\right) P'\left(r_0\left(q_1\right) + q_1\right) q_1 = \frac{d\Pi_1\left(r_0\left(q_1\right), q_1\right)}{dq_1}$$

In fact, under the assumption of convex cost for the public firm, its reaction correspondence is a contraction; i.e. $-1 \leq r_0'\left(q_1\right) \leq 0, \forall q_1 \geq 0$. Then:

$$\int_{\widehat{q}_1}^{\overline{q}_1} \frac{\partial W^d\left(r_0\left(q_1\right), q_1\right)}{\partial q_1} dq_1 > \int_{\widehat{q}_1}^{\overline{q}_1} \frac{d\Pi_1\left(r_0\left(q_1\right), q_1\right)}{dq_1} dq_1 = \Pi_1\left(\underline{q}_0, \overline{q}_1\right) - \Pi_1\left(\widehat{q}_0, \widehat{q}_1\right) > 0$$

[11] See Milgrom and Roberts (1990, Theorem 5)).

[12] GR stays for the graph of the best-reply function $r_i\left(\cdot\right)$.

where the latter inequality comes from the result in (8).

This completes the proof.[13] ■

Lemma 12 *Under the assumptions of point (a) in Lemma 3, the output of the private firm (public firm) in any Stackelberg game with both orders of moves is never smaller (never larger) than in the Pareto-dominant mixed duopoly Cournot equilibrium.*

Proof. We have to distinguish the two cases of (i) private leadership, and (ii) public leadership.

(i) Suppose, by contradiction, that there exists a point $\left(q_0^f, q_1^l\right) \in S_1$ such that $q_1^l < \overline{q}_1$. Since both $\left(q_0^f, q_1^l\right)$ and $\left(\underline{q}_0, \overline{q}_1\right)$ lay on $GR\ r_0(.)$, it follows that $q_0^f > \underline{q}_0$. Then:

$$\Pi_1\left(\underline{q}_0, \overline{q}_1\right) \geq \Pi_1\left(\underline{q}_0, q_1^l\right) > \Pi_1\left(q_0^f, q_1^l\right) \tag{10}$$

where the first inequality is due to $\overline{q}_1$ being best response to $\underline{q}_0$ and the second by the fact that $\Pi_1(.)$ is strictly decreasing in q_0. Inequality (10) contradicts the nature of Stackelberg equilibria and so it is always the case that $q_1^l \geq \overline{q}_1$ and $q_0^f \leq \underline{q}_0$.

(ii) Suppose, by contradiction, that there exists a point $\left(q_0^l, q_1^f\right) \in S_0$ such that $q_1^f < \overline{q}_1$. The first intermediate result is that

$$W^d\left(q_0^l, q_1^f\right) \leq W^d\left(r_0\left(q_1^f\right), q_1^f\right) \tag{11}$$

since $r_0\left(q_1^f\right)$ is a best response. Moreover, given that both $\left(r_0\left(q_1^f\right), q_1^f\right)$ and $\left(\underline{q}_0, \overline{q}_1\right)$ lay on $GR\ r_0(.)$, we can apply the envelope theorem as in equation (9) and obtain the following relation between welfare and profits:

$$W^d\left(\underline{q}_0, \overline{q}_1\right) - W^d\left(r_0\left(q_1^f\right), q_1^f\right) > \Pi_1\left(\underline{q}_0, \overline{q}_1\right) - \Pi_1\left(r_0\left(q_1^f\right), q_1^f\right) > 0 \tag{12}$$

The latter inequality come from the fact that

$$\Pi_1\left(\underline{q}_0, \overline{q}_1\right) \geq \Pi_1\left(\underline{q}_0, q_1^f\right) > \Pi_1\left(r_0\left(q_1^f\right), q_1^f\right)$$

where the first inequality comes from $\overline{q}_1$ being best response to $\underline{q}_0$, and the second is due to the fact that $\Pi_1(.)$ is strictly decreasing in q_0 and $r_0\left(q_1^f\right) > \underline{q}_0$.[14] So, combining inequalities (11) and (12), whenever $q_1^f < \overline{q}_1$the following relation holds

$$W^d\left(q_0^l, q_1^f\right) \leq W^d\left(r_0\left(q_1^f\right), q_1^f\right) < W^d\left(\underline{q}_0, \overline{q}_1\right)$$

that contradicts the nature of Stackelberg equilibrium. Therefore, it must be that $q_1^f \geq \overline{q}_1$and $q_0^l \leq \underline{q}_0$.

[13] Note that this result has the obvious extension that all the mixed duopoly Cournot equilibria can be Pareto ranked from the smallest to the largest in the reversed order. A similar result is obtained by (Milgrom and Roberts, 1990, Theorem 7) for 2-player supermodular games when both objectives are monotonic in the rival's action. Since the public firm's objective has not this property, we have developed a novel methodology to make comparative statics in the framework of a (domestic) mixed duopoly.

[14] Since both $\left(r_0\left(q_1^f\right), q_1^f\right)$ and $\left(\underline{q}_0, \overline{q}_1\right)$ lay on $GR\ r_0(.)$ and given the contradiction hypothesis that $q_1^f < \overline{q}_1$, it follows that $r_0\left(q_1^f\right) > \underline{q}_0$ because any selection of $r_0(.)$ is strictly decreasing.

This completes the proof. ■

Lemma 13 *Under the assumptions of point (a) in Lemma 3, the payoff of the leader in any Stackelberg equilibrium is strictly larger than in any Cournot equilibrium as far as the Pareto dominant Cournot equilibrium is interior.*

Proof. If $\left(\underline{q}_0, \overline{q}_1\right)$ is interior, the following first-order conditions for public and private firms must hold:

$$\frac{\partial W^d\left(\underline{q}_0, \overline{q}_1\right)}{\partial q_0} = 0; \qquad \frac{\partial \Pi_1\left(\underline{q}_0, \overline{q}_1\right)}{\partial q_1} = 0$$

We now analyze the two Stackelberg games.

(i) If $\left(q_0^f, q_1^l\right) \in S_1$ is also interior, then the following first-order condition for the private leader must hold:

$$\frac{\partial \Pi_1\left(q_0^f, q_1^l\right)}{\partial q_1} + \frac{\partial \Pi_1\left(q_0^f, q_1^l\right)}{\partial q_0} r_0'\left(q_1^l\right) = 0$$

where $r_0'\left(q_1^l\right)$ is a Dini derivate.[15] Since $\frac{\partial \Pi_1\left(\underline{q}_0, \overline{q}_1\right)}{\partial q_0} < 0$ by equation (5), and $r_0'\left(q_1^l\right) < 0$ in the interior by point (a) of Lemma 3, it follows that $\frac{\partial \Pi_1\left(q_0^f, q_1^l\right)}{\partial q_1} < 0$ in any interior point, and the fact that $\left(\underline{q}_0, \overline{q}_1\right) \notin S_1$ directly follows. If $\left(q_0^f, q_1^l\right)$ is not interior, then the conclusion follows from the interiority of $\left(\underline{q}_0, \overline{q}_1\right)$.

(ii) Considering public leadership, if $\left(q_0^l, q_1^f\right) \in S_0$ is also interior, the following first-order condition for the public leader must hold:

$$\frac{\partial W^d\left(q_0^l, q_1^f\right)}{\partial q_0} + \frac{\partial W^d\left(q_0^l, q_1^f\right)}{\partial q_1} r_1'\left(q_0^l\right) = 0.$$

Since the point $\left(q_0^l, q_1^f\right)$ lies on $r_1\left(q_0^l\right)$, the following first-order condition of the private firm holds:

$$P\left(q_0 + r_1\left(q_0\right)\right) + r_1\left(q_0\right) P'\left(q_0 + r_1\left(q_0\right)\right) - C_1'\left(r_1\left(q_0\right)\right) = 0$$

Then, in $\left(q_0^l, q_1^f\right)$ the price is strictly above the private firm's marginal cost and it follows that $\frac{\partial W^d\left(q_0^l, q_1^f\right)}{\partial q_1} > 0$. Being $\left(q_0^l, q_1^f\right)$ interior, $r_1'\left(q_0^l\right) < 0$ and we have the result that $\frac{\partial W^d\left(q_0^l, q_1^f\right)}{\partial q_0} > 0$. This implies that $\left(\underline{q}_0, \overline{q}_1\right) \notin S_0$. If $\left(q_0^l, q_1^f\right)$ is not interior, then the conclusion follows from the interiority of $\left(\underline{q}_0, \overline{q}_1\right)$.

This completes the proof. ■

[15] The four Dini derivates are the lim sup and lim inf of the one-sided (left and right) directional slopes starting at any point and always exist in the extended reals.

Appendix B

This Appendix includes all the proofs of the paper.

Proof of Lemma 2. Since the public firm maximizes total welfare as defined in equation (1), its objective has DDP since

$$\frac{\partial^2 W^d(q_0, q_1)}{\partial q_0 \partial q_1} = P'(q_0 + q_1) < 0 \qquad \forall q_0, q_1 \geq 0.$$

So, under the standard assumptions, any selection of the best-response correspondence of the public firm is strictly decreasing. ■

Proof of Lemma 3.

a) Following Novshek (1985), the private firm's objective has DDP when

$$\frac{\partial^2 \Pi_1(q_0, q_1)}{\partial q_0 \partial q_1} = P'(q_0 + q_1) + q_1 P''(q_0 + q_1) < 0 \qquad \forall q_0, q_1 \geq 0$$

which is equivalent to

$$P'(x) + xP''(x) < 0 \qquad \forall x \geq 0$$

while a sufficient condition for the dual SSCP is

$$P(x) P''(x) - P'^2(x) \leq 0 \qquad \forall x \geq 0$$

as defined in Amir (1996a). If one of these conditions holds, then any selection of the private firm's best-response correspondence is strictly decreasing.

b) Since the production costs of the private firm are zero, its objective reduces to $q_1 P(q_0 + q_1)$. Given that $\log P(\cdot)$ is convex by assumption, it follow that the objective is logsupermodular, i.e.

$$\frac{\partial^2 \log q_1 P(q_0 + q_1)}{\partial q_0 \partial q_1} = \frac{P(q_0 + q_1) P''(q_0 + q_1) - P'^2(q_0 + q_1)}{P^2(q_0 + q_1)} > 0.$$

So, every selection of the private firm best-response correspondence is strictly increasing.[16] ■

Proof of Theorem 4. To prove this result we need to show that both firms prefer the follower outcome to any simultaneous play solution.

(i) By point (i) of Lemma 12 we know that any $\left(q_0^f, q_1^l\right) \in S_1$ is such that $q_0^f \leq \underline{q}_0$ and $q_1^l \geq \overline{q}_1$. Moreover, from the definitions of Cournot and Stackelberg equilibria, both $\left(q_0^f, q_1^l\right)$ and $\left(\underline{q}_0, \overline{q}_1\right)$ lay on $GR\, r_0(q_1)$. Then, we can apply the *envelope theorem* and obtain the following relation between welfare and profits:

$$W^d\left(q_0^f, q_1^l\right) - W^d\left(\underline{q}_0, \overline{q}_1\right) \geq \Pi_1\left(q_0^f, q_1^l\right) - \Pi_1\left(\underline{q}_0, \overline{q}_1\right) \geq 0 \tag{13}$$

where the latter inequality comes from the property of Stackelberg equilibria (Lemma 13), and it is strict whenever $\left(\underline{q}_0, \overline{q}_1\right)$ is interior.

[16] See Amir and Grilo (1999)

(ii) By point (i) of Lemma 12 we know that any $\left(q_0^l, q_1^f\right) \in S_0$ is such that $q_0^l \leq \underline{q}_0$ and $q_1^f \geq \overline{q}_1$. Then it follows that

$$\Pi_1\left(q_0^l, q_1^f\right) \geq \Pi_1\left(q_0^l, \overline{q}_1\right) \geq \Pi_1\left(\underline{q}_0, \overline{q}_1\right) \tag{14}$$

where the first inequality comes from the fact that q_1^f is best response to q_0^l and the second is due to the private firm's objective being decreasing in q_0. Note that the latter inequality is strict whenever $q_0^l < \underline{q}_0$ that is always true when $\left(\underline{q}_0, \overline{q}_1\right)$ is interior, by Lemma 13.

The conditions of point (ii) of Proposition 1 hold and both Stackelberg equilibria are SPE of the endogenous timing game.

The last remark is about the boundary solutions. By Lemma 12, If $\underline{q}_0 = 0$, then both private and public leadership equilibria always coincide with $\left(\underline{q}_0, \overline{q}_1\right)$. This is not necessary when $\overline{q}_1 = 0$; in fact, it may well be the case that (one or both) Stackelberg equilibria do not coincide with $\left(\underline{q}_0, \overline{q}_1\right)$. So, in the first case, Cournot and both Stackelberg equilibria are SPE of the endogenous timing game. It is straightforward to see that all the equilibria coincide with the private monopoly solution. In the latter case, Cournot is a SPE if and only if both public and private leadership equilibria coincide with $\left(\underline{q}_0, \overline{q}_1\right)$, that is the public monopoly solution. ■

Proof of Theorem 6. By quasi-concavity of both objectives, the reaction correspondences of both firms are single-valued and continuous. Moreover the reaction correspondence of the public firm is strictly decreasing when interior. Hence, the mixed duopoly Cournot equilibrium $\left(\underline{q}_0, \overline{q}_1\right)$ is unique. We now prove that: (i) the private firm's payoff is strictly larger in the Cournot than in the public leadership equilibrium; while (ii) the public firm is better off in the private leadership than in the mixed-duopoly Cournot equilibrium.

(i) By point (ii) in Lemma 12 $\left(q_0^l, q_1^f\right) \in S_0$ is such that $q_1^f > \overline{q}_1$. Since $\left(q_0^l, q_1^f\right)$ and $\left(\underline{q}_0, \overline{q}_1\right)$ lie on $\overline{r}_1$, that is strictly increasing in the interior, it follows that $q_0^l > \underline{q}_0$. As a consequence, we can rank private firm's payoffs in the following way:

$$\Pi_1\left(\underline{q}_0, \overline{q}_1\right) \geq \Pi_1\left(\underline{q}_0, q_1^f\right) > \Pi_1\left(q_0^l, q_1^f\right)$$

where the first inequality follows from the property of Cournot equilibria and the second from the fact that Π_1 is strictly decreasing in q_0. Then, the private firm strictly prefers Cournot to public leadership equilibria.

(ii) We now analyze the private firm's leadership equilibrium $\left(q_0^f, q_1^l\right) \in S_1$. From the interiority assumption, as show in point (i) of Lemma 13, the following relation holds:

$$\Pi_1\left(q_0^f, q_1^l\right) > \Pi_1\left(\underline{q}_0, \overline{q}_1\right) \geq \Pi_1\left(\underline{q}_0, q_1^l\right).$$

Hence, $q_0^f < \underline{q}_0$ and $q_1^l > \overline{q}_1$ because r_0 is strictly decreasing in the interior. Then, applying the same analysis as in point (i) of Theorem 4, it follows that

$$W\left(q_0^f, q_1^l\right) > W\left(\underline{q}_0, \overline{q}_1\right).$$

The conditions of point iii) of Proposition 1 hold and the private leadership equilibrium is the unique SPE of the endogenous timing game. ■

Proof of Lemma 7. A sufficient condition to have $W^f(q_0, q_1)$ strictly supermodular (strict submodular) is $\frac{\partial^2 W^f(q_0,q_1)}{\partial q_0 \partial q_1} > (<) 0$. Since

$$\frac{\partial^2 W(q_0, q_1)}{\partial q_0 \partial q_1} = -q_1 P''(q_0 + q_1)$$

a sufficient condition for every selection of $r_0(.)$ to be strictly increasing (strictly decreasing in the interior is $P''(q_0 + q_1) < (>) 0$. ■

Proof of Theorem 8. First of all we show that there exist a Pareto dominant mixed-duopoly Cournot equilibrium. As in Lemma 11, the set C includes a point $\left(\underline{q}_0, \overline{q}_1\right)$ where the private firm produces its largest output and the public firm produces its smallest output in the set C. This is the most preferred simultaneous equilibrium by both firms since, $\forall (\widehat{q}_0, \widehat{q}_1) \in C$ different from $\left(\underline{q}_0, \overline{q}_1\right)$, we have that $\underline{q}_0 < \widehat{q}_0$ and $\overline{q}_1 > \widehat{q}_1$. Then, it is possible to show that:

$$\Pi_1\left(\underline{q}_0, \overline{q}_1\right) \geq \Pi_1\left(\underline{q}_0, \widehat{q}_1\right) > \Pi_1(\widehat{q}_0, \widehat{q}_1) \quad (15)$$

$$W^f\left(\underline{q}_0, \overline{q}_1\right) \geq W^f(\widehat{q}_0, \overline{q}_1) > W^f(\widehat{q}_0, \widehat{q}_1) \quad (16)$$

Relation (15) is in fact the same as (8) in Lemma 11. In relation (16) the first inequality is due to the fact that $\underline{q}_0$ is a best response to $\overline{q}_1$ and the latter derives from $W^f(.)$ being strictly increasing in q_1.

Moreover, by point (i) in Lemma 12 any $\left(q_0^f, q_1^l\right) \in S_1$ is such that $q_1^l \geq \overline{q}_1$ and $q_0^f \leq \underline{q}_0$. It is easy to show that any $\left(q_0^l, q_1^f\right) \in S_0$ is such that $q_1^f \geq \overline{q}_1$and $q_0^l \leq \underline{q}_0$. Suppose not: if there exists a $\left(q_0^l, q_1^f\right) \in S_0$ such that $q_1^f < \overline{q}_1$ the following relation holds:

$$W^f\left(q_0^l, q_1^f\right) \leq W^f\left(r_0\left(q_1^f\right), q_1^f\right) < W^f\left(r_0\left(q_1^f\right), \overline{q}_1\right) \leq W^f\left(\underline{q}_0, \overline{q}_1\right)$$

where the first and the third inequality come from the fact that $r_0\left(q_1^f\right)$ and $\underline{q}_0$ are best response to q_1^f and $\overline{q}_1$, respectively; while the second inequality is due to W^f being strictly increasing in q_1.

Now we show that both firms prefer the follower outcome to the Pareto dominant Cournot equilibrium.

For the private firm the following relation holds:

$$\Pi_1\left(q_0^l, q_1^f\right) \geq \Pi_1\left(q_0^l, \overline{q}_1\right) \geq \Pi_1\left(\underline{q}_0, \overline{q}_1\right)$$

where the first inequality comes from the fact that q_1^f is best response to q_0^l and the second is due to the private firm's objective being decreasing in q_0. Note that the latter inequality is strict whenever $q_0^l < \underline{q}_0$, that is always true when $\left(\underline{q}_0, \overline{q}_1\right)$ is interior.

For the public firm the following relation holds:

$$W^f\left(q_0^f, q_1^l\right) \geq W^f\left(\underline{q}_0, q_1^l\right) \geq W^f\left(\underline{q}_0, \overline{q}_1\right)$$

where the first inequality is due to q_0^f being best response to q_1^l, and the second derives from the fact that W is increasing in q_1, and it is strict whenever $\left(\underline{q}_0, \overline{q}_1\right)$ is interior.

Then, the conditions of point (ii) of Proposition 1 hold and both Stackelberg equilibria are SPE of the endogenous timing game.

The same analysis as in Theorem 4 applies to corner solutions. ■

Proof of Theorem 10. By quasi-concavity of both $W^f(.)$ and $\Pi_1(.)$, the best-response correspondences are single-valued. Moreover, $r_0(q_1)$ is strictly increasing and $r_1(q_0)$ is strictly decreasing in the interior. Hence, the Cournot equilibrium $\left(\underline{q}_0, \overline{q}_1\right)$ is unique. Since $\left(\underline{q}_0, \overline{q}_1\right)$ is interior by assumption, the following inequalities hold:

$$W^f\left(q_0^l, q_1^f\right) > W^f\left(\underline{q}_0, \overline{q}_1\right) \geq W^f\left(q_0^l, \overline{q}_1\right).$$

Then, $q_1^f > \overline{q}_1$; and, since $\left(q_0^l, q_1^f\right)$ and $\left(\underline{q}_0, \overline{q}_1\right)$ lie on r_1, it follows that $q_0^l < \underline{q}_0$. As a consequence, we can rank private firm's payoffs in the following way:

$$\Pi_1\left(q_0^l, q_1^f\right) \geq \Pi_1\left(q_0^l, \overline{q}_1\right) > \Pi_1\left(\underline{q}_0, \overline{q}_1\right)$$

where the first inequality follows from the fact that q_1^f is a best response to q_0^l and the second from the fact that Π_1 is strictly decreasing in q_0. Then, the private firm strictly prefers the public leadership to the Cournot equilibrium.

From the interiority assumptions we can show that for $\left(q_0^f, q_1^l\right) \in S_1$

$$\Pi_1\left(q_0^f, q_1^l\right) > \Pi_1\left(\underline{q}_0, \overline{q}_1\right) \geq \Pi_1\left(\underline{q}_0, q_1^l\right)$$

Hence, $q_0^f < \underline{q}_0$; and, by the fact that both points $\left(q_0^f, q_1^l\right)$ and $\left(\underline{q}_0, \overline{q}_1\right)$ lie on r_0, it follows that $q_1^l < \overline{q}_1$. It is easy to show that:

$$W^f\left(\underline{q}_0, \overline{q}_1\right) \geq W^f\left(q_0^f, \overline{q}_1\right) > W^f\left(q_0^f, q_1^l\right).$$

In fact, the first inequality follows from the fact that $\underline{q}_0$ is best response to $\overline{q}_1$ and the second is consequence of $W^f\left(q_0^f, q_1\right)$ being strictly increasing in q_1.

The conditions of point iii) of Proposition 1 hold and the public leadership is the unique SPE of the endogenous timing game. ■

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