Reliability Analysis of Systems Based on Survival Signature

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ABSTRACT: It is essential to conduct a reliability analysis of systems with multiple types of components, and deal with the uncertainties within the components failure time distribution. Moreover, finding the most "critical" component in the system at a certain time facilitates a risk management. In this paper, we introduce the survival signature into a real world hydro power plant system and solve the epistemic uncertainties in the component's failure time distribution. By implementing a relative importance index for each component, we can derive the reliability trend of the system depending on a component failure. The results show that the survival signature used on reliability analysis can help us deal with real world systems with more than one component type as well as to get bounds on system responses concerning epistemic uncertainties.

1. INTRODUCTION

System signature has been recognized as an important tool to quantify reliability of systems and networks consisting of components with random failure times, see Eryilmaz (2010). The failure times of components in coherent systems are considered as independent and identically distributed (iid), which can be regarded as a single component type. The main advantage of system signature is that it separates the structure of the system from the failure time distribution of the components. However, when it comes to real world systems with more than one component type, a reliability analysis based on system signature is pretty complex and not feasible. Therefore, the components are generally regarded as of a single type for simplification.

In order to overcome the limitations of system signature, Coolen and Coolen-Maturi (2012) presented an improved method, which called survival signature, to conduct reliability analysis of systems with more than one component type. In the case of a single component type, the survival signature is closely related to system signature, whilst in case of more than one component type, the survival signature plays its advantages of not relying on an iid assumption between the different component types. Recent developments go in the direction of survival analysis on relative complex system. Coolen et al. (2014) have shown how the survival signature can be derived from the signatures of two subsystems in both series and parallel configuration, they developed a nonparametric-predictive inference for system reliability using the survival signature. Aslett et al. (2014) presented the use of the survival signature for systems and network reliability quantification with both a nonparametric and a parametric approach. Aslett (2014) also developed a tool for structural reliability analysis within R packages.

At a further advancement, we present, herein, an application of the survival signature to a real world system. Specifically, we solve the reliability analysis of a real world hydro power plant based on survival signature. There exist epistemic uncertainties in the component failure time distribution due to a lack of data and knowledge. However, we can deal with these uncertainties by using the survival signature and survival function. In order to find out the most "critical" component to the system at a certain time, we propose the relative importance index RII.

The next section provides a brief introduction to the survival signature and survival function. In Section 3 we discuss the epistemic uncertainties in the failure times of components. The relative importance index for the system component is presented in Section 4. An example is provided in Section 5, and Section 6 closes with a discussion and conclusions.

2. SURVIVAL SIGNATURE AND SURVIVAL FUNCTION

Consider a system with $K \geq 2$ types of m components, with m_k components of type $k \in \{1,2,\dots,K\}$ and $\sum_{k=1}^{K} m_k = m$, and assume that the random failure times of the same component type are iid or exchangeable. We define the state vector of components $\underline{x} = (x_1, x_2, \cdots, x_m) \in \{0,1\}^m$ with x_i =1 if the i th component is in working sate and x_i = 0 if not. We assume that $\emptyset = \emptyset(\underline{x}) : \{0,1\}^m \to \{0,1\}$; this defines the system status based on the state vectors $\underline{x}$. $\emptyset$ is 1 if the system provides the expected function and 0 if it does not. We refer to the function $\emptyset(\underline{x})$ as the structure function of the system.

2.1. System with single type components

For single type of components, Coolen et al. (2014) defined the survival signature $\emptyset(l)$, for $l = 1,2,\dots,m$, as the probability that the system is in a working state given that l of its components are working. Since $\emptyset(l)$ is an increasing function of the number of working components, l, it is natural to assume that $\emptyset(0) = 0$ and $\emptyset(m) = 1$. Further, $\underline{x} = (x_1, x_2, \dots, x_m) \in \{0,1\}^m$ with $x_i = 1$ if the i th component functions and $x_i = 0$ if not. There are $\binom{m}{l}$ state vectors $\underline{x}$ with precisely l components x_i = 1, with $\sum_{i=1}^{m} x_i = l$, and we express the set of these state vectors as S_l. Due to the iid assumption for the failure time of the m components, all these state vectors are equally likely to occur. The survival signature of the system can then be denoted by:

$$\emptyset(l) = \binom{m}{l}^{-1} \sum_{\underline{x} \in S_l} \emptyset(\underline{x}) \qquad (1)$$

Suppose it is possible to characterize the component's failure time by means of a distribution and let $F(t)$ be its CDF. Let $C_t \in \{0,1,\dots,m\}$ denote the number of components in the system that are in working status at time t. Then, the probability of $C_t = l$ at time t is:

$$P(C_t = l) = \binom{m}{l} [F(t)]^{m-l} [1 - F(t)]^l \qquad (2)$$

Hence, the survival function of the system is:

$$P(T_s > t) = \sum_{l}^{m} \emptyset(l) P(C_t = l) \qquad (3)$$

2.2. System with multiply types components

The survival signature can be generalized with $K \geq 2$ types of components by $\emptyset(l_1, l_2, \dots, l_k)$, with $l_k = 0,1,\dots,m_k$ for $k = 1,2,\dots,K$. Let $S_{l_1,l_2,\dots,l_K}$ denote the set of all state vectors for the whole system for which $\sum_{i=1}^{m_k} x_i^k = l_k$, $k = 1,2,\dots,K$. Assume that the random failure times of components of the different types are fully independent, while are exchangeable within the same component types, hence:

$$\emptyset(l_1, \dots, l_k) = \left[\prod_{k=1}^{K} \binom{m_k}{l_k}^{-1}\right] \times \sum_{\underline{x} \in S_{l_1,\dots,l_k}} \emptyset(\underline{x}) \qquad (4)$$

Let $C_k(t) \in \{0,1,\dots,m_k\}$ denote the number of components of type k in the system which function at time t, and assume that the components of the same type have the known CDF $F_k(t)$ for type k, then:

$$P(\cap_{k=1}^{K} \{C_k(t) = l_k\}) = \prod_{k=1}^{K} P(C_k(t) = l_k) = \prod_{k=1}^{K} \binom{m_k}{l_k} [F_k(t)]^{m_k - l_k} [1 - F_k(t)]^{l_k} \qquad (5)$$

Hence, the survival function of the system with K types of components is:

$$P(T_s > t) = \sum_{l_1=0}^{m_1} \cdots \sum_{l_k=0}^{m_k} \emptyset(l_1, \ldots, l_K) P(\cap_{k=1}^{K} \{C_k(t) = l_k\}) \quad (6)$$

From Eq. (3) and Eq. (6) for the survival function with single type and multiple types of components, it becomes obvious that the systems structure is separated from the component reliability information.

In this paper, we have a strong iid assumption of failure times within same components types, which allows state vectors are equally likely to occur and $P(C_k(t) = l_k)$ to be calculated by $F_k(t)$. Moreover, we assume that the failure times of different components types are independent, as shown at the left hand side of Eq. (5).

3. EPISTEMIC UNCERTAINTY IN THE FAILURE TIMES OF COMPONENTS

In order to describe the distribution of the failure time of the components in a system, we need a sufficient amount of data. However, in many cases, data are quite limited and we can only determine a value range (e.g. as confidence interval) for the parameters of the component failure time distribution. This problem can be readily addressed with imprecise probabilities, see Beer et al. (2013). This technology has been merged with advanced Monte Carlo simulation and further stochastic techniques and implemented into OpenCossan, see Patelli et al. (2012 and 2014).

Using the framework of imprecise probabilities in form of a p-box (see Ferson (2004)), we denote the lower and upper CDF for the failure times of components of type k are $\underline{F}_k(t)$ and $\overline{F}_k(t)$, respectively. Because of the iid assumption between components of the same type k, then for a monotonic system with $l_k \in \{0,1,\ldots,m_k\}$, the lower probability for $C_k(t) = l_k$ is $\underline{P}(\cap_{k=1}^{K}\{C_k(t) = l_k\})$, and the corresponding upper probability is $\overline{P}(\cap_{k=1}^{K}\{C_k(t) = l_k\})$.

Consequently, the lower survival function of the system at time t is

$$\underline{P}(T_s > t) = \sum_{l_1=0}^{m_1} \cdots \sum_{l_k=0}^{m_k} \emptyset(l_1, \ldots, l_K) \underline{P}(\cap_{k=1}^{K} \{C_k(t) = l_k\}) \quad (7)$$

and the corresponding upper survival function is

$$\overline{P}(T_s > t) = \sum_{l_1=0}^{m_1} \cdots \sum_{l_k=0}^{m_k} \emptyset(l_1, \ldots, l_K) \overline{P}(\cap_{k=1}^{K} \{C_k(t) = l_k\}) \quad (8)$$

That is, we obtain the bounds of the survival function of a system with epistemic uncertainty on the failure times of components by using Eq. (7) to Eq. (8).

4. RELATIVE IMPORTANCE INDEX

Sensitivity analysis aims at identifying how the output of a model changes due to variations of the input. Sensitivity analysis allows one to study the relationship between components and the system, and to identify the most signification components affecting the reliability of the whole system. For local sensitivity analysis (LSA), Tarantola et al. (2012) extended the concepts to a regional sensitivity analysis (RSA) with consideration of how the input variables influence the variance of the model output. To capture sensitivities that defy a differential approach, Saltelli et al. (2000) suggested a global sensitivity analysis (GSA), enlarge the scope for sensitivity analysis in computational modelling practice. Patelli et al. (2010) presented an efficient sampling-based algorithm for the estimation of the upper bounds of the total sensitivity indices. As for epistemic uncertainties, Li et al. (2014) developed a technique called “contribution to failure probability plot” to detect the important aleatory and epistemic uncertain variables, and also measured the contribution of specific regions of these important input variables to the failure probability.

An important objective of a reliability and risk analysis is to identify those components or events that are most important (critical) from a reliability/safety point of view and that should be given priority with respect to improvements. For this purpose, we propose a relative importance index RII. Survival signature is adopted to quantify the difference between the probability that the system functions if the ith component works and the probability that the system functions if the ith component fails. The value of the difference can be defined as relative importance index $RII_i(t)$ of the ith component over the time. The measure $RII_i(t)$ expresses the system reliability improvement potential of component i along with the time. In other words, it represents the reliability loss component i fails. This measure can be used for all types of reliability definitions, and it can be used for repairable and non-repairable systems.

The equation of RII can be expressed as follows:

$$RII_i(t) = P(T_s > t) - P_i(T_s > t) \qquad (9)$$

In the above equation, $P(T_s > t)$ represents the probability that the system functions if the ith component works; $P_i(T_s > t)$ represents the probability that the system functions if the ith component fails.

From the definition of the relative importance index RII, we can see that $RII_i(t)$ is a function of time, it reveals the trend of the survival functions $P(T_s > t)$ and $P_i(T_s > t)$ of the system over the time or the values of RII of each component at a certain time. At each point in time the largest RII over all components shows the most "critical" component in the system. This helps to allocate resources for inspection, maintenance and repair in an optimal manner and depending on time over the lifetime of a system.

As a further observation, components that are in parallel in a system with the same RII, irrespective of their failure time distribution.

5. EXAMPLE OF HYDRO POWER PLANT

We consider a real-world system, a hydro power plant, as an example. The system is schematically shown in Figure 1. It can be modelled as a complex system comprising the following main twelve components: (1) control gate (CG); (2) two butterfly valves ($BV1$,$BV2$); (3) two turbines ($T1$,$T2$); (4) two generators ($G1$, $G2$); (5) three circuit breakers ($CB1, CB2, CB3$); and (6) two transformers ($TX1$,$TX2$).

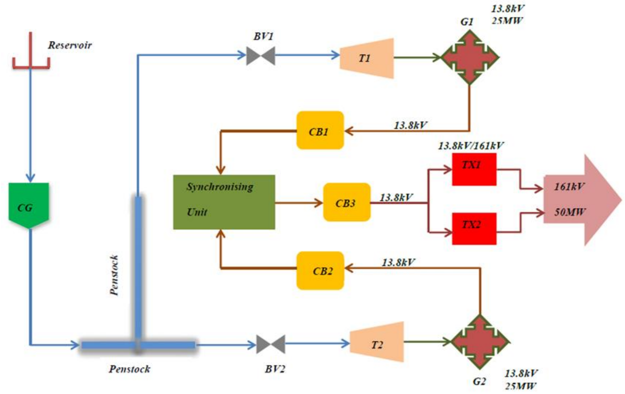

Figure 1: Schematic diagram of a hydro power plant

Suppose the same component type have the same failure time distribution. Failure type and distribution parameters for each component are listed in Table 1.

Table 1: Failure type and distribution parameters of components in a hydro power plant

Component Names	Distribution Types	Parameters (α, β) or λ	
CG	Beta	1.3	1.8
BV	Weibull	1.2	2.3
T	Exponential	0.8	
G	Beta	1.6	2.6
CB	Gamma	1.3	3.0
TX	Gamma	0.6	1.1

Let l_1, l_2, l_3, l_4, l_5 and l_6 denote CG, BV, T, G, CB and TX, respectively. Table 2 shows the survival signature of the hydro power plant.

Table 2: Survival function of the hydro power plant in Figure 1; $\emptyset(l_1, l_2, l_3, l_4, l_5, l_6) = 0$ is not shown

l_1	l_2	l_3	l_4	l_5	l_6	$\emptyset(l_1, l_2, l_3, l_4, l_5, l_6)$
1	1	1	1	2	1	1/12
1	1	1	1	2	2	1/12
1	1	1	1	3	1	1/4
1	1	1	1	3	2	1/4

1	1	1	2	2	1	1/6
1	1	1	2	2	2	1/6
1	1	1	2	3	1	1/2
1	1	1	2	3	2	1/2
1	1	2	1	2	1	1/6
1	1	2	1	2	2	1/6
1	1	2	1	3	1	1/2
1	1	2	1	3	2	1/2
1	1	2	2	2	1	1/3
1	1	2	2	2	2	1/3
1	1	2	2	3	1	1
1	1	2	2	3	2	1
1	2	1	1	2	1	1/6
1	2	1	1	2	2	1/6
1	2	1	1	3	1	1/2
1	2	1	1	3	2	1/2
1	2	1	2	2	1	1/3
1	2	1	2	2	2	1/3
1	2	1	2	3	1	1
1	2	1	2	3	2	1
1	2	2	1	2	1	1/3
1	2	2	1	2	2	1/3
1	2	2	1	3	1	1
1	2	2	1	3	2	1
1	2	2	2	2	1	2/3
1	2	2	2	2	2	2/3
1	2	2	2	3	1	1
1	2	2	2	3	2	1

The survival function of the hydro power plant with twelve components of six types is shown in Figure 2.

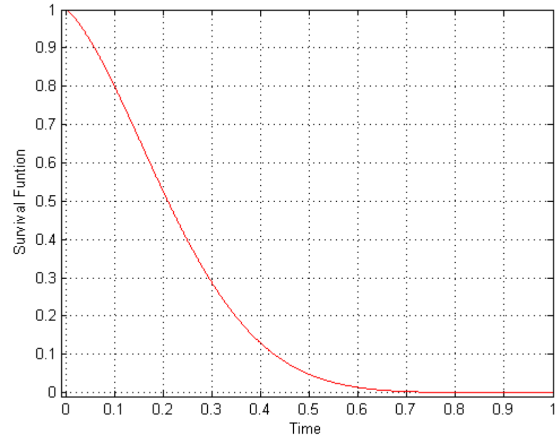

Figure 2: Survival function of the hydro power plant

Now suppose we only know intervals of the distribution parameters of the failure times according to Table 2.

Table 2: Components distribution parameters bounds of a hydro power plant

Component Names	Distribution Types	Parameters (α,β) or λ	
CG	Beta	[1.2,1.5]	[1.5,2.1]
BV	Weibull	[1.0,1.6]	[2.1,2.5]
T	Exponential	[0.4,1.2]	
G	Beta	[1.3,1.8]	[2.3,2.9]
CB	Gamma	[1.2,1.4]	[2.8,3.3]
TX	Gamma	[0.3,0.8]	[1.0,1.3]

This leads to upper and lower survival functions of the hydro power plant that reflect the epistemic uncertainties, see Figure 3.

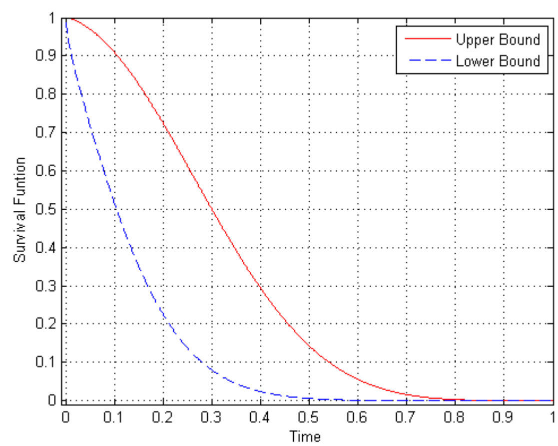

Figure 3: Upper and lower survival function of the hydro power plant

On the basis we can calculate the influence degree of each component in the hydro power plant at a certain time. We can calculate the survival function of the plant leaving out an individual component (e.g. CG, CB1, CB2, CB3, BV1, BV2, T1, T2, G1, G2, TX1 or TX2) at a time, one by one. This leads to the relative importance index $RII_i(t)$ of each component at a certain time t. The results of $P(T_s > t)$ and $P_i(T_s > t)$ are plotted in Figure 4.

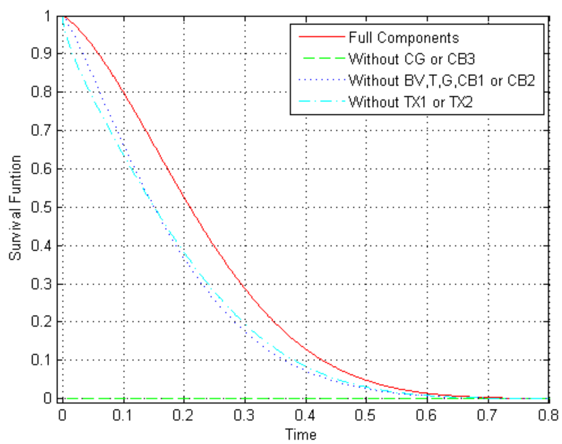

Figure 4: Survival function of the hydro power plant without component CG, BV1, BV2, T1, T2, G1, G2, CB1, CB2, CB3, TX1 or TX2

It is clearly that the survival functions of the system are the same if without component CG or CB3. If without BV1, BV2, T1, T2, G1, G2, CB1 or CB2, the system also has the same survival function. And the survival functions of the power plant are in the same situation without TX1 or TX2. In addition, the relative importance index is obtained as a dynamic value over time as illustrated in Figure 5. It reveals the trend of over the time for each component. The bigger the value of $RII_i(t)$ is, the more "critical" is the ith component to the whole system. Therefore, we can allocate human, material and financial resources in a targeted manner to the most "critical" component at a certain time.

Obviously, we can see that CG and CB3 has the same values of RII, also the relative importance indices of BV1, BV2, T1, T2, G1, G2, CB1 and CB2 have the same values, and the same applies to RII of TX1 and TX2. This reflects the location of components in a parallel configuration, they have the same importance to the whole system, no matter what the failure time types and parameters of these components are. What is more, the RII of CG and CB3 are in a decreasing trend over the time, while for BV1, BV2, T1, T2, G1, G2, CB1, CB2, TX1 and TX2, RII are increasing first, then declining along with the time.

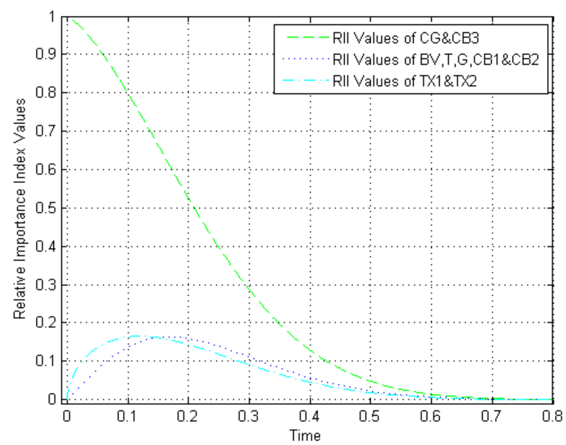

Figure 5: Relative importance index values of components CG, BV, T, G, CB1, CB2, CB3 and TX

6. CONCLUSIONS

In this paper, we have demonstrated a reliability analysis for systems with multiple components types based on survival signature.

Survival signature has been proven to be an efficient method to estimate the survival function of systems with multiple component types. This method can not only separate the system structure from the component failure time distribution, it does also not need the iid assumption between different component types. We have included the consideration of epistemic uncertainties in the component failure time distributions to reflect the lack of data to estimate the failure time distribution parameters. On the basis, we obtain lower and upper survival functions of the system. The relative importance index $RII_i(t)$ of the ith component is introduced to identify the most "critical" system component at a certain time t. This allows an optimal allocation of resources for repair, maintenance and inspection. The proposed approach has been demonstrated on a real world hydro power plant system with a reliability analysis.

Further research may focus on component failure and repair events, and investigate effects of corrective and preventive maintenance on systems.

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