Vortex morphology in Richtmyer–Meshkov-induced turbulent mixing

Ioannis W. Kokkinakis^{a,*}, Dimitris Drikakis^{b,2} and David L. Youngs^{a,3}

^aDepartment of Mechanical & Aerospace Engineering, University of Strathclyde, Glasgow, G1 1XJ, UK

^bUniversity of Nicosia, Nicosia, Cyprus

ARTICLE INFO

Keywords:

multi-component
Richtmyer–Meshkov
turbulent mixing
double-planar
inverse-chevron
vortex-structures

ABSTRACT

This study investigates the vortical structures that form and develop during the turbulent mixing induced by the Richtmyer–Meshkov instability (RMI). By using high-resolution implicit large-eddy simulations, we examine the turbulent structures that develop post-shock at a planar interface separating two distinct gases and track their evolution in time during subsequent reshocks and rarefactions. Although no coherent turbulent structures can be identified during the early phase of the mixing process following the initial shock at the interface, further deposition of baroclinic vorticity caused by subsequent reshocks and rarefactions leads to the formation of various coherent turbulent structures. At late time, when the two fluids become heavily mixed at the molecular level and the mixing width becomes sufficiently large, the coherent structures eventually break down to form very fine isotropic turbulent structures that exhibit no obvious structure or shape. The study also considers shock interactions with an inclined interface, such as an inverse chevron interface, where several additional coherent structures form independently of the RMI—these are due instead to the initial geometric inclination of the interface itself. Finally, we discuss the importance of the formation of such structures in the modelling of RMI-type mixing processes and draw several important conclusions.

1. Introduction

The Richtmyer–Meshkov (RM) instability was first identified and examined by the theoretical and experimental work carried out by Richtmyer [1] and Meshkov [2] respectively, on interfaces between two fluid layers of differing densities that are impulsively accelerated by an incident shock-wave. In the early stages of RM-induced (henceforth labelled as RMI) mixing, small surface instabilities develop rapidly and reach a nonlinear state where the mixing layer between heavy and light components develops characteristic mushroom-shaped protrusions, commonly referred to as bubbles and spikes. These bubble and spike features can later break down into fully turbulent mixing regions of flow and are thus associated with transition and turbulent mixing. Understanding how the mixing interface and associated flow structures form and develop is crucial to understanding the evolution of RMI mixing.

Obtaining a better understanding of the physical phenomena that govern RMI mixing is of particular importance to many practical processes, e.g., for the wakes of jet engines and supersonic combustion [3] as well as in astrophysics for the prediction of the evolution of supernovae [4, 5]. Another eminent example is inertial-confinement fusion (ICF), where RMI mixing plays a critical part in determining the performance of ICF capsules [6, 7, 8]. Thus, all physical processes

behind the development of RMI mixing must be established and well described for the process to be successfully modelled or to find an accurate analytical description.

Numerical simulations have pushed the boundaries of understanding of RMI turbulent mixing. One of the first works published was of Youngs [9] concerning 2D single- and multi-mode simulations for a shock-tube experiment. Over the last three decades, the increasing computational power has allowed researchers to use compressible computational fluid dynamics to simulate the 3D RMI. This has enabled researchers to investigate numerically the self-similarity properties, the influence of initial conditions on RMI and Rayleigh–Taylor induced (RTI) turbulent mixing, and late time mixing effects, e.g. Refs. [10, 11, 12, 13, 14, 15, 16, 17, 18] and references therein. Lombardini et al. [19, 20] have also analysed the Atwood and Mach number effects on shock-driven turbulent mixing flows. It should be noted that most of the existing experiments and simulations do not reach a fully turbulent regime and thus the self-similar growth of the instability remains, to some extent, a hypothesis [21]. For a thorough discussion and up-to-date literature regarding both the Richtmyer–Meshkov and Rayleigh–Taylor instabilities, the reader is referred to the work of Zhou [22, 23].

Despite the aforementioned efforts, studies that investigate the topology of turbulent structures associated with RMI mixing are limited. Only Grinstein et al. [24] are known to have made such an effort by using isosurfaces of the vorticity magnitude, which however is not generally regarded as a vortex identification scheme. In this study, we investigate the formation and evolution of various “organized” turbulent structures that develop during RMI mixing using isosurfaces of the Q -criterion [25], and highlight some important physical processes that should be considered while developing RM theories or models. One such crucial phenomenon that has been unravelled here is the formation of

*Corresponding author

✉ ioannis.kokkinakis@strath.ac.uk (I.W. Kokkinakis);
drikakis.d@unic.ac.cy (D. Drikakis)
ORCID(s): 0000-0001-9694-3764 (I.W. Kokkinakis);
0000-0002-3300-7669 (D. Drikakis)

¹Lecturer, Department of Mechanical & Aerospace Engineering, University of Strathclyde, Glasgow, UK.

²Professor and Vice President for Global Partnerships, University of Nicosia (UNIC), Cyprus; also at the Defense & Security Research Institute.

³Visiting Professor, Department of Mechanical & Aerospace Engineering, University of Strathclyde, Glasgow, UK.

vortex rings, whose subsequent interaction leads to the development of complex turbulent structures in the flow as will be revealed. It is theorized that the formation, evolution and collapse of such structures play a vital role in the self-similar growth rate of the mixing layer.

Vortex rings have been the subject of a plethora of investigations, whether incompressible or compressible, inviscid or viscous, immiscible or miscible, theoretical/analytical or numerical. Toroidal bubbles are typically encountered in multiphase flows as the result of the collapse of cavitation bubbles near a wall. One of the first attempts to understand this phenomena dates back to the efforts of Morton et al. [26], who investigated the properties of an isolated mass of buoyant fluid released from rest in stratified surroundings with the goal of applying the results to the atmosphere. The following year, Turner and Taylor [27] showed how the motion in such a “buoyant cloud” can be considered as a particular case of a buoyant vortex ring. The more general theory considers an arbitrary amount of vorticity injected into the ring as the ring forms, which greatly alters the behaviour of the ring. Walters and Davidson [28] made a detailed experimental study of the formation and evolution of toroidal buoyancy-driven bubbles. A year later, they analytically showed [29] that buoyancy forces cause an initially spherical bubble to evolve towards a toroidal shape. They theoretically analysed the first stages of the evolution of large, initially spherical bubbles by considering that the evolution of the bubble shape is driven by a competition between inertia and buoyancy. This simple inviscid and irrotational model, which neglects surface tension, describes how an initially spherical bubble becomes toroidal in shape.

Pedley [30] theoretically investigated the evolution of a toroidal bubble in both inviscid and slightly viscous fluid and showed that, although viscous effects do not alter the evolution of the ring radius that grows as the square root of time (t) to satisfy conservation of the vertical impulse, they slightly reduce the rise velocity, which then decays as $t^{1/2}$. Pedley [31] also pointed out that vortex-ring bubbles have a finite lifetime and are ultimately destroyed by surface-tension instabilities.

A numerical investigation by Bonometti and Magnaudet [32] demonstrated that the topological transition from simply to doubly connected vortex is a function of the Bond (Bo) and Archimedes (Ar) numbers, which compare the magnitude of inertial effects to those of surface-tension effects and viscous effects, respectively. Specifically, at large Archimedes numbers, viscous effects are negligible (inviscid limit), whereas at large Bond numbers, capillary (or surface tension) effects are negligible. Sreedhar and Ragab [33] warns that capturing the 3D wake dynamics is of crucial importance to understand the possible influence of wake turbulence on the bubble path and its stabilizing influence on the shape of the bubble rear.

Recently, experimental [34] and computational [35] studies have investigated whether immiscible-like flow phenomena can occur in miscible fluid processes. Both studies were conducted at relatively low speeds and in zero gravity to

isolate other flow phenomena from the dominant buoyancy-driven flow. These studies were done to investigate a century-old theory that is impossible to confirm experimentally on Earth. In the mid-1800s, the Dutch scientist Korteweg predicted that, until the molecules of miscible fluids, such as honey and water, diffuse together and become a uniform solution, these fluids would act like immiscible fluids.

Miscible fluids in microgravity was a zero-upmass experiment done in the International Space Station [34] and whose goal was to determine if interfacial phenomena observed in immiscible fluids also occur in miscible fluids. The study concluded that (i) a denser miscible fluid injected into a lighter miscible fluid can exhibit the Rayleigh–Tomotika instability and break up into small drops and (ii) an aspherical drop of the lighter miscible liquid in the heavier miscible fluid can cause the former to spontaneously assume a spherical shape. The study also demonstrated that the stream shape of the heavier miscible fluid remains constant when injected into the lighter miscible fluid. In conclusion, the study detected no other behaviour beyond simple diffusion.

Apart from the experimental work, numerical simulations based on the Navier–Stokes equations plus a Korteweg stress term [35] were performed to investigate if convection (analogous to Marangoni convection in immiscible fluids) could occur because of thermal and concentration gradients. Given realistic parameters, the numerical results indicate that measurable fluid flow can result in the absence of buoyancy-driven convection.

With regards to numerical modelling, Sreedhar and Ragab [33] used large-eddy simulation (LES) to investigate the response of longitudinal stationary vortices to random perturbations at a Reynolds number of $Re = 10^5$ based on the maximum initial tangential velocity and the core radius. Mansfield et al. [36] used a Lagrangian LES to simulate the collision of coaxial vortex rings and successfully captured several distinctive phenomena. Faddy and Pullin [37] used direct numerical simulation (DNS) to study the flow structures of a pair of counter-rotating vortices in 3D at a low Re of 10^3 and an LES at a high Re of 2×10^4 based on the circulation of the Lamb–Oseen vortex. Ren and Xu [38] utilized an LES technique to investigate a vortex ring impinging on a three-dimensional circular cylinder at a high Re of 4×10^4 based on the initial translational speed and diameter of the vortex ring. Previously, Ren and Lu [39] analysed the turbulence kinetic energy at the same Reynolds number using LES, revealing the transition from a laminar to a turbulent state. The types of vortices identified were found to play an important role in the flow-state evolution. Regarding miscible fluids, Shimokawa et al. [40] numerically solved the variable-density incompressible Navier–Stokes equations and found that circulation suppresses the instability of the vortex ring, which implies that the RT instability plays a dominant role in the instability of vortex rings and vice versa.

In shock-vortex interactions, compressible vorticity is also an important parameter from both a fundamental and practical standpoint. Detailed numerical simulations [41] and experimental work [42, 43] have examined shock structure

and vorticity amplification resulting from the interaction between shock-waves and vortex pairs. The mixing enhancement is most rapid for the case of a strong vortex ring interacting with a strong shockwave, is somewhat slower for a weak vortex ring interacting with the same shockwave and is significantly slower for the case of a strong vortex ring interacting with a weaker shockwave. Vortex rings have also been investigated in the context of acoustics [44, 45, 46], examining the effects of the vortex on the shock structure and the structure of the acoustic field generated by the shock-vortex interaction.

The success of the LES in the above studies has proven it to be a useful numerical tool for investigating the dynamics of vortex core formation, evolution and interaction with the surrounding flow-field. This study focuses on identifying the various vortex structures that develop during the mixing process proceeding an impulsive (burst-like) acceleration caused by a shockwave of a non-smooth material interface between two fluids. Inspiration is drawn here from the literature to investigate several of the distinct phenomena described above, but which have not been previously discussed with respect to RMI mixing. The scope herein is foremost to identify, at the $Re \rightarrow \infty$ limit, whether vortex rings, or any other well-defined vortex-core structures, form within the context of compressible RMI mixing. Their evolution as well as interaction with the shock-waves, mixing layer and between themselves are also examined.

The paper is organised as follows: Section 2 presents the governing equations used to describe compressible multi-component fluids. Section 3 briefly covers the iLES numerical solution used to obtain the results and §4 gives a full description of the setup of the two test cases considered herein. The results are then presented and discussed in §5 for the double-planar (§5.1) and inverse-chevron (§5.1.9) cases examined. The study finally concludes with §6, where a summary of all major observations and findings is made.

2. Governing Equations

Richtmyer–Meshkov interactions typically occur at very high Reynolds number. In this limit, we assume $Re \rightarrow \infty$ and so the viscous terms in the Navier–Stokes equations are neglected. Thus, the Euler equations used here in the context of multi-component flows involve solving the momentum and energy equations in conjunction with the five-equation quasi-conservative model of Allaire et al. [47], assuming ideal gases (i.e. $p_i = \rho RT_i$). For the two gases considered here, the system of partial differential equations consists of the following two equations for the species densities:

$$\frac{\partial z_1 \rho_1}{\partial t} + \nabla \cdot (z_1 \rho_1 \mathbf{u}) = 0 \quad (1)$$

$$\frac{\partial z_2 \rho_2}{\partial t} + \nabla \cdot (z_2 \rho_2 \mathbf{u}) = 0 \quad (2)$$

a single equation for the advection of the mixture momentum (convection),

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u} + p \mathbf{I}) = 0 \quad (3)$$

the advection of the total energy density (ρe) equation,

$$\frac{\partial \rho e}{\partial t} + \nabla \cdot [(\rho e + p) \mathbf{u}] = 0 \quad (4)$$

and finally, an additional equation for the (non-conservative) advection of the volume fraction of one of the components (i.e. component 1),

$$\frac{\partial z_1}{\partial t} + \mathbf{u} \cdot \nabla z_1 = 0 \quad (5)$$

where ρ is the mixture mass density, $\mathbf{u}$ is the mixture velocity vector, z_i is the volume fraction of species i , p is the mixture fluid pressure, e is the mixture total specific energy and $\mathbf{I}$ is the identity tensor. Note that for pure air $z_1 = 1$. Further details of the numerical method and calculation of the mixture thermodynamic parameters can be found in Ref. [48] and references therein.

3. Numerical methodology

In the present numerical study, we employ the implicit large eddy simulation (iLES) approach to solve the above set of governing equations. A thorough background literature of the iLES approach can be found in the books of Grinstein and Drikakis [49] and Grinstein et al. [50] with regards to turbulence and turbulent mixing, respectively. Similar to previous studies concerning RMI simulations, Refs. [10, 24, 48], amongst others, the limit $Re \rightarrow \infty$ is considered. Turbulent mixing is thus assumed to be solely driven by the resolved turbulent scales, at least during the initial development phase of the RMI instabilities, provided the initial interface perturbations are sufficiently resolved by the grid resolution. We also consider the case where the Schmidt number is of order unity, so the gases are considered miscible and it is assumed that homogeneous mixing occurs at the grid scale. A discussion on the effective Reynolds number and miscible fluids is provided in §5.

The numerical method solves the governing equations by using a finite-volume Godunov-type method [51]. The inter-cell numerical fluxes are calculated based on the (approximate) solution to the Riemann problem by using the reconstructed variables to the left and right of the cell interface. The iLES results presented herein use the same numerical methods as Ref. [48], wherein further details and references of the numerical details are given. In short, we use the Harten, Lax, van Leer and (the missing) contact approximate Riemann solver of Toro et al. [52]. Higher-order accuracy in smooth-flow regions is achieved by using monotonic upstream centred scheme for conservation laws (MUSCL) extrapolation (Van Leer, 1977) with the fifth-order limiter proposed by Kim and Kim [53]. The standard MUSCL extrapolation is additionally augmented by using the low-Mach limiting scheme of Thornber et al. [54], which extends the validity of Godunov-type methods for compressible solvers to at least $M \approx 10^{-4}$. Although the flow is largely compressible, low Mach mixing can occur at different stages of the flow evolution, thus requiring accurate modelling. Finally,

Figure 1: Schematic illustration of initial conditions used for (a) double-planar and (b) inverse-chevron test cases.

the discretization of the temporal derivatives is achieved by using a third-order, three-stage, strong-stability-preserving Runge–Kutta method [55].

For both the DP and IC cases, the equations are solved on a structured Cartesian mesh. The grid comprises of $1280 \times 320 \times 640$ cells in the streamwise (x -axis), spanwise (y -axis) and wall-normal (z -axis) directions, respectively, with a grid filter size of $\Delta = 1/32$ cm that has been previously shown to be sufficient to obtain grid-independent results [48]. In LES, the inertial length scales are modelled and therefore, convergence at all scales cannot be achieved. Indeed, only in the limit of DNS, does convergence of all scales and gradients exist. Thus, the investigation carried out herein is limited to assessing only the large scale behaviour and those quantities that only weakly depend on the fine scales for a sufficiently high Reynolds number flow ($Re \rightarrow \infty$).

4. Initial and boundary conditions

The experiment involves a block of dense sulphur-hexafluoride (SF_6) gas, seeded with olive oil droplets, encased in air within a tube. Figures 1(a) and 1(b) show the initial flow field for the double-planar (DP) and inverse-chevron (IC) test cases, respectively. The cases considered here are based on previous experimental work of Holder et al. [56] and Holder and Barton [57].

The domain size ($x - z$ plane) is shown in Fig. 1 and extends 0.1 meters in the spanwise direction (y -axis). In the experiment, the interfaces were created by using membranes supported on a very fine wire mesh with a grid spacing of 0.4 cm. Simulating the effects of the perturbations requires a re-

alistic method of representing the 2D interface between the two materials, as has been previously demonstrated by Hahn et al. [48]. Here, to recreate a realistic initial interfacial variation of the volume fraction (VF) of the species, we use the same numerical methods as previously [48]. The method relies on summing a number of individual wave modes, in accordance with an assumed surface-perturbation power spectrum, as originally proposed by Youngs [58]. In this investigation, the interface perturbations satisfy a power spectrum of the form $P(k) \propto k^{-2}$ for which majority of the perturbations introduced are concentrated at long wavelengths. This form of perturbation has practical importance as it is representative of the measured surface finish of an inertial confinement fusion capsule [59].

The effect of the boundary layer on the diagnostics is reduced by using the laser-sheet technique in the experiments of Holder and Barton [57]. Thus, by neglecting the viscous stresses, the thin boundary layer no longer needs to be resolved, thereby significantly reducing the computational requirements and allowing for high-resolution iLES simulations. Thus, the boundary conditions in the simulations are reflective at the upper, lower and end walls. In the streamwise direction, an extended 1D domain was used to minimize spurious reflections from the upstream outflow boundary. In the spanwise (or lateral) direction, periodic boundary conditions are applied.

All fluids are at rest, apart from the incoming shocked air (superscript *). The Mach number M_s of the shockwave can be calculated by using the Rankine–Hugoniot shock-tube relation

$$M_s = \sqrt{\frac{\gamma - 1}{2\gamma} + \frac{(\gamma + 1)}{2\gamma} \frac{p_{air}^*}{p_{air}}} \quad (6)$$

The speed u_s of the shockwave is then computed based on the speed of sound c_{air} in the pre-shocked region. Explicitly,

$$u_s = M_s c_{air} \quad (7)$$

where $c_{air} = \sqrt{\gamma p_{air} / \rho_{air}}$. The speed of the compressed air after the shockwave is

$$u_{air}^* = u_s \left(1 - \frac{\rho_{air}}{\rho_{air}^*} \right) \quad (8)$$

Finally, the density behind the shock is obtained by

$$\rho_{air}^* = \rho_{air} \frac{(\gamma_{air} + 1) M_s^2}{2 + (\gamma_{air} - 1) M_s^2} \quad (9)$$

Based on the stagnation properties of air given in Table 1, the velocity u_s of the primary shockwave is calculated to be 431.756 m/s. Note that the adiabatic index is assumed constant for both gases. For air, it is taken as $\gamma_{air} = 1.4$, whereas for SF_6 gas $\gamma_{SF_6} = 1.076$.

Figure 2: Overview of the flow development from left to right and top to bottom: snapshots at times 1.26, 1.90, 2.22, 2.70, 3.26 and 3.82 ms.

Table 1

Initial conditions of the air and SF_6 gases.

	Air 2	SF_6	Shocked Air
Density (kg/m^3)	1.184	6.34	1.703
Pressure (bar)	1.0	1.0	1.67261
Velocity (m/s)	0.0	0.0	131.575

5. Results and discussion

This section presents and analyses the results obtained for the DP and IC cases using iLES. Figure 2 depicts how at $t = 1.26$ ms the mixing layer has developed. Not surprisingly, the first (left) interface in both cases looks identical. The vortex structures to be showcased are examined at specific times that correspond to those obtained in the experiments of Holder and Barton [57], which were carried out using gases which naturally molecularly mix and hence are miscible. In the inverse chevron experiments, the turbulent mixing has been visualized using pulsed laser-sheet illumination of the Mie scattered light from seeding particles in the SF_6 region. In the iLES, the SF_6 mass per unit volume, or density (kg/m^3), can be interpreted as the numerical equivalent of the photographic images retrieved by the experiments. As explained by Holder et al. [56], the laser-sheet technique is subject to multiple scattering of the reflected light, thus the experimental pictures appear slightly diffuse and only the large-scale features can be observed clearly. Since the flow is essentially two dimensional at the largest scales, the spanwise average (y -direction) of the SF_6 density is shown here (Fig. 2) for illustration purposes and which are comparable to the experimental and computational images of Holder and Barton [57].

As the incident shockwave $M_s > 1$ transverses the shock-

tube from left to right, it ruptures the membranes separating the two fluids, leading to the formation of RM instabilities. The membranes are supported on a very fine wire mesh with a grid spacing of 0.4 cm. The initially small perturbations grow rapidly in size, leading to high-Reynolds number turbulent mixing of the two gases. The interface perturbations are further amplified by the formation of reflected waves as the primary and subsequent shockwaves travel through and interact with the material interfaces and the end wall. Solutions of the Riemann problem show that a heavy-to-light shock-contact refraction produces a reflected expansion fan and the case of a light-to-heavy refraction produces a reflected shock wave [60].

Specifically, once the incident shockwave passes the initial planar interface, it continues to travel through the SF_6 , while additional reflected shockwaves form in the compressed air travelling in the reverse direction. Eventually, the shockwave reaches the secondary interface between SF_6 and uncompressed air. The second interface for the first case considered is planar, whereas it is inclined and forms an IC shape for the second. Surface perturbations with a similar power spectrum are imposed on both material interfaces. As the shockwave moves past the IC interface and into the less-dense air, expansion waves form in the SF_6 . The primary shockwave continues downstream and is eventually reflected at the end wall. Having reversed direction, it reshocks the second interface. Simultaneously, the expansion waves created earlier reach the first interface and begin to exit the SF_6 . Additional weaker shocks and expansion waves form and interact with the relatively diffuse and (molecularly) mixed interfaces at this later stage.

Complex flow is a culmination of the different secondary waves that form when a shockwave passes through a negative –or positive– Atwood number interface relative to the direction of travel of the shockwave. In either case, the sec-

Figure 3: Approximate $x-t$ diagram for the double-planar case depicting the position of the edges of each interface extracted from the one-dimensional Favre-averaged mass-fraction streamwise profile using constraints 0.01 for the spike and 0.99 for the bubble, where 1 is pure air and 0 is pure SF_6 ; position of the (re-)shocks and expansion fans that form until $t = 3.0$ ms. Note that only the expansion fans produced by the primary shockwave are shown and for clarity, only the mean position of the compression and expansion waves is given; grey area illustrates pure SF_6 .

ondary waves that form will always travel in opposite directions. While the primary incident shockwave moves downstream (from left to right), the negative Atwood configuration creates an expansion wave. In contrast, a positive Atwood number interface produces a secondary shock. This process is then repeated by each additional shockwave, but with decreasing strength. The following moments are of particular interest in this study:

- At 1.26 ms, both interfaces have been shocked once by the primary shockwave, which is located past the second interface travelling right towards the shock tube end.
- At 1.90 ms, the first expansion fan (created when the primary shock exited the right SF_6 -air interface) has passed through interface 1 (air- SF_6), while the second SF_6 -air interface has been reshocked by the primary shockwave after it reflected off the shocktube end. The primary shockwave is positioned inside the SF_6 between the two mixing layers travelling leftwards.
- At 2.22 ms, interface 1 has been reshocked by the primary shockwave, whereas a compression wave (caused when expansion wave 1 exited the SF_6 at $t \approx 1.45$ ms) shortly followed by shockwave 2 (caused when the primary shockwave reshocks the second interface at $t \approx 1.65$ ms) have passed through the second interface.
- At 2.70 ms, interface 1 has been reshocked by the reflected shockwave 2, while the compression wave, followed shortly by the secondary expansion fan 2 (created when the primary shockwave exited the first in-

terface travelling leftwards at $t \approx 2.22$ ms) and the reflected shockwave 3 (caused when shockwave 2 reshocked interface 2 at $t \approx 2.2$ ms), have all passed through the second interface.

The evolution of the double-planar case, as described above, is best illustrated in the $x-t$ diagram of Fig. 3.

The three-dimensional turbulent structures obtained from different simulations are revealed by plotting the isosurfaces of Q criterion [25]. The Q criterion represents vortex cores by using as an indication when the vorticity prevails over the strain or, more specifically, the Euclidean norm (or magnitude) of the vorticity tensor manages to dominate that of the rate-of-strain:

$$Q = \frac{1}{2} (\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij})$$

where

$$\Omega_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right), \quad S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

are the antisymmetric and symmetric parts of the velocity gradient tensor, respectively. Note that the value of Q used is greatly influenced by the grid resolution and accuracy of the numerical method employed. Similarly, to enstrophy [48], the Q criterion is not expecting to converge for $Re \rightarrow \infty$, and therefore other studies should adjust their value accordingly. Additionally, note that the local Mach number does not exceed 0.22 anywhere in the mixing layer during the course of the simulation. Therefore, the Q criterion can provide meaningful results for the vortex topology of the reshocked layer.

Although there exist other approaches in order to extract and visualize vortical structures from a turbulent flow field,

Figure 4: Vortex core isosurfaces coloured by the mass fraction at $t = 1.26$ ms ($Q = 10^6$ s $^{-2}$).

such as the λ_2 criterion [61], it has been reported that both (Q and λ_2) criteria give similar results, free of deformation regions [62, 63]. However, based on our experience, and as reported by Dubief and Delcayre [63], the Q criterion gives the clearest picture of the flow, though it should be cautioned that none of these methods can be truly regarded as the definitive solution to the problem of vortex identification [62].

Recently, Schneider and Gauthier [64] reported the presence of such vortex rings in their large-scale numerical simulations of laminar and turbulent Rayleigh–Taylor induced (RTI) mixing between miscible fluids. The authors exploited the vortex core identification superiority of the Q criterion to gain an insight into the structure of turbulence (vortical and dissipative structures, small-scale intermittency, and homogeneity of compressible mixing layers) that ensues. Nonetheless, the investigation of vortex rings and structures within the context of miscible RTI mixing is far from being exhaustive. For example, the effects of the initial interface material perturbation [65], Atwood number [66], and unsteady acceleration with sign reversals [67], remain largely unknown.

5.1. Double planar

After the initial shock at the interfaces, the mixing between the two species remains low. At 1.26 ms, the vortex structures (Fig. 4) indicate that turbulence is chaotic, i.e., of no distinct orderly organization and, crucially, is of relatively low strength. The first white-coloured isosurfaces –going from left to right– depict the turbulent structures present at

the shocked air. Next, and in close proximity, the black isosurfaces show that the heavier SF_6 has been perturbed much more intensely than the left (air) region of the first interface. The reverse holds true at the second (right) interface, whose vortex structures shortly after shock are predominantly composed of air. It appears that the baroclinic deposition of vorticity favours the second material through which the shock-wave moves through.

The turbulence intensity remains sufficiently high in the mixing layer ($Q = 10^6$ s $^{-2}$) that the turbulence associated with the shockwave wake and expansion-fan, induced by the irregularities caused by their prior interaction with the perturbed interfaces, is not visible. At later time steps, the mixing layer turbulence intensity reaches values four-to-five orders of magnitude larger, concealing the presence of any other (weaker) turbulent-flow structures.

The material interface perturbations themselves produce some noticeable instability features, such as spikes or bubbles manifested by the VF (z_1) isosurfaces in Fig. 5, but are still at an early stage of development. The relative velocity of the bubbles or spike edges with respect to the mean interface velocity remains low –gradually decreasing after the primary shock– and is thus not expected to significantly impact the local velocity field as Fig. 4 indicates. However, a vortex ring can only form once a vorticity threshold is reached.

The passage of an expansion wave at 1.4 ms through interface 1 and the reshock of interface 2 at 1.7 ms lead to a sharp rise in the speed of the bubbles and spikes at both in-

Figure 5: Volume fraction isosurfaces $z_1 = 0.01$ (yellow, SF_6) and 0.99 (blue, air) at time $t = 1.26$ ms.

terfaces, as shown by the abrupt increase in the interfaces width (Fig. 6) and results in the further deposition of baroclinic vorticity, $(\nabla \rho \times \nabla p)/\rho^2$. The width of each mixing interface, characterized by the distance between the edges of the bubbles and spikes, has been extracted from the simulations by tracking the average 1D position of the VF at threshold values less than 0.1 (spikes) and greater than 0.9 (bubbles), where $\text{VF} = 1$ is pure air. Fig. 6 also shows that the width of the second interface increases continuously and at a greater rate after each successive reshock. This result is in sharp contrast with what happens at the second interface where, during the interval from 2.1 to 2.4 ms, it actually collapses, whereas from 2.8 to 3.1 ms, its rate of change stagnates and even begins to gradually re-collapse, albeit at a much slower rate. The same process can also be discerned from the position of the front and back of the first mixing layer in the time-space diagram given in the numerical study of Nelson and Grinstein [17]. The ramifications of the collapsing width of the first interface are expected to be of significant importance in the context of turbulence modelling of miscible-fluid shock-induced mixing studies [68]. The density across interface 1 appears to compress (Fig. 7) as one would expect between 2.1 and 2.3 ms due to the reshock, however, this is not the cause for the observed decrease in the

width of interface 1. Rather, the sudden deceleration caused by the reshock to the heavier SF_6 (see Fig. 8) causes it to act as a “wall” to the incoming air, resulting in the interface collapsing onto itself. On the other hand at interface 2, the decelerated lighter air is not sufficiently dense to hinder the denser SF_6 RM instabilities from continuing to grow, at least with regards to the spike region of the mixing layer.

After the first shockwave-interface interaction, most turbulent structure isosurfaces near the first interface are composed of pure SF_6 (black), whereas at the second interface they are composed of pure air (white). This selectivity, or preference, of the vorticity generation for the region in which the shockwave passes relative to its direction of travel, appears to be irrespective of the sign of the Atwood number. Most likely, this is caused by the rapid temporal changes in the physical properties of the shockwave as it enters a material of different physical properties, be it rapid (de-) acceleration as it moves into a (more) less dense fluid in conjunction with the spatial variations or perturbations of the material interface.

Although the temporal changes in the shockwave properties occur rapidly during its transversal through the material mixing zone, the process still results in a spatial variation that is believed to be the underlining cause for the selectiv-

Figure 6: Interface width over time; edge detection constraints used: one-dimensional Favre-averaged mass-fraction equal to 0.01 for the spike and 0.99 for the bubble, where 1 is pure air and 0 pure SF_6 .

(a)

(a)

(b)

(b)

Figure 7: Density (kg/m^3) across the mixing layer in time for interfaces 1 (top) and 2 (bottom).

Figure 8: Stream-wise velocity (m/s) across the mixing layer in time for interfaces 1 (top) and 2 (bottom).

ity of the vorticity production. In short, the perturbations of the material interface cause variations in the pressure surface of the planar shockwave, thereby producing trailing vortices. Richtmyer–Meshkov instabilities form irrespective of

the sign of the Atwood number; however, what is not widely known or well understood is that the vorticity generated—at least for the initial shock at a material interface—is typically orders of magnitude greater downstream of the interface.

Figure 9: Vortex core isosurfaces coloured by the mass fraction at $t = 1.90$ ms ($Q = 10^8$ s $^{-2}$).

5.1.1. Onset of the vortex rings

By $t = 1.90$ ms, the expansion wave has completely exited the first material interface and the second interface has been reshocked. The initial chaotic turbulent structures now appear “orderly” (Fig. 9), having obtained what looks like a ‘web’ structure composed of worm-like vortices. The intermingling between the white and black vortex cores suggests that molecular mixing between the two components and the growth of the mixing width have both commenced. Hereon, the turbulent structures formed will be examined both temporally and spatially to answer questions such as “*how do the structures evolve in time?*” and “*what pattern and shape do these structures take?*”, respectively. Finally, note that due to the additional baroclinic deposition of vorticity by the reshock at interface 2 and expansion fan at interface 1, the strength of the vortices has dramatically increased since 1.26 ms, resulting in a four-fold increase in the value of the Q criterion used to plot the turbulent-structure isosurfaces.

Common to both interfaces with different Atwood number sign, the vortex rings begin to form and develop after the secondary interface interaction with a shockwave or rarefaction wave, as evident in Fig. 9. The location of the vortex-ring structures with respect to the turbulent mixing zone is initially adjacent to the edge of the lighter of the two components. Subsequent interactions of the interface with the secondary waves cause the vortex rings associated with the RMI bubble instabilities to gradually shift into the centre of the turbulent mixing layer as is clearly evident at interface 1 at 2.22 ms (Fig. 10). By this time, the bubble vortex rings are

positioned much closer to the centre of the mixing layer than their spike-induced counterparts and, by this stage, they are thought to play a crucial role in determining the molecular diffusion and mixing width of the mixing zone.

In the proceeding analysis, the evolution of the turbulent structures is examined at times 1.90, 2.22 and 2.70 ms (Fig. 11 – Fig. 20). Again, the vortex structures are unravelled by means of the Q criterion isosurfaces, although they are of varying degrees of strength (depending on time) and are coloured according to streamwise velocity. Note that, for each figure, the left image shows an interface from the right to the left of the domain, whereas the reverse, or opposite view, is shown in the right image. Both views are given to appreciate the differences between the turbulent structures that form either side of the air-SF₆ material interface.

5.1.2. Time 1.90 ms

To this point, the first interface has been shocked (at ~ 0 ms) and an expansion wave has traversed (at ~ 1.5 ms), whereas the second interface has been shocked twice by the initial (primary) shockwave at ~ 0.8 and 1.67 ms. The resulting additional deposition of baroclinic vorticity at the interfaces has given rise to a number of small vortex structures that resemble toroidal vortex (bubble) rings. A vortex ring –the presence of which is implied by the Q criterion isosurfaces shown in the images below (Fig. 11 to Fig. 20 for the DP, and Fig. 27 to Fig. 32 for the IC test cases)– resembles a doughnut ring that spins poloidally, forming a torus-like structure. The underlining physical process behind their for-

Figure 10: Streamwise normal view of volume-fraction levels coloured by Q criterion ($Q \approx 10^9 \text{ s}^{-2}$) of interface 1 at $t = 2.22 \text{ ms}$.

mation, in this case, is due to the poloidal flow, which is set in motion, at least in part, by the shockwave-induced impulsive acceleration of the superimposed initial mixing-layer instabilities. Parallel to the toroidal vortices, the well-known spike and bubble instabilities characteristic of RMI mixing also begin to emerge. Subsequent reshocks only act to further enhance and strengthen the process.

Note that, for the first interface, the passage of the expansion fan leads to the formation of the vortex rings, whereas for the second interface the first reshock accomplishes this task. An important difference between the two events is that the expansion fan causes a decrease in pressure in the direction of travel of interface 1, whereas the reshock at interface 2 creates an impulsive pressure rise in the SF_6 that also causes a momentary negative pressure gradient. However, how the position within each interface at which the shock and expansion waves alter properties and create secondary pressure waves or discontinuities affects the vortex-ring formation and dynamics, as well as the turbulence properties across the mixing layer, remains unclear.

5.1.3. Time 2.22 ms

Another important observation is the location and composition of the vortex rings associated with the bubble-type RMIs. At both interfaces, the vortex rings in the near SF_6 edge of the turbulent mixing layer are composed of both SF_6 and air. For the first interface, this occurs at 2.22 (Fig. 16, left), whereas for the second earlier at 1.90 ms (Fig. 14, right), perhaps due to the larger deposition of baroclinic vorticity caused by the reshocks at interface 2 than by the rarefaction wave at interface 1. Tritschler et al. [16] found that the first expansion wave deposited much less energy than the reflected shockwave; however, they cautioned that Grinstein et al. [24] had previously shown that energy deposition depends heavily on the respective initial interface perturbations. Another possibility is that the more gradual and slower

acceleration caused by the rarefaction waves favours the formation of vortex rings within the *inner* edges of the mixing layer, rather than the surrounding *outer* area, as is the case for impulsively accelerated mixing layers. In addition, a more gradual formation of the vortex rings permits more time for the molecular diffusion processes taking place at the mixing zone to influence the composition of adjacent vortex rings.

5.1.4. Time 2.70 ms

By 2.70 ms, large-scale turbulent structures are no longer visible at either interface (Figs. 19 & 20) and the remaining turbulent structures are believed to be mostly remnants of the vortex-ring breakup. However, some “remnants” that are indicative of the previous orderly structure at the second interface still exist. These remnants are partly visible due to the effective colouring of the vortex-core isosurfaces by the streamwise velocity, although other flow properties may be just as (or more) effective. The remnants are most obvious on the SF_6 side of the interface (right image), where blue regions indicate a reversal in the travelling direction of large parts of the mixing layer, whereas the red interconnecting isosurfaces indicate the past presence of vortex bubble rings. Although turbulence now appears homogeneous and isotropic, this is not yet the case as indicated by the common velocity (green) of the vortex cores.

It is unclear whether the vortex rings eventually just dissipate or collapse by bursting in a process similar to that of bubble rings fragmenting into rings of spherical bubbles when, because of the Plateau–Rayleigh instability, the ring becomes thinner than a few mm. In the latter case, when the bubble reaches a certain thickness, surface-tension effects distort the bubble’s surface, pulling it apart into separate bubbles. In this case, because the strength of the Q criterion is an order of magnitude greater than previously ($t = 1.90$ and 2.22 ms), this likely indicates that the additional reshocks incurred by the vortex-rings has caused their

Figure 11: Interface 1: vortex-core isosurfaces ($Q = 10^8 \text{ s}^{-2}$) coloured by the x-velocity ($u_x \in [100, 160] \text{ m/s}$) at $t = 1.90 \text{ ms}$.

Figure 12: Interface 1: vortex-core isosurfaces ($Q = 10^8 \text{ s}^{-2}$) coloured by the volume fraction (black is SF_6 , white is air) and semi-transparent volume-fraction isosurfaces (red is 80% SF_6 , blue is 80% air) at $t = 1.90 \text{ ms}$.

sudden collapse and ‘burst’ from their well-defined 2D structure (axisymmetric) into the smaller but more energetic 3D chaotic turbulent structure observed. A counter explanation may be that the reshocks are in fact not sufficiently powerful to continue powering the vortex rings. However, destruction of the vortex rings is more likely to be the product of the col-

lapse of their powering mechanism; namely, the spike and bubble RMIs.

Lundgren and Mansour [69] studied the stability and transition to turbulence of a simple incompressible 2D bounded swirling flow with a rectangular planform and showed that this type of flow is unstable against 3D disturbances, where

Figure 13: Interface 2: vortex-core isosurfaces ($Q = 10^8 \text{ s}^{-2}$) coloured by the x-velocity ($u_x \in [10, 100] \text{ m/s}$) at $t = 1.90 \text{ ms}$.

Figure 14: Interface 2: vortex-core isosurfaces ($Q = 10^8 \text{ s}^{-2}$) coloured by the volume fraction (black is SF_6 , white is air) and semi-transparent volume-fraction isosurfaces (red is 80% SF_6 , blue is 80% air) at $t = 1.90 \text{ ms}$.

the instability produced takes the form of counter-rotating swirls perpendicular to the axis, thereby bending the vortex into a periodic wave. As these swirls grow in amplitude the primary vorticity is compressed into thin vortex layers, which develop secondary instabilities that roll up into vortex tubes. In this way, they showed how the initial 2D vor-

tex flow attains a turbulent state that is populated by intense elongated vortex tubes with weaker vortex layers spiralling around them. This explanation perfectly describes the late-time turbulent mixing zone at 2.70 ms (Figs. 19 & 20).

Maxworthy [70, 71] shows that a vortex ring becomes turbulent once the Reynolds number Γ/ν (where Γ is the

Figure 15: Interface 1: vortex-core isosurfaces ($Q = 10^8 \text{ s}^{-2}$) coloured by the x-velocity (left: $u_x \in [-30, 80] \text{ m/s}$, right: $u_x \in [-30, 140] \text{ m/s}$) at $t = 2.22 \text{ ms}$.

Figure 16: Interface 1: vortex-core isosurfaces ($Q = 10^8 \text{ s}^{-2}$) coloured by the volume fraction (black is SF_6 , white is air) and semi-transparent volume-fraction isosurfaces (red is 80% SF_6 , blue is 80% air) at $t = 2.22 \text{ ms}$.

circulation and ν is the kinematic viscosity) is greater than about 600. At this threshold, turbulence arises not only in the vortex core but also in an ellipsoidal ‘bubble’ of fluid moving with the vortex ring. Maxworthy provided experimental evidence to support the following physical picture: The co-moving turbulence bubble entrains non-turbulent fluid into

its front and ejects most of it into its wake along with some of the vorticity of the ring. A small amount of the entrained fluid is retained, causing the size of the ring core and bubble to slowly increase. The loss of circulation to the wake slows the vortex, halving the initial velocity by the time it has travelled about 10 initial diameters.

Figure 17: Interface 2: vortex-core isosurfaces ($Q = 10^8 \text{ s}^{-2}$) coloured by the x-velocity ($u_x \in [10, 70] \text{ m/s}$) at $t = 2.22 \text{ ms}$.

Figure 18: Interface 2: vortex-core isosurfaces ($Q = 10^8 \text{ s}^{-2}$) coloured by the volume fraction (black is SF_6 , white is air) and semi-transparent volume-fraction isosurfaces (red is 80% SF_6 , blue is 80% air) at $t = 2.22 \text{ ms}$.

5.1.5. Mechanism behind vortex ring formation

Shusser and Gharib [72] consider vortex-ring formation in a starting axisymmetric buoyant plume and provide a physical explanation of steady vortex rings based on the Kelvin–Benjamin variational principle: vortex rings grow provided the plume provides the energy required for a steady vortex

ring [73]. This generalizes the result obtained for piston-generated vortex rings Gharib et al. [74]. Here, we can imagine replacing the rising plume by the rising SF_6 component. The time scale found by Lundgren et al. [75], which is the ratio of the velocity of a buoyant plume after it has travelled one diameter to its diameter, is equivalent to the time

Figure 19: Interface 1: Vortex-core isosurfaces ($Q = 10^8 \text{ s}^{-2}$) coloured by the x-velocity ($u_x = \pm 50 \text{ m/s}$) at $t = 2.70 \text{ ms}$.

scale (formation time) found by Gharib et al. [74] experimentally for uniform-density vortex rings generated with a piston or cylinder arrangement. A similar universal time scale for the formation of vortex rings generated through impulsively started jets was also predicted by an analytical model [76]. In the present work, the relative velocity between the SF_6 spike and the shocked air leads to the formation of the vortex rings. As an example, the spike-RMI region of the second interface has well-defined vortex rings by time 1.90 ms, which formed shortly after the first reshock, as the spike RMIs begin to form. By examining the high-resolution iLES data, we estimate that the average size of the spikes that begin to form is approximately 10 cell widths, or 3.125 mm, whereas the average speed difference between the SF_6 spike and the surrounding air is approximately 40 m/s. This gives an estimate of the time at which the vortex rings began to form ($\sim 1.822 \text{ ms}$). Indeed, our closest available data (at 1.82 ms) indicate that the vortex rings around the tips of the spike RMIs of the second interface have begun to form; however, their strength is between one and two orders of magnitude lower in terms of the Q criterion than that at 1.90 ms. The previous-earliest results available, at 1.77 ms, show no sign of any vortex rings at interface 2; instead, most turbulent structures appear chaotic and are mostly located in the shocked air.

Shusser and Gharib [72] have previously shown that similarly to piston-generated vortex rings [74], a buoyant vortex ring pinches off from the plume when the latter can no longer provide the energy required by the former. The dimensionless time of the pinch-off (i.e. the formation number) can be reasonably well predicted by assuming that, at pinch-off,

the vortex-ring velocity exceeds the plume velocity. Because of the complex nature of the flow, in this case, a culmination of the multiple interactions between the interface-shock/rarefaction waves as well as between the vortex rings themselves, the scope of the current study is limited and so estimates of equivalent timescales are not warranted. Instead, we restrict our findings to the observation that the vortex rings do indeed linger a little while longer after the collapse of the bubble and spike RMIs, as suggested by Figs. 15 and 17. Perhaps the only exception is the bubble region of interface 2, which still shows some signs of the previous large-scale vortex rings even up until 2.70 ms (Figs. 19 & 20).

5.1.6. Considerations regarding immiscibility

At interface 1, the accelerated air moves through the centre of the bubbles formed in the mixing layer and impinges on the heavier SF_6 , which acts similarly to a solid surface. As the lighter air spreads out along the sides of the spherical cap bubbles, the flow begins to rotate, causing vortex rings to form along the inner edges (Fig. 12, left). Conversely, behind interface 1 (Fig. 12, right), spikes form from the displaced or “ejected” SF_6 mass towards the air. The cause for the ejected SF_6 here is the impact of the higher-velocity air, which is caused by the lower impulsive acceleration of the heavier SF_6 gas (larger inertia). As a result of the local velocity shear and combined with the grid diffusion and numerical dissipation, vortex rings are generated. The process described is similar to that of the experiments of Baumann et al. [77] with immiscible liquids, in which vortex-ring formation is associated with circulations generated by viscosity.

Immiscibility depends on the ratio of surface energy to

Figure 20: Interface 2: Vortex-core isosurfaces ($Q = 10^8 \text{ s}^{-2}$) coloured by the x-velocity ($u_x = \pm 50 \text{ m/s}$) at $t = 2.70 \text{ ms}$.

tension. Surfaces do not form between gases and so lack surface tension. The scientific definition of a surface is an interface that forms a common boundary between two different phases of matter (e.g. insoluble solid and a liquid), two immiscible liquids, a liquid and an insoluble gas or a liquid and vacuum. The effect of surface tension on the development of a material interface is governed by the Weber number, which can be thought of as a measure of the relative importance of the fluid's inertia compared to its surface tension. The effect of surface tension is twofold: to render the problem well-posed and to regularize the instability [78]. Kojima et al. [79] neglected surface tension in their theoretical effort to predict translating miscible drops and were unable to quantitatively reproduce certain aspects such as the rate of torus expansion. The authors reasoned that although very small, surface tension at interfaces partitioning miscible liquids exists while the system is not in equilibrium. The authors concluded by cautioning that further complexity arises when the interfacial tension between miscible liquids is considered to diminish in time as diffusion proceeds and the two liquids ultimately mix.

As aforementioned, Baumann et al. [77] demonstrated experimentally that in immiscible liquids, vortex-ring formation is associated with circulations generated by viscosity. Unlike classical vortex rings, which occur in miscible liquids at high Reynolds numbers, vortex rings in immiscible liquids do not (molecularly) diffuse. Thus, their structure is well-defined and can form even at very low Reynolds numbers, provided that surface tension is small with respect to viscous forces. On a separate note, many similarities exist between the formation and flow dynamics of miscible and immisci-

ble vortex rings. For a ring to form, the drop fluid must be much more viscous than the host fluid, and the interfacial-tension effects (assumed to be zero between miscible fluids) must be weaker than the viscous effects. However, note that the normal stress balance at the interface shows that, apart from surface tension and viscosity, two more forces are at work due to gravity and inertia. Since vortex rings can form and be observed much more easily in immiscible fluids, little effort has been devoted to investigate such structures in miscible fluids.

In immiscible fluids, toroidal vortex bubbles form as soon as a bubble cap collapses. This creates an impulsively accelerated jet but unlike the situation studied herein, the air vortex pocket also forms simultaneously as the cap bubble changes from simply to doubly connected. However, the local velocity gradient and viscosity (or shear forces) combine to transfer momentum to the doubly connected bubble region, thereby giving it the rotational speed that characterizes a toroidal vortex ring. Here, similarly, the velocity shear caused by the velocity difference between the two impulsively accelerated miscible components gives rise to the formation of the vortex rings. The fluids are not yet heavily mixed at the molecular level, as shown by the distinct colour (pure white or black) of the vortex rings that form and by the very thin layer separating the two components (Figs. 4 & 9). For the evolution of the mixing layer, the formation of the vortex rings is significant because they cause momentum –and thus kinetic energy– to transfer between the two components and create drag by transforming lateral momentum into rotational momentum.

Similarly, the numerical model based on the inviscid vor-

tex dynamics developed by Lundgren et al. [75] to describe the development of a microburst, explained how a strong shear layer forms between the heavier downward-accelerating fluid and the lighter fluid that rises around it. The vorticity in this shear layer forces the lighter fluid to penetrate through the heavier fluid from above, forming a heavy-fluid vortex ring that entrains the lighter fluid by winding. In this process, the heavier fluid initially gains momentum by accelerating in the gravitational field but as the vortex develops, momentum transfers to the lighter fluid until it mostly resides in a vigorous down flow of lighter fluid through the aperture of the vortex ring.

Note that the aim of this paper is not to provide an analytical or theoretical analysis of vortex-ring development in miscible fluids, as the uncertainties associated with the many irregularities, perturbations and vortex-ring interactions would hinder such an effort; instead, such a study would require an isolated vortex ring with appropriately defined initial conditions. Alternatively, we study and analyse herein the vortex-ring formation, structure and evolution with respect to their effect on the mixing-layer dynamics and properties, while also cautioning their presence in relation to turbulence models.

5.1.7. Interconnecting structures between vortex rings either end of the mixing layer

The vortex rings associated with either side of the mixing layer (i.e. the bubble- and spike-induced vortex rings) are interconnected by what look like long worm-like vortices. As the vortex rings run across the edges of the rim of the bubble-associated RMI instabilities, the latter of which grow in the outwards direction, they eventually collide. In regions where two bubble vortex rings meet (where vortex core lines are typically parallel to each other), they begin to interact. Due to their opposite directions of rotation, they begin to destabilize and weaken each other. In some regions, typically behind which RMI spikes, up to three bubbles may be interacting. This suggests that a strong correlation exists between the number of spikes and the number of bubbles that form. It is not clear whether the bubble vortex rings determine the development and number of spike vortex rings or vice versa; however, the initial instability perturbation of the material interface is more likely to be responsible for creating regions of the mixing zone that 'favour' the development of one instability over the other. This is further supported by the observation that spikes do not form where the bubble RMI penetration is the largest, and vice versa.

In addition, in these triply connected bubble-vortex-ring regions, a streamwise column of vortex cores can be typically found, rising out of the streamwise normal plane and towards the vortex rings associated with spike RMIs. They appear particularly clear in Fig. 14, left at the second interface (1.90 ms). They are composed primarily of the lighter of the two components, air in this case, and are thus believed to form due to shear forces generated by the velocity difference between the air and the heavier SF_6 component. The exact origin and importance of this "web" of interconnect-

ing vortex cores is not yet well understood; however, they are believed to form near the region where two bubble RMIs collide, forming "peaks" of SF_6 material and trace along these peaks as the heavier component merges into the spike RMIs at the triple-point juncture. For illustration purposes, the streamwise-orientated worm-like vortices of the inner web structure can be isolated from the remaining vortex structures by imposing a minimum threshold value for the helicity (the streamwise vorticity works equally well), as depicted in Fig. 21.

In order to elucidate the mechanism behind these streamwise vortex tubes, it is necessary to obtain a better understanding of their origin, as well as their relationship with the vortex rings located on both sides of the mixing layer. Further research should be carried out based on the tracking of Lagrangian material surfaces (Lagrangian scalar field) whose isosurface is a vortex surface, using multi-scale geometric analysis [80]. The approach is capable of extracting from a turbulent field the temporal evolution of material surfaces with quantitative multiscale and multi-directional statistical geometry successfully in a number of cases, i.e., isotropic turbulence [81], simple transitional flows such as Taylor–Green and Kida–Pelz flows [82], and fully developed channel flows [83]. Due to the interactions and breakdown of the vortices, which can cause vortex re-connections and severe topological changes, the deviation between the material surface and the vortex surface should be taken into account. The vortex-surface-field method proposed by Yang and Pullin [82] has proved promising in characterising the evolution of vortex surfaces with topological changes [84].

5.1.8. On the energy spectrum and anisotropy of the mixing layer

In the discussion to follow, all RMI studies referenced investigated the shockwave interface interaction at the same Atwood number sign as that of interface 1 of the current double-planar setup. Weber et al. [85] observed in their experiment that the peak of the density-specific volume correlation initially shifts towards the light-gas side, but later moves towards the centre of the mixing zone. This is consistent with the large part of the turbulent structures found on the SF_6 side of interface 1 after the shock at 1.26 ms, as seen in Fig. 4 earlier. The greater level of turbulence enhances turbulence diffusion between the components on that side of the interface, leading eventually to the smaller density-specific volume correlation value observed experimentally near the heavier component. By 1.90 ms, most turbulent structures have also shifted within the centre of the mixing layer (Fig. 9), however, in contrast to Weber et al. [85], the enhanced turbulence is the culmination of the interaction of the rarefaction wave with interface 1 and so it remains unclear whether the re-positioning of the peak value would otherwise occur. Note that the experiment, similar to the iLES herein, imposed a broadband initial condition on the material interface. This is particularly relevant since both Grinstein et al. [24] and Hahn et al. [48] reported that energy deposition depends strongly on the respective initial interface per-

Figure 21: Vortex-core isosurfaces ($Q = 10^8 \text{ s}^{-2}$) coloured by the helicity ($H = \pm 5 \times 10^6 \text{ m}^2/\text{s}^2$) at $t = 1.90 \text{ ms}$. Note that only the vortex-core regions that satisfy $|H| \geq 5 \times 10^7 \text{ m}^2/\text{s}^2$ appear on the right.

turbations, with the short wavelength, or narrowband spectrum, material-interface perturbation acquiring greater TKE during the initial early-time interaction with the reshock and rarefaction waves.

Grinstein et al. [24] also observed that the velocity fluctuations in the mixing zone become almost isotropic when the initial interface is perturbed by short wavelengths. The superposition of the short wavelengths either (i) accelerates the breakdown of the vortex rings by destabilizing their rotational velocity or other circulation, (ii) results in smaller and weaker vortex rings that dissipate sooner or (iii) results in a combination of both, with (iii) being the most likely scenario.

The RMI typically results in inhomogeneous anisotropic unsteady turbulence, with the exception of late-time mixing of an initially narrowband perturbed interface [24]. Nonetheless, homogeneous isotropic turbulence is often used to describe and quantify the RMI mixing-layer turbulence. However, it now becomes clear that the vortex-core isosurfaces largely determine the mixing-layer properties, depending on the time and location of the measurement. As depicted in Figs. 22 and 23, neither interface manages to obtain a fully isotropic mixing zone, because the anisotropy, although decreasing with time, steadily converges towards an asymp-

totic value of ~ 3 to 4 at late time. However, in their numerical study, Tritschler et al. [16] found that, in contrast to the anisotropy, where the reshock does not contribute to the isotropisation, the mixing zone becomes significantly more homogeneous after reshock, as observed in the temporal evolution of the volume average of the density-specific volume correlation in the inner mixing zone. As evident by the mass-fraction distribution of the vortex rings in Fig. 12 and Fig. 16 (Fig. 14 and Fig. 18) for interface 1 (and 2), the multiple reshocks cause the bubble-side (or SF_6 -dominant side of each interface) vortex rings to partially transgress through the corresponding interface. This transgression is thought to be caused by the difference in acceleration and deceleration between vortex rings and mixing layer. Subsequent reshocks cause the fluids to become increasingly mixed, leading to the collapse of the RMI spikes and bubbles, followed shortly by the collapse of the vortex rings. By 3.26 ms (Figs. 19 & 20), only small-scale turbulent vortex core structures prevail. It is thought that the mechanism behind the vortex ring destruction are both (i) the decorrelation of the translational velocity between the ring and axisymmetric jet flow (spikes) velocity [86], and (ii) the collapse of the RMI structures which deliver the energy at a rate required for steady vortex ring existence as according to the Kelvin–Benjamin variational

Figure 22: Ratio of streamwise to spanwise turbulence kinetic energy (K_x/K_y) as a function of time and component mass fraction within the mixing layer for interfaces 1 (top) and 2 (bottom).

principle [73].

Perhaps one of the most serious effects of the vortex rings is their influence on the estimate of energy spectra at the mixing interface. Note that, although inhomogeneous in three dimensions, for a planar interface the resulting turbulence is in fact homogeneous in the 2D streamwise normal ($y-z$) planes. In the analysis of RMI mixing, the turbulence-energy spectrum within the mixing layer is typically obtained by using either multiple streamwise normal slices of the fluctuating velocity field (and in some cases density field as well) or a single slice at the centre of the mixing zone.

For example, Thornber et al. [87] found a $k^{-5/3}$ or k^{-2} scaling range after reshock of a narrowband or broadband interface perturbation, respectively. They evaluated their radial spectra at the centre of the mixing zone, where very few vortex rings would have affected the measurement of the energy spectrum because, at the moment of the first reshock, most vortex rings are near the extremities of the mixing zone. It remains unclear whether the presence of vortex rings at the edges of the mixing layer influenced the turbulence properties near the centre of the mixing zone. It is also uncertain whether the different scaling behaviour observed by Hill et al. [12] and Lombardini et al. [20], who found in their multicomponent LES at finite Reynolds numbers a $k^{-5/3}$ scaling in the centre of the mixing zone, can be attributed to the ef-

Figure 23: Ratio of streamwise to wall-normal turbulence kinetic energy (K_x/K_z) as a function of time and component mass fraction within the mixing layer for interfaces 1 (top) and 2 (bottom).

fects of viscosity on the dynamical behaviour of the vortex-rings formation and evolution.

In another approach, Cohen et al. [88] and Tritschler et al. [16] obtained the energy spectrum across multiple planes within the turbulent mixing zone. Both studies found a $k^{-6/5}$ scaling range for the single-shock RMI averaged over several transverse slices within the mixing zone. This indicates a slower decay of TKE, which is expected when including slices closer to the mixing-layer edges, where the vortex rings exert their strongest effects. The vortex rings act as an effective mechanism of transfer of mean kinetic energy towards the growth of the RMIs, thus decreasing turbulence dissipation.

We assume that in the spanwise directions, (i) turbulence is homogeneous, (ii) mean-flow velocities are zero (apart from the streamwise direction) and thus (iii) mean-flow-variable gradients in those directions are also zero. Consequently, the time accurate, spatial mean Favre averaged 1D turbulence kinetic energy (K) profile is obtained by:

$$K = \left\langle \frac{1}{2} \rho \left[(u_x - \tilde{u}_x)^2 + u_y^2 + u_z^2 \right] \right\rangle_{yz} / \langle \rho \rangle_{yz}$$

where the brackets $\langle \cdot \rangle$ denote an ensemble average and the tilde $\tilde{\cdot}$ denotes Favre averaging ($\tilde{u}_i = \langle \rho u_i \rangle / \langle \rho \rangle$). The ratio of the turbulence kinetic energy in the streamwise direction

(K_x) to those in the wall-normal (K_z) and spanwise (K_y) directions reveals the level of turbulence anisotropy in the resulting mixing layer. A much larger (or smaller) ratio than one suggests greater anisotropy, whereas unity itself is characteristic of isotropic turbulence. The maximum degree of anisotropy at both interfaces gradually converges to ~ 5 at late time, as Figs. 22 and 23 suggests. In addition, the turbulence magnitude in the wall-normal and spanwise directions shows little to no difference at either interface; however, the degrees of anisotropy obtained by each when first shocked differ greatly as a result of the different sign of the dot product of the density and pressure gradients. At the first interface, a lower-density fluid (shocked air) is accelerated into a heavier fluid (SF_6), whereas the opposite occurs at the second interface. The impulsively accelerated heavier fluid may penetrate deeper into the lighter fluid, as manifested by the narrow spikes, thus leading to a much larger streamwise velocity.

Conversely, when the lighter fluid is accelerated into the heavier fluid, the latter acts as a ‘wall’ and does not allow the lighter fluid to penetrate as much as in the opposite scenario. The result—in the wall-normal and spanwise directions—is a wider spread of the lighter fluid that manifests as wide “bubbles”, thus preventing the streamwise velocity from increasing to any great extent compared with those of the remaining two directions. It is unclear whether Rayleigh–Taylor-induced mixing also “plateaus” at a maximum anisotropy degree of 5 for the same Atwood number, or whether it even varies over time. An understanding of this mechanism is paramount when implementing and testing mixing models since such processes can greatly influence the evolution of the mixing layer.

Finally, further analysis needs to be carried out in order to verify whether or not the sharp increase observed in the anisotropy at $t \approx 2.2$ ms near the air limit ($\text{VF} \approx 0.9$) in Figs. 22 and 23 left, particularly evident at interface 1, is due to the vortex rings structure. Similarly for the fourfold increase observable near the SF_6 limit of interface 2 beginning at $t \approx 2.1$ ms (in Figs. 22 & 23 right).

5.1.9. Effective Reynolds number

An important aspect of the current simulations is the $Re \rightarrow \infty$ assumption. It has been shown by Hahn et al. [48] that integral quantities, such as the total-mix, converge at the chosen grid resolution. However, it is anticipated that the smallest possible resolved structures will be grid dependent. Since the initial condition is grid resolved, it is anticipated that refining the grid further should capture only finer scale structures, in addition to those already present. The interaction between the bigger, resolved scales, and the finer unresolved scales, which are present only on the finer grid, will eventually have an increasing impact on the solution as the mixing layer evolves, particularly once the RMI structures begin to “break down”.

The initial condition is identical to that of Hahn et al. [48], i.e., a broadband type perturbation using wavelengths in the range of $\lambda_{\min} = 0.5$ cm to $\lambda_{\max} = 5.0$ cm. For the cho-

sen grid resolution, the smallest initial material perturbation is resolved by $16\Delta x$, whereas for the finer mesh this is doubled to $32\Delta x$. In order to investigate the effect of grid resolution, we have carried out simulations on a grid with twice the number of cells in each direction, i.e. $2560 \times 640 \times 1280$ ($\sim 2.1 \times 10^9$ cells). In Fig. 24, the vortex structures resolved at times 1.90, 2.22 and 2.70 ms on the two different grid resolutions are plotted side by side for comparison purposes. In the case of the finer resolution, we find that (qualitatively) similar vortex rings structure are present as on the coarser grid, with finer details being resolved as anticipated.

On closer inspection of the fine grid results (Fig. 25), it is noticed that the larger vortex rings, which are also resolved on the coarse grid, are covered by ‘new’ smaller diameter vortex rings that either encircle the larger vortex rings, or replace the thicker vortex ring by many smaller ones. Additionally, the diameter of the larger vortex tubes is smaller on the finer grid. This is attributed to the sharper shockwave in conjunction with the grid dependent initial material interface ($3\Delta x$).

In order to further investigate the effect of grid resolution, an estimate of the effective Reynolds number is obtained. Using homogeneous decaying turbulence (HDT) in a triply periodic cube, Zhou and Thornber [89] investigated three of the most popular approaches in estimating the effective viscosity, hence the effective Reynolds number, of implicit large eddy simulations. Since Richtmyer–Meshkov induced flow can be viewed as a HDT after the passage of the shock, any of the approaches chosen is applicable in the context of the double-planar simulations carried out herein. Here, we chose to use the model based on the inner-viscous length scale [90, 91, 92, 93], for which the effective Reynolds number is given by:

$$Re_{\text{eff}} = (50\delta/\lambda_v)^{4/3} \quad (10)$$

where δ is the outer-length scale of the flow. The reason for choosing this model is twofold. Firstly, it allows for a spatial variation of the effective Reynolds number to be obtained and secondly it does not require domain-wide volume integrals quantities that are conserved. As defined in Ref. [93], the inner-viscous scale, λ_v , is the lower limit of the inertial range and can be estimated as a multiple of (but not at) the Kolmogorov length scale.

In the case of iLES, λ_v is assumed to be the smallest scale resolvable for a given numerical method and grid resolution. Zhou and Thornber [89] used a similar numerical method in their study (5th-order MUSCL with low Mach correction for spatial reconstruction, a 3rd order SSP-RK for time integration and the HLLC Riemann solver), and estimated a value for λ_v of $10\Delta x$. Since this estimate appears consistent with other findings of the code used herein (see [94]), the same value has also been adopted here. Note that the use of the low Mach correction reduces any weakly time dependence due to non-exact invariance of the numerical dissipation and the Mach number of the mixing layer.

A value for the outer-length scale, δ , is required in order to compute the effective Reynolds number (Re_{eff}). Fol-

Figure 24: Effect of grid resolution on the vortex structures resolved; interface 2 front view; first row $1280 \times 320 \times 640$, second row $2560 \times 640 \times 1280$.

Figure 25: Effect of grid resolution; zoom-in of the vortex structures resolved at interface 2 and $t = 1.90$ ms.

Figure 26: Effect of grid resolution on the estimated effective Reynolds number, Re_{eff} , at interfaces 1 (top) and 2 (bottom) at different time instances.

following Zhou and Thornber [89], this is obtained, for HDT, by taking δ equal to the integral length scale (L), which is calculated from the intercept of the longitudinal turbulence kinetic energy (TKE) spectrum:

$$L = \pi / (u''_{rms})^2 E(k = 0) \quad (11)$$

The intercept of the energy spectral density (ESD), $E(k = 0)$, is obtained according to the following procedure. First, the 2D discrete Fourier transform (DFT) of each velocity component Favre fluctuation, $u''(y, z)$, at each yz -plane along the x -direction is calculated, denoted here as $\mathcal{F}_{yz}(k_y, k_z)$. The 2D energy spectral density (ESD) is simply obtained

according to $E_{yz}(k_y, k_z) = |\mathcal{F}_{yz}(k_y, k_z)|^2$. A radial integration of the 2D-ESD is then carried out to obtain the 1D-ESD of each velocity component:

$$E(k) = \frac{2}{\pi} \int_0^{\pi/2} E_{yz}(k \cos \theta, k \sin \theta) d\theta \quad (12)$$

where $k = \sqrt{k_y^2 + k_z^2}$ and $k_{max} = 1/(2\Delta x)$. Finally, the 1D-ESD of each fluctuating velocity component are summed to obtain the 1D-TKE-ESD, $E(k)$, from which the intercept

Figure 27: Isosurfaces of Q criterion ($Q = \times 10^9 \text{ s}^{-2}$) coloured by the wall-normal vorticity ($\omega_z = \pm 7 \times 10^4 \text{ s}^{-1}$) at the inclined interface at (left) $t = 2.22 \text{ ms}$ and (right) $t = 2.70 \text{ ms}$.

at $k = 0$ is obtained by linearly interpolating the values at $k = 1$ and 2.

Similar to previous iLES studies of multi-component mixing [48, 95, 96], the present iLES is conducted assuming that the Reynolds number is high enough so that the flow is beyond the mixing transition, as defined by Dimotakis [90], in order for the effect of the Schmidt number to become important. According to experimental results [90, 97] and references therein, the mixing transition corresponds to $Re \approx 10^4$. Thus, the results shown here are applicable to high Reynolds number mixing since the effective Reynolds number is found to be of order 10^5 , as Fig. 26 indicates.

Furthermore, Zhou [93] defined a “*minimum state*” as being “the lowest Reynolds number turbulent flow where all the modes in the energy containing scales will only interact with modes in the same spectral range or those within the inertial range”. The critical Reynolds number of the minimum state is quoted as being $Re^* \approx 1 \times 10^5$. The lowest estimated effective Reynolds number found in the set of results presented herein is indeed very close to this value, and is obtained at the first interface on the coarser grid at time $t = 2.70 \text{ ms}$.

The effective Reynolds number is found to be approximately twice as large on the finer mesh, consistently across both interfaces and different time instances. Due to the dependence of the inner-viscous scale, λ_v , on the grid resolu-

tion, it is expected that Re_{eff} will scale by a factor of 2.56 approximately for every doubling of mesh resolution per direction, according to Eq. (10). The slight offset observed around this scaling factor value is attributed to differences in the integral length scale ($\delta \equiv L$) value between the two mesh resolutions, and more specifically, the obtained values of u''_{rms} and $E(k = 0)$ according to Eq. (11).

Nonetheless, it is appreciated that if the effect of finite Reynolds number is required, DNS is essential, as it has been observed for Rayleigh–Taylor mixing studies using both iLES and DNS by Youngs [96].

5.2. Inverse chevron

In the inverse-chevron (IC) case, at the second interface a large ‘penetrating’ jet forms in the centre while large bubbles develop near the top and bottom walls. The first interface is less perturbed and remains similar to the DP case. The IC inclination of the second interface may be viewed as a large-scale perturbation that heavily alters the mixing process, giving rise to a number of flow features that do not form in planar interfaces, although such features are not restricted to planar interfaces. Since the first interface remains planar, any alteration of its turbulent structures with respect to the DP case can be attributed to the inclined second interface. The interaction between the inclined material interface and the shock and rarefaction waves is expected to alter to some

Figure 28: Isosurfaces of Q criterion ($Q = \times 10^9 \text{ s}^{-2}$) coloured by the spanwise vorticity ($\omega_y = \pm 7 \times 10^4 \text{ s}^{-1}$) at the inclined interface at (left) $t = 2.22 \text{ ms}$ and (right) $t = 2.70 \text{ ms}$.

degree the turbulence and mixing characteristics of the first interface, particularly post shock.

This section is concerned primarily on the inclined material interface. Plotting the Q criterion reveals a series of counter-rotating vortex rings (CRVRs) at different critical locations of the turbulent mixing zone. Such rings have been previously observed by Dora et al. [98] to form ahead of a primary compressible vortex ring at the open end of a shock tube at sufficiently high Mach number. Earlier studies asserted that the CRVRs form because of the embedded shock strength, although experimental particle-image-velocimetry (PIV) measurements and high-order numerical simulations show that CRVRs do not form in the absence of a Mach disk in the sonic under-expanded jet behind the primary vortex ring. As observed by Rikanati et al. [99] in shockwave Mach reflection, a Kelvin–Helmholtz-type (KH-type) shear-flow instability of the slipstream, which originates from the triple point of the Mach disk and subsequent eddy pairing, is responsible for the formation of CRVRs. In addition, both the strength of the Mach disk and its duration of persistence result in an exit impulse that determines the number of CRVRs formed [98].

At the tilted interface, CRVR formation is attributed to a different physical process. First, two regions are identified where CRVRs form: The first region is at the front of the SF_6 spike tip that forms at the midstream after the first and

second reshock of the inclined interface. The second region occurs at the same time but is located near the vicinity of the (slip) wall. The study of Lundgren and Mansour [69] is perhaps more relevant here than in the DP case. Although the stability and transition to turbulence were studied for a simple incompressible 2D bounded swirling flow, this flow still demonstrates how 3D disturbances can cause counter-rotating swirls perpendicular to the axis, thereby bending the vortex ring into a periodic wave. These waves develop secondary instabilities, which form secondary vortex rings around the primary vortex ring. For transition to occur, Maxworthy [70, 71] has shown that a vortex ring becomes turbulent once the Reynolds number Γ/ν exceeds ~ 600 . In the present simulations, because the physical (kinematic) viscosity $\nu \rightarrow 0$, all such phenomena associated with transition and turbulence of vortices in high-Reynolds-number flows are expected to manifest.

At 2.22 ms, where the SF_6 mass ejects from the wall-normal midpoint, two different groups of CRVRs form, as seen in Figs. 27 and 28. The primary CRVRs are believed to be the inner, thicker vortex rings that are highlighted by the spanwise vorticity (blue and red) in Fig. 28. These are believed to form due to the KH instability arising from the velocity shear between the air and the ejecting SF_6 spike. The 3D perturbations combined with $Re \rightarrow \infty$ cause these primary vortex rings to destabilize and thus form ahead of

Figure 29: Vortex-core isosurfaces ($Q = 10^7 \text{ s}^{-2}$) coloured by the volume fraction z_1 (white for pure air, black for pure SF_6) at (left) $t = 2.22 \text{ ms}$ and (right) $t = 2.70 \text{ ms}$.

the secondary (semi) vortex rings, which are highlighted by the wall normal vorticity in Fig. 27 (blue and red).

Note that the secondary CRVRs persist in the flow longer than the primary CRVRs, which by 2.70 ms begin to transition and break up (Figs 27 & 28, right) and are visible at least until the ejecting SF_6 column impacts the solid surface. The speed and time when the SF_6 spike reaches the solid wall end is thought to be influenced early on by (i) the strength of the two primary CRVRs, (ii) the amount of turbulence and thus mixing caused by their break up and (iii) the number and strength of the secondary CRVRs that form. Both primary and secondary CRVRs are believed to entrain air in a defined orderly vortex motion that reduces the (chaotic) turbulent mixing between the two components. It is yet to be shown whether, in regions where vortex structures emerge, turbulence diffusion and thus molecular mixing are greatly over-predicted by turbulence models that assume isotropic diffusion (validity of the gradient-diffusion approximation). One of the problems with such vortex structures is that they effectively lead to regions of large TKE but low diffusion, something that cannot be effectively modelled by current turbulence models.

The majority, if not almost all, of the vortex structures, are primarily composed of the lighter of the two components (Fig. 29). In comparison with the second interface of the DP case at 2.22 ms (Fig. 17, right), the wall-normal velocity and shear forces that develop at the inclined interface between the SF_6 and air slightly increase the vortex-structure compo-

sition of the SF_6 (Fig. 29, left). Surprisingly, by 2.70 ms, the extent of the vortex structures that consist of greater amounts of SF_6 has in fact reduced (no black isosurfaces); however, more structures now consist of some SF_6 (grey) rather than none (white) as before at 2.22 ms. This is independent of the magnitude of the wall-normal velocity that develops on the turbulent vortex structures, which is relatively high in some regions and directed towards the heavier SF_6 (Fig. 30). Nonetheless, these regions do not contain more SF_6 than any of the other turbulent structures present in the IC plane.

Another interesting group of counter-rotating vortices form at the IC plane near the slip walls. These appear as “*tornadoes*” that reach the solid boundary but have a distinct feature that separates them from any other vortex structure examined thus far. They not only counter rotate, as the wall-normal vorticity clearly demonstrates in Fig. 31, but also appear to alternate material, as suggested by Fig. 29, left. This phenomenon is most peculiar and currently has no good explanation.

Although not vortex rings, these vortex tubes are also plagued by the same secondary instabilities that form during the transition to vortex-ring turbulence. Upon closer examination of the vortex tubes at 2.70 ms (see Fig. 30, right and particularly the expanded view provided in Fig. 31, right), a series of vortex rings appear to have formed and encircled the vortex tubes, which suggests that the primary vortex tubes constitute on-going transitions and will soon break up into finer-scale turbulent worm-like vortices, likely resulting

Figure 30: Isosurfaces of Q criterion ($Q = 10^7 \text{ s}^{-2}$) coloured by the wall-normal velocity ($u_z = \pm 50 \text{ m/s}$) at the inclined interface at (left) $t = 2.22 \text{ ms}$ and (right) $t = 2.70 \text{ ms}$.

Figure 31: Isosurfaces of Q criterion ($Q = 10^8 \text{ s}^{-2}$) coloured by the wall-normal vorticity ($\omega_z = \pm 5 \times 10^4 \text{ s}^{-1}$) at the inclined interface at (left) $t = 2.22 \text{ ms}$ and (right) $t = 2.70 \text{ ms}$.

in a sudden increase of local molecular mixing.

A visual comparison of (i) the turbulent structures at the first planar interface between the DP case at 2.22 ms (Fig. 15, left) and 2.70 ms (Fig. 19, top left) with (ii) the IC case at the same times (Fig. 32), reveals a surprising similarity con-

sidering the differences in the shape, structure and strength of the secondary shock and rarefaction waves. Although a slight inclination or bulging is evident in the IC case, this appears to have very little effect on the turbulent structures that have formed thus far. Later it will be shown that, for the IC

Figure 32: Isosurfaces of Q criterion ($Q = 10^8 \text{ s}^{-2}$) coloured by the x -velocity at interface 1; $t = 2.22 \text{ ms}$, $u_x \in [-30, 80] \text{ m/s}$ (left); $t = 2.70 \text{ ms}$, $u_x \in [-50, 50] \text{ m/s}$ (right).

case, the first interface begins to bulge significantly at later times, although by 2.70 ms (Fig. 32, right) some noticeable differences already appear in the upper and lower near-wall regions, particularly the negative (blue)-shift in the stream-wise velocity of the turbulent structures.

6. Conclusions

We present a preliminary iLES study to investigate the flow physics associated with Richtmyer–Meshkov-induced (RMI) mixing and to gain a better understanding of the mixing processes driving the development of a material interface width. Several vortical structures are identified to form during RM-induced mixing. Such vortices are shown to crucially affect the mixing width growth rate and are believed to affect the mixedness of the evolving interface. The 2D nature of the vortex structures, in contrast to 3D isotropic turbulence, unavoidably reduces the turbulence kinetic energy dissipation. Such vortical structures can withhold their form over a greater period and significantly reduce the molecular diffusion rate between the two fluids. In addition, the prolonged lifetime of such vortices may play a crucial role in the rapid re-allocation of materials within the mixing width and might explain the inability of turbulence models to capture the abrupt changes near the extremities of the mixing zone in some spatial profiles.

The additional deposition of baroclinic vorticity at the reshock phase is crucial in the development and strength of vortex structures. However, once such vortex structures form and the two fluids become sufficiently mixed on a molecular level, additional reshocks lead to their demise, resulting in the appearance of finer 3D worm-like turbulence structures.

Initially, the 2D structures that form appear to be composed of vortex rings that closely resemble toroidal vortex bubble rings typically encountered in immiscible fluids. It is found that these tend to be larger in diameter near the SF_6 (heavy-fluid/bubble) region of the mixing zone and smaller, but finer, near the air (light-fluid/spike) region.

The presence of these vortical structures might be responsible for the reduction in accuracy observed by turbulence models during reshocks, as most are calibrated either using Rayleigh–Taylor or single shocked Richtmyer–Meshkov induced mixing. Though such structures have been reported in other numerical studies in RTI mixing [64], it remains largely unknown what their impact is overall on the calibration and accuracy of mixing models.

The iLES data reveal that the late time reshock of the left interface causes a collapse of the bubbles and spikes formed by the initial shock. This later prevents the development of any RM instabilities and abruptly changes the self-similar growth rate of the interface, which might even

become nonlinear [100], and on which the development of most turbulence models is theoretically based on. This could potentially explain the change in the integral width exponent growth rate θ ($W \propto \tau^\theta$) observed at late time RMI mixing by Thornber [101] while investigating the impact of finite domain size, which naturally limits the growth of the large scales. Some caution however is warranted regarding the numerical nature of the results. The combined effect of the employed numerical method and grid resolution in resolving such vortical structures, particularly at late time, might impose a strict limit on how fine or small the resolved formed vortex rings can be.

The large scale perturbation of the second plane in the inverse-chevron case significantly restrains vortex rings from forming. However, the first (planar) interface only begins to be affected by the inclination of the second interface from time 2.22 ms onwards. Up until then, the developing vortex rings structure is found to be very similar to the double-planar case despite the inclination of the initial stronger reshocks to the interface. It is thought that the inclination of the second interface promotes Kelvin–Helmholtz rather than Richtmyer–Meshkov induced type mixing, which appears to impede the formation of vortex rings. Nonetheless, several distinct counter-rotating vortex ring (CRVR) groups are identified, the most important of which form in the protruding large-scale spike and the other near the wall-normal boundaries with important ramifications in turbulence modelling.

Future work should quantify the effect of the reshocks and rarefactions on the development and strength of the vortices that form, in conjunction with the time and strength of the reshock(s) and length of the mixing width at the time that the reshock occurs. Additionally, the resulting vortex structures can be examined in relation to the time of the reshock by differentiating the pre-reshock development time of the shocked interface, similarly to the experimental work carried out by Reilly et al. [102]. The angle of the material interface may also play a fundamental role in the vortex structure that develops, and can be examined by imposing a similar initial condition as that by McFarland et al. [103].

Acknowledgements

The authors would like to thank EPSRC for the computational time made available on the UK supercomputing facility ARCHER via the UK Turbulence Consortium grant (EP/L000261/1).

References

- [1] R. D. Richtmyer, Taylor instability in shock acceleration of compressible fluids, *Communications on Pure and Applied Mathematics* 13 (1960) 297–319.
- [2] E. E. Meshkov, Instability of the interface of two gases accelerated by a shock wave, *Fluid Dynamics* 4 (1972) 101–104.
- [3] J. Yang, T. Kubota, E. E. Zukoski, Applications of shock-induced mixing to supersonic combustion, *AIAA Journal* 31 (1993) 854–862.
- [4] W. D. Arnett, J. N. Bahcall, R. P. Kirshner, S. E. Woosley, Supernova 1987A, *Annual Review of Astronomy and Astrophysics* 27 (1989) 629–700.
- [5] A. S. Almgren, J. B. Bell, C. A. Rendleman, M. Zingale, Low Mach Number Modeling of Type Ia Supernovae. I. Hydrodynamics, *The Astrophysical Journal* 637 (2006) 922–936.
- [6] R. L. Holmes, G. Dimonte, B. Fryxell, M. L. Gittings, J. W. Grove, M. Schneider, D. H. Sharp, A. L. Velikovich, R. P. Weaver, Q. Zhang, Richtmyer–Meshkov instability growth: experiment, simulation and theory, *Journal of Fluid Mechanics* 389 (1999) 55–79.
- [7] P. Amendt, J. D. Colvin, R. E. Tipton, D. E. Hinkel, M. J. Edwards, O. L. Landen, J. D. Ramshaw, L. J. Suter, W. S. Varnum, R. G. Watt, Indirect-drive noncryogenic double-shell ignition targets for the National Ignition Facility: Design and analysis, *Physics of Plasmas* 9 (2002) 2221–2233.
- [8] M. Brouillette, The Richtmyer–Meshkov Instability, *Annual Review of Fluid Mechanics* 34 (2002) 445–468.
- [9] D. L. Youngs, Numerical simulation of turbulent mixing by Rayleigh–Taylor instability, *Physica D: Nonlinear Phenomena* 12 (1984) 32–44.
- [10] D. Youngs, Numerical simulation of mixing by Rayleigh–Taylor and Richtmyer–Meshkov instabilities, *Laser and Particle Beams* 12 (1994) 725–750.
- [11] D. Oron, L. Arazi, D. Kartoon, A. Rikanati, U. Alon, D. Shvarts, Dimensionality dependence of the Rayleigh–Taylor and Richtmyer–Meshkov instability late-time scaling laws, *Physics of Plasmas* 8 (2001) 2883–2889.
- [12] D. J. Hill, C. Pantano, D. I. Pullin, Large-eddy simulation and multiscale modelling of a Richtmyer–Meshkov instability with reshock, *Journal of Fluid Mechanics* 557 (2006) 29.
- [13] O. Schilling, M. Latini, High-order WENO simulations of three-dimensional reshocked Richtmyer–Meshkov instability to late times: dynamics, dependence on initial conditions, and comparisons to experimental data, *Acta Mathematica Scientia* 30 (2010) 595–620.
- [14] B. Thornber, D. Drikakis, D. L. Youngs, R. J. R. Williams, The influence of initial conditions on turbulent mixing due to Richtmyer–Meshkov instability, *Journal of Fluid Mechanics* 654 (2010) 99–139.
- [15] B. Thornber, D. Drikakis, D. L. Youngs, R. J. R. Williams, Physics of the single-shocked and reshocked Richtmyer–Meshkov instability, *Journal of Turbulence* 13 (2012) N10.
- [16] V. K. Tritschler, B. J. Olson, S. K. Lele, S. Hickel, X. Y. Hu, N. A. Adams, On the Richtmyer–Meshkov instability evolving from a deterministic multimode planar interface, *Journal of Fluid Mechanics* 755 (2014) 429–462.
- [17] N. J. Nelson, F. F. Grinstein, Effects of initial condition spectral content on shock-driven turbulent mixing, *Physical Review E* 92 (2015) 013014.
- [18] T. Oggian, D. Drikakis, D. L. Youngs, R. J. R. Williams, Computing multi-mode shock-induced compressible turbulent mixing at late times, *Journal of Fluid Mechanics* 779 (2015) 411–431.
- [19] M. Lombardini, D. J. Hill, D. I. Pullin, D. I. Meiron, Atwood ratio dependence of Richtmyer–Meshkov flows under reshock conditions using large-eddy simulations, *Journal of Fluid Mechanics* 670 (2011) 439–480.
- [20] M. Lombardini, D. I. Pullin, D. I. Meiron, Transition to turbulence in shock-driven mixing: a Mach number study, *Journal of Fluid Mechanics* 690 (2012) 203–226.
- [21] S. I. Abarzhi, Review of nonlinear dynamics of the unstable fluid interface: conservation laws and group theory, *Physica Scripta* T132 (2008) 014012.
- [22] Y. Zhou, Rayleigh–Taylor and Richtmyer–Meshkov instability induced flow, turbulence, and mixing. I, *Physics Reports* 720–722 (2017) 1–136.
- [23] Y. Zhou, Rayleigh–Taylor and Richtmyer–Meshkov instability induced flow, turbulence, and mixing. II, *Physics Reports* 723–725 (2017) 1–160.
- [24] F. F. Grinstein, A. A. Gowardhan, A. J. Wachtor, Simulations of Richtmyer–Meshkov instabilities in planar shock-tube experiments, *Physics of Fluids* 23 (2011) 034106.
- [25] J. C. R. Hunt, A. A. Wray, P. Moin, Eddies, streams, and convergence zones in turbulent flows, in: *Center for Turbulence Research*:

- Proceedings of the Summer Program, volume -1, 1988, pp. 193–208.
- [26] B. R. Morton, G. I. Taylor, J. S. Turner, Turbulent gravitational convection from maintained and instantaneous sources, *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* 234 (1956) 1–23.
- [27] J. S. Turner, G. I. Taylor, Buoyant vortex rings, *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* 239 (1957) 61–75.
- [28] J. K. Walters, J. F. Davidson, The initial motion of a gas bubble formed in an inviscid liquid part 1. the two-dimensional bubble, *Journal of Fluid Mechanics* 12 (1962) 408–416.
- [29] J. K. Walters, J. F. Davidson, The initial motion of a gas bubble formed in an inviscid liquid. Part 2. The three-dimensional bubble and the toroidal bubble, *Journal of Fluid Mechanics* 17 (1963) 321–336.
- [30] T. J. Pedley, The stability of rotating flows with a cylindrical free surface, *Journal of Fluid Mechanics* 30 (1967) 127–147.
- [31] T. J. Pedley, The toroidal bubble, *Journal of Fluid Mechanics* 32 (1968) 97–112.
- [32] T. Bonometti, J. Magnaudet, Transition from spherical cap to toroidal bubbles, *Physics of Fluids* 18 (2006) 052102.
- [33] M. Sreedhar, S. Ragab, Large eddy simulation of longitudinal stationary vortices, *Physics of Fluids* 6 (1994) 2501–2514.
- [34] J. A. Pojman, N. Bessonov, V. Volpert, M. S. Paley, Miscible fluids in microgravity (MFMG): A zero-upmass investigation on the international space station, *Microgravity - Science and Technology* 19 (2007) 33–41.
- [35] J. A. Pojman, Y. Chekanov, V. Wyatt, N. Bessonov, V. Volpert, Numerical simulations of convection induced by Korteweg stresses in a miscible polymer-monomer system: Effects of variable transport coefficients, polymerization rate and volume changes, *Microgravity Science and Technology* 21 (2009) 225–237.
- [36] J. R. Mansfield, O. M. Knio, C. Meneveau, Dynamic LES of Colliding Vortex Rings Using a 3D Vortex Method, *Journal of Computational Physics* 152 (1999) 305–345.
- [37] J. M. Faddy, D. I. Pullin, Flow structure in a model of aircraft trailing vortices, *Physics of Fluids* 17 (2005) 085106.
- [38] H. Ren, C. Xu, Three-dimensional numerical simulation of a vortex ring impacting a bump, *Theoretical and Applied Mechanics Letters* 4 (2014) 032004.
- [39] H. Ren, X. Lu, Large eddy simulation of a vortex ring impinging on a three-dimensional circular cylinder, *Theoretical and Applied Mechanics Letters* 3 (2013) 032007.
- [40] M. Shimokawa, R. Mayumi, T. Nakamura, T. Takami, H. Sakaguchi, Breakup and deformation of a droplet falling in a miscible solution, *Physical Review E* 93 (2016) 062214.
- [41] J. L. Ellzey, E. S. Oran, J. M. Picone, Interaction of a shock with a compressible vortex., *AIP Conference Proceedings* 208 (1990) 928–933.
- [42] B. Cetegen, J. Hermanson, Mixing characteristics of compressible vortex rings interacting with normal shock waves, *Combustion and Flame* 100 (1995) 232–240.
- [43] H. Zare-Behtash, K. Kontis, N. Gongora-Orozco, Experimental investigations of compressible vortex loops, *Physics of Fluids* 20 (2008) 126105.
- [44] J. L. Ellzey, M. R. Henneke, J. M. Picone, E. S. Oran, The interaction of a shock with a vortex: Shock distortion and the production of acoustic waves, *Physics of Fluids* 7 (1995) 172–184.
- [45] T. Ukai, H. Zare-Behtash, K. Kontis, S. Obayashi, Three-dimensional shock wave distortion in shock-square vortex loop interaction, *Experimental Thermal and Fluid Science* 79 (2016) 85–90.
- [46] T. Murugan, S. De, C. L. Dora, D. Das, Numerical simulation and PIV study of compressible vortex ring evolution, *Shock Waves* 22 (2012) 69–83.
- [47] G. Allaire, S. Clerc, S. Kokh, A Five-Equation Model for the Simulation of Interfaces between Compressible Fluids, *Journal of Computational Physics* 181 (2002) 577–616.
- [48] M. Hahn, D. Drikakis, D. L. Youngs, R. J. R. Williams, Richtmyer–Meshkov turbulent mixing arising from an inclined material interface with realistic surface perturbations and reshocked flow, *Physics of Fluids* 23 (2011) 046101.
- [49] F. F. Grinstein, D. Drikakis, Computing Turbulent Flow Dynamics With Implicit Large Eddy Simulation, *Journal of Fluids Engineering* 129 (2007) 1481.
- [50] F. F. Grinstein, A. J. Wachtor, J. R. Ristorcelli, Coarse Grained Simulation and Turbulent Material Mixing, in: *Whither Turbulence and Big Data in the 21st Century?*, Springer International Publishing, Cham, 2017, pp. 401–420. doi:10.1007/978-3-319-41217-7_21.
- [51] S. K. Godunov, A difference scheme for numerical computation of discontinuous solutions of equations of fluid dynamics, *Matematicheskii Sbornik* 47 (1959) 271–306.
- [52] E. F. Toro, M. Spruce, W. Speares, Restoration of the contact surface in the HLL-Riemann solver, *Shock Waves* 4 (1994) 25–34.
- [53] K. H. Kim, C. Kim, Accurate, efficient and monotonic numerical methods for multi-dimensional compressible flows, *Journal of Computational Physics* 208 (2005) 527–569.
- [54] B. Thornber, A. Mosedale, D. Drikakis, D. Youngs, R. Williams, An improved reconstruction method for compressible flows with low Mach number features, *Journal of Computational Physics* 227 (2008) 4873–4894.
- [55] S. Gottlieb, C.-W. Shu, E. Tadmor, Strong Stability-Preserving High-Order Time Discretization Methods, *SIAM Review* 43 (2001) 89–112.
- [56] D. A. Holder, A. V. Smith, C. J. Barton, D. L. Youngs, Shock-tube experiments on Richtmyer–Meshkov instability growth using an enlarged double-bump perturbation, *Laser and Particle Beams* 21 (2003) 411–418.
- [57] D. A. Holder, C. J. Barton, Shock Tube Richtmyer–Meshkov Experiments: Inverse Chevron and Half Height, in: S. B. Dalziel (Ed.), *9th International Workshop on the Physics of Compressible Turbulent Mixing*, Cambridge University Press, Cambridge, UK, 2004, p. 9.
- [58] D. L. Youngs, Three-dimensional numerical simulation of turbulent mixing by Rayleigh–Taylor instability, *Physics of Fluids A: Fluid Dynamics* 3 (1991) 1312–1320.
- [59] C. W. Barnes, S. H. Batha, A. M. Dunne, G. R. Magelssen, S. Rothman, R. D. Day, N. E. Elliott, D. A. Haynes, R. L. Holmes, J. M. Scott, D. L. Tubbs, D. L. Youngs, T. R. Boehly, P. Jaanimagi, Observation of mix in a compressible plasma in a convergent cylindrical geometry, *Physics of Plasmas* 9 (2002) 4431–4434.
- [60] M. Lombardini, D. I. Pullin, Startup process in the Richtmyer–Meshkov instability, *Physics of Fluids* 21 (2009) 044104.
- [61] J. Jeong, F. Hussain, On the identification of a vortex, *Journal of Fluid Mechanics* 285 (1995) 69.
- [62] R. Cucitore, M. Quadrio, A. Baron, On the effectiveness and limitations of local criteria for the identification of a vortex, *European Journal of Mechanics - B/Fluids* 18 (1999) 261–282.
- [63] Y. Dubief, F. Delcayre, On coherent-vortex identification in turbulence, *Journal of Turbulence* 1 (2000) N11.
- [64] N. Schneider, S. Gauthier, Visualization of Rayleigh–Taylor flows from Boussinesq approximation to fully compressible Navier–Stokes model, *Fluid Dynamics Research* 48 (2016) 015504.
- [65] A. Banerjee, M. J. Andrews, 3D Simulations to investigate initial condition effects on the growth of Rayleigh–Taylor mixing, *International Journal of Heat and Mass Transfer* 52 (2009) 3906–3917.
- [66] A. Banerjee, W. N. Kraft, M. J. Andrews, Detailed measurements of a statistically steady Rayleigh–Taylor mixing layer from small to high Atwood numbers, *Journal of Fluid Mechanics* 659 (2010) 127–190.
- [67] D. Aslangil, A. Banerjee, A. G. W. Lawrie, Numerical investigation of initial condition effects on Rayleigh–Taylor instability with acceleration reversals, *Physical Review E* 94 (2016) 053114.
- [68] B. M. Haines, F. F. Grinstein, J. D. Schwarzkopf, Reynolds-averaged Navier–Stokes initialization and benchmarking in shock-driven turbulent mixing, *Journal of Turbulence* 14 (2013) 46–70.
- [69] T. S. Lundgren, N. N. Mansour, Transition to turbulence in an elliptic vortex, *Journal of Fluid Mechanics* 307 (1996) 43–62.

- [70] T. Maxworthy, The structure and stability of vortex rings, *Journal of Fluid Mechanics* 51 (1972) 15–32.
- [71] T. Maxworthy, Turbulent vortex rings, *Journal of Fluid Mechanics* 64 (1974) 227–240.
- [72] M. Shusser, M. Gharib, A model for vortex ring formation in a starting buoyant plume, *Journal of Fluid Mechanics* 416 (2000) 173–185.
- [73] T. B. Benjamin, The alliance of practical and analytical insights into the nonlinear problems of fluid mechanics, in: P. Germain, B. Nayroles (Eds.), *Applications of Methods of Functional Analysis to Problems in Mechanics: Joint Symposium IUTAM/IMU Held in Marseille, September 1–6, 1975*, volume 503, Springer Berlin Heidelberg, 1976, pp. 8–29. doi:10.1007/BFb0088744.
- [74] M. Gharib, E. Rambod, K. Shariff, A universal time scale for vortex ring formation, *Journal of Fluid Mechanics* 360 (1998) 121–140.
- [75] T. S. Lundgren, J. Yao, N. N. Mansour, Microburst modelling and scaling, *Journal of Fluid Mechanics* 239 (1992) 461.
- [76] K. Mohseni, M. Gharib, A model for universal time scale of vortex ring formation, *Physics of Fluids* 10 (1998) 2436–2438.
- [77] N. Baumann, D. D. Joseph, P. Mohr, Y. Renardy, Vortex rings of one fluid in another in free fall, *Physics of Fluids A: Fluid Dynamics* 4 (1992) 567–580.
- [78] D. I. Pullin, Numerical studies of surface-tension effects in nonlinear Kelvin–Helmholtz and Rayleigh–Taylor instability, *Journal of Fluid Mechanics* 119 (1982) 507–532.
- [79] M. Kojima, E. J. Hinch, A. Acrivos, The formation and expansion of a toroidal drop moving in a viscous fluid, *Physics of Fluids* 27 (1984) 19.
- [80] I. Bermejo-Moreno, D. I. Pullin, On the non-local geometry of turbulence, *Journal of Fluid Mechanics* 603 (2008) 101–135.
- [81] Y. Yang, D. I. Pullin, I. Bermejo-Moreno, Multi-scale geometric analysis of Lagrangian structures in isotropic turbulence, *Journal of Fluid Mechanics* 654 (2010) 233–270.
- [82] Y. Yang, D. I. Pullin, On Lagrangian and vortex-surface fields for flows with Taylor–Green and Kida–Pelz initial conditions, *Journal of Fluid Mechanics* 661 (2010) 446–481.
- [83] Y. Yang, D. I. Pullin, Geometric study of Lagrangian and Eulerian structures in turbulent channel flow, *Journal of Fluid Mechanics* 674 (2011) 67–92.
- [84] D. I. Pullin, Y. Yang, Whither vortex tubes?, *Fluid Dynamics Research* 46 (2014) 061418.
- [85] C. R. Weber, N. S. Haehn, J. G. Oakley, D. A. Rothamer, R. Bonazza, An experimental investigation of the turbulent mixing transition in the Richtmyer–Meshkov instability, *Journal of Fluid Mechanics* 748 (2014) 457–487.
- [86] M. Shusser, M. Gharib, Energy and velocity of a forming vortex ring, *Physics of Fluids* 12 (2000) 618–621.
- [87] B. Thornber, D. Drikakis, D. L. Youngs, R. J. R. Williams, Growth of a Richtmyer–Meshkov turbulent layer after reshock, *Physics of Fluids* 23 (2011) 095107.
- [88] R. H. Cohen, W. P. Dannevik, A. M. Dimits, D. E. Eliason, A. A. Mirin, Y. Zhou, D. H. Porter, P. R. Woodward, Three-dimensional simulation of a Richtmyer–Meshkov instability with a two-scale initial perturbation, *Physics of Fluids* 14 (2002) 3692–3709.
- [89] Y. Zhou, B. Thornber, A Comparison of Three Approaches to Compute the Effective Reynolds Number of the Implicit Large-Eddy Simulations, *Journal of Fluids Engineering* 138 (2016) 070905.
- [90] P. E. Dimotakis, The mixing transition in turbulent flows, *Journal of Fluid Mechanics* 409 (2000) 69–98.
- [91] Y. Zhou, B. A. Remington, H. F. Robey, A. W. Cook, S. G. Glendinning, A. Dimits, A. C. Buckingham, G. B. Zimmerman, E. W. Burke, T. A. Peyser, W. Cabot, D. Eliason, Progress in understanding turbulent mixing induced by Rayleigh–Taylor and Richtmyer–Meshkov instabilities, *Physics of Plasmas* 10 (2003) 1883–1896.
- [92] Y. Zhou, H. F. Robey, A. C. Buckingham, Onset of turbulence in accelerated high-Reynolds-number flow, *Physical Review E* 67 (2003) 056305.
- [93] Y. Zhou, Unification and extension of the similarity scaling criteria and mixing transition for studying astrophysics using high energy density laboratory experiments or numerical simulations, *Physics of Plasmas* 14 (2007) 082701.
- [94] P. Tsoutsanis, I. W. Kokkinakis, L. Könözy, D. Drikakis, R. J. Williams, D. L. Youngs, Comparison of structured- and unstructured-grid, compressible and incompressible methods using the vortex pairing problem, *Computer Methods in Applied Mechanics and Engineering* 293 (2015) 207–231.
- [95] D. L. Youngs, The density ratio dependence of self-similar Rayleigh–Taylor mixing, *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 371 (2013) 20120173–20120173.
- [96] D. L. Youngs, Rayleigh–Taylor mixing: Direct numerical simulation and implicit large eddy simulation, *Physica Scripta* 92 (2017) 74006.
- [97] A. W. Cook, P. E. Dimotakis, Transition stages of Rayleigh–Taylor instability between miscible fluids, *Journal of Fluid Mechanics* 457 (2002) 410–411.
- [98] C. L. Dora, T. Murugan, S. De, D. Das, Role of slipstream instability in formation of counter-rotating vortex rings ahead of a compressible vortex ring, *Journal of Fluid Mechanics* 753 (2014) 29–48.
- [99] A. Rikanati, O. Sadot, G. Ben-Dor, D. Shvarts, T. Kuribayashi, K. Takayama, A secondary small-scale turbulent mixing phenomenon induced by shock-wave Mach-reflection slip-stream instability, in: K. Hannemann, F. Seiler (Eds.), *Shock Waves*, Springer Berlin Heidelberg, Berlin, Heidelberg, 2009, pp. 1347–1352. doi:10.1007/978-3-540-85181-3_89.
- [100] G. Dimonte, M. Schneider, Density ratio dependence of Rayleigh–Taylor mixing for sustained and impulsive acceleration histories, *Physics of Fluids* 12 (2000) 304–321.
- [101] B. Thornber, Impact of domain size and statistical errors in simulations of homogeneous decaying turbulence and the Richtmyer–Meshkov instability, *Physics of Fluids* 28 (2016) 045106.
- [102] D. Reilly, J. McFarland, M. Mohaghar, D. Ranjan, The effects of initial conditions and circulation deposition on the inclined-interface reshocked Richtmyer–Meshkov instability, *Experiments in Fluids* 56 (2015) 168.
- [103] J. A. McFarland, J. A. Greenough, D. Ranjan, Simulations and Analysis of the Reshocked Inclined Interface Richtmyer–Meshkov Instability for Linear and Nonlinear Interface Perturbations, *Journal of Fluids Engineering* 136 (2014) 071203.