A new static method of calibration for low-cost laser triangulation systems

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Abstract

As a non-contact high-efficiency method, laser triangulation has long been studied by many scholars. Building such a system usually requires calibration where system parameters are found or amended to achieve the targeted precision. Traditional calibration methods require using either precise movement or high precision gauge object that are accompanied with extra manufacturing cost and auxiliary equipment/devices and hence increase the overall cost as well as prevent widespread applications, for instance, due to required environment conditions to be fulfilled. A new static method was, therefore, proposed to calibrate a system without need of a motion or a high precision gauge object to be deployed. The method relies on preparation of the data on a scalar compound target with a geometric estimator as quality indicator. Based on this method, three versions of a low-cost prototype were built, which achieved sub-millimeter precision. Although the precision is sufficient for a measurement often used manufacturing large panels, higher precision is achievable with additional work which is to be undertaken in the future. The proposed method makes tradeoff between precision and cost. This was followed by testing on the measurement of a small panel part as application-case study. The proven feasibility of the method proposed suggested potential of developing low-cost, inline industrial system (large or small) for inspecting sheet parts during manufacturing, especially panel component manufacturing, including that from metal sheets and composite materials.

Keywords: measurement, inspection, laser triangulation, modelling, calibration, panel manufacturing

1. Introduction

Manufacturing of panel parts, such as car panels, architecture dome parts and aircraft skin components from metal sheets and from composite sheets, often needs to inspect the parts formed, either inline or offline, in order to quantify the forming-errors, which would form a basis for the following tooling-reconfiguration or machine re-setting to reduce or eliminate forming errors in following production or manufacturing trials. An

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Figure 1: Illustration of laser triangulation

example of such applications is forming panel components with multi-points forming tools [1, 2] where the forming errors identified through the inspection could be corrected by adjusting the heights of pin-heads, until the desired panel forms/quality are achieved. Incorporating an efficient inspection system into such a production process could speed up the tooling resetting and even realize a real, inline forming-error compensation procedure during production. Several inspection methods may be considered for developing such an inspection system [3–6] while laser triangulation is considered to be more convenient to be adopted for creating such a system for inspection of panel-type parts.

Laser triangulation is a method of obtaining range data. The basic principle is illustrated in Fig. 1 — a laser beam, a camera, and the dedicatedly designed positioning between those two are the key elements of this technique. The object, the laser, and the camera form a triangle and the depth information can be obtained via computation using the position of the laser spot captured by the camera. Based on the same principle, many versions of variations have been made in order to meet different applications.

Instead of using a single camera, binocular systems have also been developed. Davis et al. utilised mirrors to achieve a stereo image of the sampling system [7]. Trucco et al. in [8] used two cameras mainly for coverage of the shadowed area as well as for data consistency. Structured light were also used in triangulation [9] for an immediate merit that the data could be quickly retrieved in a large field. It is very common today, that gantry systems are adopted for laser triangulation measurement equipment. This kind of design has been reported in literature [10–12], where the positioning between the laser and the camera was dedicatedly maintained. Compared to static positioning, authors of the literature [13] and [14] changed the optical path of the laser by mirrors while another [7] freed the position of laser by using a stereo 3D-reconstruction. Franca et al. [15] proposed a system where some dedicated movement of both the camera and the laser source would be possible.

To explain the laser triangulation method, let point $B(x, y, z)$ be the real world coordinate of a laser spot on the surface of an object, and $B'(x', y')$ be the corresponding spot in the image captured by the camera as illustrated by Fig. 2. One solves for some of the three coordinates of B from x', y' . A suitable approach for obtaining the solution must be deployed for the specific design of a triangulation system. As shown in [16] and in the late sections of this paper, a fundamental fact is that knowing only B' is not enough for solving all unknown components of the point B . Intrinsic parameters of the camera, position of the laser and the camera, are indispensable for solving the coordinate of B . Hence, calibration, a process determining all the hidden parameters, must be conducted before a triangulation system could function correctly.

The calibration process is not trivial. Besides what was reported in [8], where the Trucco et al. proposed a direct calibration method, which essentially established a look-up table avoiding all the difficulties of determining the intrinsic parameters, the most of the work, nowadays, concern model-based calibrations. That is, trying to figure out the parameters that are needed for the solution. Existing calibration methods may be categorised mainly as either tracking a moving gauge object (as that used in the literature [8]) or tracking the movement of the triangulation system with a static reference object (as that used in [14]). A combination of both approaches can also be found in the literature [12].

These calibration methods could be serving well if one has sufficient budget for implementation. Besides the spec of the hardware, such as resolution of a camera, we propose a new calibration method which is of much lower cost for implementation by featuring a new static calibration method for a multi-laser triangulation system where auxiliary devices for motion tracking, precise gauge object are not needed, and hence it is easy to be implemented in a production environment with tight budget. Based on this method, three prototypes have already been built with use of a webcam and a CCD camera. Measuring a formed panel part was also performed to confirm the suitability of the method and the system. Sub-millimeter precision has been achieved with potential for further improvements.

2. Background

This work was part of the INNOVATE UK project AUTOMAN, which develops a die-less forming machine featuring reconfigurable multi-point forming tool [17]. The targeted markets of the formed 3D panel products vary from car manufacturing to architecture and the maximum size of the products was set to $500 \times 500\text{mm}^2$. Commercial compact laser scanners, like Micro-Epsilon 2910-100, cover a linear range around 200mm maximum, and it requires an auxiliary gantry system to cover all the range. To reduce the potential cost while making the inspection online, as a trial, it was decided to use multi-point laser triangulations for millimetre precision. For prototypes built in Strathclyde lab, the size of the product is limited to $120 \times 120\text{mm}^2$. The fixture for camera and laser source was designed with the integration of the triangulation measurement system into the multi-point tooling as the first priority and existing calibration methods can hardly be applied with reasonable cost. This leads to the development of the calibration method presented in this paper.

3. Modeling and Method

3.1. Definitions and Notations

A simplified model of a laser triangulation system is illustrated by Fig. 2. Let C be the projective center of the camera, S the screening plane for imaging. In general, S can be different from the imaging sensor inside of the camera and here it is assumed to be the sampled bitmap image. S can be regarded as that by which an affine coordinate transformation is applied to the imaging sensor. Let the distance between C and S be d . In Fig. 2, d is positive. However, there is no mathematical restriction on d other than $d \neq 0$. If d is negative the model becomes a typical pinhole camera. Use x, y, z for the real world coordinates, and x', y' the coordinates on S . For a real-world point $B(x, y, z)$, denote its image on S by $B'(x', y')$. Let $A(x_d, 0, 0)$ on the x -axis be the point such that $\overrightarrow{AC}$ is perpendicular to plane S (in some work like [18], $\overrightarrow{AC}$ is also called as *principle ray*). The intersection of $\overrightarrow{AC}$ with S , denoted by O' is the origin of the coordinate system on S . The $x'y'$ -coordinate system will be used for the image space with x' -axis, opposite to the y -axis and y' -axis parallel to $\overrightarrow{AC} \times \overrightarrow{O'A'}$.

Figure 2: Projection model

3.2. Known and Unknown Variables

In this section known and unknown variables are to be identified for later discussion. As illustrated by Fig. 2, the projective centre C is on the xOz plane. From a modelling aspect point of view, it can also be understood as that the x axis is determined by the position of the camera. Suppose a fixture to be used and this positioning is realized, the z axis can then be chosen at an arbitrary position along the x axis between A and C . Although different choices of the z axis would change x_d, x_c , the difference $x_d - x_c$, which is intrinsic as long as the laser beam and camera are fixed, remains invariant. As

will be seen later, $x_d - x_c$ is related to the estimation of the calibration quality. With the z axis determined, x_d is known, because point A can be marked on the xOy plane with the help of the camera. x_c and z_c are generally unknown as the cameras nowadays available on the market are usually encapsulated units and it is practically difficult to tell the the exact position of the projective center. Laser beams are straight lines whose equation can be written as

$$(x, y, z) = (x_0, y_0, z_0) + t(\alpha, \beta, \gamma), \text{ where } t \in \mathbb{R}. \quad (1)$$

The point (x_0, y_0, z_0) on a given laser beam is known because the position of the laser source is determined by design. The direction vector (α, β, γ) can be found out in the calibration process and hence will be regarded as known. This means that the equation of each laser beam is known. As a remark, (x, y, z) , the coordinate of a laser spot is constrained by equation (1) of the corresponding laser beam and this essentially means that it is safe for one to think two of x, y, z are known. For laser triangulation devices where the laser beam is set perpendicular to the xOy plane [19], the only unknown component is z . x' and y' are retrieved from the image by software and hence, are known. The included angle between $\overrightarrow{CA}$ and $\overrightarrow{zO}$ is known by the device design and is denoted by θ . Table 1 summarizes the discussion above.

Known parameters	Unknown parameters
$x_d, x_0, y_0, z_0, \alpha, \beta, \gamma, x', y'$	d, t, x_c, z_c, θ

Table 1: Known and unknown variables

4. The Calibration Process

This section presents the calibration method and a short discussion on how to use the calibrated device for measurement.

4.1. Assumptions

The calibration of a triangulation system consists of two parts: one is the calibration of the camera, and the other is the calibration of the triangulation system [13]. Calibration of the camera helps compensating for the non-uniformity in both axes, the non-perpendicular positioning of axes, and the cylindrical distortion. Similar to what was reported in [20] and [21], it is assumed that the camera itself is calibrated in advance. Finally, instead of using a corner, let the origin of the $x'O'y'$ system be at the centre of the captured image.

4.2. The Modelling Computation

Following symbols and notations shown in Fig. 2, the coordinate of D is calculated as

$$\begin{aligned}\overrightarrow{OD} &= \overrightarrow{OC} + \overrightarrow{CD} = (x_c, 0, z_c) + \frac{|\overrightarrow{CA}|}{|\overrightarrow{CB}|} \frac{\overrightarrow{CB}}{\cos \angle BCA} \\ &= (x_c, 0, z_c) + \frac{(x_d - x_c)^2 + z_c^2}{(x - x_c)(x_d - x_c) - z_c(z - z_c)} (x - x_c, y, z - z_c) \\ &= (x_1, y_1, z_1),\end{aligned}$$

where x_1, y_1, z_1 are placeholders and are functions of x_c, z_c, z . Now $\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD} = (x_1 - x_d, y_1, z_1)$, and it holds

$$-\frac{x'}{y_1} = \frac{d}{\sqrt{(x_d - x_c)^2 + z_c^2}} \quad (2)$$

$$\frac{y'}{\frac{z_c}{\sqrt{(x_c - x_d)^2 + z_c^2}}(x_1 - x_d) + \frac{-x_c + x_d}{\sqrt{(x_c - x_d)^2 + z_c^2}}z_1} = \frac{d}{\sqrt{(x_d - x_c)^2 + z_c^2}}. \quad (3)$$

Although equation (2) and (3) look tedious, especially with placeholders x_1, y_1, z_1 ; x' and y' are actually rational to x, y, z . That is, simplified x', y' are of the form

$$\frac{k_1x + k_2y + k_3z + k_4}{k_5x + k_6y + k_7z + k_8} \quad (4)$$

where $k_1, k_2, \dots, k_8$ are constants determined by other variables. Because the laser spot detection algorithm works on the sub-pixels, x', y' values are in real numbers.

4.3. The Calibration Procedure

The calibration consists of four steps:

STEP I: Prepare a piece of paper with grids of known dimensions and place it onto the area of a plane for the measurement. The grid paper provides readings for the points from the xOy plane. Align the camera and the grid paper so that the y' axis overlays on the x axis. This can be confirmed via the captured images from the camera.

STEP II: Find out x_d with the help of the camera. For each laser beam, find out its equation as described by (1). The coordinate of the light source can be used for the point (x_0, y_0, z_0) in equation (1), which can be obtained from the mechanical design. Then one can read the position of the corresponding laser spot on the grid paper $(\overline{x_0}, \overline{y_0}, 0)$. Then direction vector is the difference of these two points. In a multi-laser-points system, subscripts will be used to distinguish different laser beams. For the i^{th} laser beam, the following will be used for its equation

$$(x_{0,i}, y_{0,i}, z_{0,i}) + t\vec{\nu}_i,$$

where $\vec{\nu}_i = (x_{0,i}, y_{0,i}, z_{0,i}) - (\overline{x_{0,i}}, \overline{y_{0,i}}, 0)$ denotes the direction vector.

STEP III: Find several points P_i on the xOy plane. For point P_i record its coordinate (x_i, y_i) and its coordinate on the image (x'_i, y'_i) . The recorded coordinates of the laser spot in STEP II can be used for this step, and in this case one needs to find out the coordinates on the image.

STEP IV: With all the data obtained from STEP I to III conduct the calibration method in the following section to find x_c, z_c, d .

4.4. The Calibration Method

For points on the xOy plane the followings can be derived from equations (2) and (3)

$$\begin{aligned} x' &= x'(x, y, 0) = -\frac{\sqrt{(x_d - x_c)^2 + z_c^2}}{(x - x_c)(x_d - x_c) + z_c^2} d y \\ y' &= y'(x, y, 0) = \frac{(x - x_d)}{x_c^2 - x_c x_d + x(x_d - x_c) + z_c^2} d z_c. \end{aligned}$$

These are for the points on the grid paper. With the data set (x_i, y_i, x'_i, y'_i) obtained in STEP III, nonlinear regression can be conducted to find out the hidden variables x_c, z_c, d . However, with reading errors the fitting outputs are usually of poor quality. Instead we propose to use the model

$$f(x, y) = \lambda x' + y',$$

where $\lambda \in \mathbb{R}$ is a constant. Various fitting methods could be used once a λ is chosen. For example, the Levenberg-Marquardt method is easy to implement and can be found in many software toolboxes, like MATLAB or Mathematica. To estimate the appropriateness of λ and the quality of the result, use the geometric positioning constraint:

$$z_c \tan \theta = x_d - x_c. \quad (5)$$

Let $\delta = |z_c \tan \theta - (x_d - x_c)|$ and ideally it should be zero. Pragmatically, one might try different λ to minimize δ . As a remark, generally it is impossible to make δ zero; and it is safe to stop the process if δ is within the range of the mechanical error.

4.5. Measurement

After calibration, the numerical estimations of x_c, z_c, d are obtained, and so are all $k_1, \dots, k_8$ in (4) for both $x' = x'(x, y, z)$ and $y' = y'(x, y, z)$. Now the device is ready for making measurements. Assume the position of the i^{th} laser spot is identified to be (x'_i, y'_i) on the image with some proper algorithm. To find out the real-world coordinate of the laser spot, one needs to solve for x_i, y_i, z_i using x'_i, y'_i . In the equation of the i^{th} laser beam

$$(x, y, z) = (x_{0,i}, y_{0,i}, z_{0,i}) + t \vec{V}_i,$$

x, y, z are functions of t and in the equations (2) and (3), $x' = x'(x, y, z)$ and $y' = y'(x, y, z)$ are also functions of t . Then one finds t_0 minimizing

$$(x' - x'_i)^2 + (y' - y'_i)^2.$$

Using the laser beam equation and t_0 , the coordinates x_i, y_i and z_i can be computed. As a remark, the laser beam is generally not perpendicular to the base surface, thus instead of saying the height of the spot, the word “position” is more appropriate.

4.6. Comments on Laser Spot Detection

To detect the position of the laser spot on the obtained image from the camera, a window based search algorithm was implemented for both simplicity and performance purpose. The coordinate of the laser spot changes as the underlying object surface changes and the movement is described by equations (2) (3). In the laboratory configuration, the change in x' is insignificant. Therefore the search windows are in the form of portrait rectangles. To speed up the process, the rectangles are encoded with lookup tables prioritizing the vertical direction (the y' direction) and then symmetrically spread to horizontal sides. The position of laser is calculated using its geometric center. Because cheap LEDs are non-uniform in luminance, every laser has its own configuration which contains thresholds for red, green, and blue channels respectively.

5. Examples and Real World Tests

5.1. A Numerical Example

Given a camera with parameters $d = 3$, $z_c = 10$, $x_c = -2$, $\theta = 45^\circ$, then by equation (5) it can be found out that $x_d = 8$. Suppose in a calibration process, one reads $\tilde{x}_d$ to be 8.2, that is, with 0.2 as an error. In STEP III, nine points are obtained and the data is summarized in Table 2. For testing purpose random errors up to 5% is added to the readings on image. With $\lambda = 10$ the result is found to be:

$$\tilde{x}_c \approx -1.842 \quad \tilde{z}_c \approx 10.799 \quad \tilde{d} \approx 3.099$$

As can be seen from the result, the first measurement error in $\tilde{x}_d$ is partially compensated for by $\tilde{x}_c$. This means that the difference $x_d - x_c$ in equations (2) and (3) are also stabilized.

x_i, y_i	x'_i, y'_i	x_i, y_i	x'_i, y'_i
(7,-2)	(0.443676,-0.165689)	(7,0)	(-0.00495479,-0.133413)
(7,2)	(-0.445904,-0.157642)	(8,-2)	(0.436016,-0.0249103)
(8,0)	(-0.00377886,0.0124215)	(8,2)	(-0.41833,-0.00190769)
(9,-2)	(0.409582,0.133562)	(9,0)	(-0.00270297,0.162003)
(9,2)	(-0.399135,0.135805)		

Table 2: Data with error relatively up to 5%

5.2. Initial Prototypes

Two prototypes of multi-laser triangulation systems were built initially and calibrated using the method presented in this paper, and Fig. 3(a) and (b) illustrate 3D model of the first and second prototype (low-cost systems) respectively: the first one had 4 laser beams and the second features a flexible configuration initially with 9 laser beams (a maximum of 25 laser beams possible). Both prototypes were built for testing; the second one uses a SONY XCG-C130 CCD camera (Part 3 in Fig. 3(b)), replacing Logitech C310 webcam used in the first one (Part 3 in Fig. 3(a)), with improved laser source replacing laser pointers (Part 5 in Fig. 3(a)) by laser LED diodes (Part 5 in Fig. 3 (b)). Both prototypes feature a rotating structure (Part 2) for angular adjustment, and a marked

base plate (Part 1) for positioning the grid paper for calibration. Part 4 of both designs are fixtures of the laser sources. The second prototype has lens (Part 6 in Fig. 3(b)) attached to the camera, which gives the user an ability to adjust the field of view and focus. Fig. 3(c) is a captured image after the calibration STEP I, using the second second prototype built. It shows clearly that the laser beam is not perpendicular to the base surface. Fig. 3(d) is a captured image from the software in a real measurement of a metal workpiece. For presentation purpose the brightness of the captured image is adjusted. After calibration, the second prototype was tested together with use of a 10mm gauge block, and the result showed sub-millimeter precision achieved in all nine laser beams.

Figure 3: First two prototypes and readings (a). Prototype with 4 laser pointers; (b). Prototype with 9 laser LED diodes; (c). Calibration STEP I; (d). Measuring trial on a metal part

5.3. Testing System for Panel Measurement

An updated version of the second prototype was built with a view to testing it for panel measurement, associated with the development of a multi-points tooling sheet

forming system reported in [22]. Fig. 4 is an overview of the 25-laser LED prototype with all lasers on it and also shows the screenshots of the software developed. The calibration process used 4 points (Table. 3). In the design, x_d was set to 223.5mm and θ to $\pi/4$ as the nominal values. At $\lambda \approx 0.009$, δ gets close to its minimum value 0.00563391 and the fitting results are

$$x_c \approx -16.7932\text{mm} \quad z_c \approx 240.288\text{mm} \quad d \approx 2437.67\text{mm}$$

A test against a 10mm gauge block shows that all 25 laser beams provide measurement at sub-millimeter precision (Fig. 5).

Figure 4: System test for metal gauge block measurement

x_i, y_i (mm)	x'_i, y'_i	x_i, y_i (mm)	x'_i, y'_i
(287, 70.7)	(-451, 284)	(287, -69.3)	(440, 285)
(157, 70.7)	(-595, -391)	(157, -59.3)	(504, -392)

Table 3: 4 point calibration

Fig. 6 presents photos showing the sheet-forming tool (Fig. 6(a)) used for forming sheet metal panel parts and inspection setup with 25 laser LED diodes (Fig. 6(b)). A bent sheet metal parts was initially used to test the inspection system (Fig. 6(c)). After a part is formed through multi-points tooling, the upper part of the tooling was lifted up to leave a gap allowing the inspection frame to be moved in for the inspection. The frame was designed for an open structure for easy operation, e.g. the whole frame laterally slides in, or the laser-holding plate is rotated, in order to position the measurement frame correctly under the upper tooling and above the panel part or lower tooling.

Fig. 7(a) shows a sample part measured using the testing system. The coordinates of the sampled data are given in Table. 4. The data was imported to the Mathematica

Figure 5: Errors against the 10mm gauge block measurement

Figure 6: Forming tooling and inspection set-up (a). Forming tool system; (b) Inspection set-up. (c) Test on the measurement of a bent sheet metal part

software for visualization as shown in Fig. 7(b). During the sampling process, the brightness was adjusted manually (Fig. 8). The panel was later examined by a CMM machine, and the data was imported to the Mathematica software and visualized (Fig. 7(c)).

5.4. Further Analysis

The estimated covariance matrix of the parameters is computed in this section. Let $\vec{p} = (x_c, z_c, d)^T$. Then on the plane of $z = 0$,

$$y' = y'(x, y; \vec{p}) = y'(x, y, 0) = \frac{(x - x_d)}{x_c^2 - x_c x_d + x(x_d - x_c) + z_c^2} d z_c.$$

Figure 7: Sample metal sheet part and the result

(280.8, 55.1, 2.8)	(281.8, 30.0, 3.0)	(280.7, 4.3, 4.2)	(278.8, -25.1, 3.5)	(279.9, -59.9, 2.3)
(245.0, 53.1, 5.3)	(247.6, 26.2, 7.8)	(242.7, -6.1, 8.2)	(250.1, -33.3, 6.7)	(246.5, -63.3, 3.2)
(222.3, 57.5, 4.1)	(226.8, 22.5, 8.1)	(223.7, -8.1, 8.6)	(222.0, -29.6, 7.4)	(215.8, -57.8, 4.3)
(190.2, 52.5, 4.4)	(189.4, 33.7, 7.7)	(187.6, -3.6, 8.0)	(186.8, -30.0, 7.8)	(178.0, -62.7, 3.7)
(171.1, 58.0, 3.7)	(163.7, 27.2, 4.0)	(162.9, -1.2, 4.1)	(161.5, -33.8, 3.2)	(159.0, -54.7, 2.8)

Table 4: Sampled points (mm)

For samples $(y'_i, x_i, y_i), i = 1, \dots, n$, the Taylor expansion is

$$\vec{Y}(\vec{p}) = (y'_1 \dots y'_n)^T \approx \vec{Y}(\vec{p}_*) + J(\vec{p} - \vec{p}_*)$$

where $\vec{p}_*$ is the best parameter found. Let J denote the Jacobian matrix

$$J = \begin{pmatrix} \frac{\partial y'_1}{\partial x_c} & \frac{\partial y'_1}{\partial z_c} & \frac{\partial y'_1}{\partial d} \\ \vdots & \vdots & \vdots \\ \frac{\partial y'_n}{\partial x_c} & \frac{\partial y'_n}{\partial z_c} & \frac{\partial y'_n}{\partial d} \end{pmatrix}_{n \times 3}.$$

Let the variance of the error be σ^2 , then V , the covariance matrix of $\vec{p}$, is

$$V = \sigma^2 (J^T J)^\dagger$$

Figure 8: Brightness adjusted

where $M^\dagger$ is the Moore-Penrose inverse of the matrix M [23][24]. In the example of 5.3, $\sigma^2 \approx 1.0006$ and the covariance matrix of $(x_c, z_c, d)^T$ is

$$V \approx \begin{pmatrix} 0.150136 & 0.454702 & 0.0148004 \\ 0.454702 & 7.09249 & 0.0448377 \\ 0.0148004 & 0.0448377 & 0.00145902 \end{pmatrix}.$$

Although in 5.3 the choice of λ was 0.009, which seems to be irrelevant, the importance of λ may be explained as follows: when λ is 0, the calibration only uses y'_i and ignores x'_i . In this case, the same data gives the estimation

$$x_c \approx 82.6198\text{mm} \quad z_c \approx 218.733\text{mm} \quad d \approx 1569.76\text{mm},$$

and $x_d - x_c \approx 140.88\text{mm}$. This estimation undermines the underlying model because it implies $\theta \approx 32.7^\circ$ which is far away from the nominal 45° . Thus a calibration process without consideration of λ is equivalent to that ignoring the geometric condition and generally gives unsatisfactory measurement results in low-cost systems where the assembly error must be considered.

6. Discussion

The precision of the final result after calibration does depend on many aspects of the system. Camera resolution, size of laser spot, mechanical errors and the system components and assembly are the most significant factors. The laser LEDs used in the low-cost systems built have large spot sizes. With the help of the grid paper, the diameters of the laser spots are found to be varying between about 1.8mm and 3.6mm and the area between about 2.9mm² and 9.1mm². Angular misalignment caused by the assembly error is clearly observable in Fig. 3(c) and by a simple calculation the included angle ψ (Fig. 9) between the actual laser beam and the z axis, which is the nominal direction, is up to 4.5° . For each laser source, (x_0, y_0, z_0) is intentionally chosen to be A , the centre of the fixture hole. A rough computation shows that the maximum error in each $x_{0,i}, y_{0,i}$ is up to 0.78mm. The spatial direction of ψ varies with no obvious bias on the xOy plane which means that the supporting plate of the laser LEDs can be regarded as horizontal. Thus, comparing to the error in $x_{0,i}, y_{0,i}$ the error in $z_{0,i}$ is relatively quite small.

Figure 9: Dimension of the LED installation hole and illustration of angular misalignment

Besides the hardware which might be limited by some reasons such as budgets, it is worth pointing out that the choice of points in STEP III affects the calibration result and hence, the precision of the system. In section 5.3, it is shown that the system is able to achieve a sub-millimeter precision by only using 4 points in the STEP III during the calibration. More points can be used for the calibration process. Table 5 contains 25 points that was prepared in the same calibration process presented in section 5.3. Taking $x_d = 223.5$ and $\theta = \pi/4$ as before, the minimal value of $\delta \approx 0.00708181$ is achieved at around $\lambda \approx 0.163$, and in this case $x_c \approx -27.3368$, $z_c \approx 250.844$, and $d \approx 2546.63$. Generally, larger δ can be interpreted as a larger angular misalignment of the camera.

x_i, y_i (mm)	x'_i, y'_i	x_i, y_i (mm)	x'_i, y'_i	x_i, y_i (mm)	x'_i, y'_i
(281, 55.2)	(-359, 261)	(282, 30.2)	(-197, 266)	(281, 4.7)	(-32, 263)
(279, -24.8)	(158, 254)	(280, -59.5)	(382, 259)	(245, 53.2)	(-370, 110)
(247.7, 26.4)	(-184, 119)	(242.5, -6.2)	(43, 95)	(250.4, -33.3)	(235, 132)
(246.5, -63.3)	(435, 114)	(222.5, 57.7)	(-421, 1)	(227.5, 22.4)	(-162, 22)
(224.2, -8.3)	(60, 5)	(222.3, -29.3)	(215, -4)	(215.8, -57.6)	(425, -36)
(190.3, 52.5)	(-409, -180)	(189.5, 24.4)	(-192, -183)	(187.5, -3.5)	(28, -193)
(186.7, -29.8)	(235, -199)	(177.6, -62.7)	(506, -255)	(171.5, 58.2)	(-474, -294)
(163.8, 27.2)	(-226, -346)	(163, -1.1)	(11, -353)	(161.5, -33.8)	(284, -362)
(159, -54.5)	(462, -379)				

Table 5: 25 point calibration

7. Conclusions and Future Work

A new method of measurement with multi-laser-beams has been developed, being enabled with novel calibration of the laser triangulation systems. The fitting model combines two components of the obtained coordinates and utilizes the geometric constraints as an estimator. The laboratory prototypes have validated this method and the low-cost realization has achieved sub-millimeter precision.

The method avoids the use of precise movements of both camera and the reference object and hence, greatly reduces the cost of a triangulation system for measure-

ment/inspection. As such, it is of great potential for popular uses when low-cost measuring/inspection systems are to be introduced. Potential applications include that for sheet-metal forming, and panel manufacturing in general.

The calibration method supports the design of multi-point forming system like one that was developed in the AUTOMAN project where the underlying triangulation system can be extended with little extra cost. For example to cover the $500 \times 500\text{mm}^2$ with 2500 points, the additional cost on LEDs and peripheral electronics could be less than 3% of a standard high-precision triangulation system with no extra cost on the calibration process.

Further work will be carried out on the analysis using different fitting methods, choosing or giving suggestions on more appropriate λ (ref subsection 4.4). Detailed error analysis and uncertainty budget will be the next important work in order to improve the overall precision.

8. Acknowledgements

The authors would like to acknowledge the support from INNOVATE UK and EPSRC on their support to the reported research through AUTOMAN project.

Author's Contributions

Y. Zhao: Proposed the method of calibration and design of the laboratory prototypes, developed the software and conducted the tests, and drafted the manuscript.

T. Mohamed: Conducted the design of early laboratory prototypes and worked with Y. Zhao on the integration into the multi-point tooling forming system.

S. Yang: Provided multi-point tooling formed samples for measurement tests.

Y. Qin: Project supervisor.

Conflict of Interest

All authors have no conflict of interest to report.

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