

# General Perturbation Method for Satellite Constellation Deployment using Nodal Precession

Ciara N. McGrath\* and Malcolm Macdonald†  
*University of Strathclyde, Glasgow, United Kingdom, G1 1XJ*

## Nomenclature

$a$	=	mean semi-major axis, km
$A_{prop}$	=	propulsion system acceleration, $m/s^2$
$F$	=	thrust, N
$h$	=	altitude with respect to mean Earth radius, km
$i$	=	inclination, deg
$J_2$	=	coefficient of the Earth's gravitational zonal harmonic of the 2nd degree, —
$m_{dry}$	=	satellite dry mass, kg
$m_p$	=	propellant mass, kg
$R_e$	=	mean Earth radius, km
$t$	=	time, sec
$u$	=	argument of latitude, deg
$\Delta V$	=	change in velocity, m/s
$\mu$	=	standard gravitational parameter of Earth, $m^3/s^2$
$\Omega$	=	right ascension of the ascending node, deg
$\omega_e$	=	angular velocity of Earth, rad/s

## I. Introduction

**T**HE dawn of “New Space” in recent years is changing the landscape of the space industry. In particular, the shift to smaller satellites, requiring shorter development times and using off-the-shelf-components and standardized buses, has led to a continuing reduction in spacecraft cost [1, 2]. However, launch costs remain extremely high and frequently dominate the total mission cost. Additionally, many small satellites are designed to operate as part of a larger constellation, but traditional launch methods require a different dedicated launch for each orbit plane to be populated. This need for multiple costly launches can stifle, and even prohibit, some missions requiring numerous orbit planes as

---

\*Research Associate, Department of Mechanical and Aerospace Engineering, 75 Montrose Street, Glasgow, G1 1XJ, AIAA Member.

†Professor, Department of Mechanical and Aerospace Engineering, 75 Montrose Street, Glasgow, G1 1XJ, and AIAA Associate Fellow.

the launch cost increases beyond what can be justified for the mission. As of 2014, most Smallsats, including CubeSats, have been launched on opportunistic ‘rideshare’ or ‘piggy-back’ launches [3], in which the spacecraft shares its launch with other craft, often as a secondary payload. This has the advantage of providing a cheaper launch but restricts the operator’s choice of orbit [4], which will affect the system performance.

An alternative deployment strategy, patented in 1993 [5], proposed launching all of the satellites in a constellation into a single orbit plane and then maneuvering them to different altitudes, using the perturbing effects caused by the Earth’s non-spherical gravitational field to achieve the desired separation through the right ascension of the ascending node. The FORMOSAT-3/COSMIC mission successfully demonstrated this method in 2006 [6, 7], and the upcoming FORMOSAT-7/COSMIC-2 mission will also use this deployment technique [8, 9]. The FORMOSAT-3/COSMIC satellites were deployed by launching them into a low altitude orbit and exploiting the natural perturbation of the Earth’s non-spherical gravitational field to produce the desired orbit plane separation. They were then raised to their target altitude using low-thrust propulsion, with the maneuvers timed to obtain the desired spacing between the satellites. This method requires a relatively small amount of propellant compared with traditional high-thrust plane-change maneuvers but comes at the expense of requiring a longer deployment time. However, it may be possible to provide a partial service during the deployment phase, depending on the mission requirements.

Previous work investigating the efficient deployment of satellite constellations from a single launch has tended to focus on the use of differential drag techniques [10, 11]. For satellites with high-thrust propulsion capabilities, Baranov et al. [12] propose the use of impulsive maneuvers to enter and exit an intermediate orbit that can be used to obtain the desired satellite spacing. Crisp et al. [4] considered the use of low-thrust propulsion for deployment by differential nodal precession. This work considered satellites in circular orbits and proposed using in-plane, low-thrust maneuvers to change the satellite altitude, creating a difference in the orbit period and nodal drift rate of the satellites. The authors considered the effect of the Earth’s first gravitational zonal harmonic, also known as  $J_2$ , as well as atmospheric drag, and were able to accurately recreate the deployment of the FORMOSAT-3/COSMIC constellation. However, they used a semi-analytical orbit propagator to perform the analysis, limiting the general insights that could be obtained. Jenkins et al. [13] considered the deployment of a constellation of CubeSats equipped with low-thrust propulsion using a similar technique as that used by Crisp et al. They modeled the maneuvers using a variation of parameters method employing Gauss’ planetary equations and included the influence of both  $J_2$  and atmospheric drag. Integrating these equations over time and applying a differential evolution algorithm allowed them to find the minimum time and minimum propellant solutions to separate two spacecraft through 180 deg right ascension of the ascending node. However, this work assumed that only one of the two spacecraft would maneuver, and that this spacecraft would raise its altitude. This does not consider the possibility of one spacecraft lowering its altitude, or both spacecraft maneuvering simultaneously, meaning the results are specific to the case presented and provide little insight into the broader problem.

The long time period required to deploy a satellite constellation using nodal precession and the high cost associated

with launching satellite constellations using traditional launch methods means mission designers must consider a high stakes trade-off between launch cost and deployment time. Additionally, the launch strategy chosen will have inherent implications for the cost of operations, system reliability, mission risk, and financial returns. For example, the mission designer may wish to consider a staged deployment of their system to generate initial revenue, whilst enabling them to adapt to unexpected variation in consumer demand [14]. Informed consideration of such complex scenarios requires insights that would be challenging to obtain using numerical analysis methods without significant computational effort and time commitment. Thus, a method that can be rapidly solved to explore the full solution space and provide insights into the interplay among the deployment maneuvers selected for each satellite would be extremely valuable. To this end, this note extends the general perturbation method developed in [15] and applies it to the challenge of constellation deployment using nodal precession. The previously developed method provides a fully-analytical solution to a restricted Lambert rendezvous problem that considers low-thrust, circular-to-circular, coplanar transfers using tangential thrust, and includes secular central body perturbations of the first zonal harmonic,  $J_2$ ; it is of note that it would be straightforward to include the secular effects of higher order central body disturbance terms, were it deemed desirable. The effect of other disturbing perturbations are not considered, with the exception of atmospheric drag, which is indirectly accounted for by assuming that atmospheric drag compensation maneuvers are used to maintain a constant altitude during any phases in which no active altitude changing maneuvers are performed. By considering the perturbing effects of only  $J_2$  and drag, this solution is applicable for satellites in low-Earth orbit regimes where these are the dominant perturbations. Additionally, the spacecraft are assumed to maintain a circular orbit and constant inclination throughout the maneuvers. These assumptions are supported by the findings of Refs. [13], which concluded that out-of-plane thrusting was significantly less efficient than in-plane thrusting and hence unsuitable for the deployment of a CubeSat constellation. These simplifications allow for a fully analytical solution to be formed, enabling insights into the broader challenge of constellation deployment using differential nodal precession to be considered, including the effectiveness of both altitude-lowering and -raising maneuvers, the effect of inclination on the maneuver, the use of simultaneous maneuvers, and the corresponding propellant distribution between spacecraft.

## II. Method

The general perturbation method presented in [15] provides analytical expressions for the right ascension of the ascending node (RAAN) and argument of latitude (AoL) of a satellite after it has performed a so-called 3-phase maneuver. This maneuver consists of an initial thrusting phase in which the satellite increases or decreases its altitude relative to its initial orbit, referred to as phase 1. This altitude change is performed using continuous low-thrust with constant acceleration. During the second phase, referred to as phase 2, the satellite is assumed to maintain a constant altitude by thrusting to counteract the effect of atmospheric drag. In phase 3, the satellite moves to the target altitude using continuous low-thrust with the same constant acceleration as in phase 1. Although all applied thrust is in-plane,

by moving the satellites to different altitudes the natural perturbing force of the oblate central body and the change in the spacecraft orbit period can be exploited to change the satellite RAAN and AoL. The analytical expressions for the RAAN and AoL of a satellite post-maneuver are expressed in [15] as a function of the satellite's initial orbit parameters, the final altitude of the satellite, the change in velocity required for the maneuver,  $\Delta V_{total}$ , and the time required for the maneuver,  $t_{total}$ . Assuming that the initial orbit parameters and final altitude are known, this gives

$$\Omega_{total} = f(\Delta V_{total}, t_{total}) \quad (1)$$

$$u_{total} = f(\Delta V_{total}, t_{total}) \quad (2)$$

where  $\Omega_{total}$  and  $u_{total}$  are the RAAN and AoL of the satellite, respectively, at the end of the maneuver.

The total change in velocity required for the maneuver,  $\Delta V_{total}$ , includes the amount required to change the satellite altitude in phase 1,  $\Delta V_1$ , and phase 3,  $\Delta V_3$ , as well as that required for atmospheric drag compensation during phase 2,  $\Delta V_{drag}$ . This gives

$$\Delta V_{total} = \Delta V_{alt} + \Delta V_{drag} \quad (3)$$

where  $\Delta V_{alt} = \Delta V_1 + \Delta V_3$ . The required  $\Delta V_{drag}$  is calculated as described in [15] assuming a static atmosphere with an exponential atmospheric density model implemented using the CIRA-72 atmospheric model [16]. The effect on the atmospheric density of varying solar activity is not considered. These assumptions will have a limited effect on the accuracy of the analysis for shorter maneuvers but may become significant for longer maneuvers. In particular, excluding atmospheric rotation means that the difference in drag effects for differently inclined orbits is not captured [17]. Additionally, the solar cycle can have a significant effect on the atmospheric drag experienced by the spacecraft by changing the atmospheric density as discussed in Ref. [18]. However, the 22 year solar cycle is significantly longer than the time required for the maneuvers considered within this note and, as such, the variation in density experienced during a maneuver is expected to be small. Including a more accurate atmospheric density model could improve the accuracy of the presented method; indeed, the method presented by Kerr and Macdonald in Ref. [18] enabled the effect of the solar cycle to be included in an analytical atmospheric density model and could be incorporated if desired.

If the 3-phase maneuver is to be used to deploy a constellation of satellites, it is necessary to consider the separation through RAAN and AoL that can be obtained between the satellites. This is simply the difference between the RAAN or AoL of the satellites post-maneuver and is described as

$$\Omega_{dif} = \Omega_{totalA} - \Omega_{totalB} \quad (4)$$

$$u_{dif} = u_{totalA} - u_{totalB} \quad (5)$$

where the subscripts *totalA* and *totalB* denote the value for satellite A or B at the end of the maneuver, respectively. If only one satellite is to maneuver, then the  $\Delta V_{alt}$  of the non-maneuvering satellite can be fixed as zero and the natural change in RAAN and AoL, including the effects of  $J_2$ , will still be captured.

Applying the general perturbation method to the deployment of a constellation of satellites using nodal precession requires consideration of a number of unique factors. These factors include whether moving the satellite through a positive or negative RAAN or AoL angle will be more efficient in a given scenario, the interplay between the change in RAAN and AoL that will occur during the maneuver, and how to best balance the  $\Delta V$  used across multiple maneuvering satellites. These factors are investigated in the remainder of this section for a spacecraft with a propulsive acceleration of  $\pm 1.1667 \times 10^{-4} \text{m/s}^2$ . This acceleration is based on a 3 kg spacecraft (e.g., a 3U CubeSat [19, 20]) equipped with the electrospray propulsion system, developed by the Space Propulsion Laboratory of the Massachusetts Institute of Technology (MIT), which produces a nominal thrust of  $350 \mu\text{N}$  [21, 22]. For all cases considered in this section it is assumed that the maneuvering satellites begin and end their maneuvers at the same altitude, although this need not be the case and as long as both the initial and final altitude of the spacecraft are specified there will be no change to the formulation of the presented solution. The parameters given in Table 1 are used for all analyses throughout this note and orbit parameters are selected arbitrarily to provide illustrative examples of the technique, unless otherwise stated.

**Table 1 Analysis parameters.**

Parameter	Symbol	Value	Units
Gravitational Parameter	$\mu$	$3.986 \times 10^{14}$	$\text{m}^3/\text{s}^2$
Radius of Earth	$R_e$	$6.371 \times 10^3$	km
Coefficient of $J_2$ for Earth	$J_2$	$1.0827 \times 10^{-3}$	—
Angular velocity of Earth	$\omega_e$	$7.2921 \times 10^{-5}$	rad/s

### A. Achieving a Positive or Negative Change in RAAN and AoL

Considering the change in RAAN and AoL that can be achieved by a maneuvering satellite relative to a non-maneuvering reference satellite with the same initial orbit parameters provides an insight into the effectiveness of the proposed maneuver technique. By using the 3-phase maneuver to either lower or raise the satellite altitude, a positive or negative rate of change of RAAN and AoL relative to a non-maneuvering reference satellite can be obtained, depending on the orbit parameters. Figure 1 (a) shows the RAAN separation,  $\Omega_{dif}$ , obtained between a maneuvering satellite and a

non-maneuvering reference, both at an initial mean altitude of 595 km. The results presented consider a 7 day maneuver using a  $\Delta V_{alt}$  of 30 m/s (i.e.,  $\Delta V_1 = \Delta V_3 = 15$  m/s) over a range of orbit inclinations for both an altitude-lowering and an altitude-raising maneuver. The time and change in velocity considered for this example are selected arbitrarily to demonstrate the trends seen in the maneuver performance as the orbit parameters are varied. Note that RAAN is undefined at 0 deg and 180 deg inclination and so the values for these inclinations should be disregarded. These results show that when using an altitude-lowering maneuver for prograde orbits the relative RAAN change of the maneuvering satellite will be negative, while for retrograde orbits the change in RAAN will be positive. The opposite is true for a satellite-raising maneuver. Also clear is that as the orbit inclination approaches 90 deg the RAAN separation approaches zero. This result is as expected considering the Lagrange planetary equations for rate of change of RAAN with only the secular effects of  $J_2$  for a circular orbit. Taking the expressions given in Ref. [23] and simplifying as required gives

$$\frac{d\Omega}{dt} = -\frac{3\sqrt{\mu}R_e^2J_2}{2a^{\frac{7}{2}}}\cos(i) \quad (6)$$

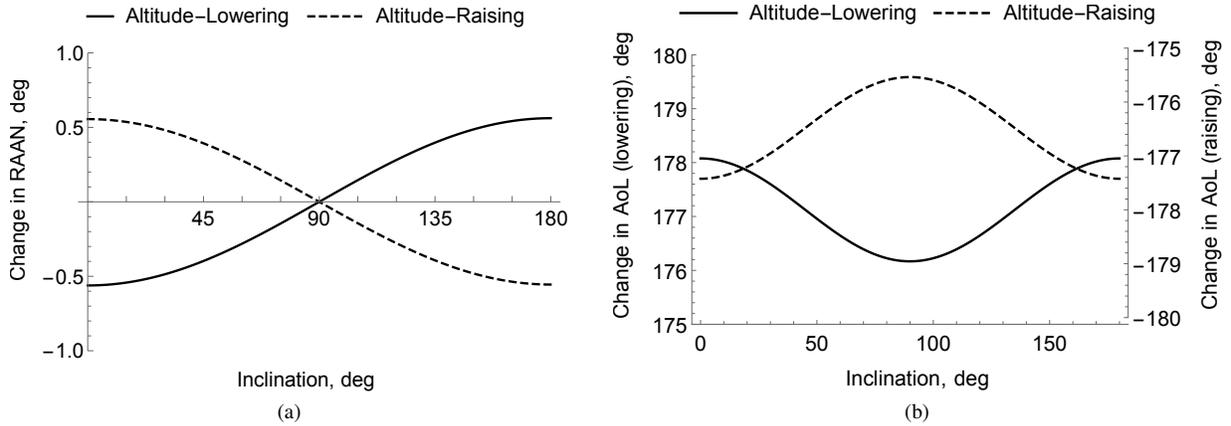
The presence of the  $\cos(i)$  term in equation (6) defines this behaviour in which the achievable change in RAAN will decrease as the orbit inclination approaches 90 deg.

Figure 1 (b) shows a similar analysis considering the change in AoL. Contrary to the RAAN, the AoL change will be positive for both prograde and retrograde orbits for an altitude-lowering maneuver, while for an altitude-raising maneuver the change in AoL will be negative for all inclinations. Additionally, the orbit inclination has relatively little effect on the change in AoL obtained as the influence of the Earth's oblateness is relatively small in comparison to the effect of the change in orbit period. This, again, is as expected when considering the Lagrange planetary equations for rate of change of AoL with only the secular effects of  $J_2$  for a circular orbit. Taking the expressions given in Ref. [15] and simplifying as required gives

$$\frac{du}{dt} = \sqrt{\frac{\mu}{a^3}} + \frac{3\sqrt{\mu}R_e^2J_2}{4a^{\frac{7}{2}}}(4 - 5\sin^2(i)) \quad (7)$$

From (7), it can be seen that the change in orbit period, described by the  $\sqrt{\frac{\mu}{a^3}}$  term, is significantly larger than the subsequent contribution of the effect of  $J_2$ . Additionally, the  $\sin(i)$  term can be seen to define the shape of the curve seen in Figure 1 (b), such that the magnitude of the change in AoL is maximised at 90 deg inclination.

Figure 2 (a) shows the  $\Delta V_{alt}$  and corresponding maneuver time for contours of positive and negative RAAN change for a satellite with a mean altitude of 595 km and an inclination of 45 deg. This figure only shows results for which the maneuver time is sufficient to produce the corresponding  $\Delta V_{alt}$ . It can be seen in Fig. 2 (a) that for a satellite in a prograde orbit, a negative change in RAAN, corresponding to an altitude-lowering maneuver, requires less time than a positive change using the same  $\Delta V_{alt}$ , and that the difference between the two increases as the required RAAN

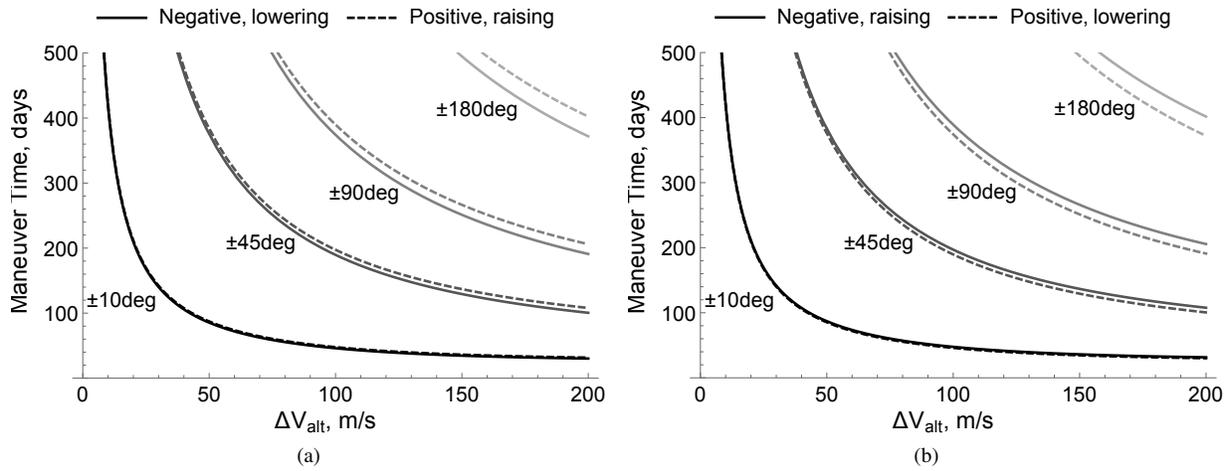


**Fig. 1** Change in (a) RAAN and (b) AoL as a function of inclination for both altitude-lowering and -raising maneuvers using 30 m/s  $\Delta V_{alt}$  and a maneuver time of 7 days.

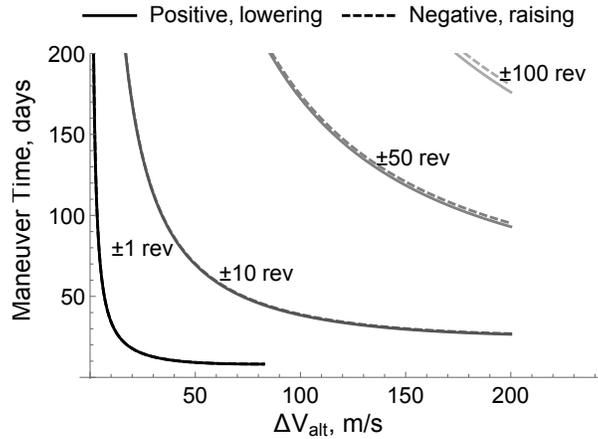
separation increases. Figure 2 (b) shows similar contours but for a satellite with an inclination of 135 deg. In this case, a positive change in RAAN, achieved by lowering the satellite altitude, is more efficient. Figure 3 shows the results for changing the AoL of a satellite inclined at 45 deg; in this case the results are found to be the same for both prograde and retrograde orbits. A positive change in AoL, corresponding to an altitude-lowering maneuver, requires less time to achieve than a negative change in AoL for the same  $\Delta V_{alt}$ . These results demonstrate that if the goal of maneuvering is to maximize the separation between satellites then it will be more efficient to do so using an altitude-lowering maneuver than using an altitude-raising maneuver; whether this corresponds to a positive or negative change of the parameter in question will be determined by whether the orbit is prograde or retrograde. This, again, is as expected when considering equations (6) and (7). Due to the presence of the semi-major axis,  $a$ , in the denominator of both expressions, it is clear that the influence of  $J_2$  and of the change in orbit period, will increase at lower altitudes; thus it is expected that an altitude lowering maneuver would produce a higher change in RAAN and AoL than a corresponding altitude raising maneuver. Additionally, for a given orbit and total change in velocity, there will be a minimum time in which a given change in RAAN or AoL can be achieved. The analytical solution presented thus provides a fast method to assess the applicability of the proposed deployment technique in a given scenario.

## B. Interaction between Right Ascension of the Ascending Node and Argument of Latitude Change

Successful deployment of a constellation will require that the satellites be placed in the desired orbit planes, corresponding to a separation in RAAN, as well as spaced correctly within the orbit planes, corresponding to a separation in AoL. As the satellites perform both the RAAN and the AoL separation maneuvers by changing altitude, it is impossible to vary one without affecting the other. However, due to the relatively short time required to change the satellite's AoL compared with its RAAN, as can be seen in Figures 1, 2 and 3, it is assumed that the RAAN change can be



**Fig. 2 Comparison of maneuver time required to obtain a positive or negative change in right ascension of the ascending node as a function of  $\Delta V_{alt}$  for (a) a prograde orbit and (b) a retrograde orbit.**



**Fig. 3 Comparison of maneuver time required to obtain a positive or negative change in argument of latitude as a function of  $\Delta V_{alt}$ .**

performed first and that the AoL change can be completed in the target orbit plane with minimal impact on the RAAN. However, there will be some unavoidable change in RAAN, referred to herein as ‘parasitic RAAN drift’, and this must be quantified to justify this assumption. From examination of equation (6) and Fig. 1, it can be determined that rate of change of RAAN, and hence the greatest accumulation of parasitic RAAN drift, will occur when the spacecraft is at a low altitude and a 45 deg inclination. As such, to justify the assumption that only a small parasitic drift in RAAN will occur when changing the AoL, a presumed worst-case scenario is assessed in which a single maneuvering spacecraft at a 45 deg inclination and a mean altitude of 200 km is considered. The parasitic RAAN drift is calculated for this spacecraft as it changes its AoL by 360 deg; this is assumed to be the maximum value that any satellite would need to change its AoL by during a deployment scenario. The general perturbation solution in the form given by equation (5) is used with the assumption that satellite B does not maneuver. Rearranging gives the time required to achieve a given

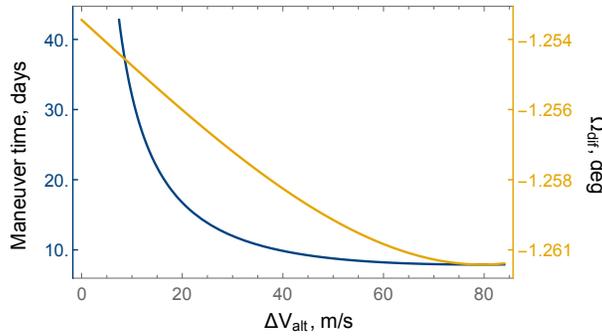
AoL separation,  $u_{dif}$ , as a function of the  $\Delta V$  used by the maneuvering satellite as

$$t_{total} = f(u_{dif}, \Delta V_{alt}). \quad (8)$$

The corresponding change in RAAN,  $\Omega_{dif}$ , that will occur in this time can be described by combining equations (4) and (8), giving

$$\Omega_{dif} = f(u_{dif}, \Delta V_{alt}). \quad (9)$$

Thus, for  $u_{dif} = 360$  deg both the maneuver time,  $t_{total}$ , and the corresponding change in RAAN,  $\Omega_{dif}$ , can be described as a function of only  $\Delta V_{alt}$ . Figure 4 shows that the maximum parasitic RAAN drift that would occur as the AoL is changed by 360 deg will have a magnitude less than 1 deg for all  $\Delta V_{alt}$  values to 84 m/s; for  $\Delta V_{alt}$  values greater than this, there is insufficient time to produce the necessary velocity change and so these results are not considered. These results validate the assertion that changing the satellite AoL will cause only a small corresponding change in RAAN. It should be noted that the results here are dependent on the orbit parameters selected and as such provide only an illustrative example. However, the results for a specific scenario can be easily calculated using the presented method. The analytical nature of the general perturbation method uniquely enables this insight into the interdependence between the change in RAAN and AoL obtained using the 3-phase maneuver.



**Fig. 4** Maneuver time (blue) and corresponding parasitic right ascension of the ascending node drift (yellow) as a function of  $\Delta V$  required for altitude change for a change in argument of latitude of 360 deg.

### C. Distribution of $\Delta V$ between Maneuvering Satellites

To establish the most propellant-efficient means of separating a constellation of satellites through RAAN or AoL, consider a simple case of two satellites. Obtaining a desired separation between them through either RAAN or AoL can be achieved by both satellites maneuvering in ‘opposite directions’ (i.e., one satellite raises its altitude and the other satellite lowers its altitude) to maximize the difference in rate of change of RAAN and AoL between the two satellites in the shortest time. However, as demonstrated in Section A, an altitude-lowering maneuver will result in a greater

change in rate of change of RAAN and AoL than an altitude-raising maneuver using the same  $\Delta V$ . As such, the most propellant-efficient maneuver strategy may be one in which both satellites maneuver using different amounts of  $\Delta V$ .

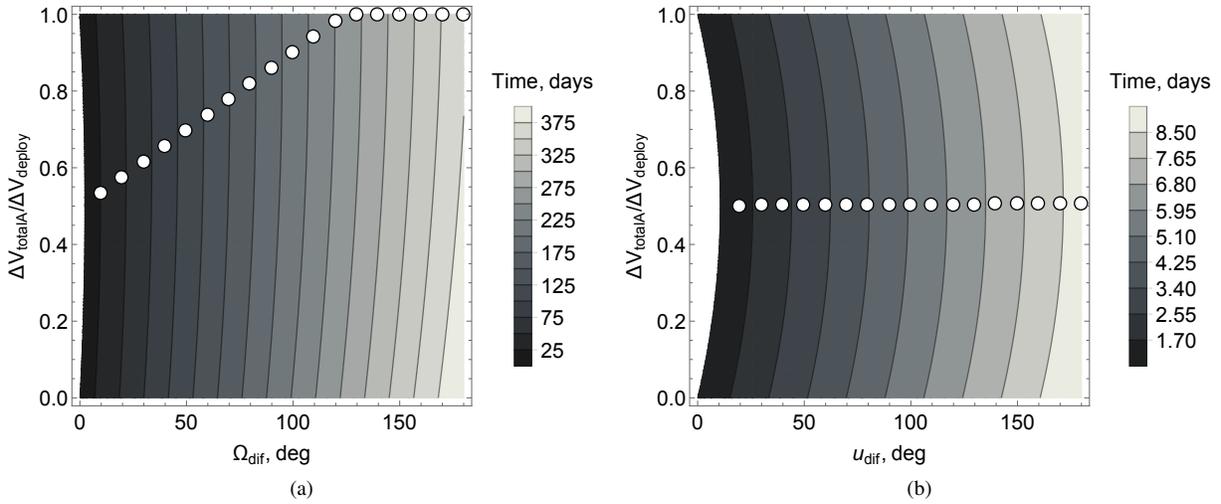
In order to identify the most efficient maneuver strategy to separate two satellites, equation (4) is used with the constraint set that the two spacecraft, satellites A and B, must finish their maneuver at the same time but can use a different total  $\Delta V$  such that

$$\Omega_{totalA} = f(\Delta V_{totalA}, t_{total}) \quad (10)$$

$$\Omega_{totalB} = f(\Delta V_{totalB}, t_{total}). \quad (11)$$

where satellite A uses an altitude-lowering maneuver and satellite B uses an altitude-raising maneuver. The change in AoL is treated similarly. Let the total  $\Delta V$  to be used for separation be  $\Delta V_{deploy}$ , such that  $\Delta V_{deploy} = \Delta V_{totalA} + \Delta V_{totalB}$ .

Figure 5 shows the time required to obtain a given (a) RAAN separation,  $\Omega_{dif}$ , and (b) AoL separation,  $u_{dif}$ , between two satellites as a function of the proportion of  $\Delta V_{deploy}$  used by satellite A. These results are calculated for two satellites with an initial mean altitude of 595 km and an inclination of 45 deg. The RAAN separation is calculated for a fixed  $\Delta V_{deploy}$  of 200 m/s and the AoL separation is calculated for a fixed  $\Delta V_{deploy}$  of 20 m/s, and solutions are only shown for cases in which the required  $\Delta V$  can be obtained in the allotted time. The dots show the distribution of  $\Delta V$  between the two spacecraft that will obtain a given separation in the minimum time. Figure 5 (a) shows that for smaller RAAN separations the optimum distribution of  $\Delta V$  is close to 50/50, with a slightly higher proportion being given to satellite A, which uses an altitude-lowering maneuver. As the desired separation increases, the proportion of  $\Delta V$  to be used by satellite A for minimum time deployment also increases. For separations greater than approximately 130 deg, in this case, the minimum time deployment will be one in which only satellite A maneuvers. Figure 5 (b) shows that for all AoL separations  $< 180$  deg, the optimal distribution of  $\Delta V$  between the satellites is approximately 50/50 for this case. This is because, due to the relatively short total maneuver time, it is more beneficial to obtain the altitude separation between the satellites quickly, by maneuvering both satellites simultaneously. Conversely, when considering RAAN separation, the long maneuver times mean that it is preferable to achieve the maximum difference in rate of change of RAAN between the satellites by primarily, or only, using a satellite lowering maneuver, as this increased efficiency compensates for the longer time required to achieve the initial separation. The specific distribution will, of course, be dependent on the orbit and spacecraft parameters and the available  $\Delta V$ , and so this provides an illustrative example only. However, the analysis demonstrates the valuable insights that can be gained through the use of the general perturbation method, and the described method can be easily applied to other cases, as will be demonstrated in Section IV.



**Fig. 5 Time required to achieve a separation in (a) right ascension of ascending node and (b) argument of latitude as a function of the proportion of  $\Delta V_{deploy}$  used by satellite A, with dots indicating the minimum time solution for a given separation.**

### III. Validation by Comparison with FORMOSAT-3/COSMIC Constellation Deployment

In order to validate the accuracy and applicability of the proposed general perturbation method for constellation deployment, the deployment of the FORMOSAT-3/COSMIC constellation is analyzed. FORMOSAT-3/COSMIC is a Global Positioning System (GPS) radio occultation mission that performs global atmospheric measurements contributing to climate monitoring and weather forecasting [6, 7]. The constellation consists of six satellites, dubbed FM1 – FM6, that were launched to a 516 km altitude circular parking orbit at 72 deg inclination. These satellites were then raised to an altitude of 800 km with the maneuvers timed to achieve a 30 deg RAAN separation between the satellites; this corresponds to a 3-phase maneuver with no phase 1 and with the satellites launched directly to the phase 2 altitude. The order of maneuvering was as follows: FM5, FM2, FM6, FM4, FM3, FM1. The six satellite maneuvers were carried out over an 18 month period in 2006 and 2007. One of the satellites, FM3, experienced a solar array deployment failure and could not complete the orbit-raising maneuver. The other five satellites all reached the target altitude and achieved the desired RAAN separation [7].

#### A. Deployment Time Calculated using General Perturbation Method

The FM5 satellite was the first of the FORMOSAT-3/COSMIC satellites to be raised to the target altitude. Using FM5 as a reference satellite for all other maneuvers, equation (4) can be used to express the desired RAAN of all other satellites relative to FM5 as a function of time, the given orbit parameters, the  $\Delta V_{alt}$ , and the propulsion system acceleration,  $A_{prop}$ . In order to account for the fact that there is no phase 1, the time for this phase is set to zero. It should be noted that this method assumes that the spacecraft thrusts continuously during the altitude-raising maneuver; any pauses in thrusting, for operational reasons, for example, will result in a longer maneuver time. In reality, the power

requirements of the thrusters may limit the ability of the spacecraft to thrust when in eclipse, for example, meaning that continuous thrusting may not be possible. Thus, the calculated solution can be considered the minimum possible deployment time for a maneuver of this type.

The FORMOSAT-3/COSMIC satellites are equipped with a blow-down mono-propellant hydrazine propulsion system that produces a thrust between 1.1 N (at beginning of life) and 0.2 N (at end of life) [7]. This gradual change in thrust and spacecraft mass over time will result in a varying acceleration profile; however, the general perturbation method assumes a constant acceleration and so an average acceleration is estimated for the satellites. With a satellite dry mass of 54 kg and an average propellant usage across all maneuvered satellites of 4.65 kg of an available 6.65 kg [7], this average acceleration can be estimated as

$$A_{prop} = \frac{F_{max} + F_{min}}{2(m_{dry} + m_p)} \quad (12)$$

where  $F_{max}$  and  $F_{min}$  are the maximum and minimum available thrusts respectively,  $m_{dry}$  is the satellite dry mass and  $m_p$  is the average mass of the on-board propellant during the maneuver, calculated as

$$m_p = \frac{m_{p0} + m_{p1}}{2} \quad (13)$$

where  $m_{p0}$  is the maximum available propellant mass and  $m_{p1}$  is the average propellant mass across all satellites remaining at the end of the maneuver. In this case,  $m_{p1}$  is 2 kg giving an acceleration of 0.018 m/s<sup>2</sup> and 0.004 m/s<sup>2</sup> at beginning and end of life, respectively, and an average acceleration,  $A_{prop}$ , of  $\pm 0.0111$  m/s<sup>2</sup>.

Assuming circular orbits are maintained throughout the maneuver, and assuming a small propellant mass flow rate and a small propellant mass fraction, the change in velocity required to raise the satellites' altitudes from the launch altitude,  $h_{start}$ , to the target altitude,  $h_{end}$ , can be approximated [24] using

$$\Delta V = \left| \sqrt{\frac{\mu}{h_{end} + R_e}} - \sqrt{\frac{\mu}{h_{start} + R_e}} \right| \quad (14)$$

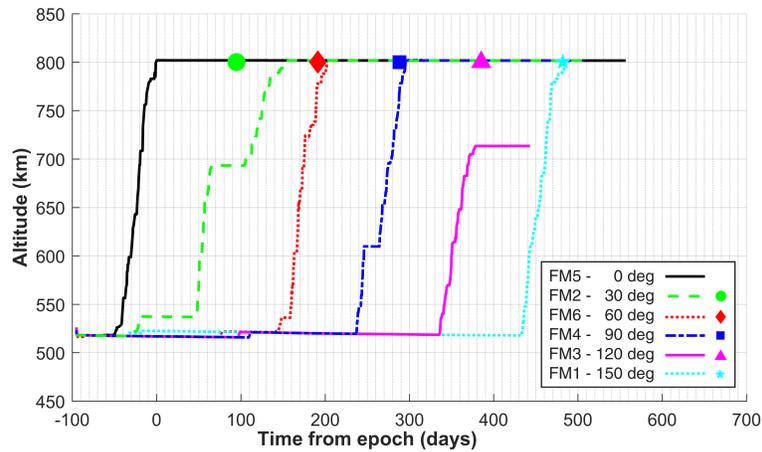
as 151.7 m/s, allowing the final RAAN separation to be described as a function of time only. Solving for the desired satellite separations gives the results shown as point markers in Fig. 6, where the epoch is taken as the time that FM5 reaches the target 800 km altitude.

## B. Comparison with Two-Line Element Data

Using the two-line element (TLE) data of the FORMOSAT-3/COSMIC satellites it is possible to track the satellites through their actual deployment maneuvers, as shown by the lines in Fig. 6, and thus to determine the true time required to achieve the desired RAAN separation. The results calculated using the general perturbation method are shown as

point markers in Fig. 6 for comparison. It should be noted that the satellite positions derived from the TLE data are in the true equator mean equinox (TEME) coordinate system and, as such, are not directly comparable with the results produced by the general perturbation method. However, the inaccuracies arising from the use of the general perturbation method to propagate the satellites' positions over the lengthy deployment time will be significantly larger than those errors arising from the use of the alternate coordinate frame, which are expected to be of the order  $< 0.5$  km for LEO satellites [25].

Comparison with the TLE data shows that the general perturbation method accurately predicts the time required to achieve the given RAAN separation for satellites FM6, FM4 and FM1. FM4 and FM1 have a difference in the predicted total maneuver time of 3 days and 6 days, respectively, when compared to the TLE data, giving an error of less than 2%. FM6 has a difference of 11 days compared to the TLE data, which is less than 6% error. The prediction of the deployment time for FM2 gives an error of 62 days; this can be explained by the approximately 40 day pause at 700 km altitude during its maneuver. This pause was due to a change in the mission parameters that originally called for a 24 deg RAAN separation between orbit planes in order to facilitate a shorter deployment [7]. However, due to better than expected attitude control performance in the injection orbit, this requirement for a shorter deployment time was relaxed and a 30 deg RAAN separation, which would give improved mission performance, was decided upon instead. FM3 cannot be used for comparison as it never reached the target orbit altitude, but the trend of the TLE data before failure suggests that it likely would have reached its target position close to the predicted time. While the consideration of other influences would give improved results, the current method is considered sufficiently accurate to predict the required minimum time for constellation deployment with expected errors of  $< 10\%$  for deployments up to at least 500 days.



**Fig. 6 Time to deploy FORMOSAT-3/COSMIC satellites. Lines show actual deployment as derived from TLE data; point marks show deployment time predicted using general perturbation method.**

## IV. Global Coverage Constellation Deployment Case Study

A responsive fire monitoring constellation designed to provide global fire detection and with the ability to maneuver to provide targeted coverage of fire outbreaks is considered in [15]. The proposed constellation consists of 24 3U CubeSats [19, 20] in four evenly spaced orbit planes with the orbit parameters and spacecraft parameters given in Table 2. The acceleration used here is based on a 3 kg spacecraft equipped with the electrospray propulsion system developed at MIT, as described in Section II. Deploying this constellation using traditional methods would require four individual launches, one per plane, each carrying six 3 kg satellites. A traditional deployment, therefore, could be extremely costly compared to the development and manufacturing costs of the satellites; as such, it is of interest to consider an alternate deployment strategy such as that used by the FORMOSAT-3/COSMIC constellation discussed in Section III. It is assumed that the mission will last a total of eight years from launch and that any time spent to deploy the constellation will result in a reduction in useful mission time, though it is likely that a partial service could be provided during deployment. An end-of-life de-orbit strategy, and any associated  $\Delta V$  requirements, is not considered, as a constellation of CubeSats at 550 km altitude should naturally deorbit within 25 years in line with best-practice guidelines [18]. This investigation provides a demonstration of the insights that can be gained through the use of the presented analysis method, as well as an example of the advantages and disadvantages of differing deployment strategies for this particular scenario. While some results obtained in this investigation may be applicable to other scenarios, it is of note that orbit and spacecraft parameters will impact the results obtained. As such, this case should be considered an illustrative example, and a demonstration of how the method can be used to quickly obtain insights into the challenge of spacecraft constellation deployment.

**Table 2 Fire response constellation parameters.**

Parameter	Value	Units
Propulsion acceleration	$\pm 1.1667 \times 10^{-4}$	m/s <sup>2</sup>
Inclination	60	deg
Constellation mean altitude	550	km
RAAN spacing between orbit planes	90	deg
In-plane spacing between satellites	60	deg
Coefficient of drag	2.2	—
Satellite mass	3	kg
Satellite cross-sectional area	0.03	m <sup>2</sup>

### A. Costing

The *Electron* launch vehicle, developed by Rocket Lab Ltd [26], at the time of writing, offers flights to 500 km sun-synchronous orbit (SSO) at a list price of \$240,000 per 3U CubeSat, with a fairing capable of housing up to 24 3U CubeSats [27]. This system will be used as a baseline for comparing the different constellation deployment strategies.

The fire monitoring constellation to be deployed requires a 60 deg inclination and, as such, is not compatible with the current *Electron* launch offerings to SSO. However, in the absence of alternative publicly available launch costs, it is assumed that a dedicated launch to the desired 60 deg inclination, 550 km altitude orbit using a similar vehicle could be procured at the same cost as the currently available launch to SSO. If the satellites are to be deployed in-orbit using the 3-phase maneuver, it is desirable to initially launch to a lower altitude, as was done by the FORMOSAT-3/COSMIC constellation, to minimize the propellant required to obtain the desired separation. In this case, it is assumed that a launch to 500 km altitude with the desired 60 deg inclination would be used and could be procured for the same cost as currently advertised for the SSO launch.

## **B. Traditional Launch**

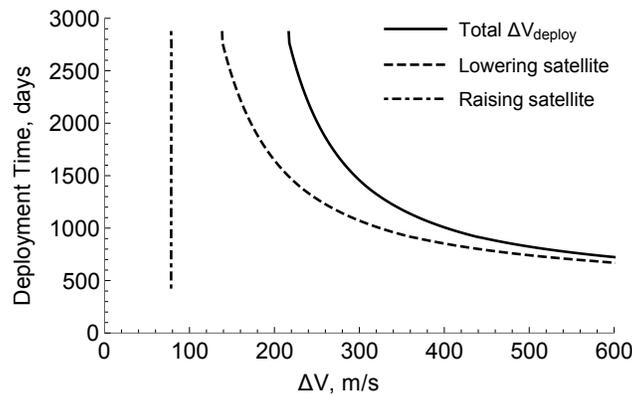
If traditional launch methods were to be used to deploy the constellation then four dedicated launches would be required. As it is assumed that a dedicated launch would be necessary due to the specific orbit requirements, the cost per launch is taken to be the cost of launching the *Electron* launch vehicle at the full capacity of 24 3U CubeSats even though only six would be on-board each vehicle. Based on the launch cost of \$240,000 per 3U CubeSat, this is estimated to cost \$5.76 million per launch for a total launch cost of \$23 million for all four planes. It is possible that the remaining slots on-board the vehicle could be sold to other parties as rideshare opportunities reducing this cost, but for the purposes of this work the maximum cost of \$ 5.76 million per launch is assumed. Assuming one launch per quarter, in line with Rocket Lab's current launch schedule, the constellation could be fully deployed within a year using traditional launch methods. The amount of  $\Delta V$  required for atmospheric drag compensation for an eight year mission in the case of direct injection to 550 km altitude is calculated as 51 m/s per spacecraft using the method described in Section II, noting that those satellites launched later would require slightly less  $\Delta V$  for this purpose. After injection, the satellites will need to be distributed within the orbit planes. This distribution would be done in the same manner as in the case of a single launch with in-orbit deployment and is discussed in Section IV.F.

## **C. Single Launch with In-Orbit Deployment**

A single *Electron* launch to 500 km altitude at 60 deg inclination is assumed to be capable of carrying 24 3U CubeSats. Thus, it would be possible to deploy the full fire monitoring constellation at 500 km altitude on a single launch vehicle at a cost of \$5.76 million. From here, the satellites must maneuver to reach the target altitude of 550 km; this will require a minimum  $\Delta V$  of 27.5 m/s per satellite as calculated using equation (14).

In order to minimize the time to total constellation deployment, first consider the satellites in the orbit plane that will be displaced through the greatest positive change in RAAN relative to the launch injection point, plane 4, and those satellites in the plane that will be displaced through the greatest negative change in RAAN, plane 1. As the orbits are prograde, the satellites in plane 1 must perform an altitude lowering maneuver, and the satellites in plane 4 must

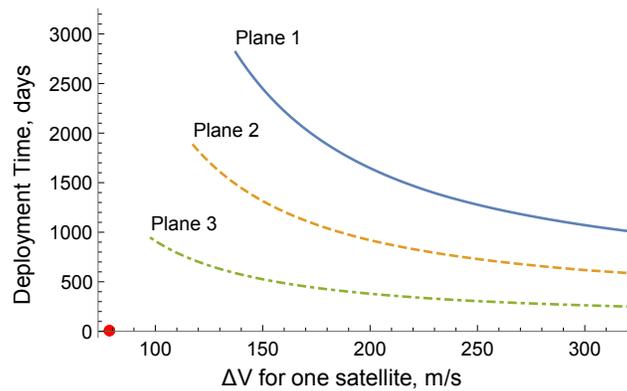
perform an altitude raising maneuver, as discussed in Section II.A. The separation,  $\Omega_{dif}$ , between these two planes once deployed will be 270 deg and the minimum deployment time will occur when the satellites in plane 1 and plane 4 reach their target orbit planes simultaneously. Considering one satellite in plane 1 and one satellite in plane 4, assume that there is a set  $\Delta V$  available for the deployment maneuver,  $\Delta V_{deploy}$ , that is to be shared between these two satellites. Using a similar analysis as in Section II.C, the distribution of  $\Delta V$  between the two satellites that gives the minimum time deployment for a given  $\Delta V_{deploy}$  can be calculated and is shown in Fig. 7. These results show that the optimal distribution of  $\Delta V_{deploy}$  is one in which the satellite performing the raising maneuver always uses the minimum possible  $\Delta V$  of 27.5 m/s to raise its orbit altitude to 550 km, without going beyond this target altitude. This occurs because the altitude-lowering maneuver is significantly more efficient than the altitude raising maneuver, as was discussed in Section II.A, and thus using the majority of the available  $\Delta V_{deploy}$  for the lowering maneuver increases efficiency. The additional  $\Delta V$  required for atmospheric drag compensation during the 8 year mission for the satellites in plane 4 is calculated as 51 m/s using the method described in Section II, giving a total required  $\Delta V$  of 78.5 m/s per satellite. The minimum possible deployment time for one satellite in plane 4 and one satellite in plane 1 using a total  $\Delta V_{deploy}$  of <400 m/s is approximately 1005 days, with the altitude lowering satellite using 322.5 m/s  $\Delta V$  for the mission.



**Fig. 7 Time to achieve a right ascension of the ascending node separation of 270 deg between two satellites as a function of  $\Delta V$ .**

Having established that the most efficient deployment will be when the satellites in plane 4 use the minimum  $\Delta V$  to maneuver, it can be concluded that the most efficient deployment of the satellites in planes 1–3 relative to plane 4 will occur when these satellites all use lowering maneuvers. Knowing this, the possible maneuvers for the satellites in planes 1–3 can be evaluated. The time and  $\Delta V$  required to deploy one satellite in each of the four orbit planes is shown in Fig. 8, where the time and  $\Delta V$  required by the satellite in plane 4 is fixed and indicated by the red dot at 78.5 m/s  $\Delta V$ . The  $\Delta V$  shown includes the  $\Delta V$  required for the deployment maneuver and that required for atmospheric drag compensation throughout the maneuver and for the remainder of the 8 year mission. The results in Fig. 8 are plotted for  $\Delta V$  values up to an arbitrary maximum of 322 m/s per satellite as a single, illustrative example; however, the method could be

similarly used to analyse the deployment of a constellation with any  $\Delta V$  capacity.



**Fig. 8 Maneuver time and  $\Delta V$  per satellite to deploy four orbit planes from a single launch.**

If the satellites in plane 1 were to use the maximum  $\Delta V$  of 322 m/s then these satellites could be deployed in approximately 1005 days; this is the minimum time in which the full constellation can be deployed for this maximum  $\Delta V$ . If the goal is to minimize time to total constellation deployment, then the  $\Delta V$  used by the satellites in the other planes can be reduced while still ensuring that they reach their target orbit at the same time as the satellites in plane 1. This solution gives a minimum constellation deployment time while also minimizing the total  $\Delta V$  used across the constellation by ensuring the synchronous arrival of all satellites in their target orbit planes. This is referred to as the “minimum time synchronous arrival” scenario and the corresponding times and  $\Delta V$  values for this deployment strategy are given in Table 3 under the heading “Sync. arrival”. It is of note that for this solution, even using the minimum possible  $\Delta V$ , the satellites in plane 3 will reach their target orbit more quickly than the satellites in plane 1 due to the smaller change in RAAN required by the plane 3 satellites to reach their final orbit. Also shown in Table 3 are the deployment times that would be required for each orbit plane if all satellites used the minimum possible  $\Delta V$  to reach the target orbit; this is referred to as the “minimum  $\Delta V$ ” scenario. Also given is the deployment time for each plane if all satellites were to use the maximum  $\Delta V$  of 322 m/s, with the exception of the satellites in plane 4 that use the minimum possible  $\Delta V$ ; this is referred to as the “maximum  $\Delta V$ ” scenario. Note that all  $\Delta V$  values given in Table 3 exclude any  $\Delta V$  to be used for in-plane deployment.

#### **D. Single Launch to a Higher Altitude**

Due to the long deployment times seen in Section IV.C Table 3, and hence the large proportion of  $\Delta V$  required for drag compensation, it is of interest to consider initially launching the constellation to a higher altitude orbit. For this, it is assumed that the spacecraft are launched to an initial altitude of 600 km; this is 50 km higher than the final mission orbit, as compared with launching into an orbit that was 50 km lower as in Section IV part C. It is assumed that this launch could deliver the 24 3U CubeSats to orbit for \$5.76 million, the same cost as a launch to 500 km altitude.

**Table 3** Maneuver time and  $\Delta V$  for a range of deployment scenarios from a single *Electron* launch.

Plane	Parameter	Sync. arrival	Min. $\Delta V$	Max. $\Delta V$	Units
1	Time	1005	2816	1005	days
	$\Delta V$ per satellite	322	137	322	m/s
2	Time	1005	1878	586	days
	$\Delta V$ per satellite	185	118	322	m/s
3	Time	940	940	249	days
	$\Delta V$ per satellite	98	98	322	m/s
4	Time	3	3	3	days
	$\Delta V$ per satellite	78	78	78	m/s

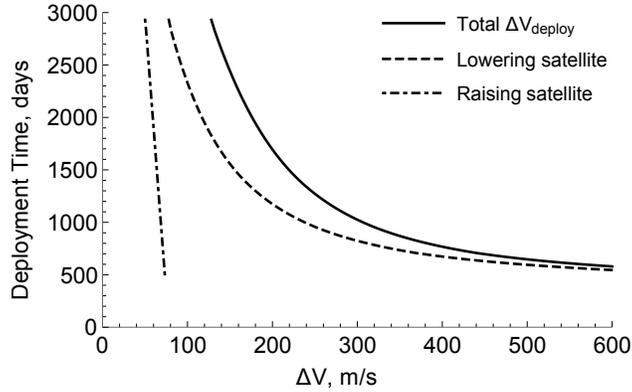
The distribution of  $\Delta V$  for this case is shown in Fig. 9. The minimum possible deployment time for the two satellites using a total  $\Delta V_{deploy}$  of  $< 400$  m/s is approximately 760 days; this is 245 days less than the case using a lower injection orbit. This reduction in deployment time occurs because less  $\Delta V$  is required for atmospheric drag compensation and so a greater proportion of the 400 m/s can be used to change the altitudes of the satellites. For all maneuvers considered, it is most efficient for the raising satellite to remain in the injection orbit at 600 km altitude until the necessary time has passed to achieve the desired RAAN change with all other satellites lowering their altitudes. The increase in  $\Delta V$  required by the ‘raising’ satellite seen in Fig. 9 as the deployment time shortens occurs as the spacecraft would spend a larger proportion of its mission in the final mission orbit, which is at a lower altitude and thus experiences more atmospheric drag. This demonstrates the complex, non-intuitive nature of the problem, and highlights the benefit of an analytical approach that can provide an overview of the trade-space.

In this case, the minimum time synchronous arrival scenario for a maximum  $\Delta V$  of 322 m/s per satellite would require that the satellites in plane 1 use a  $\Delta V$  of 322 m/s, those in plane 2 use 190 m/s, those in plane 3 use 95 m/s and those in plane 4 use 71 m/s. All satellites in planes 1 – 3 would, in this case, maneuver to an altitude lower than the final mission altitude for phase 2 of the maneuver, reaching 419 km, 476 km, and 538 km respectively. This results in a total deployment time of 776 days, which is 229 days less than required for a launch to 500 km altitude.

### E. Two Launches with In-Orbit Deployment

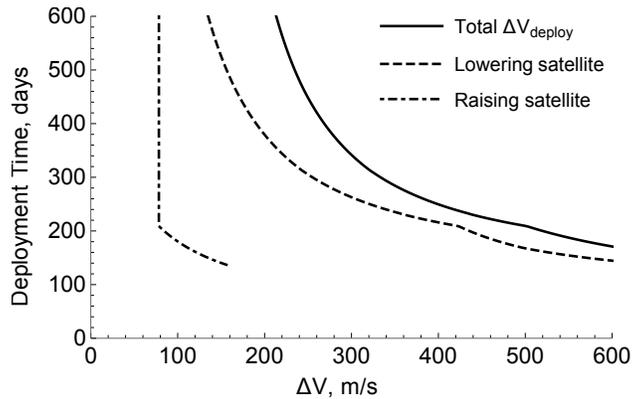
An alternative launch strategy to reduce the constellation deployment time could be to use two *Electron* launches, each carrying 12 satellites to be deployed into two orbit planes separated by 90 deg in RAAN. Assuming that the cost per launch remains constant at \$5.76 million, even though each vehicle is only at half capacity, the total launch cost is estimated as \$11.52 million. In line with the current predicted launch schedule, it is assumed that these launches could be procured 3 months apart.

Using a similar analysis as in Section II.C, two satellites initially launched to 500 km altitude and requiring a separation of 90 deg RAAN are considered. The distribution of  $\Delta V$  that gives the minimum time deployment for a



**Fig. 9 Time to achieve a right ascension of the ascending node separation of 270 deg between two satellites initially at 600 km altitude as a function of  $\Delta V$ .**

given total  $\Delta V_{deploy}$  is calculated for these two satellites and shown in Fig. 10. The change in the slope of the  $\Delta V_{deploy}$  curve at approximately 500 m/s indicates a change in the optimal distribution of  $\Delta V$ . For maneuvers using  $< 500$  m/s  $\Delta V_{deploy}$  the satellite performing the raising maneuver should always use the minimum possible  $\Delta V$  of 78 m/s. For  $\Delta V_{deploy}$  values  $> 500$  m/s, however, the optimal proportion to be assigned to the raising satellite increases. This is because beyond this point the  $\Delta V$  required for atmospheric drag compensation by the lowering satellite has increased sufficiently to offset the increased efficiency of the lowering maneuver. The minimum possible deployment time for these two satellites using a total  $\Delta V_{deploy}$  of  $< 400$  m/s is approximately 250 days. In this case, the lowering satellite requires 322 m/s  $\Delta V$  for the mission.

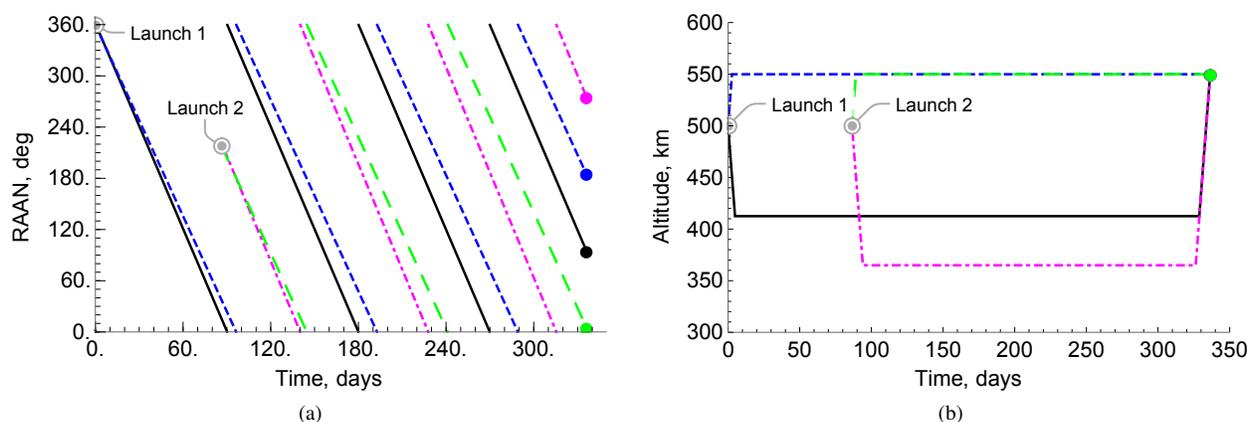


**Fig. 10 Time to achieve a right ascension of the ascending node separation of 90 deg between two satellites as a function of  $\Delta V$ .**

Assuming the satellites released from the second launch vehicle were to deploy in the minimum time of 250 days, the minimum constellation deployment time would be 340 days, once the 90 day gap between the two launches is accounted for. As such, the satellites deployed on the first launch could use a smaller amount of  $\Delta V$  and reach their final orbits at the same time as those on the second launch. From the results shown in Fig. 10, separating two satellites

through 90 deg from a single injection point in 340 days requires approximately 303 m/s  $\Delta V_{deploy}$ , with the raising satellite using 78 m/s and the lowering satellite using 225 m/s.

Finally, the injection point of the second launch relative to the first must be determined, to ensure that the satellites on the second launch will be positioned correctly relative to the satellites on the first launch after all satellites have finished maneuvering. To do this, equation (1) is used to determine the absolute position of all satellites post-maneuver. From this, the necessary offset for the second launch can be determined. In this case, an offset of 217.5 deg between the two injection planes is necessary to execute the most efficient deployment. The RAAN and altitude of the satellites throughout the full deployment scenario are shown in Fig. 11 with the markers at 340 days indicating the system state once deployment has been completed.



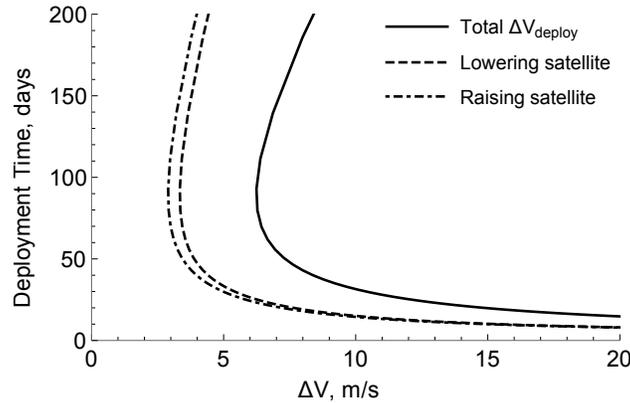
**Fig. 11 Evolution of (a) right ascension of the ascending node and (b) altitude of satellites in all four orbit planes during deployment from two *Electron* launches.**

#### F. Satellite Placement Within the Orbit Plane

Distributing the satellites through AoL within the orbit plane is assessed in a similar manner to the orbit plane placement. In this case the satellites begin the maneuvers at the target 550 km altitude. Using the method described in Section II.C the optimum distribution of  $\Delta V$  between the two satellites requiring the greatest positive and negative change in AoL, respectively, is calculated. Assuming that all satellites begin at the same location, the desired separation between these furthest satellites will be 300 deg. The optimum distribution of  $\Delta V$  is shown in Fig. 12. These results show that for a  $\Delta V_{deploy} < 20$  m/s the optimal distribution of  $\Delta V$  is one in which both satellites use almost equal amounts of  $\Delta V$ , with the lowering satellite using slightly more than the raising satellite. This occurs because, as discussed in Section II.C, for such a short maneuver it is more beneficial to obtain a separation in altitude quickly by maneuvering both satellites. The minimum possible deployment time for these two satellites using a total  $\Delta V_{deploy}$  of  $< 20$  m/s is approximately 15 days, with the lowering satellite using 10.2 m/s  $\Delta V$ , and the raising satellite using 9.8 m/s.

Once the deployment  $\Delta V$  for the two satellites furthest from the maneuver start point has been selected, the other

satellites can be placed relative to these positions, using either a raising or lowering maneuver, depending on which is more efficient. Assuming that all satellites finish maneuvering at the same time, the most efficient deployment in this case will occur when three satellites use raising maneuvers requiring 9.8 m/s, 5.6 m/s and 1.7 m/s, respectively, and three satellites use lowering maneuvers requiring 2.7 m/s, 6.6 m/s and 10.2 m/s, respectively.



**Fig. 12** Time to achieve an argument of latitude separation of 300 deg between two satellites as a function of  $\Delta V$ .

## G. Results and Discussion

A comparison of the traditional deployment method, single launch with in-orbit deployment to both 500 km and 600 km, and two launches with in-orbit deployment is given in Table 4. The results for the single launch are for the minimum time synchronous arrival scenario; this corresponds to the first column in Table 3. Note that these results do not include any  $\Delta V$  required for in-plane phasing as this is the same for all strategies. The results in Table 4 show that deployment from a single *Electron* launch vehicle can offer a theoretical reduction in launch costs of 75% when compared with traditional launch methods. However, this comes at the cost of an increased deployment time of 2.75 times that of a traditional launch and requires 2874 m/s more  $\Delta V$  across the constellation if launched to 500 km. A launch to 600 km altitude could reduce the time to deploy to 2.13 years for a similar amount of  $\Delta V$ . Whether such deployment times are reasonable will be dependent on the specific mission being considered. CubeSats launched between 2000 – 2012 are shown in Ref. [28] to have operational lives ranging from tens of days to several thousands of days; the proposed deployment strategy may be of value to those missions with longer operational lives. In addition, by using two sequential *Electron* launches to 500 km altitude the constellation could in fact be deployed more quickly than it could when using traditional methods for a cost of \$11.5 million. This strategy requires 2994 m/s more  $\Delta V$  across the constellation than traditional launch methods. However, if the cost of incorporating the necessary propulsion systems and launching the required propellant mass is < \$11.5 million, then this deployment method will offer a cost saving compared with traditional launch. It is also of note that as the launch vehicles in the two-launch case have additional remaining payload capacity, the mass of the satellites could be increased, through the inclusion of additional propellant

or capability, at no additional cost. Also of note is that these deployment methods suggest the use of different amounts of  $\Delta V$  for deployment of each satellite; this will result in the satellites in the constellation having different amounts of  $\Delta V$  remaining on board post-deployment. This disparity in  $\Delta V$  available for orbit maintenance and operation would likely result in different mission lifetimes for each of the spacecraft. Depending on the proposed mission lifetime, orbit properties, and  $\Delta V$  capacity of the spacecraft this may not be an issue, and the effects could be mitigated by initially launching spacecraft with disparate  $\Delta V$  capacity or by planning for staggered replacement of spacecraft as they reach the end of their mission life. Having identified the possible deployment options it will be up to the mission designer to trade-off all competing options and select the launch that provides the best solution for their specific mission.

**Table 4 Summary of deployment times, costs and required  $\Delta V$  for each deployment method.**

Deployment Method	Time, years	Total $\Delta V$ , m/s	Cost, million \$
Traditional Launch	1	1224	23.04
Single Launch to 500 km	2.75	4098	5.76
Single Launch to 600 km	2.13	4068	5.76
Two Launches to 500 km	0.9	4218	11.52

## V. Conclusions

General perturbation methods can be used to solve the restricted circular-to-circular, co-planar, low-thrust Lambert problem to describe deployment of a satellite constellation through right ascension of the ascending node and argument of latitude from a single launch injection point. If the goal of a mission is to achieve a separation in either right ascension of the ascending node or argument of latitude between a maneuvering satellite and a non-maneuvering reference then a 3-phase maneuver in which the satellite altitude is lowered in phase 1 using a constant acceleration, maintains a constant altitude in phase 2, and then raises its altitude in phase 3 using a constant acceleration is more efficient than a similar maneuver in which the satellite altitude is raised in phase 1 and lowered in phase 3. For a given scenario and total change in velocity it is possible to identify the distribution of propellant between maneuvering satellites that will result in a minimum time deployment. The minimum change in velocity required to deploy a given spacecraft constellation can also be established. The speed and flexibility of the general perturbation method enables a trade-off between time to deploy and change in velocity required to be rapidly performed, providing an unprecedented insight into the problem. The analytical nature of the method allows for the interplay among the satellite deployment maneuvers to be examined and the impact that the maneuvers used for one orbit plane have on the other orbit planes assessed. Initially launching to a higher altitude than the mission orbit can reduce the deployment time for a given change in velocity due to the reduction in atmospheric drag compensation required. The proposed deployment technique could reduce launch costs significantly compared to traditional launch methods and, although the maneuver time can be lengthy, the time to total constellation deployment can be shorter than traditional launch methods due to the possible wait time between

subsequent launches.

## Acknowledgments

This research was supported by the Engineering & Physical Sciences Research Council (EPSRC) [grant number EP/M506643/1]; and the Air Force Office of Scientific Research (AFOSR) [grant number FA9550-18-1-0147]. There is no data underpinning this publication; all results can be fully reproduced using the methods described in this article.

## References

- [1] Lal, B., de la Rosa Blanco, E., Behrens, J. R., Corbin, B. A., Green, E. K., Picard, A. J., and Balakrishnan, A., “Global Trends in Small Satellites,” Tech. Rep. IDA Paper P-8638, IDA Science and Technology Policy Institute, July 2017.
- [2] Bouwmeester, J., and Guo, J., “Survey of worldwide pico-and nanosatellite missions, distributions and subsystem technology,” *Acta Astronautica*, Vol. 67, No. 7-8, 2010, pp. 854–862. doi:10.1016/j.actaastro.2010.06.004.
- [3] Wekerle, T., Pessoa Filho, J. B., Costa, L. E. V. L. d., and Trabasso, L. G., “Status and trends of smallsats and their launch vehicles—An up-to-date review,” *Journal of Aerospace Technology and Management*, Vol. 9, No. 3, 2017, pp. 269–286. doi:10.5028/jatm.v9i3.853.
- [4] Crisp, N., Smith, K., and Hollingsworth, P., “Launch and deployment of distributed small satellite systems,” *Acta Astronautica*, Vol. 114, 2015, pp. 65–78. doi:10.1016/j.actaastro.2015.04.015.
- [5] King, J. A., and Beidleman, N. J., “Method and apparatus for deploying a satellite network,” US Patent 5,199,672, April 1993.
- [6] Fong, C.-J., Huang, C.-Y., Chu, V., Yen, N., Kuo, Y.-H., Liou, Y.-A., and Chi, S., “Mission results from FORMOSAT-3/COSMIC constellation system,” *Journal of Spacecraft and Rockets*, Vol. 45, No. 6, 2008, pp. 1293–1302. doi:10.2514/1.34427.
- [7] Fong, C.-J., Shiau, W.-T., Lin, C.-T., Kuo, T.-C., Chu, C.-H., Yang, S.-K., Yen, N. L., Chen, S.-S., Kuo, Y.-H., and Liou, Y.-A., “Constellation deployment for the FORMOSAT-3/COSMIC mission,” *Institute of Electrical and Electronics Engineers Transactions on Geoscience and Remote Sensing*, Vol. 46, No. 11, 2008, pp. 3367–3379. doi:10.1109/TGRS.2008.2005202.
- [8] da Silva Curiel, A., Lambert, M., Liddle, D., Sweeting, S. M., Chu, C.-H. V., Fong, C.-J., and Chang, G.-S., “Introduction to FORMOSAT-7/COSMIC-2 mission,” *27th Annual American Institute of Aeronautics and Astronautics and Utah State University Conference on Small Satellites, SSC13-VI-1*, 2013.
- [9] Cook, K., Fong, C.-J., Wenkel, M. J., Wilczynski, P., Yen, N., and Chang, G., “FORMOSAT-7/COSMIC-2 GNSS radio occultation constellation mission for global weather monitoring,” *Aerospace Conference*, Institute of Electrical and Electronics Engineers, 2013. doi:10.1109/AERO.2013.6497317.
- [10] Leppinen, H., “Deploying a single-launch nanosatellite constellation to several orbital planes using drag maneuvers,” *Acta Astronautica*, Vol. 121, 2016, pp. 23–28. doi:10.1016/j.actaastro.2015.12.036.

- [11] Foster, C., Mason, J., Vittaldev, V., Leung, L., Beukelaers, V., Stepan, L., and Zimmerman, R., “Constellation phasing with differential drag on Planet Labs satellites,” *Journal of Spacecraft and Rockets*, Vol. 55, No. 2, 2018, pp. 473–483. doi:10.2514/1.A33927.
- [12] Baranov, A. A., Grishko, D. A., and Mayorova, V. I., “The features of constellations formation and replenishment at near circular orbits in non-central gravity fields,” *Acta Astronautica*, Vol. 116, 2015, pp. 307–317. doi:10.1016/j.actaastro.2015.06.025.
- [13] Jenkins, M. G., Krejci, D., and Lozano, P., “CubeSat constellation management using Ionic Liquid Electrospray Propulsion,” *Acta Astronautica*, Vol. 151, 2018, pp. 243–252. doi:10.1016/j.actaastro.2018.06.007.
- [14] De Weck, O. L., Neufville, R. D., and Chaize, M., “Staged deployment of communications satellite constellations in low earth orbit,” *Journal of Aerospace Computing, Information, and Communication*, Vol. 1, No. 3, 2004, pp. 119–136. doi:10.2514/1.6346.
- [15] McGrath, C. N., and Macdonald, M., “General perturbation method for satellite constellation reconfiguration using low-thrust maneuvers,” *Journal of Guidance, Control, and Dynamics*, 2018. doi:10.2514/1.G003739.
- [16] Committee on Space Research, “CIRA 1972: COSPAR International Reference Atmosphere 1972,” Akademie-Verlag, Berlin, 1972.
- [17] Sterne, T. E., “Effect of the rotation of a planetary atmosphere upon the orbit of a close satellite,” *American Rocket Society Journal*, Vol. 29, No. 10, 1959, pp. 777–782. doi:10.2514/8.4901.
- [18] Kerr, E., and Macdonald, M., “Incorporating solar activity into general perturbation analysis of atmospheric friction,” *Journal of Guidance, Control, and Dynamics*, Vol. 41, No. 6, 2017, pp. 1320–1336. doi:10.2514/1.G003229.
- [19] Hutputtanasin, A., and Tooria, A., “CubeSat Design Specification,” Technical specification, California Polytechnic State University, 2005.
- [20] Rankin, D., Kekez, D. D., Zee, R. E., Pranajaya, F. M., Foisy, D. G., and Beattie, A. M., “The CanX-2 nanosatellite: expanding the science abilities of nanosatellites,” *Acta Astronautica*, Vol. 57, No. 2-8, 2005, pp. 167–174. doi:10.1016/j.actaastro.2005.03.032.
- [21] Krejci, D., Mier-Hicks, F., Thomas, R., Haag, T., and Lozano, P., “Emission characteristics of passively fed electrospray microthrusters with propellant reservoirs,” *Journal of Spacecraft and Rockets*, Vol. 54, No. 2, 2017, pp. 447–458. doi:10.2514/1.A33531.
- [22] Mier-Hicks, F., and Lozano, P. C., “Electrospray thrusters as precise attitude control actuators for small satellites,” *Journal of Guidance, Control, and Dynamics*, Vol. 40, No. 3, 2017, pp. 642–649. doi:10.2514/1.G000736.
- [23] Vallado, D. A., and McClain, W. D., *Fundamentals of astrodynamics and applications*, 3<sup>rd</sup> ed., Microcosm Press & Springer, 2007. pp. 642–667.
- [24] Vallado, D. A., and McClain, W. D., *Fundamentals of Astrodynamics and Applications*, 3<sup>rd</sup> ed., Microcosm Press & Springer, 2007. pp. 373–389.

- [25] Seago, J., and Vallado, D., “Coordinate Frames of the US Space Object Catalogs,” *Astrodynamics Specialist Conference*, American Institute of Aeronautics and Astronautics, 2000, p. 105. doi:10.2514/6.2000-4025.
- [26] “Rocket Lab Ltd: Electron,” <https://www.rocketlabusa.com/electron/>, 2017. Accessed: 16/02/2018.
- [27] “Rocket Lab Ltd: Launch Cost,” <https://www.rocketlabusa.com/book-my-launch/>, 2017. Accessed: 16/02/2018.
- [28] Swartwout, M., “The first one hundred cubesats: A statistical look,” *Journal of Small Satellites*, Vol. 2, No. 2, 2013, pp. 213–233.