Multi-objective robust early stage ship design optimisation under uncertainty utilising surrogate models

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Abstract

The paper addresses the parametric design and multi-objective optimisation of ships under uncertainty, applying the Holistic Optimisation Design Approach. The methodology starts with the creation of a detailed parametric model that captures both the external and internal geometric characteristics of the vessel, along with the integration of a number of numerical tools, purposely developed in order to determine each variant's performance. This allows the evaluation of a multitude of merit functions and design constraints, all part of the optimisation problem. In contrast to previous approaches, the uncertainties introduced by the physical and financial environment are incorporated in the model, providing a more realistic representation of the solution space to the decision maker. The developed methodology is applied on a case study of a Ro-Pax ship, optimised in the concept design phase with regard to the minimisation of the total resistance, required freight rate and building cost.

Keywords: parametric design; ship design; multi-objective; optimisation; uncertainty;
surrogate model

1 Introduction

The ever-increasing complexity and competitiveness in the shipping industry has led the shipbuilding industry to push the boundaries in the field of ship design (Brett et al., 2018). Changes in the latter have been induced by the societal demand for safer and cleaner ships, followed by the introduction of new, stricter regulations. The ship design methodologies have evolved from sequential, iterative processes (Evans, 1958) to more complex ones, such as the decision- or set-based methodologies introduced by Mistree et al. (1990) and Parsons and Singer (1999), all the way to more the most recent and sophisticated holistic design approach (Papanikolaou, 2010). Andrews et al. (2018) present in great detail the latest developments in the ship design industry. The challenges faced by the designers nowadays call for a multi-disciplinary, multi-objective approach to the ship design optimisation problem.

Comparing the novel work of (Murphy et al., 1963) or (Nowacki et al., 1970) with today's holistic approaches (Papanikolaou, 2018), it is evident that vast advances have been achieved in the past five decades. The main enabling factor was the introduction of computer-aided design (CAD), followed by computer-aided engineering (CAE) and the significant reduction of the computational fluid dynamics (CFD) and Finite Element Analysis (FEA) costs. They all benefited from the exponential increase of the available computational power which is still following Moore's law (Moore, 2018). Having all that capacity in place, the main challenge is to set up a methodology that analyses the ship design problem to its components, evaluates them accurately (enough) and then synthesises their output in a comprehensive, informative,

consistent, transparent and understandable way to the designer or the decision maker in general. In this paper, the authors present a flavour of such an approach, using a parametric model developed in the CAE platform used in the framework of the European Union funded HOLISHIP project (HOLISHIP, 2016-2020), integrated with techno-economic databases, calculation and optimisation modules and software tools.

2 Background

2.1 Ship design optimisation

Papanikolaou and Nowacki present very lively the history of ship design optimisation, underlining the differences of the contemporary holistic approach related to previous methods (Papanikolaou, 2018). Its main advantage is that the multi-objective optimisation of ship design problem is solved considering simultaneously (holistically) all design aspects of the ship system and not as a collection of part (Papanikolaou, 2018). This is the approach investigated and further developed currently in the framework of the European Union funded HOLISHIP project (HOLISHIP, 2016-2020). Based on previous works by the authors (Priftis et al., 2018a, Priftis et al., 2018b), the methodology starts with the creation of a detailed parametric model that captures both the main particulars and the internal compartmentation of the vessel, along with the integration of a number of numerical tools, purposely developed in order to determine each variant's performance. This allows the evaluation of a multitude of merit functions and design constraints, all part of the optimisation problem. The gap which the authors have identified is lack of a way to capture the inherent uncertainties introduced by the physical and financial environment and their corresponding numerical models, missing

the opportunity to provide a more pragmatic representation of the solution space to the decision maker.

2.2 Uncertainty

2.2.1 Uncertainty in design

In most engineering problems, the design parameters are known to a certain extent. In order to be more accurate, in some cases, design parameters are described by a probability function instead of a single value. Brefort and Singer (Brefort and Singer, 2018) characterise this as epistemic uncertainty that stems from many sources, including the limited information of a vessel model, the difficulty identifying precise design performance targets and the difficulty to identify the precise bounds on the validity of the analysis tools. Therefore, it becomes clear that a level of uncertainty is introduced from the process itself in the solution of the design problem. Nikolopoulos and Boulougouris (Nikolopoulos and Boulougouris, 2018) have investigated three categories of uncertainty, coming from the environment, the market and the method. The impact of uncertainty is significant, as many design characteristics are not properly defined and the response of the model is ambiguous. If these uncertainties and their effects are captured properly and early enough, the required error margin arising along the design process can be minimised.

2.2.2 Robust optimisation

An optimal design can be characterised as robust when its quality does not deviate much from optimality when small changes in the problem occur (Branke et al., 2008). Traditional

optimisation methodologies that overlook uncertainty tend to produce non-robust, over-optimised designs that do not correspond to reality. On the contrary, robust optimisation methods take input uncertainties in the design variables, as well as uncertainties within the system or model into account and try to find optimal solutions in a noisy, uncertain environment. The results of a robust optimisation end up being insensitive to variations occurring in their operating environment (Steponavice and Miettinen, 2012). Contrary to single objective optimisation problems, robustness in multi-objective optimisation is complicated, as it has to be defined for all Pareto solutions. The incorporation of uncertainty in the problem increases the number of the optimisation objectives. Consequently the decision making process becomes convoluted.

There have been several studies, which investigate the effect of uncertainty on design optimisation through various approaches. One option is to minimise the expectation of the general loss in the performance of the solution (De Groot, 1970). Trosset et al. used the most conservative approach in robust optimisation, the “minmax” approach, which is the minimisation of the worst possible case for the objective function (Trosset et al., 2003). Vasile and Di Carlo combined the “minmax” approach with the use of surrogate models and inflationary differential evolution (Vasile and Di Carlo, 2015). Erfani and Utyuzhnikov introduced a tuneable robust function as an objective in the optimisation problem, which enables the control of the robustness extent by designer preferences (Erfani and Utyuzhnikov, 2011).

In ship design, uncertainty was first introduced utilising approaches, such as real options analysis, found in the financial sector (Erikstad and Rehn, 2015). For instance, Dixit focused on shipping applications, where several decisions related to the shipping market were

evaluated (Dixit, 1989). Other applications involve the analysis of available options in shipbuilding contracts (Hoegh, 1998) and in naval ship design (Gregor, 2003). Another way to solve the optimisation problem under uncertainty is through the utilisation of probabilities. Stochastic optimisation introduces the probability distributions of design parameters, extending the deterministic optimisation problem to improve the robustness of the solutions. Applications in the marine industry include vehicle routing problems (Fagerholt et al., 2010), simulation-driven optimisation of bulk carrier design (Nikolopoulos and Boulougouris, 2018) and mission-based ship design under arctic sea ice conditions (Choi et al., 2015).

3 Proposed methodology

3.1 Problem definition

The general optimisation problem is defined as (Equations 1, 2):

$$\min_{\mathbf{x} \in X} \mathbf{f}(\mathbf{x}, \mathbf{y}), \quad \text{given } \mathbf{y} = \hat{\mathbf{y}} \in Y \quad (1)$$

$$\text{subject to } g_n(\mathbf{x}, \hat{\mathbf{y}}) \leq 0, n = 1, \dots, N \quad \text{and} \quad h_m(\mathbf{x}, \hat{\mathbf{y}}) = 0, m = 1, \dots, M \quad (2)$$

Where $\mathbf{x}$ is the design variables vector, $\mathbf{y}$ is the design parameters vector, independent of the designer choice, and $\mathbf{f}$, g_n and h_m are the optimisation objective, inequality and equality constraint functions. Many of design parameters are in fact uncertain in real-life applications and may differ from the design conditions $\hat{\mathbf{y}}$. Therefore, the design parameters vector is assumed as a stochastic process with probability density function $p(\mathbf{y})$ and expected value (mean) (Equation 3):

$$\bar{\mathbf{y}} := \mu(\mathbf{y}) = \int_{\mathbf{y}} \mathbf{y} p(\mathbf{y}) d\mathbf{y} \quad (3)$$

The design variables are assigned upper and lower limits by the designer and change during the optimisation loop, while the design parameters are independent of the designer choice and are assigned a probability density function and mean value for the solution of the optimisation problem.

Figure 1. Computational setups

In this paper, the effort is focused on the elimination of the uncertainty effects by identifying the uncertain design parameters $\mathbf{y}$ throughout the design methodology and analysing their effects in the optimisation stage through the establishment of uncertainty region for the optimisation objectives $\mathbf{f}$. These regions –shown in an indicative graph in Figure 1– define the limits of the attained values for each objective, depending on the uncertain parameters of the problem and they have a significant impact on the decision making phase, where the decision maker can easily identify the limits of the objective functions $\mathbf{f}$ for the solutions located on the Pareto front. Hence, the robustness of the selection process in identifying the optimal design is increased.

Figure 2. Design optimisation procedure

3.2 Input

The process begins with the definition of the parametric design and continues with the construction of the ship model. The setup for surrogate models is implemented afterwards. Throughout this process the design variables, objective functions and constraints of the problem are determined. The multi-objective optimisation setup follows, for which genetic algorithms are utilised. The setup is run and the results are analysed in the multi-criteria decision making process. (Figure 2).

The parametric hull is produced within CAESES® (Friendship Systems, 2018), and consists of four main parts; the main frame, aft body, fore body, and main deck. Most parts of the hull, such as the bulbous bow, the shape and fullness of key design curves (e.g. design waterline, flat of bottom, flat of side), as well as the main frame, are controlled by design parameters (e.g. fullness of bulbous bow, fullness of main deck curve, flare of the hull surface at the design waterline, etc.). The designer can choose which of these parameters are used as design variables for the particular optimisation problem. Overall, 70 design parameters affect the overall shape of the hull. Once the initial hull is defined, a Lackenby transformation is applied (Lackenby, 1950). A hydrostatic and a sectional area curve calculation are performed. These are used as input to the Lackenby transformation. By adjusting the prismatic coefficient (C_P) and longitudinal centre of buoyancy (LCB) through two design parameters; the percent variation of C_P (δC_P) and the percent variation of LCB (δLCB), the final hull is produced. This process allows shifting sections aft and fore, while fairness optimised B-Splines are utilised (Abt and Harries, 2007).

Once the final shape of the hull is created in CAESES®, an IGES file, which contains the information related to its geometry, is generated and imported to NAPA® (NAPA, 2018) to continue with the rest of the design process. A list of in-house developed macros is run to

generate the complete ship model. In particular, macros that generate surfaces for the internal arrangement are run first. The position of the surfaces is linked to the design parameters mentioned in the previous paragraph (e.g. frame spacing, breadth, depth). Using these surfaces as limits, the required rooms for the definition of a complete ship model are defined. Afterwards, an internal arrangement is formed by assigning the appropriate purposes to each room (e.g. fuel tank, water ballast tank, engine room, etc.). The designer controls the values used for several design parameters related to the definition of the internal spaces, such as the height of superstructure decks and the frame spacing.

Apart from the geometry, operational input is required to be set. The distance of the round trip the ship operates on, the expected lifetime of the ship, the number of port calls and time spent at port, the service speed, the wind speed, as well as the load factors for passengers and vehicles are used as input, depending on the case the optimisation needs to be solved for. In addition, financial parameters, such as the discount rate and fuel price, are specified by the designer, depending on the case study.

3.3 Objectives and constraints

Two different setups for the computations performed to provide results for the objectives and constraints of the problem are defined (Figure 1). The first one (“main setup”), which provides a deterministic solution, is run during the main optimisation phase. The uncertainty of the various design parameters is not taken into account and fixed values are used to get the results. The second setup (“uncertainty setup”) is identical to the first; however, random values replace the fixed ones. In order to identify the uncertainty regions for the objective functions, several scenarios are examined, where the uncertain parameters are assigned

random values based on their probability distributions (Table 1). Considering that the uncertainty regions should be relatively confined and not too broad –otherwise the caption of uncertainty would be impossible– the second setup is utilised only for the designs located on the Pareto front, following the main optimisation phase, so as to reduce the overall computational time of the study.

Table 1: Uncertain parameters

Parameter	Probability distribution	Minimum value	Maximum value	Average value	Standard deviation
Diesel oil price (\$/t)	Normal	308.93	1058.2	721.67	200.52
Fuel oil price (\$/t)	Normal	188.18	759.61	488.81	149.83
Variance	Uniform	0.95	1.05	N/A	N/A
Ship speed (Knots)	Uniform	25	29	N/A	N/A
Wind speed (Knots)	Normal	0.00	34.50	11.08	4.79

Information regarding the calculation of the objectives and constraints examined in the present optimisation problem is provided below.

The *total resistance* is estimated as a sum of the calm water resistance, added wave resistance and wind resistance.

The first resistance component is calculated using two methods; Holtrop & Mennen 84 method (Holtrop and Mennen, 1978) and CFD through Star-CCM+® (Siemens, 2017). The former is utilised as a quick alternative to estimate the calm water resistance during the NAPA® computations, while the latter provides a more detailed estimation.

According to Holtrop & Mennen 84 method, resistance is calculated as the sum of the frictional, the wave-making and wave-braking resistance, the additional pressure resistance introduced by the bulbous bow and the immersed transom, the resistance of the appendages and the model-ship correlation resistance. The dimensions of the appendages (e.g. bilge keels) are set by the designer and used as input in the calculation. The more accurate CFD calculations are done in model scale so as to keep the number of mesh cells –and consequently the computational time– low. The results are then converted to full scale, using the ITTC procedure (ITTC, 2017). The parametric hull is translated to model scale to reduce the required computational time and the required numerical input for the Star-CCM+® model setup is provided by a CAESES® feature. For instance, the model's properties (mass, centre of gravity, moment of inertia, wetted surface), solver parameters, stopping criteria and coordinates of the volumetric mesh refinements mentioned below are calculated based on the parameters of the problem and provided as output by the developed CAESES® feature. Within Star-CCM+®, macros are used, responsible for the various actions required to define the computational setup. The process starts with the mesh generation, including the definition and formation of the necessary volumetric mesh refinements on areas close to the hull surface to refine the free water surface area. The latter is performed parametrically, depending on the parameters affecting the hull shape. Afterwards, the physical and numerical setup is defined. The method utilised is the Volume of Fluid in conjunction with the k- ϵ turbulence model to simulate the flow characteristics. The simulation results are obtained after setting the solver

parameters and stopping criteria –controlled by parameters accessible to the user. During the post-processing phase, the results corresponding to the model are converted to full-scale to obtain the calm water resistance for the design to be optimised (Figures 3, 4).

Figure 3. CFD calculation domain and mesh generation

Figure 4. Wave pattern during CFD calculations in Star-CCM+® (base model)

The utilisation of two methods of variable fidelity levels for the calm water resistance estimation is ultimately used as an indicative way to quantify the uncertainty levels associated with the resistance estimation. A parameter calculating the difference of the calm water resistance value between the Holtrop Mennen 84 method and the CFD calculations is introduced. The smaller the difference, the higher the confidence levels for the estimation of the calm water resistance. However, the case where both methods fail to provide an accurate result cannot be detected.

The added wave resistance is calculated using the STAWAVE-1 method (Boom et al., 2013). According to this method, added wave resistance is calculated as follows (Equation 4):

$$R_{AWL} = 1000 \cdot \frac{1}{16} \cdot \rho \cdot g \cdot H_{W,1/3}^2 \cdot B \cdot \sqrt{\frac{B}{L_{BWL}}} \quad (4)$$

Where ρ is the sea water density, g is the gravitational acceleration, $H_{W,1/3}$ is the significant wave height, B is the ship's breadth and L_{BWL} is the length of the bow on the waterline to 95% of maximum breadth. The significant wave height is calculated as follows:

$$H_{W,1/3} = 0.21 \cdot \frac{(1.026 \cdot V_{10})^2}{g} \quad (5)$$

Weather data are used based on the operational profile of the ship (i.e. on which region the ship operates) to identify the wind speed at 10m above sea surface (V_{10}) used in equation 5.

Finally, the wind resistance is calculated according to Jensen (Jensen, 1994). The sum of the three resistance components is used to calculate the required main engine's power. The latter is computed by applying the propulsive η_D and shaft η_S efficiency. Due to lack of propeller

298 data at this design phase, estimation for the efficiency factors are used, proposed by
299 Schneekluth and Bertram (1998).
300

301 For the “main setup”, a mean value is used for the wind speed, which is based on historical
302 weather data for the selected regions of operation. However, for the “uncertainty setup”, run
303 for the designs located on the Pareto front, random wind speed values are used to calculate
304 the added wave and wind resistance, which are based on a probability distribution according
305 to the same historical weather data.
306

307 The calculation of the *lane metres* is performed based on the internal arrangement. A custom
308 NAPA® macro calculates the length of each lane in the available car decks, taking into
309 account the limits of the relevant rooms in the ship model.
310

311 *Lightship* is divided into three categories; the steel weight, outfitting weight, and machinery
312 weight. The first weight group is computed using the Schneekluth and Müller-Köster
313 methods (Papanikolaou, 2014). Outfitting and machinery weights are calculated using
314 empirical formulae, taking as input several parameters, such as the main dimensions of the
315 ship, as well as the main engine’s power (Papanikolaou, 2014).
316

317 Even though the methods utilised for this step are empirical approaches and thus an
318 approximation of the exact values, effort is put to get as accurate results as possible. In this
319 context, the application of a correction factor that captures the difference between the
320 lightweight estimation during the concept design stage and the real lightweight, which value
321 becomes available after the construction phase, can minimise the uncertainty associated with
322 the weight estimation during the early stages of ship design. Based on previous studies

(Taggart, 2003), a range of $\pm 6.5\%$ around the mean lightweight value is used in the “uncertainty setup”, where the uncertainty regions for the objective functions are identified.

A custom macro responsible for the *deadweight analysis* is run in NAPA®, generating the necessary values for the determination of the loading case examined, based on the specified operational profile. Deadweight is calculated in such way to achieve an overall displacement matching the design draught, which is part of the design variables set. An allowance of $\pm 10\%$ of the design draught value marks the limit the optimisation algorithm takes into account to evaluate the validity of each design variant (more information is given below in Section 4, where the constraints of the optimisation round are outlined in Table 4).

Taking into account the results of the deadweight analysis, a *load case* is defined, where both passengers and cars are loaded to the maximum level, provided that the intact stability criteria according to the IMO A.749/A.167 guidelines are not violated (IMO, 1993).

The *required freight rate (RFR)* is calculated using custom NAPA® macros. This value indicates the minimum rate that evens the properly discounted ship’s expenses. In this study, the RFR represents the passenger ticket in economy class. Other instances (e.g. passenger ticket in cabin, car fare, truck fare, etc.) are related to the base RFR value by introducing ratios obtained from existing shipping companies’ pricing plans. The method used to estimate the RFR has derived from the SAFEDOR project (SAFEDOR, 2005-2009). There are several groups that are calculated in order to estimate the RFR, including the building and running costs, as well as the and on board, decommissioning, passenger and vehicle earnings. The building cost depends on the parameters related to the steel, outfit and machinery weight, while the running cost is related to the operational profile of the vessel.

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349 Several uncertain parameters are associated with the estimation of the RFR. On the one hand,
350 the fuel cost and the mean ship speed in three distinct operational phases (normal weather
351 conditions, adverse weather conditions and navigation in port), as well as the amount of time
352 the ship operates in each of these phases affect the operation costs. On the other hand, the
353 load factors for passengers and vehicles in three distinct phases (low, mid and high season)
354 affect the on board, passenger and vehicle earnings. For the deterministic solution of the
355 problem, obtained by running the “main setup”, mean values are used for the estimation of
356 the RFR based on historical data, while for the “uncertainty setup”; random values are
357 assigned based on the probability distribution of each parameter. Hence, different values for
358 the RFR are obtained which constitute its uncertainty region.

359

360 The methodology incorporates two recent developments in the international maritime
361 regulations, which are going to affect future ship designs and particularly specific ships types
362 such as Ro-Pax ships.

363

364 One major driver is the requirement for the minimisation of their Green House Gas (GHG)
365 emission footprint, which has been expressed by introduction of the *Energy Efficiency Design*
366 *Index (EEDI)* (IMO, 2012a, IMO, 2012c, IMO, 2012b). The EEDI relates the GHG emissions
367 of a ship to her transportation work and is in fact an indicator of a vessel’s energy efficiency.
368 The requirement for new ships started with some baseline values in 2013, which will be
369 lowered successively in three steps until 2025. Furthermore, IMO has published its plan for
370 50% reduction of its GHG emissions by 2050 in line with the Paris Agreement (Comer et al.,
371 2018). The determination of EEDI is based on a rather complicated looking (but indeed
372 simple) formula, while it is required that the calculated value is below a reference line set by

the IMO regulation for the specific ship type and size. Using custom NAPA® macros, both the attained and the required values for the three implementation phases are computed.

In a parallel development, new safety regulations are being introduced, as a *new generation of intact stability criteria* is currently being prepared by the International Maritime Organisation (IMO) (IMO, 2018). The introduction of ships with newly developed characteristic and operation modes has challenged the assumption that the current criteria are sufficient to prove their stability. Hence, the new criteria will be performance-based and will address five modes of stability failure; parametric roll, pure loss of stability, excessive acceleration, stability under dead ship condition and surf-riding/broaching. As far as Ro-Pax ships are concerned, parametric roll is considered one of the most important modes of stability failure (Peters et al., 2011). Pure loss of stability failure mode should also be investigated, as the considerable flare found in the aft and fore parts of a Ro-Pax hull results in significant changes in the waterplane area as the ship sails through waves. These changes may result in a large roll angle or even capsize (Peters et al., 2011). Likewise, excessive acceleration failure mode should be checked in Ro-Pax cases, due to high superstructures found in such kind of ships. Hence, the draft criteria of level 1 and 2 for excessive acceleration, pure loss of stability and parametric roll failure modes according to SDC 6/5 (IMO, 2018) are included in this methodology.

These checks are performed using custom made programmes (or “features”, as they are called in CAESES®), taking the hull geometry as input and the results of the computations described above as output. Level 1 checks are meant to be simple and conservative, in order to quickly detect any vulnerability to each of the three failure modes. Level 2 checks are more complex, thus less conservative, taking into account more design- and operation-related

aspects in order to determine whether the ship is vulnerable to either of the examined failure modes. For each failure mode, several features are developed within CAESES®, connecting various external software to evaluate certain parameters required to perform these computations. Maxsurf® Stability (Bentley Systems, 2014) is run to produce the necessary metacentric height (GM) values for various wave conditions, while Matlab® (Mathworks, 2014) is used to calculate the roll amplitude, where complex equations must be solved.

3.4 Use of surrogate models

Some of the calculations described above require substantial computational resources, which, if run for every design variant throughout the optimisation, render the whole process extremely time-consuming. For instance, the calculation of calm water resistance in model scale for one design using CFD on High Performance Computer (HPC) facilities (2 x Intel Xeon Gold 6138 20 core 2.0GHz CPU's, 192GB RAM) take around 30 minutes, while the level 1 and 2 checks for the IMO SGIS criteria for one design require around 15 minutes on a standard desktop computer (Intel Core i7 4790 3.60GHz CPU, 8GB RAM). In addition, a large number of design samples are needed in order to quantify the uncertainty in the design process described above. For these reasons, surrogate models are employed. Designs of experiments (DoE) are undertaken for selected design variables using the Latin Hypercube Sampling (LHS) method (McKay et al., 1979). LHS is preferred over the standard Monte-Carlo method, as the former is proven more efficient than the latter in exploring the design space, producing more reasonable results (Aistleitner et al., 2012). The results are used to create response surfaces, which are used in the formal optimisation to evaluate the produced design variants. In this case, three surrogate models are created; (a) for the IMO SGIS criteria, (b) for the CFD output and (c) for the uncertainty quantification. The evaluation of

these response surfaces takes a few seconds and consequently the substantial savings in computational power become apparent.

All the runs required for this purpose are performed within CAESES® in collaboration with Dakota® (Sandia, 2018) software. Taking advantage of Dakota® incremental sampling feature, the number of design variants in each DoE required to build the response surface is kept to a minimum. This is achieved due to the ability to run an initial sample set, check the quality of the produced response surface and, in case the performance of the latter is inadequate, perform further sampling on the same setup to get better estimates of the mean and variance values and to obtain a more comprehensive sample coverage.

The response surfaces are built using the Kriging method (Cressie, 1990) and their accuracy is checked by comparing additional variants that are not contained in the training set with the corresponding results from direct simulations.

3.5 Multi-objective optimisation

The formal optimisation phase is run using a selected algorithm. CAESES® provides a selection of multi-objective optimisation algorithms, including the Non-dominated Sorting Genetic Algorithm II (NSGA-II), the Multi-Objective Simulated Annealing (MOSA) algorithm and the option to connect Dakota® to CAESES® and use it as external optimisation software. The design variables and their extents, as well as the objectives of the optimisation required to be minimised are selected and the inequality and equality constraints, which check the validity of the produced variants, are set.

The results of a multi-objective optimisation define the Pareto front of the non-dominated designs. For those designs an uncertainty quantification phase takes place, where DoEs are run for each design variant. The surrogate model containing information for the uncertainty quantification is evaluated to provide results for the objective functions and identify their uncertainty region. The design variables used in this case are only those associated with uncertainty and their values derive from their probability distributions. The designer selects the amount of scenarios investigated for each design variant. After the completion of the simulation runs, the standard deviation for the examined objectives is calculated. The lowest standard deviation for the objectives is the most preferable condition, since this is translated to a “sturdier”, more robust design, which performance is not affected by the change of the uncertain parameters. Hence, the sum of the standard deviation for the optimisation objectives is calculated relatively to their mean value and is taken into account during the decision making phase. As the decision maker needs to select one design, Multi Attribute Decision Making (MADM) is applied. Case scenarios are defined by the decision maker, to determine the optimal of the top solutions to the problem. In each case scenario the significance of each objective is acknowledged differently by assigning specific “weights” following the utility functions technique of decision making theory (Sen and Yang, 1998).

4 Case study

The methodology described above is applied on a case study of a Ro-Pax ship, optimised in the concept design phase with regard to the minimisation of the total resistance, RFR and building cost. The base model is defined taking into account an existing vessel with similar characteristics, such as the main dimensions and internal compartmentation. The values our model produces (e.g. deadweight, lane metres, lightship, required main engine power) are

checked and validated by comparing them with the existing ship data. The parametric hull for the base model is presented in figures 5 and 6, while the ship model constructed in NAPA® is illustrated in figure 7. The operational profile selected for the case study is presented in table 2.

Figure 5. Parametric hull geometry in CAESES® (base model)

Figure 6. Parametric hull geometry in CAESES® (base model, bodyplan view)

Figure 7. Ship model in NAPA® (base model)

Table 2. Operational profile

Roundtrip route distance (nm)	366
Operational time (years)	25
Number of ports	2
Average time at port (h)	1.5
Average operational speed (Knots)	27

For the formal optimisation phase, NSGA-II is utilised (Deb et al., 2002). In particular, 125 generations are created, having a population size of 12, each. This results in a total of 1500 produced design variants. The mutation and crossover probabilities are set to 0.1 and 0.9 respectively. The design variables and their range are presented in table 3, while the design constraints are presented in table 4. Four objectives are selected to be minimised in this case study:

- Total resistance
- RFR
- Structural building cost
- Calm water resistance uncertainty estimation between the two examined methods (δ_{RCW})

Each of these objectives affects the design in a different way. For instance, minimisation of the resistance would push for slender, longer hull forms. Minimisation of the structural building cost would call for more compact designs in terms of length. The RFR is an objective which takes into account many parameters, such as the building cost, operational profile and financial parameters, such as the fuel price.

In addition, each objective is associated with different aspects of ship design. For instance, total resistance is related to finance and environment, as it affects the required power which is associated with both the operational costs (fuel consumption) and the compliance with environmental regulations (EEDI). Minimisation of the structural building cost would be desirable by shipyards which are responsible for the steel construction.

The objectives are selected in such way to get a more in-depth understanding of how each one influence the rest, as well as how the optimal set of design variables is ultimately selected. Nevertheless, the user can omit, add or substitute any of them according to the objectives of the case study.

Table 3. Design variables

Design variable	Min. value	Max. value
Y-extent of bilge (m)	2	4
Z-extent of bilge (m)	2	4
Double bottom (m)	0.76	2.00
Number of passenger decks	4	6
Number of Ro-Ro decks	2	3

Design draught (m)	5	9
δC_P	-0.02	0.02
δLCB	-0.02	0.02
Number of lanes	6	10
Length between perpendiculars (m)	150	180

519

520 Table 4. Constraints

Design constraint	Value
IMO A.749/A.167 criteria	= 1 (pass)
Excessive acceleration criteria	= 1 (pass)
Parametric roll criteria	= 1 (pass)
Pure loss of stability criteria	= 1 (pass)
“Attained/Required” EEDI ratio	≤ 1
$ \text{Draught}_{\text{Design}} - \text{Draught}_{\text{Load Case}} $	$\leq 10\%$

521

522 Taking into account the definition of the RFR calculation model described in, steel weight is
523 used as a measure to describe the building cost. Despite the increased outfitting weight
524 observed in passenger vessels, compared to merchant ships, the steel weight accounts for
525 roughly 50% of the total lightship weight. In this case study, the minimisation of the steel
526 weight is interpreted as a way to reduce the building cost.

527

528 The newly introduced regulations might affect the optimal ship design. For instance, slender
529 hull forms might be preferred when one considers minimising the attained EEDI value, due to
530 lower resistance values and consequently lower power requirements. However, such hull
531 forms could potentially be unfeasible solutions to a problem where the IMO Second

Generation Intact Stability (SGIS) criteria are taken into consideration, due to the vulnerability of slender bodies in intact stability.

For the uncertainty quantification phase which follows the NSGA-II run, 50 scenarios are checked to calculate the uncertainty regions of the total resistance, the RFR and the steel weight.

As far as the decision making phase is concerned, three case scenarios are defined (Table 5) and the utility function illustrated in equation 6 is calculated to identify the best designs. Before applying the weights w , each utility function value is normalised based on the best and worst performance of the design variants in each objective.

$$U = w_{R_{Total}} u(R_{Total}) + w_{RFR} u(RFR) + w_{W_{Steel}} u(W_{Steel}) + w_{\delta_{RCW}} u(\delta_{RCW}) + w_{\sigma_{RT,RFR,W_{Steel}}} u(\sigma_{RT,RFR,W_{Steel}}) \quad (6)$$

In scenario 1, the five objectives (the four main objectives and the sum of the standard deviation deriving from the uncertainty quantification phase) are considered equally important; hence, each one is assigned a weight at saturation of 20%. On the other hand, in scenario 2, the total resistance, the RFR and the steel weight are chosen to be more significant for the decision maker (designer, operator) by assigning to them a weight of 25% each, whereas the uncertainty indicators (δ_{RCW} and $\sigma_{RT,RFR,W_{Steel}}$) are assigned a weight of 12.5% each. Similarly, in scenario 3, the uncertainty indicators are assigned a weight of 35% each, while each of the remaining takes up 10% of the total. After obtaining the results of the main optimisation run, the data are normalised according to the scenarios. Next, the normalised data are ranked to find the optimal variant of the model. The maximum score that can be

achieved after this process for each design in each case scenario is 1, whereas the lowest is 0.

In all cases, a specific variant dominates in every scenario.

Table 5. Case scenarios – weight at saturation

Objective	Scenario 1	Scenario 2	Scenario 3
Total resistance	20%	25%	10%
RFR	20%	25%	10%
Steel weight	20%	25%	10%
δ_{RCW}	20%	12.5%	35%
$\sigma_{RT,RFR,WSteel}$	20%	12.5%	35%

5 Discussion of results

Before proceeding to the optimisation results, some essential information about the base model is presented in order to have a clear perspective of the initial hull (Tables 6, 7).

Table 6. Base model design variable values

Design variable	Base model
Y-extent of bilge (m)	3
Z-extent of bilge (m)	3
Double bottom (m)	1.5
Number of passenger decks	5
Number of Ro-Ro decks	2
Design draught (m)	7.1

δC_P	0
δLCB	0
Number of lanes	8
Length between perpendiculars (m)	162.85

Table 7. Base model objective values

Objective	Base model
Total resistance (KN)	1999
RFR (\$)	41.86
Steel weight (t)	6346
δ_{RCW} (KN)	351
$\sigma_{RT,RFR,WSteel}$	0.417

5.1 Surrogate models evaluation

The setup of the DoEs run to generate the required response surfaces is presented in table 8.

Two rounds of DoEs are run in order to exploit the incremental feature and save on computational time.

Table 8. Design of experiments

Purpose	Designs (Round 1)	Designs (Round 2)
IMO SGIS criteria	150	150
CFD output	150	150
Uncertainty quantification	500	500

The typical deviations for each response surface are presented in table 9. For illustration purposes, the accuracy of response surfaces for the CFD output and one objective examined in the uncertainty quantification phase are presented in figures 8 and 9 by comparing some designs within the analysed design space.

Table 9. Response surface accuracy

Purpose	Typical deviation
IMO SGIS criteria	0.92%
CFD output	0.49%
Uncertainty quantification	0.71%

Figure 8. Accuracy of the response surface for the CFD output

Figure 9. Accuracy of the response surface for the uncertainty quantification

5.2 Multi-objective optimisation

Following the NSGA-II run and the evaluation of the results, the designs located on the Pareto front were identified and the uncertainty setup was run. The relative standard deviation for the examined objectives was calculated and the designs were ranked during the decision making phase. Three designs were identified as the best, named Des0903, Des1417 and Des1419. All three of them lie on the Pareto fronts illustrated in figures 10 and 13. Des1417 ranked first in the first two scenarios, while Des0903 was first in scenario 3 ranking, which is uncertainty-oriented. On the other hand, Des1419 seems to be an all-around solution as it ranked second in all three scenarios (Table 10). Nevertheless, in the three examined scenarios, the top ranked designs are the same, which illustrates the strength of the identified Pareto front.

Table 10. Scenarios rankings

Scenario 1	Scenario 2	Scenario 3
------------	------------	------------

Des1417	0.7918	Des1417	0.7760	Des0903	0.8927
Des1419	0.7907	Des1419	0.7549	Des1419	0.8622
Des0903	0.7861	Des0399	0.7334	Des1417	0.8235
Des0399	0.7587	Des0903	0.7328	Des0860	0.8189
Des0860	0.7568	Des0706	0.7314	Des0399	0.8091
Des0706	0.7545	Des1084	0.7261	Des1084	0.8073
Des1084	0.7531	Des0860	0.7258	Des0706	0.8007
Des0505	0.7454	Des0505	0.7246	Des0565	0.7920
Des0839	0.7384	Des0839	0.7221	Des0505	0.7870
Des0565	0.7369	Des0422	0.7177	Des0839	0.7710

605

606 A set of graphs containing the relation between the optimisation objectives is presented

607 below. The Pareto front is demonstrated by a solid black line in each case.

608

609 As far as the relation between the total resistance and the RFR is concerned, two groups of

610 design variants can be identified, the main one ranging between 30 and 60 \$/passenger and a

611 smaller one in the range of 90 – 110 \$/passenger. This is can be explained by the value of the

612 total lane metres in each design variant, as the designs characterised by higher RFR values

613 are primarily those which feature 6 or 7 lanes, while the designs with 8 or more lanes

614 achieved lower RFR values. The optimisation algorithm pushed the design space to the

615 region of 1600 – 2000 KN, as far as the total resistance is concerned, with most of the designs

616 achieving such numbers for the total resistance. All three identified improved designs

617 perform much better with regard to the resistance, while Des1419 and Des0903 achieved a

618 value of less than 40 \$/passenger for the RFR. A slight rise in the RFR can be observed as the

619 total resistance increases (Figure 10).

Figure 10. Total resistance vs. RFR

Figure 11. RFR vs. Steel weight

Figure 12. Steel weight vs. Total resistance

Figure 13. δ_{RCW} vs. RFR

As far as the uncertainty levels related to the optimisation process, the results are presented in figure 14. The relative standard deviation for the examined objectives is shown for the Pareto designs and all three identified improved variants achieved among the best values for that additional objective. In fact, Des0903 was the best performer, i.e. the most robust design of the case study.

Figure 14. Relative standard deviation

653

654 The design variable values and the objective responses of the three improved designs can be
 655 found in tables 11 and 12. The values can be compared with the base model information
 656 given in tables 6 and 7.

657

658 Table 11. Improved designs design variable values

Design variable	Des0903	Des1417	Des1419
Y-extent of bilge (m)	2.914	3.846	3.745
Z-extent of bilge (m)	3.460	3.971	2.796
Double bottom (m)	1.295	1.436	0.775
Number of passenger decks	5	4	5
Number of Ro-Ro decks	2	2	2
Design draught (m)	5.256	5.005	5.005
δC_p	-0.01667	-0.01403	-0.01497
δLCB	-0.01468	0.00364	-0.01199
Number of lanes	8	8	8
Length between perpendiculars (m)	163.424	164.404	160.826
Lane metres (m)	1813	1745	1731

659

660 Table 12. Improved designs objective values

Objective	Des0903	Des1417	Des1419
Total resistance (KN)	1697	1632	1648
RFR (\$)	37.58	41.88	37.55
Steel weight (t)	6138	5716	6070

δ_{RCW} (KN)	4	48	19
$\sigma_{RT,RFR,WSteel}$	0.400	0.436	0.417

Some general observations can be made regarding the differences between the base model and the improved designs. Firstly, the number of passenger and Ro-Ro decks seems to be constant among these designs. The same applies to the number of lanes, since all of the three identified improved variants feature 8. Des903 and Des1419 feature the lowest RFR values of the three identified best designs, while their steel weight is higher than the one of Des1417. On the other hand, the former designs feature an extra passenger deck, which can be translated to a greater passenger capacity, explaining the lower RFR value.

By analysing the results, it becomes clear that the selection of an optimal design is a complicated process. On the one hand, Des1417 is considered the best option when the total resistance, RFR and steel weight are declared the most significant objectives (scenario 2). However, if uncertainty is introduced to the decision making process, Des0903 ranks in the first place (scenario 3). This is a typical outcome in robust design optimisation, where the optimal design might not be the most robust. Des0903 achieved the highest consistency in its performance with regard to the examined objectives, irrelevant of the changes in the uncertain parameters, compared to the rest of the designs. On the other hand, Des1417 ranked first in a scenario where uncertainty is not taken seriously into account.

The improved designs are compared with the base model below in figures 15, 16 and 17, showing the differences in the hull lines.

Baseline (gray)
Des0903 (black)

Figure 15. Base model vs. Des0903

Baseline (gray)
Des1417 (black)

Figure 16. Base model vs. Des1417

Figure 17. Base model vs. Des1419

6 Summary

A method for a multi-objective robust early stage ship design optimisation under uncertainty has been presented. The method captures the effect of the uncertain parameters of the problem by examining several scenarios, during which the uncertain parameters are assigned random values according to given probability distributions and the response of the ship model is analysed. The uncertainty is measured by calculating the standard deviation of the generated values for the objectives of the optimisation problem and is included as an additional objective in the decision making process. In addition, the prediction of the calm water resistance is examined using two methods of varying fidelity levels and the difference between the results is monitored to achieve greater coherence.

The flexibility of the methodology setup should be mentioned, thanks to the connectivity capabilities of CAESES®. While NAPA® is utilised to obtain some results within the

process, the required output to evaluate the objective functions can be obtained by similar software platforms (e.g. Maxsurf®). In addition, the objective functions of the methodology are not limited to the ones described in this paper. For instance, the definition of a complete ship model included in the presented methodology allows the addition of damage stability in the list of the objective functions (Paterson et al., 2018a, Paterson et al., 2018b).

In order to save on computational time, two measures are taken. Firstly, the uncertainty quantification takes place only for the designs located on the Pareto front, since the uncertainty regions for the objectives is expected to be relatively confined to render the capture of uncertainty possible. Secondly, surrogate models are utilised for several computations, for which the level of accuracy is checked before the formal optimisation phase begins to ensure the reliability of the produced values is satisfactory.

The method is applied on a case study where a Ro-Pax vessel is optimised with regard to the total resistance, RFR and building cost (in terms of minimisation of the steel weight). The results of the study demonstrate the dilemma decision makers are confronted with in a robust optimisation problem. Two best designs are identified, one performing better with regard to the three aforementioned objectives, and one showing higher levels of robustness. The former might be the obvious solution when uncertainty is not taken into account, however, the latter can be a safer choice for the decision maker as the smaller deviations from the average performance indicates a more robust design that is not affected by the change of the uncertain parameters of the problem.

Overall, the study indicates that uncertainty has a significant impact on ship design. When considered during an optimisation study, uncertainty affects the results and complicates the

decision related to the selection of the optimal design. The paper highlights the need to incorporate uncertainty in the solution of the ship design optimisation problem and proposes a method to accomplish that while preventing the significant increase of the computational time.

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