Streak Growth in High-Speed Boundary Layers: Assessment through the Compressible Boundary Region Equations

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ABSTRACT

Streamwise vortices and the associated streaks evolve in boundary layer flows over flat or concave surfaces as a result of various disturbances initiated in the upstream or from the wall surface. Following the transient growth, they can lead to secondary instabilities and early transition to turbulence via bursting processes. It is desirable, therefore, to accurately predict and efficiently control the growth of these disturbances in an attempt to reduce the associated frictional drag. In high-speed boundary layers, additional complications are involved due to the compressibility and thermal effects, the level of contribution of which scales with the Mach number. In this work, we study streaks in high-speed boundary layers via the numerical solution to the full nonlinear compressible boundary region equations, which is the high Reynolds number asymptotic extension of the Navier-Stokes equations in the assumption that the streamwise wavenumber of the disturbances is much smaller than the wall-normal and spanwise wavenumbers (this is characteristic to boundary layer streaks). The base flow is excited by freestream disturbances imposed at the upstream boundary, and the growth of the streaks is studied using bi-global stability analysis.

1. Introduction

Streaks formation in pre-transitional boundary layer flows over flat or curved surfaces occur when the height of roughness elements exceeds a certain critical value, or the amplitude of the free-stream disturbances is greater than a given threshold. The stream-wise component of velocity exhibit elongated streaky features, characterized by adjacent regions of acceleration (high-speed streaks) and deceleration (low-speed streaks) of fluid particles. Elongated streaks in the form of streamwise (Görtler) vortices also appear inside a boundary layer flow along a concave surface due to the imbalance between radial pressure gradients posed by the wall, and centrifugal forces. For highly curved walls, for example, vortex formation occurs more rapidly and can significantly alter the mean flow causing the laminar flow to breakdown into turbulence. Under certain conditions, Görtler vortices can be efficient precursors to transition: they consist of counter-rotating streamwise vortices that grow at a certain rate, depending on the surface curvature and the receptivity of the boundary layer to environmental disturbances and surface imperfections. In boundary layers over surfaces of small to medium curvature, these vortices can significantly alter the mean flow and cause the laminar flow to breakdown. It was recognized in many studies that boundary layer streaks are important in the path of transition to turbulence in a laminar flow. Many previous studies and results indicated that it is the transient part of the disturbance that dominates the growth of streaks or other three-dimensional disturbances that lead to breakdown, so any effective method of control must focus on restricting the development of the transient modes.

In the compressible regime, bypass transition remains largely unexplored. A number of experiments have been conducted to establish the gross correlation between the transition Reynolds number and freestream turbulence level (FST). They showed that the transition position shifts significantly depending on both freestream turbulence level (Dryden [1], Schneider [8]) and surface roughness (e.g. Pate [5]). However, there exist only a few investigations of the detailed physics underlying such correlation. The experiments of Kendall [2] provide much information concerning supersonic boundary-layer transition under the influence of high level FST. A salient feature is that fluctuations over a wide frequency range undergo substantial growth within the boundary layer.

The focus of this work is on the investigation of streamwise vortices and the associated streaks that develop in high-speed boundary layers, over both flat and curved surfaces, by using the full nonlinear compressible boundary region equations, which is a high Reynolds number asymptotic extension of the Navier-Stokes equations in the assumption that the streamwise wavenumber of the disturbances is much smaller than the wall-normal and spanwise wavenumbers This set of equations is parabolic in the streamwise direction, allowing for a straightforward marching procedure to be applied along the streamwise direction.

2. Scalings and Governing Equations

We consider a compressible boundary layer flow over a flat or concave surface excited by upstream freestream disturbances or surface non-uniformities. The span-wise length scale, Λ^*, is in the same order of magnitude as the local boundary-layer thickness. The air is treated as a perfect gas so that the sound speed in the free-stream $c^*_\infty = \sqrt{\gamma R T^*_\infty}$, where $\gamma = 1.4$ is the ratio of the specific heats, and $R = 287.05 Nm/(kgK)$ is the universal gas constant; Mach number is assumed to be of order one.

All dimensional spatial coordinates (x^*, y^*, z^*) are normalized by the spanwise separation λ^*, while the dependent variables by their respective freestream values, except the pressure, which is normalized by the dynamic pressure. Reynolds number based on the spanwise separation, Mach number and Prandtl number are defined as $R_\lambda = \rho^*_\infty V^*_\infty \lambda^*/\mu^*_\infty, Ma = V^*_\infty/a^*_\infty, Pr = \mu^*_\infty C_p/k^*_\infty$, where μ^*_∞, a^*_∞ and k^*_∞ are dynamic viscosity, sound speed and thermal conductivity, respectively,

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and C_p is the specific heat at constant pressure.

The flow is divided into four regions as in Leib et al. [3] or Marensi et al. [4]: a) region I in proximity to the leading edge, where the flow is assumed inviscid and the disturbances are treated as small perturbations of the base flow; b) region II is the boundary layer in the vicinity of the leading edge, with the thickness much smaller than the spanwise separation associated with the freestream disturbances; c) region III is the viscous region that follows in the downstream of region II; here, the boundary layer thickness is in the same order of magnitude as the spanwise separation d) region IV is above region III, and the flow is assumed again inviscid since the viscous effects are negligible.

The focus of this paper is on region III, where the streamwise wavenumber of disturbances are expected to be small, and the flow is governed by the compressible boundary region equations. These equations can be derived from the full compressible Navier-Stokes equations based on the above assumption: the streamwise distance can be scaled as $x = \bar{x}/R_\lambda$ while the other two coordinates are kept the same $y = \bar{y}$, $z = \bar{z}$, and the time as $t = \bar{t}/R_\lambda$. Also, the cross-flow components of velocity are expected to be small compared to the streamwise component, and variations of pressure are expected to be very small. This suggest the introduction of the scaling of dependent variables as: $u = \bar{u}$; $v = \bar{v}/R_\lambda$; $w = \bar{w}/R_\lambda$; $\rho = \bar{\rho}$; $p = \bar{p}/R_\lambda^2$; $T = \bar{T}$; $\mu = \bar{\mu}$; $k = \bar{k}$;. Plugging these into the full Navier-Stokes equations, and performing on order of magnitude analysis, the boundary region equations are obtained in the form

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\vec{V} = 0$$

$$\rho\frac{Du}{Dt} = \nabla_c\cdot(\mu\nabla_c u)$$

$$\rho\frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial y}\left[\frac{2}{3}\mu\left(3\frac{\partial v}{\partial y} - \nabla\cdot\vec{V}\right)\right] + \frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu\nabla_c\cdot\vec{V}\right)$$

$$\rho\frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial z}\left[\frac{2}{3}\mu\left(3\frac{\partial w}{\partial z} - \nabla\cdot\vec{V}\right)\right] + \frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial z}\right) + \frac{\partial}{\partial y}\left(\mu\nabla_c\cdot\vec{V}\right)$$

$$\rho\frac{De}{Dt} = \frac{1}{(\gamma-1)M_\infty^2 Pr}\nabla_c\cdot(k\nabla_c T) + \mu\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right],$$

where ∇_c is the crossflow nabla operator: $\nabla_c = \partial/\partial y\vec{j} + \partial/\partial z\vec{k}$. This set of equations is parabolic in the streamwise direction and elliptic in the spanwise direction. Appropriate initial/upstream and boundary conditions are necessary to close the problem; these conditions are the same as those used by Ricco & Wu [7].

3. Results

In this section, we show results from a flat-plate boundary layer excited by an upstream vortical disturbance with the amplitude being 3% from the base flow. Two Mach numbers are considered, 3 and 6, corresponding to the spanwise wavelengths of 4 and 2 mm, respectively. In figure 1, contour of temperature in (y, z) cross-planes at several different stramwise locations are plotted for both adiabatic (a and c) and isothermal (b and d) wall conditions. As was found in previous studies (see for example, Spall & Malik [6]), the isothermal wall seems to be more destabilizing than the adiabatic wall; also, as the Mach number is increased, the strength of the streamwise vortices decreases slightly. This is quantitatively illustrated in figure 2, where the energy of the vortices (left) and the growth rate of energy (right) are plotted against the streamwise coordinate. Another observation inferred from figure 2 is that the energy corresponding to the isothermal wall (in black) reaches saturation earlier than the adiabatic wall.

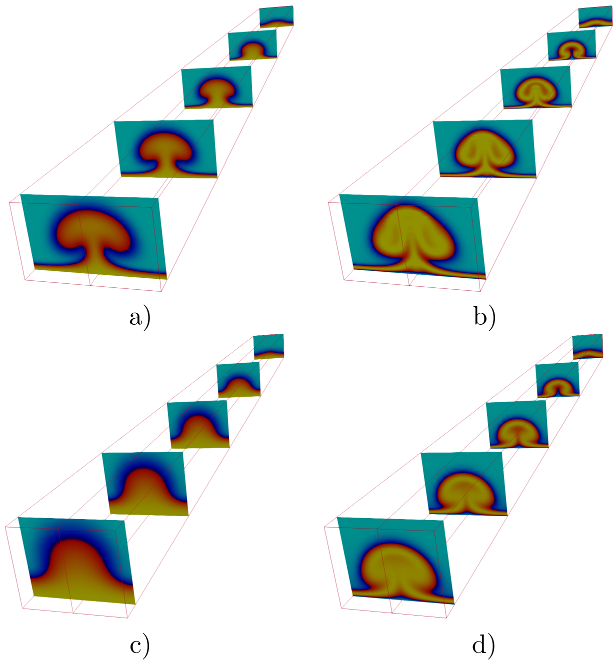

Fig. 1 Contours of: a) streamwise velocity; b) temperature.

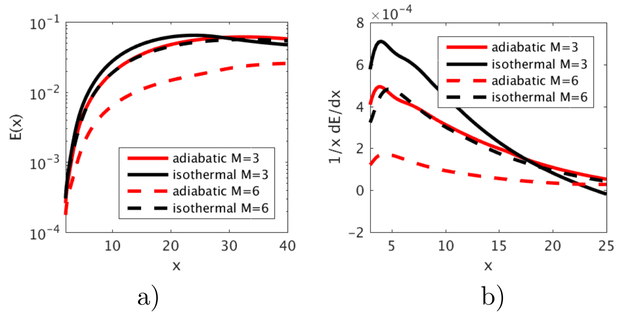

Fig. 2 Energy as a function of x: a) wall deformations; b) wall transpiration.

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