IN-PLANE AND OUT-OF PLANE FAILURE OF AN ICE SHEET USING PERIDYNAMICS

Bozo Vazic*

PeriDynamics Research Centre
University of Strathclyde
Glasgow, UK

Erkan Oterkus **Selda Oterkus**

PeriDynamics Research Centre
University of Strathclyde
Glasgow, UK

ABSTRACT

When dealing with ice structure interaction modeling, such as designs for offshore structures/icebreakers or predicting ice cover's bearing capacity for transportation, it is essential to determine the most important failure modes of ice. Structural properties, ice material properties, ice-structure interaction processes, and ice sheet geometries have significant effect on failure modes. In this paper two most frequently observed failure modes are studied; splitting failure mode for in-plane failure of finite ice sheet and out-of-plane failure of semi-infinite ice sheet. Peridynamic theory was used to determine the load necessary for in-plane failure of a finite ice sheet. Moreover, the relationship between radial crack initiation load and measured out-of-plane failure load for a semi-infinite ice sheet is established. To achieve this, two peridynamic models are developed. First model is a 2 dimensional bond based peridynamic model of a plate with initial crack used for the in-plane case. Second model is based on a Mindlin plate resting on a Winkler elastic foundation formulation for out-of-plane case. Numerical results obtained using peridynamics are compared against experimental results and a good agreement between the two approaches is obtained confirming capability of peridynamics for predicting in-plane and out-of-plane failure of ice sheets.

Keywords: Ice; Fracture; Peridynamics; Winkler elastic foundation.

1. INTRODUCTION

Modelling ice-structure interaction is a very difficult process. First of all, many different factors such as strain-rate, temperature, applied-stress, salinity, grain-size, confining pressure and porosity have significant influence on ice material response. Furthermore, macro-scale modeling may not be sufficient to capture the full physical behavior because the micro-scale effects may have a significant effect on macroscopic material behavior. Hence, it is necessary to utilize a multiscale methodology. To represent the macroscopic ice behavior accurately, Finite Element Method (FEM) has been widely used in the literature. Within FEM framework, various techniques can be used to model crack propagation including extended finite element method (XFEM) and cohesive zone model (CZM). However, a universally accepted CZM failure model is not currently available and the crack propagation may have mesh dependency. Although, the mesh dependency problem can be overcome by XFEM, enrichment process may lead to an algebraic system with a large number of unknowns which is difficult to solve numerically. Furthermore, since FEM is based on classical continuum mechanics, its formulation do not contain a length scale parameter. Hence, FEM is incapable of capturing phenomenon at the micro-scale. Hence, other techniques should be utilized at the micro-scale and linked to FEM simulation. However, it is not straightforward to obtain a smooth transition between different approaches at different scales. By taking into account all these challenging issues, a new continuum mechanics formulation, peridynamics, can be used for modelling ice failure. Peridynamics is classified as a non-local continuum mechanics formulation and it does not contain spatial derivatives in its formulation. Hence, it is very suitable to predict crack initiation and propagation occurring within the material as the material is subjected some external loading condition. Furthermore, due to its non-local character, it can capture the phenomenon at multiple scales.

There exists significant number of studies in the literature focusing on in-splitting failure of ice sheets. Moreover,

* Corresponding author (bozo.vazic@strath.ac.uk)

there is a large volume of experimental studies done in the field or multiple lab tests. Timco [1] has done a series of indentation and penetration tests on a floating sheet of ice. During these tests, splitting of an ice sheet was identified and named as “radial cracks”. Although this was an extensive study, it still lacked information on the size and geometry of the ice sheet since most of the effort was focused on indentation rate and aspect ratio. Similarly, Grape and Schulson [2] investigated the influence of lateral confinement on failure patterns and indentation pressure. Most of the aforementioned studies when used for splitting problems were obscured by many ambiguities, until Dempsey et al. [3] conducted a series of in-situ fracture tests on edge-cracked square plates (sized from 0.5-80 m) together with nonlinear fracture mechanics (NLFM) analysis. This study provides a clear picture on splitting loads and the scale effect on fracture toughness and ice strength. Another paper that is very important is by Lu et al. [4] where the authors conducted an extensive research on splitting failure of an ice sheet together with conducting an exhaustive overview of different methods used to calculate the splitting loads, such as, LEFM, Cohesive Zone, and Plastic limit theory.

On the other hand, out-of-plane failure of an ice sheet has also been studied extensively in the literature either experimentally or theoretically including Ashton [5], Kerr [6], Langhorne et al. [7], Michel [8], Sodhi [9] and Squire et al. [10]. The focus of this research has been mainly related to the interaction of sloping structures and ice in ice infested waters. In arctic marine environment offshore structures and ships have specific design. For offshore structures those are usually a sloped pylons and for ships/icebreakers specific shape of the bow. The reason behind sloping geometry is to introduce a vertical load on the edge of the incoming ice flow in order to force the ice into a bending failure mode. Bending failure mode induced in such a way can be described as some type of out-of-plane failure mode. For all intents and purposes, numerical ice sheet models are usually represented as an infinitely long thin plate resting on a Winkler elastic foundation.

In this study, two specific examples are investigated; in-plane failure and out-of-plane failure of an ice sheet subjected to an edge load. These two cases can be derived from the same real life example – an ice sheet interacting with a sloping structure. As it was described by Lu et al. [11] only initial contact between the ice sheet and the sloping structure is taken into account (see Fig 1). When ship’s hull interacts with an ice sheet a complex stress condition will develop which can be represented by four different load components as shown in Fig. 1. Loads in y direction are ones inducing the in-plane failure of an ice sheet. The load component along z-direction is responsible for the out-of-plane failure of ice sheet. The in-plane compressive stress within the ice sheet increases due to the load component along x-direction. Although ice failure condition and the fracture patterns will be affected by loads in all three directions, interactions between loads are ignored and they are decoupled into an in-plane and out-of-plane problem. Hence, in this study a simplified decoupling approach is utilized by constraining the in-plane failure problem to only F_y load and out-of-plane failure problem to F_z load. For the in-plane failure problem, a 2 dimensional bond based peridynamic model is implemented and for out-of-plane failure problem, 2 dimensional plate is modeled as a Mindlin plate resting on a Winkler elastic foundation. The idea behind this study is to use peridynamic theory to predict ice splitting load for in-plane fracture of finite ice sheet. Moreover, the relationship between radial crack initiation load and measured out-of-plane failure load corresponding to the eventual failure of the ice sheet is established.

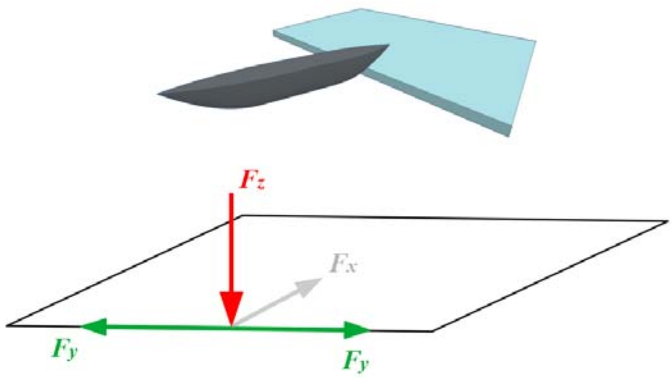

Fig. 1 Contact and load description

2. PERIDYNAMIC THEORY

Silling [12] developed Peridynamic (PD) theory as an alternative continuum mechanics formulation to classical continuum mechanics (CCM). As opposed to localized concept of CCM, non-local interactions exist amongst material points in PD theory. Therefore, material points which are far from each other can interact with each other if they are within their interaction range, called horizon, (see Fig. 2). All material points, **x'**, within the horizon, H_x, of the material point, **x**, can be considered as the members of the family of the material point, **x**. Moreover, PD theory uses displacements rather than derivatives of displacements. Therefore, the governing equation of the material point, **x** can be written in the form an integro-differential equation:

$$\rho\ddot{\mathbf{u}}(\mathbf{x},t) = \int_{H_{\mathbf{x}}}(\mathbf{f}(\mathbf{x}'-\mathbf{x},\mathbf{u}'-\mathbf{u}) - \mathbf{f}'(\mathbf{x}'-\mathbf{x},\mathbf{u}'-\mathbf{u}))dV' + \mathbf{b}(\mathbf{x},t) \quad (1)$$

and this equation is always valid even with the existence of discontinuities in the structure such as cracks. In Eq. (1), $\mathbf{u}$, $\mathbf{b}$ and ρ represent the displacement vector, body load, and mass density, respectively. "Dot" symbol represents the time derivative and $\mathbf{f}$ denotes the peridynamic force density vector representing the force that the material points $\mathbf{x}'$ and $\mathbf{x}$ exerting on each other. The relative position of the material points $\mathbf{x}'$ and $\mathbf{x}$ with respect to each other and their relative displacements are defined, respectively, as:

$$\boldsymbol{\xi} = \mathbf{x}' - \mathbf{x} \quad (2)$$

and

$$\boldsymbol{\eta} = \mathbf{u}(\mathbf{x}',t) - \mathbf{u}(\mathbf{x},t) \quad (3)$$

In the original bond based peridynamic formulation, peridynamic force for an elastic and isotropic material can be written as

$$\mathbf{f}(\boldsymbol{\eta},\boldsymbol{\xi}) = \frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{|\boldsymbol{\xi}+\boldsymbol{\eta}|} f(|\boldsymbol{\xi}+\boldsymbol{\eta}|,\boldsymbol{\xi}) \quad (4)$$

where $f(|\boldsymbol{\xi}+\boldsymbol{\eta}|,\boldsymbol{\xi})$ is a scalar function which depends on the bond constant, c and the bond stretch, s as

$$f(|\boldsymbol{\xi}+\boldsymbol{\eta}|,\boldsymbol{\xi}) = cs \quad (5)$$

The bond stretch can be expressed as

$$s = \frac{|\boldsymbol{\xi}+\boldsymbol{\eta}| - |\boldsymbol{\xi}|}{|\boldsymbol{\xi}|} \quad (6)$$

The bond constant c, can be written as a function of elastic modulus, E, and horizon size δ as:

$$c = \frac{2E}{A\delta^2}\ (1D); \frac{9E}{\pi h\delta^3}\ (2D);\ \frac{12E}{\pi\delta^4}\ (3D) \quad (7)$$

where h, A and δ represent the thickness, cross-sectional area and horizon size, respectively. In PD theory, the material damage is included as part of the constitutive relationship by introducing a failure parameter, so that if the stretch is greater than a critical value, failure parameter reduces to the peridynamic force value to zero. Hence, the peridynamic interaction (bond) between two initially interacting material points is eliminated (broken).

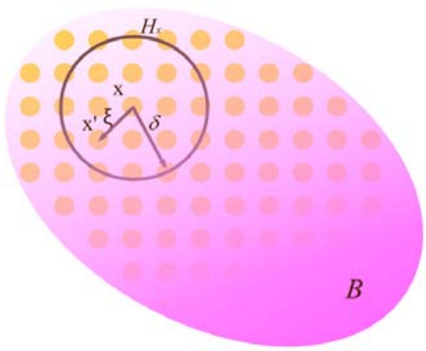

Fig. 2 The material point $\mathbf{x}$ interacts with other material points inside its horizon H_x

In order to numerically solve PD governing equation provided in Eq. 1, meshless method is widely used. Therefore, Eq. 1 can be represented in a discrete form as

$$\rho\ddot{\mathbf{u}}(\mathbf{x}_{(k)},t) = \sum_{i=1}^{N}\mathbf{f}(\mathbf{u}(\mathbf{x}_{(i)},t) - \mathbf{u}(\mathbf{x}_{(k)},t), \mathbf{x}_{(i)} - \mathbf{x}_{(k)})V_{(k)} + \mathbf{b}(\mathbf{x}_{(k)},t) \quad (8)$$

where i is the family member of the main material point, k and N is the number of family members inside the horizon of the material point k.

3. 2D BOND BASED PERIDYNAMIC MODEL FOR IN-PLANE FAILURE OF AN ICE SHEET

As mentioned earlier, in-plane splitting failure of an ice sheet has been extensively studied in the literature. There is a large volume of experimental studies done in the field or multiple lab tests. Amongst these studies, experiments that dealt directly with splitting of the ice sheet were only done by Dempsey et al. [3]. In their in-situ fracture tests, a series of edge-cracked square plates (sized from 0.5-80 m) were torn apart by using a hydraulically driven flatjack. Moreover, they studied the effect of ice sheet size on fracture properties (such as the fracture toughness, traction and separation law for ice, and evolution of the failure process zone) using known histories of load-displacement information and relevant crack opening displacement. The idea behind this analysis stems from Lu et al. [4] where the main goal is to obtain the ice splitting load utilizing analytical and numerical methods and compare the results with the experimental results of Dempsey et al. [3]. Difference between Lu et al. [4] and this study is in methods used for comparison with the experimental data. They used several methods, such as CZM + weight function, LEFM + weight function, Plastic limit theory etc., where in this study 2D Bond Based Peridynamic model was applied.

Table 1 Test model dimensions and peridynamic inputs

	L = 3 [m]	L = 10 [m]	L = 30 [m]	L = 80 [m]
A [m]	0.3	3	9	24
dx [m]	0.015	0.05	0.15	0.4
dt [s]	1	1	1	1
Num. PD points	40000	40000	40000	40000
$\sigma_{load}[kPa/s]$	12.965	7.101	4.1	2.511

Plate geometry and crack size are defined in Fig. 3. Parameters used to generate peridynamic model for this study is given in Table 1, with a fixed horizon size of δ=3dx, where dx is the distance between material points. Ice is modelled as an isotropic material with Young's modulus of E=5GPa with ice thickness h=1.8m. For this example sea ice is considered as a brittle material. Mode-I fracture energy of sea ice is given as 15 J/m^2.

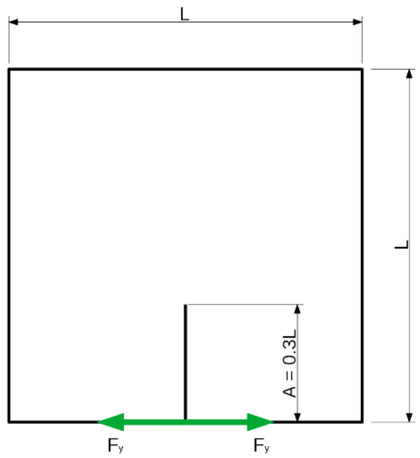

Fig. 3 Model outline for in-plain failure of ice sheet.

Loading rate was obtained the same way as it was done by Lu et al. [4]. In this study, point load was assumed as the loading with a loading rate equivalent to the loading rate of the flatjack. From Figure 6 of the reference [13], the loading rate of the flatjack can be obtained as 0.41 kPa/s for a size of 30×30m^2 ice sheet. It is also assumed that as the size of the ice sheet decreases, loading rate increases. In other words, the smallest ice sheet will have the fastest loading rate (in our case 3 m). This leads us to the following relationship:

$$\dot{\sigma}_{load_3} = \dot{\sigma}_{load_{30}} * \sqrt{\frac{30}{3}} = 1.2965 \ \ [kPa/s] \tag{9}$$

In our case loading rate was increased by factor 10 as it can be seen from Table 1. This was done in order to decrease computational time.

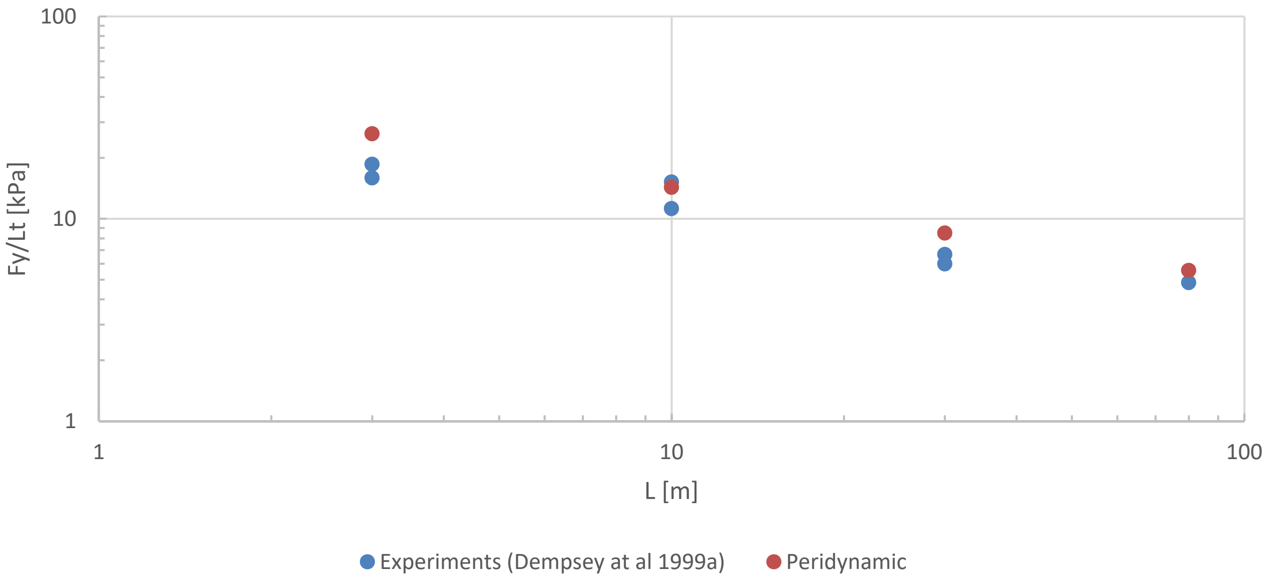

Fig. 4 Splitting load comparison between PD results and experiments

As it can be seen from the Fig. 4 results obtained for the splitting load of the ice plate from the 2D Bond Based Model have a relatively good agreement with the experimental data. Furthermore, peridynamic results agree well when compared to LFEM + weight function' and `CZM + weight function' results from Lu et al. [4], shown in Fig. 14.

4. PERIDYNAMIC MINDLIN PLATE ON WINKLER FOUNDATION MODEL FOR OUT-OF PLANE FAILURE OF AN ICE SHEET

This out-of-plane failure case was originally considered by Lu et al. [11], where the ice plate interacting with the conical body is simplified by representing the ice sheet with a thin plate resting on an Winkler elastic foundation subjected to an evenly distributed edge pressure inside a half circular area. In their study, Lu et al. [11] tried to determine the size of the ice sheet which can be considered either a finite size ice sheet or a semi-infinite ice sheet. The distinction between finite and semi-infinite sheet sizes was classified by following definition:

1) finite size ice sheet collapses due to radial cracks

2) semi-infinite ice sheet collapses by first occurrence of radial cracks followed by development of circumferential cracks, where following relationship holds (introduced by Kerr [6] in Eq. (77)) $F_{Z,B,semi}^{test} \approx 1.6 F_{Z,R0,semi}$

   a. $F_{Z,B,semi}^{test}$ is measured out-of-plane bending failure load for a semi-infinite ice sheet

   b. $F_{Z,R0,semi}$ is maximum load needed to initiate the radial crack (see [11])

Authors were able to establish the difference between size of finite and semi-finite ice sheet by studying only finite size ice sheet which they theoretically formulated and then solved it numerically. What they didn't show were actual fracture patterns for the different size ice sheets and also didn't establish the relationship between maximum load required to initiate the radial crack and measured out-of-plane bending failure load for a semi-infinite ice sheet. Fracture patterns have been studied in a separate paper, see Vazic et al. [14]. In our work we have tried to capture the relationship described by Kerr [6] analyzing several different ice sheet sizes.

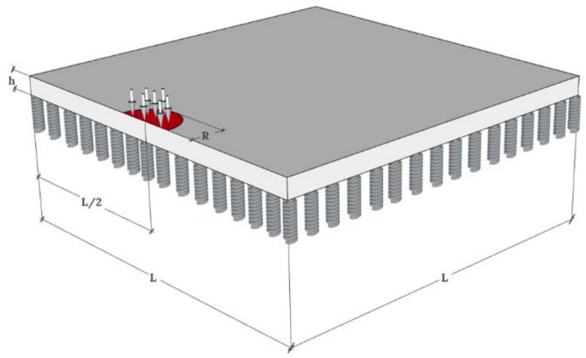

Fig. 5 Model outline for out-of-plain failure of ice sheet [14]

In Lu et al. [11] the ice plate is modelled by using Kirchhoff's plate bending theory, and elastic foundation was defined with Winkler foundation model, where the Kirchhoff's plate bending theory was solved with Finite Element Method (FEM). For the finite size ice sheet, Lu et al. [11] defined two length scales including the characteristic length l (given in Eq. 10), and the physical length L, where physical length is defined as $L = n*l$ with n being a non-dimensional factor. Hence, the characteristic length can be written as

$$l = \sqrt[4]{\frac{D}{k}} \tag{10}$$

where D is the flexural rigidity of the ice sheet defined as

$$D = \frac{Et^3}{12(1-\nu^2)} \tag{11}$$

and k is the foundation modulus for the fluid base, i.e.

$$k = \rho_w g \tag{12}$$

with ρ_w and g represent the fluid density and gravitational acceleration, respectively.

This study is based on Mindlin plate formulation developed by Diyaroglu et al. [15] and Mindlin plate resting on Winkler foundation developed by Vazic et al. [16]. The formulation is capable of analyzing Mindlin plates resting on an elastic Winkler foundation with damage prediction capability. Moreover, the direct solution approach [17] is used to obtain the solution in static conditions rather than using widely adapted Adaptive Dynamic Relaxation (ADR) scheme [18].

According to Lu et al. [11] and compared to Nevel's solution [19] normalized radial crack initiation load is approaching solution of a semi-infinite plate when the ice sheet size is $L \geq 4l$. Effectively, this implies that square finite ice sheet can be approximated as a semi-infinite ice sheet when its physical size is 4 times bigger than its characteristic length l. For our analysis this means that plates bigger than $4l$ should mirror the relationship established by Kerr [6] - $F_{Z,B,semi}^{test} \approx 1.6 F_{Z,R0,semi}$

Within our example, we have considered several semi-infinite ice sheet lengths. Ice sheet length is defined by non-dimensional factor n = 5,7,9,12,15 where $L = n*l$. Load area radius representing the sloping structure load is set to $R = 0.2*l$ = 0.086 m. The thickness of the plate is h = 0.01m (Fig. 5).

Ice is modelled as an isotropic material with Young's modulus of E = 5.5 GPa and shear modulus of G = 2.0625 GPa. The distance between material points is dx = 0.01935 m. The horizon size is chosen as $\delta = 3dx$. Winkler foundation stiffness k is set to k = 1.0055 N/m which roughly approximates water behavior. For this example sea ice is considered as a brittle material. Mode-I fracture toughness of sea ice is given as 0.06 $MPam^{1/2}$ [20]. To the authors knowledge there is no available value for Mode-III fracture toughness of sea ice in the current literature. We assumed Mode-III toughness to be 7 times greater than Mode-I by comparing the ratios to other brittle materials such as PMMA.

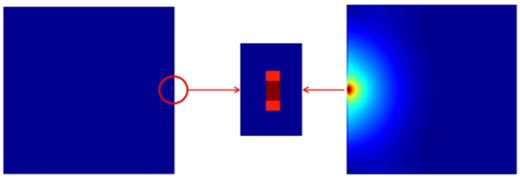

Fig. 6 Damage initiation [14]

In order to establish initial damage so that crack propagation can occur and also satisfy "first crack condition" (so that we can measure radial crack initiation load, $F_{Z,R0,semi}$ we broke 5% of bonds with the highest Von Mises stress (Fig. 6). This percentage was chosen because if more than 5% of bonds are broken, unstable fracture is observed and if less than 5% of bonds are broken, crushing behavior is observed, which is followed by unstable fracture. Initial load is set to 0 and then small increments of resultant body load $\Delta\tilde{b} = 0.1 N/m$ are induced to obtain a stable crack growth.

It can be clearly seen from the Fig. 7 that if the ice sheet length increases and by doing so better approximates semi-infinite behavior, the results are approaching the relation established by Kerr [6]. This means that peridynamic Mindlin plate on Winkler foundation model not only captures correct fracture patterns presented in Vazic et al. [14] but also is able show correct relation between measured out-of-plane bending failure load for a semi-infinite ice sheet $F_{Z,B,semi}^{test}$ and maximum load needed to initiate the radial crack in an ice sheet $F_{Z,R0,semi}$

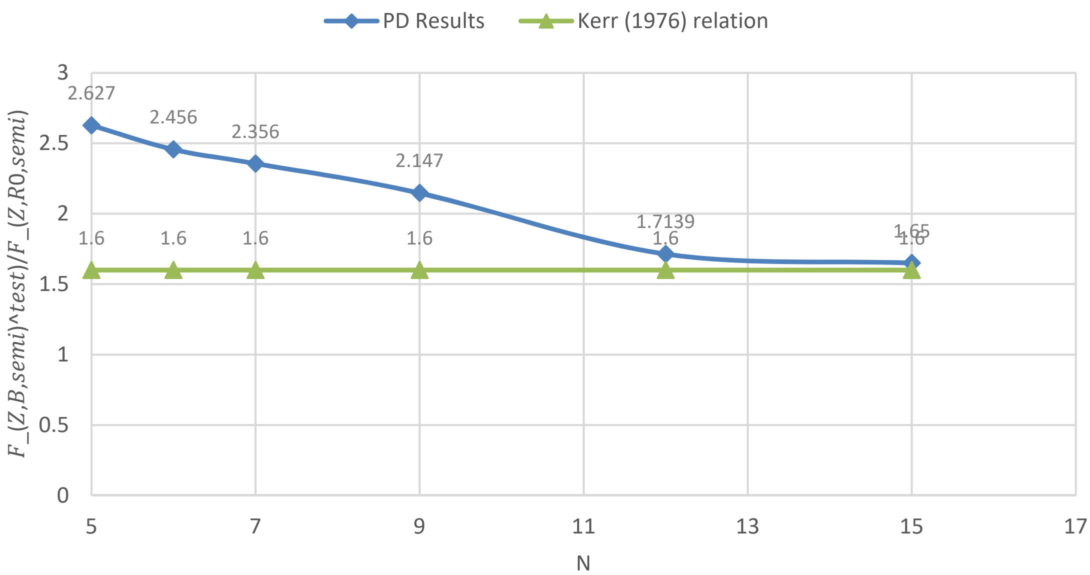

Fig. 7 Comparison between Kerr's relation and PD results for different size semi-infinite plates

5. CONCLUSION

In this study, two different peridynamic models for ice fracture are presented. First model is a 2 dimensional Bond Based model which is used to calculate in-plane splitting loads for several differently sized ice plates, where it was shown that the model has a good agreement with the experimental data. Second model is Mindlin plate resting on a Winkler type elastic foundation used to prove the relation established by Kerr [6]. As it can be seen from the results Mindlin plate model is able to establish such relation if the plate length approaches semi-infinite ice sheet size.

REFERENCES

1. Timco, G. W. "Indentation and penetration of edge-loaded freshwater ice sheets in the brittle range."*Journal of Offshore Mechanics and Arctic Engineering* **109.3** (1987): 287-294.

2. Grape, Johan A., and Erland M. Schulson. "Effect of confining stress on brittle indentation failure of columnar ice." *International Journal of Offshore and Polar Engineering* **2.03** (1992).

3. Dempsey, J. P., R. M. Adamson, and S. V. Mulmule. "Scale effects on the in-situ tensile strength and fracture of ice. Part II: First-year sea ice at Resolute, NWT." *International journal of fracture* **95.1-4** (1999): 347.

4. Lu, Wenjun, Raed Lubbad, and Sveinung Løset. "In-plane fracture of an ice floe: A theoretical study on the splitting failure mode." *Cold Regions Science and Technology* **110** (2015): 77-101.

5. Ashton, G. H. E. "River and Lake Ice Engineering Water Resources Publications." *Littleton, CO* (1986).

6. Kerr, Arnold D. "The bearing capacity of floating ice plates subjected to static or quasi-static loads." *Journal of glaciology* **17.76** (1976): 229-268.

7. Langhorne, P. J., K. J. L. Stone, and C. C. Smith. "The bearing capacity of saline ice sheets: centrifugal modelling." *Canadian geotechnical journal* **36.3** (1999): 467-481.

8. Michel, Bernard. "Ice mechanics." (1978).

9. Sodhi, Devinder S. "Breakthrough loads of floating ice sheets." *Journal of cold regions engineering* **9.1** (1995): 4-22.

10. Squire, Vernon A., et al. *Moving loads on ice plates*. Vol. **45**. Springer Science & Business Media, 2012.

11. Lu, Wenjun, Raed Lubbad, and Sveinung Løset. "Out-of-plane failure of an ice floe: radial-crack-initiation-controlled fracture." *Cold Regions Science and Technology* 119 (2015): 183-203.

12. Silling, Stewart A. "Reformulation of elasticity theory for discontinuities and long-range forces." *Journal of the Mechanics and Physics of Solids* **48.1** (2000): 175-209.

13. Mulmule, S. V., and J. P. Dempsey. "Scale effects on sea ice fracture." *Mechanics of Cohesive-frictional Materials: An International Journal on Experiments, Modelling and Computation of Materials and Structures* **4.6** (1999): 505-524.

14. Vazic, B., E. Oterkus, and S. Oterkus. "Peridynamic approach for modelling ice-structure interactions." *Trends in the Analysis and Design of Marine Structures: Proceedings of the 7th International Conference on Marine Structures (MARSTRUCT 2019, Dubrovnik, Croatia, 6-8 May 2019)*. CRC Press, 2019.

15. Diyaroglu, C., et al. "Peridynamics for bending of beams and plates with transverse shear deformation." *International Journal of Solids and Structures* **69** (2015): 152-168.

16. Vazic, Bozo, Erkan Oterkus, and Selda Oterkus. "Peridynamic model for a Mindlin plate resting on a Winkler elastic foundation." *Journal of Peridynamics and Nonlocal Modeling* (2019).

17. Bobaru, Florin, et al. "Convergence, adaptive refinement, and scaling in 1D peridynamics." *International Journal for Numerical Methods in Engineering* **77.6** (2009): 852-877.

18. Kilic, B., and E. Madenci. "An adaptive dynamic relaxation method for quasi-static simulations using the peridynamic theory." *Theoretical and Applied Fracture Mechanics* **53.3** (2010): 194-204.

19. Nevel, Donald E. *A semi-infinite plate on an elastic foundation*. No. RR-136. COLD REGIONS RESEARCH AND ENGINEERING LAB HANOVER NH, 1965.

20. Schulson, Erland M., and Paul Duval. *Creep and fracture of ice*. 2009.