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REVIEW OF AVAILABLE PROBABILISTIC MODELS OF THE CRACK GROWTH PARAMETERS IN THE PARIS EQUATION

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ABSTRACT

Crack growth rate parameters of the Paris equation are crucial inputs in the engineering critical assessment (ECA) of structures containing flaws. In fracture mechanics based reliability analysis, probabilistic models of these parameters are often used. Despite the considerable body of research in this area, there is significant variability among available models. This paper reviews the current available models in the literature and addresses areas requiring further research with a view to assisting probabilistic flaw assessment. The effect of the existing variability in crack growth model parameters is investigated by fracture mechanics analysis of a case study crack.

NOMENCLATURE

A	Parameter in crack growth equation
A_1	Parameter in stage A crack growth equation
A_2	Parameter in stage B crack growth equation
CT	Compact Tension (test specimen)
m	Parameter in crack growth equation
m_1	Parameter in stage A crack growth equation
m_2	Parameter in stage B crack growth equation
HAZ	Heat Affected Zone
SENB	Single Edge Notch Bending (test specimen)
Std.	Standard deviation
Y	Geometry factor in stress intensity solution

INTRODUCTION

The prediction of fatigue life can be performed by the S-N curve approach or the fracture mechanics approach. For fatigue life assessment of structures containing known or postulated defects the fracture mechanics method is more appropriate. Fracture mechanics uses a crack growth model commonly described by Paris-Erdogan equation given by the equation (1) below, which relates change in stress intensity factor range (ΔK) to change in crack growth rate ($\boldsymbol{da/dN}$):

$$\frac{\mathrm{da}}{dN} = A(\Delta K)^m \quad (1)$$

m and A are material constant.

The Paris equation has been traditionally described by a single slope line although recently a bilinear model has been widely used. The BS 7910 [1] recommended model is the bilinear model, while the simplified single model is cited as well. Both models are schematized in figure 1. The models also include a stress intensity factor threshold value (ΔK_{th}) below which crack growth will not occur.

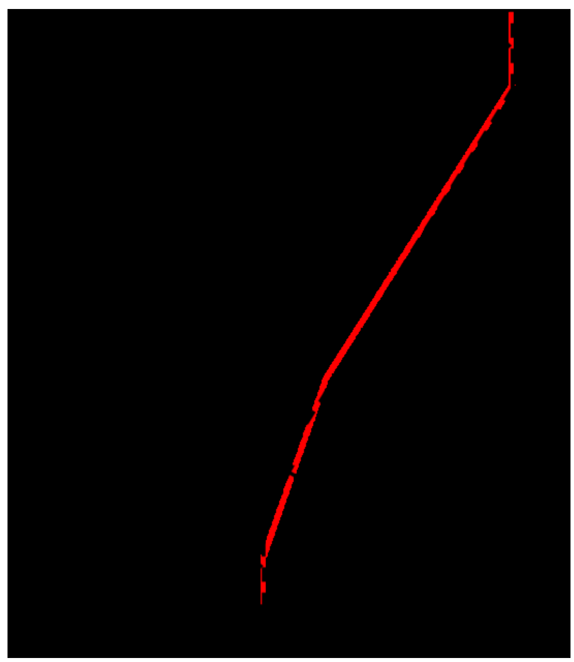

FIGURE 1: SCHEMATIC OF CRACK GROWTH MODEL BY PARIS LAW

A common engineering assessment approach is to use conservative assumptions of parameters (best estimate +2 standard deviations). For more advanced structural assessments,

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reliability-based methods can be used to determine the time variant predicted reliability which corresponds to some certain target reliabilities. In both deterministic and probabilistic cases, the crack growth parameters and their scatters need to be quantified. These parameters depend on a number of factors such as material type, loading ratio, and the environment (air, marine, etc.).

A strong relationship between m and A has been observed by a number of studies, and efforts have been made to provide models which quantify uncertainties in these parameters and based on the observed relationship. In the forthcoming sections the available models are reviewed and, where possible, the models are compared and their significance on structural integrity assessment is examined.

In crack growth assessment, stress intensity factor is presented in $N.mm^{-3/2}$ or MPa√m, crack growth rate (d*a*/d*N*) in mm/cycle or m/cycle units, and the logarithm base is either 10 or the natural. In this paper, for consistency all units are transformed to $N.mm^{-3/2}$, mm/cycle, and the natural logarithm base.

SINGLE SLOPE CRACK GROWTH MODEL

There are two methods available for modeling the crack growth variability using random variables A and m.

The first model treats both m and A as correlated random variables. It is based on a number of studies which have stablished the strong correlation between m and A. A review of these studies can be found in the paper by Cortie and Garrett [2]. The relations between m and A are estimated by analyzing crack growth curves from individual tests. For each test, a pair of m and A values is obtained. Each m and A pairs are then fitted into an appropriate curve using chosen curve fitting methods.

The second approach is more commonly practiced in reliability analysis [3] and treats m as a deterministic parameter and A as a random variable. Unlike the previous approach, in this method, crack growth measurements from all tests are pooled together and regression analysis provides a best estimate value of m. The mean value of A and an uncertainty measure (i.e. standard deviation) is estimated, as well. In reliability analysis using this method, A is represented by a lognormal distribution using estimated mean and standard deviations. Here available models for both approaches are studied.

m and A are correlated variables

Gurney [4] analyzed 56 test data of ferrous steel including structural steel, high strength steel, weld metal and HAZ tested in air. The stress ratios are reported to be around zero (R≈0). The m values for structural steel were ranging from 2.4 to 4, and for high strength steel from 1.7 to 2.7.

Tanaka and Matsuoka [5] tested four ferrous steel specimen and analyzed results in addition to the data from [6]and [7]. The pooled dataset comprises 200 test specimens at various stress ratios in air and at temperatures ranging from room temperature to elevated temperatures up to 600 ℃.

Cortie and Garrett [2] tested 18 test specimens of various stress ratios in air environment and at temperatures ranging from room temperature to elevated temperatures up to 550 ℃. Cortie and Garrett [2] combined this data with the data from Gurney [4], 36 test data reported by [8] and the data collected by [9] in the course of a 'round-robin' testing exercise tested in 15 laboratories. BS 7910:2013 [1] recommends the equations given by Cortie and Garrett [2] and Tanaka [5]. More recently, the UK Health and Safety Executive (HSE) [10] published results of tests prepared by the University of Aberdeen and Genesis Oil and Gas Consultants Limited from testing 30 welded and non-welded SENB and CT test specimens. The details of the test programs are listed in Table 1. All available studies only cover crack growth in an air environment.

Investigator	Sample size	Stress ratio	Material	Environment
Gurney [4]	Around 56 visually counted from the figure	Around 0	Ferrous steel, High strength steel, Weld and HAZ	Air
Cortie and Garrett [2]	4 different datasets	Various	Various Steel alloys	Air environment including elevated temperature
Tanaka and Matsuoka [5]	200	Various	Ferrous	Air
HSE [10]	15 Weld and 15 non-welded	0.2	Welds not Heat treated, CT and SENB	Air

Table 1: SUMMARY OF TEST CONDITIONS

Since there is a high correlation between the logarithm of A and m, the relationship between them is commonly represented by a log-linear equation with the general form:

$$\ln(A) = c + b * m \tag{2}$$

c and b values given by the above studies are listed in Table 2:

Author	c	b
Gurney [4]	-8.936	-6.797
Cortie and Garrett [2]	-7.381	-7.283
Tanaka and Matsuoka [5]	-8.356	-7.036
HSE (Non-Weld) [10]	-8.48	-6.91
HSE (Weld) [10]	-8.27	-7.3

Table 2: LINE INTERCEPTS (c) AND SLOPES (b) FROM LITERATURE

Lines plotted using parameters from Table 2 are shown in Figure 2 and Figure 5. All lines except HSE (welds) [10] generally show close magnitudes at m values between 2.4 to 3.6. Particularly, the equations given by Cortie and Garrett [2] and HSE (Non-Weld) [10] exhibit very close effects. m values between 2.4 to 3.6, according to Gurney [4], are common for structural steels.

One observation is that the HSE (Weld) [10] relation shows slower crack growth than the HSE (Non-Weld) [10] model. It should be noted that welded test specimens in [10] study only include cracks growing in the weld center line, and HAZ cracks are not analyzed.

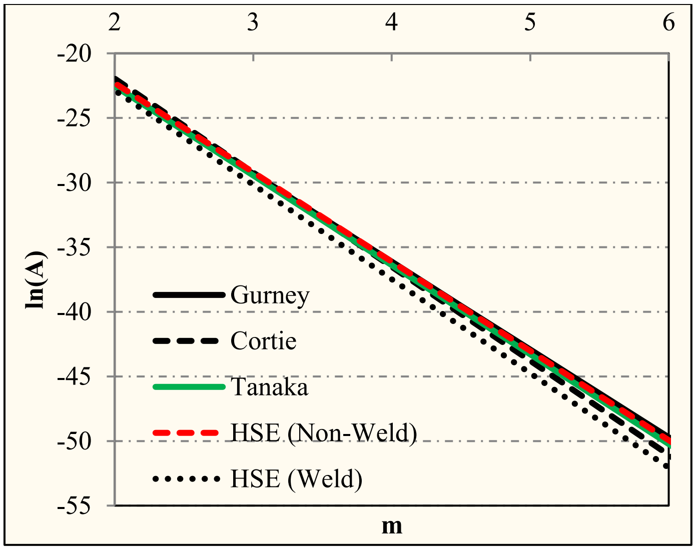

FIGURE 2: RELATION BETWEEN m AND A

m deterministic and A dependent variable

The available models can be categorized into those that are derived from analysis of test results and are accompanied by test data results and information about test (material, loading rate, loading environment, etc.) and models provided by standards (i.e. DNV and BSI). The latter could be based on both published and unpublished data. The available models for steels in air and marine environment with free corrosion are given in Table 3 and Table 4, respectively.

Author	m	Mean ln(A)	Std. ln(A)	Mean + 2Std.
Johnston [11] based on Gurney [4]	3	-29.31	0.24	-28.83
DNV (1984) [12]	3.1	-29.84	0.55	-28.74
Snijder (weld) [13]	3.07	-29.16	0.31	-28.54
Snijder (plain steel) [13]	2.8	-27.76	0.23	-27.3
New DNV (Weld) [14]	3	-29.33	0.51	-28.31
New DNV (Metal) [14]	3	-29.33	0.25	-28.83
OTH-511 (R>0.5) [15]	3	-29.02	0.37	-28.28
OTH-511 (R≈0.1) [15]	3	-29.52	0.35	-28.82
BS7910 [1] (based on [15])	3	Not provided		-28.28

Table 3: UNCERTAINTY IN PARIS PARAMETER A FOR STEEL IN AIR

For crack growth models in an air environment, the following studies are available:

A comprehensive study published by UK Health and Safety Executive (HSE), referred to as OTH-511 report [15], provides parameters for stress ratio R≈0.1 and R>0.5. The former is applicable to crack growth in base metal or stress relieved welds, while the latter is applicable to the as-welded condition, where a high amount of residual stress may be present. BS 7910 [1] does not provide a best estimate value for A; an upper bound (mean+2Std) is recommended, instead. The BS 7910 [1] crack growth models are based on OTH-511 [15], thus the mean and Standard deviation values of ln(A) from OTH-511 [15] may be used for probabilistic crack growth modeling. The upper bound A values suggested by New DNV [14] are similar to those estimated by OTH-511 [15], although the best estimates and standard deviations differ. Johnston [11] analyzed the data from Gurney [4] and estimated values similar to New DNV [14] for parent metal.

Snijder [13] analyzed the data from Maddox [16] and proposes considerably different m and A values for welds. The parameters recommended by DNV (Veritas, 1984) [12] are also significantly different from other models.

The effect of the choice of the determinist m model is illustrated by plotting crack growth rate against stress intensity factor (ΔK) in figure 3 for ΔK ranges between 300 to 1100 $N.mm^{-3/2}$.

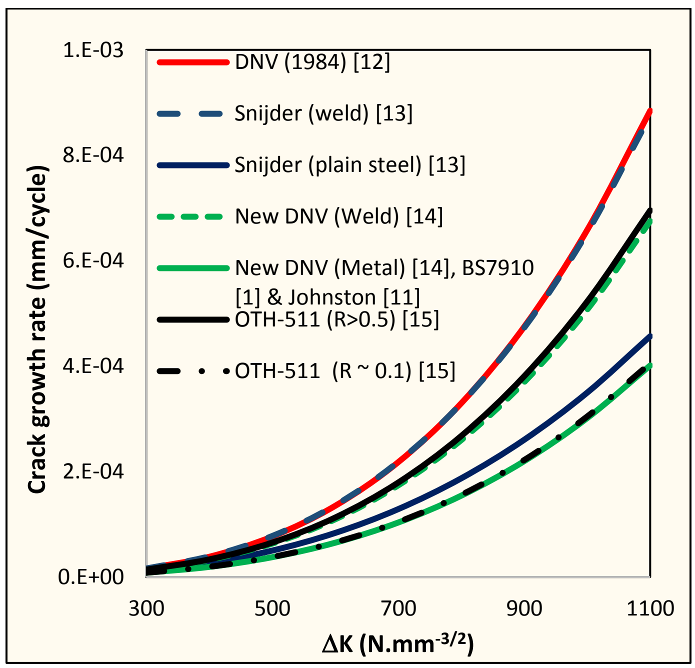

FIGURE 3: COMPARISON OF CRACK GROWTH RATES

Additionally, crack growth analysis of a 10.2 mm through thickness center crack in an infinite plate (Y=1) under constant amplitude stress of 16 MPa was conducted. The crack growth curves are shown in Figure 4.

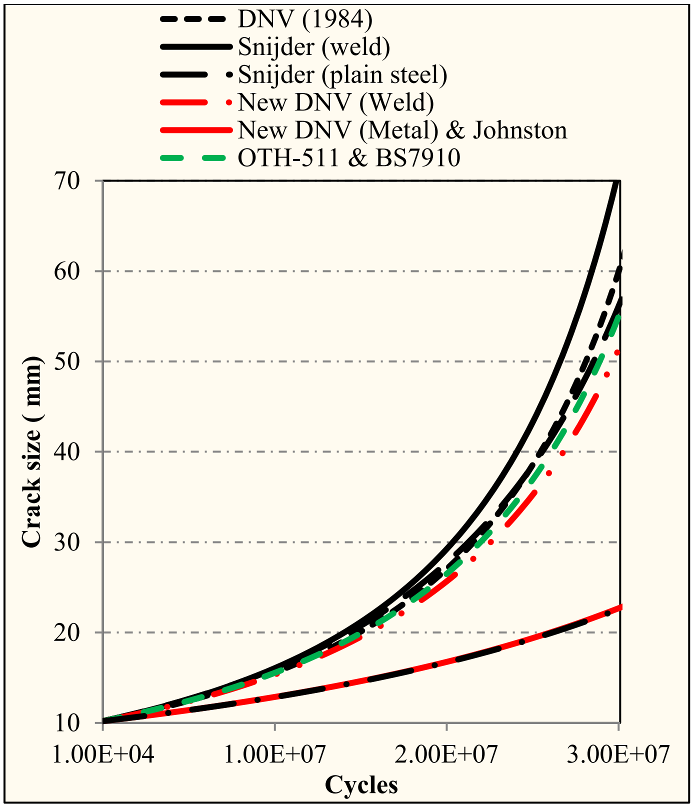

FIGURE 4: CRACK GROWTH FOR A CASE STUDY CRACK

It is observed that there is a reasonably good agreement between the models, except for the models recommended by Snijder [13] which show considerably faster crack growth. DNV (1984) [12] has now been withdrawn, and at the time that this paper is written, a new DNV document recommends using updated values [14].

There is a small number of single slope crack growth models available in the literature for a marine environment: the DNV [14] model and the old version of the DNV (1984) [12]. BS 7910 [1] recommends a value of m identical to that recommended by new DNV document [14], but best estimate value and uncertainty measures are not provided. The models are given in Table 4.

Author	m	Mean ln(A)	Std. ln(A)	Mean + 2Std.
DNV (1984) [12]	3.5	-31.01	0.76	-29.49
New DNV [14]	3	-27.81	0.507	-26.79
BS 7910 [1]	3	Not provided		-26.79

Table 4: UNCERTAINTY IN *A* PARAMETER FOR STEEL IN MARINE ENVIRONMENT WITH FREE CORROSION

Comparison between the two approaches

As previously mentioned, the parameter estimation scheme for the two approaches are different. Unlike the method based on correlation between m and A, in the deterministic method, crack growth data from all tests are pooled and a best estimate value of m is calculated. Here, an attempt has been made to compare the two models:

Figure 5 shows mean values of the parameter A, their corresponding recommended deterministic m values and m-ln(A) lines drawn using the correlated parameters method. Apart from Snijder (weld) model and OTH-511 (weld) the rest of the models show reasonably good agreement.

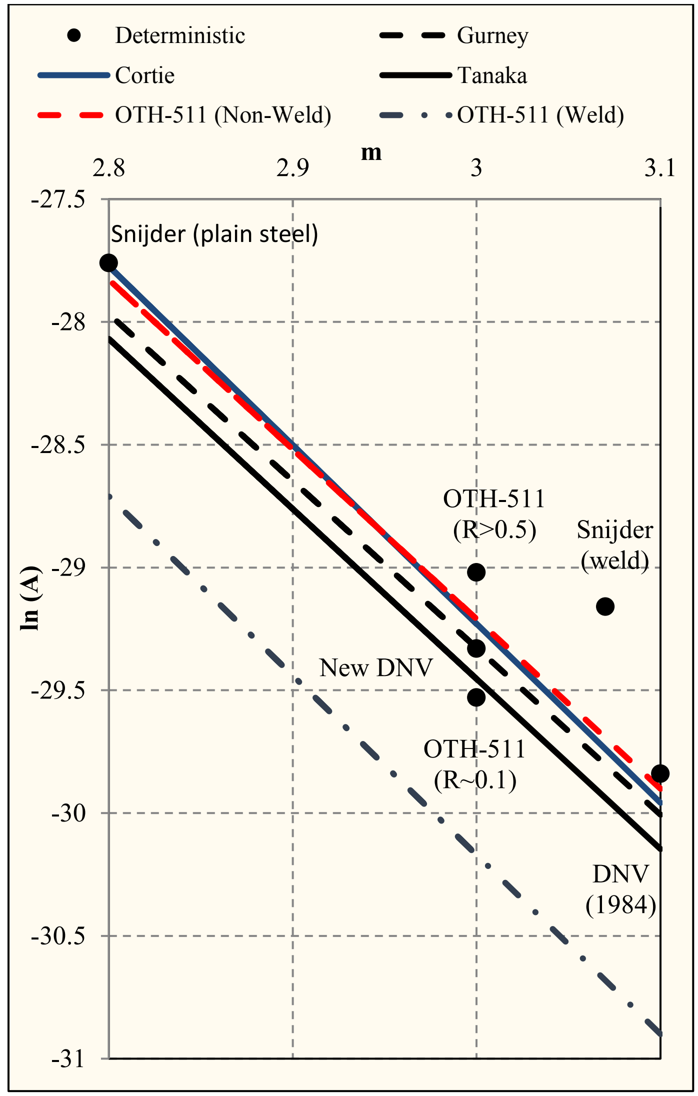

FIGURE 5: COMPARISON BETWEEN TWO APPROACHES

BILINEAR CRACK GROWTH MODEL

A two-stage crack growth model gives reduced crack growth rate at lower K.

The only available two-stage model is the OTH-511 [10] model, as shown in Table 5. The main text of BS 7910 [1] recommends identical values (Tables 10 and 11 of [1]) to those estimated by OTH-511 [10].

Environment	Stage	R	m	ln(A)	Std. ln(A)
In air	A	<0.5	8.16	-59.68	0.64
		>0.5	5.1	-39.88	0.74
	B	<0.5	2.88	-28.55	0.27
		>0.5	2.88	-28.17	0.39
Freely corroding marine environment	A	<0.5	3.42	-31.14	0.52
		>0.5	3.42	-30.56	0.58
	B	<0.5	1.3	-15.88	0.21
		>0.5	1.11	-14.38	0.14
Marine CP -850mV	A	<0.5	8.16	-59.68	0.64
		>0.5	5.1	-39.88	0.74
	B	<0.5	2.67	-25.99	0.47
		>0.5	2.67	-25.84	0.61
Marine CP -1100mV	A	>0.5	8.16	-59.68	0.64
		<0.5	5.1	-39.88	0.74
	B	>0.5	1.4	-16.71	0.26
		<0.5	1.4	-16.76	0.33

Table 5: UNCERTAINTY IN PARIS PARAMETER A FOR BILINEAR MODEL

MODELS RECOMMENDED BY BS 7910 ANNEX K

Annex K of BS 7910 [1] provides guidance on reliability assessment of structures containing flaws [17].

This annex recognizes both approaches that were discussed above.

For the approach in which m and A are assumed to be correlated, the equations by Cortie and Garrett [2] and Tanaka and Matsuoka [5] are given as example models.

For the deterministic m approach, the Snijder model and DNV (1984) [12] are recommended, although DNV (1984) [12] has been withdrawn and the values given by DNV [14] appear to be the updated recommended values.

The crack growth parameters recommended in the main text of BS 7910 [1] are identical to those estimated by OTH-511 [10], although, for the single slope model, the main clauses of BS7910 [1] provide only an upper-bound value of A, without any mention of mean value and its standard deviation. This is insufficient for probabilistic modelling. One possible recommendation could be adding the probabilistic single slope crack growth models proposed by OTH-511 [10], which is used by the main text of the BS 7910 [1] to provide upper-bound values of A.

For bilinear crack growth, a table similar to Table 5 is adopted by Annex K.

The new version of BS7910 will have a number of changes in annex K. Apart from editorial amendments, there are number of changes in the content, including updating clauses on setting target levels of reliability as well as the table which provides examples of target reliability levels. "Fracture Assessment: Level I reliability analysis" is removed from the annex, as well.

Clauses related to Paris parameters are also revised: all citations are checked against their sources and a number of them have been updated. Further, guidance is added on choosing the appropriate statistical method for estimation of a probability distribution which represents fracture toughness.

Table K.1 which provides uncertainty in Paris parameter A using single slope crack growth model has been updated. The table will be numbered as table K.2 in the 2019 version. In previous version the Snijder model [13] for plain steel were given as crack growth in welds by mistake. This issue has been fixed now. The unit system of Table K.1 has been added, as well.

Table K.3 which provides uncertainty in Paris parameter A using bilinear crack growth model has been modified by adding logarithm of A values to make it consistent with table K.1 (now K.2 in new version). Table K.2 of 2013 version will be numbered as table K.3 in the 2019 version. The reference for this table has also been corrected in the 2019 version.

DISCUSSION

The crack growth rate parameters of the Paris equation are crucial inputs in the engineering critical assessment (ECA) of structures containing flaws, and are widely used in integrity assessment of various structures across a number of engineering disciplines [18] & [19]. Two methods are available to model the crack growth variability through the use of random variables m and A:

m and A are correlated variables. -This model is currently limited to crack growth of steel and welds in air. There is a good agreement between equations relating m and A and the deviations between models are due to natural randomness of test data between analyzed datasets, variation in sample sizes and variations in stress ratios. The equation proposed by Cortie and Garrett [2] includes four independent datasets and is the most conservative model. This equation is recommended by BS 7910 Annex K, which also recommends the equation given by Tanaka and Matsuoka [5] .

Models proposed by the main clauses of BS 7910 [1] are identical to those derived from the OTH-511 [10] study. The models proposed by the most recent DNV document [14] have a very good agreement with the literature, but the source of the data is not provided by DNV [14].

m is deterministic and A is a variable. In most probabilistic crack growth calculations, only A is modelled as a variable. The models were compared through a case study crack growth analysis using upper bound A values, and good agreement was observed. The model provided by Snijder [13] is the most conservative model. Values recommended by DNV (1984) [12] result in the second most conservative crack growth prediction. This document has been withdrawn and a new document from DNV [14] is consistent with the main clauses of BS 7910 [1], and OTH-511 [10], however, DNV (1984) [12] values are currently still recommended by BS 7910 Annex K [1]. Since a number of studies have been conducted in the intervening period, it is not surprising that the values have changed, therefore, one recommendation would be to update the values recommended by DNV (1984) in annex K.

The only available bilinear crack growth model is the model estimated by OTH-511 [10]. The main clauses of BS 7910 [1] recommend similar values (Tables 10 and 11 of [1]) to those estimated by OTH-511 [10].

RECOMMENDATIONS FOR FUTURE WORK

As a result of the review presented above, a number of recommendations for future work can be made for probabilistic crack growth parameters.

- Probabilistic crack growth prediction by treating m parameter as a random variable and A as a dependent variable using existing models is limited to the single slope crack growth model in air environment. Developing similar models for crack growth in a marine environment without sufficient corrosion protection, and also bilinear crack growth models, can be advantageous.
- Current models make no distinction between parameters of weld crack growth in weld metal and those in HAZ. Fatigue cracks are more likely to appear at the weld toe (located in HAZ); however, the occurrence of fabrication defects in weld metal is also likely [20]. Further studies to quantify possible variations of crack growth parameters between these two regions would be beneficial.
- Stress ratio at the crack is a key influencing factor here. Stress ratio is affected considerably by weld residual stresses. The current assumption is that in the as-welded condition, residual stresses are high enough to increase the stress ratio above 0.5. This could be further improved by employing probabilistic residual stress models, which currently do not exist.
- Threshold stress intensity factor (ΔK_{th}) is another key input in crack growth prediction and there is a need for probabilistic modelling of the threshold.
- In this paper, upper bound crack growth parameters recommended by a number of studies and standards were compared using a crack growth case study. This study can be further enhanced by a probabilistic crack growth study, which is not covered in this paper.

CONCLUSIONS

A comprehensive review of uncertainties in the probabilistic models of crack growth parameters in the Paris law has been presented. This has shown that, whilst substantial progress has been made, there is considerable scope for further work to improve existing models.

There is a need to investigate the effect of choice of crack growth model on predictive time variant fatigue reliability of structures containing flaws. Developing models for probabilistic crack growth in marine environment condition and for bilinear model using the approach that treats m and A as correlated variables is also beneficial.

This would enable the wider application of probabilistic fatigue crack growth methods to integrity assessment and maintenance and is expected to result in enhanced safety.

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