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SOLAR SAIL TRAJECTORIES AT THE EARTH-MOON LAGRANGE POINTS

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ABSTRACT

This paper investigates displaced periodic orbits at linear order in the circular restricted Earth-Moon system, where the third massless body is a solar sail. These highly non-Keplerian orbits are achieved using an extremely small sail acceleration. The solar sail Earth-Moon system differs greatly from the Earth-Sun system as the Sun line direction varies continuously in the rotating frame and the equations of motion of the sail are given by a set of nonlinear non-autonomous ordinary differential equations. By introducing a first-order approximation, periodic orbits are derived analytically at linear order. These approximate analytical solutions are utilized in a numerical search to determine displaced periodic orbits in the full nonlinear model. The importance of finding such displaced orbits is to obtain continuous communications between the equatorial regions of the Earth and the polar regions of the Moon. As will be shown, displaced periodic orbits exist at all Lagrange points at linear order.

1. INTRODUCTION

A solar sail is propelled by reflecting solar photons and therefore can transform the momentum of the photons into a propulsive force. Solar sailing technology appears as a promising form of advanced spacecraft propulsion, which can enable exciting new space-science mission concepts such as solar system exploration and deep space observation. Although solar sailing has been considered as a practical means of spacecraft propulsion only relatively recently, the fundamental ideas are by no means new (see McInnes [1] for a detailed description).

Solar sails can also be utilised for highly non-Keplerian orbits, such as closed orbits displaced high above the ecliptic plane (see Waters and McInnes [2]). Solar sails are especially suited for such non-Keplerian orbits, since they can apply a propulsive force continuously. This allows some exciting and unique trajectories. In such trajectories, a sail can be used as a communication satellite for high latitudes. For example, the orbital plane of the sail can be displaced above the orbital plane of the Earth, so that the sail can stay fixed above the Earth at some distance, if the orbital periods are equal. Orbits around the collinear points of the Earth-Moon system are also of great interest because their unique positions are advantageous for several important applications in space mission design (see e.g. Szebehely [3], Roy [4], Vonbun [5], Thurman et al. [6], Gómez et al. [7, 8]).

In the recent years several authors have tried to determine more accurate approximations (quasi-Halo orbits) of such equilibrium orbits [9]. The orbits were first studied by Farquhar [10], Farquhar and Kamel [9], Breakwell and Brown [11], Richardson [12], Howell [13, 14]. Halo orbits near the collinear libration points in the Earth-Moon system are of great interest, particulary around the L_1 and L_2 points because their unique positions. However, a linear analysis shows that the collinear libration points L_1, L_2, and L_3 are of the type $saddle \times center \times center$, leading to the instability in their vicinity, whereas the equilateral equilibrium points L_4, and L_5 are stable ($center \times center \times center$). Although the libration points L_4, and L_5 are naturally stable and require a small acceleration, the disadvantage is the longer communication path length from the lunar pole to the sail. It is essential to note that the equilateral libration points L_4 and L_5 of the Earth-Moon system can be found to be unstable if the gravitational effect of the sun is included (see Szebehely [3]). If the orbit maintains visibility from Earth, a spacecraft on it (near the L_2 point) can be used to provide communications between the equatorial regions of the Earth and the lunar poles. The establishment of a bridge for radio communications is crucial for forthcoming space missions, which plan to use the lunar poles. McInnes [15] first investigated a new family of displaced solar sail orbits near the Earth-Moon libration points. In Baoyin and McInnes [16, 17, 18] and McInnes [15, 19], the authors describe the new orbits which are associated with artificial lagrange points in the Earth-Sun system. These artificial equilibria have potential applications for future space physics and Earth observation missions. In McInnes and Simmons [20], the authors investigate large new families of solar sail orbits, such as Sun-centered halo-type trajectories, with the sail executing a circular orbit of a chosen period above the ecliptic plane. In our study, we will demonstrate the possibility of such trajectories in the Earth-Moon system. As will be shown in the present study, there exist a new family of solar sail displaced periodic orbits in the Earth-Moon restricted three-body problem.

The first-order approximation is introduced for the linearized system of equations. The Laplace transform is used to produce the first-order analytic solution of the out-of plane motion. We find families of periodic orbits above the plane at linear order. It will be shown for example that, with a suitable sail attitude control program, a 100 km displaced, out-of-plane trajectory around the L_4 point may be executed with a sail acceleration of only 1.8×10^{-3} mms^{-3} (see Figure 2).
This paper is organized as follow: Section 2 provides the mathematical expressions describing the motion of the sail in the circular restricted three-body problem. Section 3 is devoted to the study of the periodic orbits around the Lagrange points in the Earth-Moon system. The periodic solutions to the linearized equations of motion are derived analytically. Section 4 is concerned with the numerical computation around the libration points L_3, L_4 and L_5 in the Earth-Moon system. Finally some numerical results are presented to illustrate our approach.

2. SYSTEM MODEL

In this work, we will assume that m_1 represents the larger primary (Earth), m_2 the smaller primary (Moon) and we will be concerned with the motion of the sail that has negligible mass ($m_1 > m_2$). It is always assumed that the two more massive bodies (primaries) are moving in circular orbits about their common center of mass and the mass of the third body is too small to affect the motion of the two more massive bodies. The problem of the motion of the third body is the circular restricted three-body problem (CRTBP).

In order to develop any mathematical model without loss of the generality, it is useful to introduce some parameters that are characteristics of each particular three-body system. This set of parameters is used to normalize the equations of motion. The unit mass is taken to be the total mass of the system $(m_1 + m_2)$ and the unit of length is chosen to be the constant separation between m_1 and m_2. Under these considerations the masses of the primaries in the normalized system of units are $m_1 = 1 - \mu$ and $m_2 = \mu$, with $\mu = m_2/(m_1 + m_2)$ (see Figure 1).

2.1. Equations of Motion

The vector dynamical equation for the solar sail in a rotating frame of reference is described by

$$\frac{d^2\boldsymbol{r}}{dt^2} + 2\boldsymbol{\omega} \times \frac{d\boldsymbol{r}}{dt} + \nabla U(\boldsymbol{r}) = \boldsymbol{a}, \qquad (1)$$

where $\boldsymbol{\omega} = \omega\hat{z}$ ($\hat{z}$ is a unit vector pointing in the direction of z) is the angular velocity vector of the rotating frame and $\boldsymbol{r}$ is the position vector of the solar sail relative to the center of mass of the two primaries. The three-body gravitational potential $U(\boldsymbol{r})$ and the solar radiation pres-

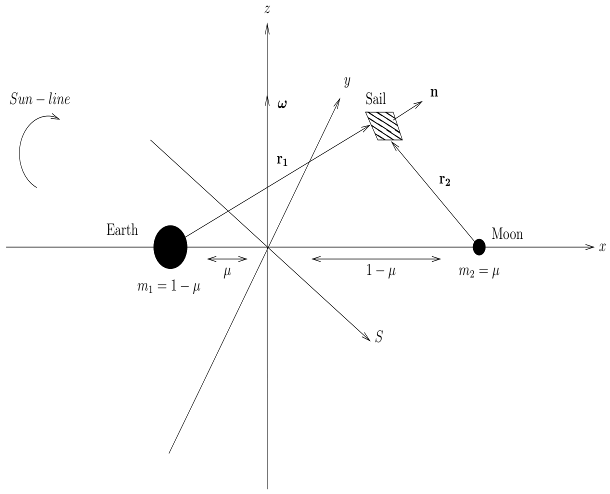

Figure 1. Schematic geometry of the Earth-Moon restricted three-body problem.

sure acceleration $\boldsymbol{a}$ are defined by

$$\begin{aligned} U(\boldsymbol{r}) &= -\left[\frac{1}{2}|\boldsymbol{\omega} \times \boldsymbol{r}|^2 + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}\right], \\ \boldsymbol{a} &= a_0(\boldsymbol{S} \cdot \boldsymbol{n})^2\boldsymbol{n}, \end{aligned} \qquad (2)$$

where $\mu = 0.1215$ is the mass ratio for the Earth-Moon system. The sail position vectors w.r.t. m_1 and m_2 respectively, are defined as $\boldsymbol{r}_1 = [x + \mu, y, z\ ]^T$ and $\boldsymbol{r}_2 = [x - (1 - \mu), y, z]^T$, and a_0 is the magnitude of the solar radition pressure force exerted on the sail. The unit normal to the sail $\boldsymbol{n}$ and the Sun-line direction are given by

$$\boldsymbol{n} = \left[\ \cos(\gamma)\cos(\omega_\star t) \quad -\cos(\gamma)\sin(\omega_\star t) \quad \sin(\gamma)\ \right]^T,$$

$$\boldsymbol{S} = \left[\ \cos(\omega_\star t) \qquad -\sin(\omega_\star t) \qquad 0\ \right]^T,$$

where $\omega_\star = 0.923$ is the angular rate of the Sun line in the corotating frame in a dimensionless synodic coordinate system.

2.2. Linearized system

We now want to investigate the dynamics of the sail in the neighborhood of the libration points. The libration points are the equilibrium solutions of the restricted three-body problem, which describes the motion of a particle (very small mass) under the gravitational attraction of two massive bodies.

We denote the coordinates of the equilibrium point as $\boldsymbol{r}_L = (x_{L_i}, y_{L_i}, z_{L_i})$ with $i = 1, \cdots, 5$.

Let a small displacement in $\boldsymbol{r}_L$ be $\delta\boldsymbol{r}$ such that $\boldsymbol{r} \to \boldsymbol{r}_L + \delta\boldsymbol{r}$. We will not consider the small annual changes in the inclination of the Sun line with respect to the plane of the system.

The Taylor-series expansion about $\boldsymbol{r}_L$ is used as defined below to determine the linear variational equations

$$\begin{aligned} g(\boldsymbol{r}_L+\delta\boldsymbol{r}) &= g(\boldsymbol{r}_L) \qquad (3)\\ &\quad+\sum_{n=1}^{\infty}\frac{1}{n!}\frac{\partial^{(n)}g(\boldsymbol{r})}{\partial\boldsymbol{r}^{(n)}}\bigg|_{\boldsymbol{r}=\boldsymbol{r}_L}(\delta\boldsymbol{r})^n,\\ &= g(\boldsymbol{r}_L)+\frac{\partial g(\boldsymbol{r})}{\partial\boldsymbol{r}}\bigg|_{\boldsymbol{r}=\boldsymbol{r}_L}\delta\boldsymbol{r}+O(\delta\boldsymbol{r}^2). \qquad (4)\end{aligned}$$

Therefore, the linear equations for the solar sail are

$$\frac{d^2\delta\boldsymbol{r}}{dt^2}+2\boldsymbol{\omega}\times\frac{d\delta\boldsymbol{r}}{dt}+\nabla U(\boldsymbol{r}_L+\delta\boldsymbol{r})=\boldsymbol{a}(\boldsymbol{r}_L+\delta\boldsymbol{r}), \quad (5)$$

and retaining only the first-order term in $\delta\boldsymbol{r}=[\delta x,\delta y,\delta y]^T$ in a Taylor-series expansion, the gradient of the potential and the acceleration can be expressed as

$$\begin{aligned} \nabla U(\boldsymbol{r}_L+\delta\boldsymbol{r}) &= \nabla U(\boldsymbol{r}_L)+\frac{\partial\nabla U(\boldsymbol{r})}{\partial\boldsymbol{r}}\bigg|_{\boldsymbol{r}=\boldsymbol{r}_L}\delta\boldsymbol{r} \qquad (6)\\ &\quad+O(\delta\boldsymbol{r}^2),\\ \boldsymbol{a}(\boldsymbol{r}_L+\delta\boldsymbol{r}) &= \boldsymbol{a}(\boldsymbol{r}_L)+\frac{\partial\boldsymbol{a}(\boldsymbol{r})}{\partial\boldsymbol{r}}\bigg|_{\boldsymbol{r}=\boldsymbol{r}_L}\delta\boldsymbol{r} \qquad (7)\\ &\quad+O(\delta\boldsymbol{r}^2).\end{aligned}$$

It is assumed that $\nabla U(\boldsymbol{r}_L)=0$, and the acceleration is constant with respect to the small displacement $\delta\boldsymbol{r}$, so that

$$\frac{\partial\boldsymbol{a}(\boldsymbol{r})}{\partial\boldsymbol{r}}\bigg|_{\boldsymbol{r}=\boldsymbol{r}_L}=0. \quad (8)$$

The linear variational system associated with the libration points at $\boldsymbol{r}_L$ can be determined through a Taylor series expansion by substituting Eqs. (7) and (8) into (5)

$$\frac{d^2\delta\boldsymbol{r}}{dt^2}+2\boldsymbol{\omega}\times\frac{d\delta\boldsymbol{r}}{dt}+K\delta\boldsymbol{r}=0, \quad (9)$$

where the time-dependant matrix K is defined as

$$K=\left[\frac{\partial\nabla U(\boldsymbol{r})}{\partial\boldsymbol{r}}\bigg|_{\boldsymbol{r}=\boldsymbol{r}_L}-\boldsymbol{a}(\boldsymbol{r}_L)\right]. \quad (10)$$

Using the matrix notation the linearized equation about the libration point (Equation (9)) can be represented by the homogeneous linear system $\dot{\mathbf{X}}=A(t)\mathbf{X}$, where the state vector $\mathbf{X}=(\delta\boldsymbol{r},\delta\dot{\boldsymbol{r}})^T$, and for which the coefficients of the matrix $A(t)=A(t+T)$ are periodic functions of time with period $T=2\pi/\omega_\star$.

The Jacobian matrix $A(t)$ has the general form

$$A(t)=\begin{pmatrix} 0_3 & I_3\\ K & \Omega\end{pmatrix}, \quad (11)$$

where I_3 is a identity matrix, and

$$\Omega=\begin{pmatrix} 0 & 2 & 0\\ -2 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}. \quad (12)$$

For convenience the sail attitude is fixed such that the sail normal vector $\boldsymbol{n}$, which is a unit vector that is perpendicular to the sail surface, points always along the direction of the Sun line with the following constraint $\boldsymbol{S}\cdot\boldsymbol{n}\geq 0$. Its direction is described by the pitch angle γ relative to the Sun-line, which represents the sail attitude.

2.2.1. *Collinear libration points*

The collinear libration points are found on the line joining both primaries.

By making the transformation $\boldsymbol{r}\rightarrow\boldsymbol{r}_L+\delta\boldsymbol{r}$ and retaining only the first-order term in $\delta\boldsymbol{r}=(\xi,\eta,\zeta)^T$ in a Taylor-series expansion, the linearized nondimensional equations of motion relative to the collinear libration points can be written as

$$\begin{aligned} \ddot{\xi}-2\dot{\eta}-U^o_{xx}\xi &= a_\xi, \qquad (13)\\ \ddot{\eta}+2\dot{\xi}-U^o_{yy}\eta &= a_\eta, \qquad (14)\\ \ddot{\zeta}-U^o_{zz}\zeta &= a_\zeta, \qquad (15)\end{aligned}$$

where U^o_{xx}, U^o_{yy}, and U^o_{zz} are the partial derivatives of the gravitational potential evaluated at the collinear libration points, and the solar sail acceleration is defined in terms of three auxiliary variables a_ξ, a_η, and a_ζ.

The acceleration components are given by

$$\begin{aligned} a_\xi &= a_0\cos(\omega t)\cos^3(\gamma), \qquad (16)\\ a_\eta &= -a_0\sin(\omega t)\cos^3(\gamma), \qquad (17)\\ a_\zeta &= a_0\cos^2(\gamma)\sin(\gamma). \qquad (18)\end{aligned}$$

2.2.2. *Equilateral libration points*

The triangular libration points are in the plane of motion of the primaries and they form an equilateral triangle with the two primaries. Those points are quasi-stable in the Earth-Moon system and can be used as parking regions at which no stationkeeping is needed (see [21]).

In a similar fashion, recalling the linearized equations of motion obtained in the Equation (9) describing the behavior of the system in the vicinity of the Lagrange points, it can be easily shown that the the linear variational equations of motion in component form the triangular points then become

$$\begin{aligned} \ddot{\xi}-2\dot{\eta} &= U^o_{xx}\xi+U^o_{xy}\eta+a_\xi, \qquad (19)\\ \ddot{\eta}+2\dot{\xi} &= U^o_{xy}\xi+U^o_{yy}\eta+a_\eta, \qquad (20)\\ \ddot{\zeta} &= U^o_{zz}\zeta+a_\zeta, \qquad (21)\end{aligned}$$

3. ANALYTICAL APPROACH

Considering the dynamics of motion near the collinear libration points We may choose a particular periodic solution in the plane of the form (see Farquhar [22])

$$\begin{aligned} \xi(t) &= \xi_0\cos(\omega_\star t), \qquad (22)\\ \eta(t) &= \eta_0\sin(\omega_\star t). \qquad (23)\end{aligned}$$

By inserting Equations (22) and (23) in the differential equations, we obtain the linear system in ξ_0 and η_0,

$$\begin{cases} \Big(U^o_{xx} - \omega_\star^2\Big)\xi_0 - 2\omega_\star\eta_0 = a_0\cos^3(\gamma), \\ -2\omega_\star\xi_0 + \Big(U^o_{yy} - \omega_\star^2\Big)\eta_0 = -a_0\cos^3(\gamma). \end{cases} \quad (24)$$

Then the amplitudes ξ_0 and η_0 are given by

$$\xi_0 = a_0\frac{\Big(U^o_{yy} - \omega_\star^2 - 2\omega_\star\Big)\cos^3(\gamma)}{\Big(U^o_{xx} - \omega_\star^2\Big)\Big(U^o_{yy} - \omega_\star^2\Big) - 4\omega_\star^2}, \quad (25)$$

$$\eta_0 = a_0\frac{\Big(-U^o_{xx} + \omega_\star^2 + 2\omega_\star\Big)\cos^3(\gamma)}{\Big(U^o_{xx} - \omega_\star^2\Big)\Big(U^o_{yy} - \omega_\star^2\Big) - 4\omega_\star^2}, \quad (26)$$

and we have the equality

$$\frac{\xi_0}{\eta_0} = \frac{\omega_\star^2 + 2\omega_\star - U^o_{yy}}{-\omega_\star^2 - 2\omega_\star + U^o_{xx}}. \quad (27)$$

Then the trajectory will be an ellipse centered on the collinear libration points. We can find the required radiation pressure be solving the equation (25)

$$a_0 = \cos^{-3}(\gamma)\left[\frac{\omega_\star^4 - \omega_\star^2(U^o_{xx} + U^o_{yy} + 4) + U^o_{xx}U^o_{yy}}{U^o_{yy} - 2\omega_\star - \omega_\star^2}\right]\xi_0.$$

By applying the Laplace transform, the uncoupled out-of-plane ζ-motion defined by the equation (15) can be solved. The transform version is obtained as

$$s^2Z - s\xi_0 - \dot{\xi}_0 - U^o_{zz}Z = \frac{a_0\cos^2(\gamma)\sin(\gamma)}{s}, \quad (28)$$

$$(s^2 - U^o_{zz})Z = \dot{\xi}_0 + s\xi_0 + \frac{a_0\cos^2(\gamma)\sin(\gamma)}{s}, \quad (29)$$

also

$$Z(s) = \frac{1}{s^2 - U^o_{zz}}\left(\dot{\xi}_0 + s\xi_0 + \frac{a_0\cos^2(\gamma)\sin(\gamma)}{s}\right). \quad (30)$$

The frequency of the out-of-plane motion is given by solving the equation

$$s^2 - U^o_{zz} = 0,$$

where $s_{1,2} = \pm i\sqrt{|U^o_{zz}|} = \pm i\omega_\zeta$.

Using Mathematica, we can find the inverse Laplace transform, which will be the general solution of the out-of-plane component

$$\begin{aligned} \zeta(t) &= \zeta_0\cos(\omega_\zeta t) + \dot{\zeta}_0|U^o_{zz}|^{-1/2}\sin(\omega_\zeta t) && (31) \\ &\quad + a_0\cos^2(\gamma)\sin(\gamma)|U^o_{zz}|^{-1}[U(t) - \cos(\omega_\zeta t)], \\ &= U(t)a_0\cos^2(\gamma)\sin(\gamma)|U^o_{zz}|^{-1} && (32) \\ &\quad + \dot{\zeta}_0|U^o_{zz}|^{-1/2}\sin(\omega_\zeta t) \\ &\quad + \cos(\omega_\zeta t)[\zeta_0 - a_0\cos^2(\gamma)\sin(\gamma)|U^o_{zz}|^{-1}], \end{aligned}$$

where the nondimensional frequency is defined as

$$\omega_\zeta = |U^o_{zz}|^{1/2}$$

and $U(t)$ is the unit step function.

Specifically for the choice of the initial data $\dot{\zeta}_0 = 0$, equation (32) can be more conveniently expressed as

$$\begin{aligned} \zeta(t) &= U(t)a_0\cos^2(\gamma)\sin(\gamma)|U^o_{zz}|^{-1} && (33) \\ &\quad + \cos(\omega_\zeta t)[\zeta_0 - a_0\cos^2(\gamma)\sin(\gamma)|U^o_{zz}|^{-1}]. \end{aligned}$$

The solution can be made to contain only the periodic oscillatory modes at an out-of-plane distance

$$\zeta_0 = a_0\cos^2(\gamma)\sin(\gamma)|U^o_{zz}|^{-1}. \quad (34)$$

Furthermore, the out-of-plane distance can be maximized by an optimal choice of the sail pitch angle determined by

$$\frac{d}{d\gamma^\star}\cos^2(\gamma^\star)\sin(\gamma^\star) = 0, \quad (35)$$

$$\gamma^\star = \tan^{-1}(2^{-1/2}), \quad (36)$$

$$\gamma^\star = 35^\circ.264. \quad (37)$$

Following the idea already presented for the collinear points, since the particular solution in the plane cannot satisfy the linear ODEs for the triangular points, the subsequent discussion is to find the solutions that satisfy the differential equations.

Assume that a solution to the linearized equations of motion (19-21) is periodic of the form

$$\xi(t) = A_\xi\cos(\omega_\star t) + B_\xi\sin(\omega_\star t), \quad (38)$$

$$\eta(t) = A_\eta\cos(\omega_\star t) + B_\eta\sin(\omega_\star t), \quad (39)$$

where A_ξ, A_η, B_ξ and B_η are free parameters to be determined.

By substituting Equations (38) and (39) in the differential equations, we obtain the linear system in A_ξ, A_η, B_ξ and B_η,

$$\begin{cases} -(\omega_\star^2 + U^o_{xx})B_\xi + 2\omega_\star A_\eta - U^o_{xy}B_\eta = 0, \\ -U^o_{xy}A_\xi + 2\omega_\star B_\xi - (\omega_\star^2 + U^o_{xx})A_\eta = 0, \\ -(\omega_\star^2 + U^o_{xx})A_\xi - U^o_{xy}A_\eta - 2\omega_\star B_\eta = a_0\cos(\gamma)^3, \\ -2\omega_\star A_\xi - U^o_{xy}B_\xi - (\omega_\star^2 + U^o_{yy})B_\eta = -a_0\cos(\gamma)^3. \end{cases}$$

Thus, the linear system may be solved to find the coefficient A_ξ, B_ξ, A_η and B_η, which will satisfy the ODEs.

For convenience, define

$$\mathbf{x} = \begin{bmatrix} A_\xi & B_\xi & A_\eta & B_\eta \end{bmatrix}^T, \quad \mathbf{A} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix},$$

and

$$\mathbf{b} = \begin{bmatrix} 0 & 0 & a_0\cos^3(\gamma) & -a_0\cos^3(\gamma) \end{bmatrix}^T,$$

where the submatrices of $\mathbf{A}$ are

$$A_1 = \begin{bmatrix} 0 & -\omega_\star^2 - U^o_{xx} \\ -U^o_{xy} & 2\omega_\star \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 2\omega_\star & -U^o_{xy} \\ -\omega_\star^2 - U^o_{yy} & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -\omega_\star^2 - U^o_{xx} & 0 \\ -2\omega_\star & -U^o_{xy} \end{bmatrix},$$

$$D_1 = \begin{bmatrix} -U^o_{xy} & -2\omega_\star \\ 0 & -\omega_\star^2 - U^o_{yy} \end{bmatrix}.$$

We have in matrix form $\mathbf{Ax} = \mathbf{b}$, and the solution to the linear system is given by

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}.$$

The coefficients A_ξ, A_η, B_ξ and B_η are amplitudes that characterize the orbit.

As mentioned before (for the collinear points),the out-of-plane motion (equation (21))is decoupled from the in-plane equations of motion ((19)-(20)), hence the solution of the third equation is given by equation (33). Therefore, the required sail acceleration for a fixed distance can be given by

$$a_0 = \frac{\zeta_0 |U^o_{zz}|}{\cos(\gamma)^2 \sin(\gamma)}. \tag{40}$$

4. ONE-MONTH ORBITS

This section is concerned with the numerical computation of displaced periodic orbits around the lagrange points in the Earth-Moon system.

As will be shown, there exist displaced orbits at all Lagrange points. For example, the numerical nonlinear results for the Lagrange points L_3 (Figure 2), L_4 (Figure 4), and L_5 (Figure 6) demonstrate, that displaced periodic orbits appear in their vicinity with a period of 28 days (synodic lunar month).

Furthermore, the numerically integrated nonlinear (solid line) equations match the linear analytic solutions (dashed line) for a small displaced orbit (Figure 3, 5, and 7).

The collinear libration points L_1, L_2, and L_3 are unstable equilibria, which implies the instability of any trajectory around these points, such that the exact computation of the displaced orbits is required for its maintenance.

5. CONCLUSIONS

In this work we have presented a novel family of displaced periodic orbits at linear order using solar sail propulsion.

Using the linearized equations of motion around the Lagrange points, periodic orbits that are displaced can be derived, which will be interesting for future mission design. Even though these numerical results for L_3, L_4 and L_5 offer insight into the dynamics of the system, it has to be noted that the Earth and the lunar pole can only be viewed if the out-of-plane displacement is large enough compared to the radius of the moon. However, these results demonstrate that displaced periodic orbits exist at all

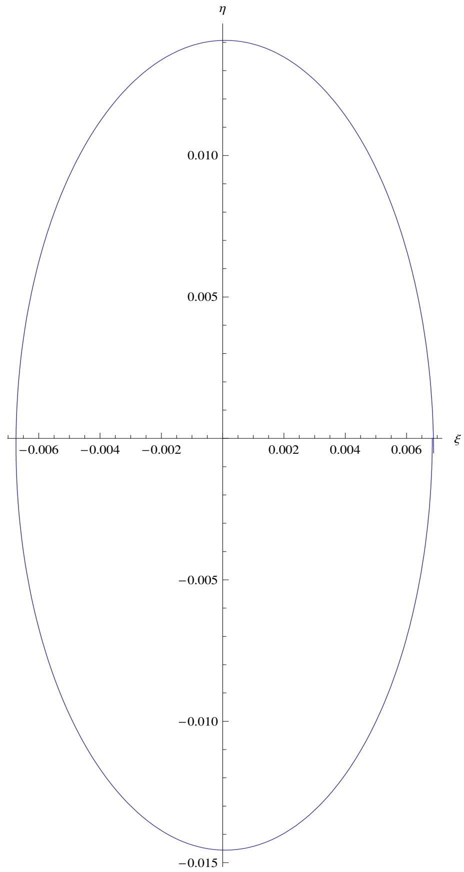

Figure 2. Periodic Orbits at linear order around L_3.

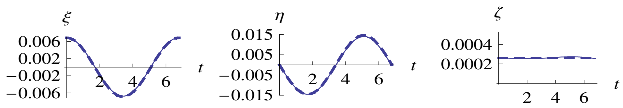

Figure 3. Comparison between the analytical (dashed line) and nonlinear (solid line) results (L_3).

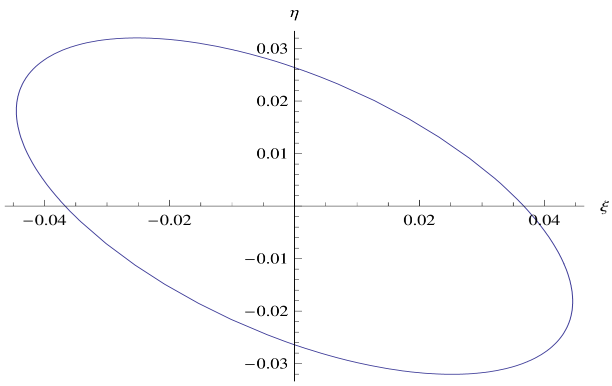

Figure 4. Periodic Orbits at linear order around L_4.

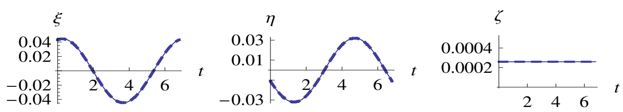

Figure 5. Comparison between the analytical (dashed line) and nonlinear (solid line) results (L_4).

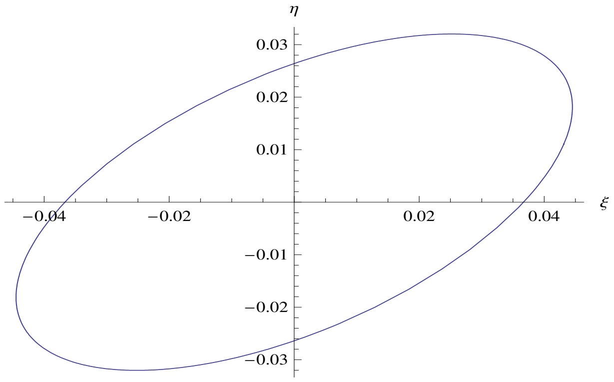

Figure 6. Periodic Orbits at linear order around L_5.

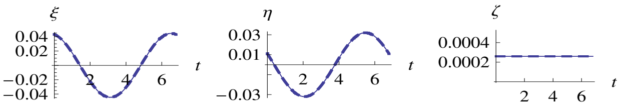

Figure 7. Comparison between the analytical (dashed line) and nonlinear (solid line) results (L_5).

Lagrange points at linear order. Due to the instability of the L_1 and L_2 points of the Earth-Moon system, future work will be focussed on linear control techniques to the problem of tracking and maintaining the solar sail on prescribed orbits.

6. ACKNOWLEDGMENTS

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