ASIAN JOURNAL OF CIVIL ENGINEERING (BHRC) VOL. 14, NO. 1 (2013)
PAGES 1-16

BRIDGE VIBRATIONS EFFECTIVELY DAMPED BY MEANS OF TUNED LIQUID COLUMN GAS DAMPERS

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Received: 20 January 2012; **Accepted**: 5April 2012

ABSTRACT

Sealed tuned liquid column gas dampers, i.e. with a gas spring effect taken into account, TLCGD, are ideally suited to increase the effective structural damping of bridges when vibrating in the critical low frequency band. The evident features of this type of absorbers are no moving mechanical parts, cheap and easy implementation, low maintenance costs and simple modification of the natural frequency, and even of the damping properties (by means of built-in orifice plates). Modal tuning in the design stage by means of the analogy to the classical mechanical damper of the spring-mass-dashpot type, TMD, is subsequently followed by fine-tuning in state space, rendering the absorber parameter (frequency and damping) optimal and, when designing smaller units in parallel action, yields the control even more robust. The equilibrium gas-pressure is the main control parameter to optimize the absorber frequency when the volume of the individual gas vessel above the liquid-gas interface is properly selected. U- or V-shaped TLCGD with horizontal extension maximized, are proposed to reduce dominating horizontal vibrations of long-span bridges (including pedestrian bridges) and, as worked out in detail, in the case of the cantilever method of bridge construction, to increase the allowable maximum length of the cantilever, despite of wind-gusts. An alternative design, VTLCGD provides the control force vertically, and thus counteracts dominating vertical, traffic-induced vibrations. Sealing of the piping system in that case is a necessary condition since an equilibrium gas-pressure difference is essential for the vertical action of the displaced fluid. The horizontal length of this absorber is kept to a minimum. Illustrative examples are described to convincingly approve the un-revealed action of the liquid absorber.

Keywords: Vibration absorber, horizontal (torsional) or vertical bridge vibration, cantilever method of bridge construction, footbridge, den hartog and state space optimization, robust control

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1. INTRODUCTION

Structural damping of bridges is extremely low. Since relative motions are small, the direct application of dashpots and/or frictional dampers requires magnification and thus a complicating design. To concentrate the energy consumed from the vibrating bridge, mechanical dampers are properly modally tuned (TMD) and applied at optimal locations, see [1] for their general design. The quite expensive reconstruction of the Millennium Footbridge in London is described in [2], where both, dampers and TMD are used to increase the effective structural damping beyond its cut-off value of the synchronization effect observed in the excitation process of pedestrian bridges, for the latter see [3] and [4]. The expensive TMD substituted with tanks and sloshing fluid, TLD, solved the problems of the vibration prone Toda Park Foot Bridge, [5]. In [6-8], the (classical) U- or V- shaped tuned liquid column damper (TLCD) has been developed as an absorber superior to the TLD, with applications to reduce the horizontal vibrations at extremely low frequencies of tall buildings. In [9] a novel design is invented by sealing the piping system and thus providing finite gas volumes above the liquid interfaces, thereby creating an additional restoring force by the resulting gas spring effect in parallel action to gravity for the liquid oscillation in the tuned liquid column gas damper, TLCGD. Even active control by injection and removal of gas above the liquid column has been proposed, thus creating an ATLCGD as the cheap counterpart to the ATMD. For both, see again [9]. In the passive mode, a sealed piping system with gas pressure in the equilibrium state properly adjusted extends the frequency range of application of TLCGD just limited by the maximum allowable relative speed of the fluid of about 12 m/s for an intact interface between fluid and gas, [10]. In [11] the detailed model of the TLCGD interacting with the bridge in coupled oblique bending-torsional motion was developed, paralleled by laboratory testing, see also [12]. This investigation includes both, the Millennium Foot Bridge and the Toda Park Foot Bridge, besides other bridges, and illustrates the effective damping action of the TLCGD. Simulation results are presented for the main span of the Millennium Footbridge. Tuning of TLCGD is done in two steps. By means of an analogy between TMD and TLCGD in the design stage, worked out in detail in [9] and [12], modal tuning is performed by a transformation of the classical Den Hartog [13] formulas for optimizing the TMD. In a second step, fine-tuning in state space with the Den Hartog parameters as starting values is recommended. Its application during the cantilever method of bridge construction to reduce wind-excited vibrations is worked out in this paper with final use of the absorbers for effectively damping the long-span bridges. Further, the novel pipe-in-pipe alternative design of a VTLCGD that provides the control force vertically, and thus counteracts dominating vertical, traffic-induced vibrations is described in some detail for a single-span steel bridge. See again [10] for a summary and the limits of application of these liquid absorbers.

2. MECHANICAL MODEL OF A CONTINUOUS BRIDGE WITH A SINGLE LIQUID COLUMN DAMPER ATTACHED

Bridges with low structural damping are forced to, more or less coupled, oblique bending and torsional vibrations. Intensity of excitation increases with the action of traffic flow, trains moving sinusoidal on their tracks and/or at critical speed, gusty wind, dense population of walking pedestrians and runners. If lateral horizontal and torsional motions dominate, liquid column dampers tuned with respect to frequency and energy absorption (TLCGD) are ideally suited to increase the effective structural damping of the bridge. The mechanical model is developed in steps, starting with the in-plane rigid body motion of the cross-section of the bridge where a TLCGD is attached: substructure synthesis is applied with the control force considered, likewise to the action of a TMD. If vertical vibrations dominate, the alternative design of a VTLCGD provides the vertically acting control force.

Extending the more or less coupled, flexural and torsional partial differential equations of motion of a continuous beam, - this simple model of a straight bridge is applied here, see e.g. Nowacki [14], to include oblique bending in horizontal *y*-direction and vertical *z*-direction, with deflections of the center of stiffness *v* and *w* respectively, $\vartheta \ll 1$ denotes the cross-sectional rotation, - yields three coupled, self-explaining equations, see again [12] for details; $m=\rho A$ is the mass per unit of length, EJ the flexural and GJ_T the torsional rigidity of the bridge, $\delta(x-\xi)$ is the Dirac-Delta function and $x=\xi$ is the (optimal) position of the TLCGD providing the control force components $F_{y'}$, $F_{z'}$,

$$EJ_z v_{,xxxx} + m(\ddot{v} - d\ddot{\vartheta}) = (-F_{y'} + F_{z'}\vartheta)\delta(x-\xi) + p_y(x,t) \tag{1a}$$

$$EJ_y w_{,xxxx} + m(\ddot{w} + c\ddot{\vartheta}) = (-F_{y'}\vartheta - F_{z'})\delta(x-\xi) + p_z(x,t) \tag{1b}$$

$$EA_{\varphi\varphi}\vartheta_{,xxxx} - GJ_T\vartheta_{,xx} + m\left[e^2\ddot{\vartheta} + c\ddot{w} - d\ddot{v}\right] = (-M_{Ax} + F_{y'}d_A)\delta(x-\xi) + m_x(x,t) \tag{1c}$$

In Eq. (1) the bridge axis through the center of mass is assumed off symmetry and thus, the squared radius of gyration with respect to the stiffness center C_S becomes $e^2 = c^2 + d^2 + I_0/A$, Figure 1. Prescribed distributed loads are in general forces per unit of length $p_y(x,t)$, $p_z(x,t)$ and possibly a specific torsional moment $m_x(x,t)$. Substituting the single term Ritz approximation (or modal shape function),

$$v(x,t) = Y(t)\chi(x),\ w(x,t) = Y(t)\phi(x),\ u_T(x,t) = e\vartheta(x,t) = Y(t)\psi(x) \tag{2}$$

into Eqs. (1a-c) that are pre-multiplied with the shape functions and subsequently added and integrated over the span to render generalized conditions of orthogonality, see again

[12]. Thus the modal equation results when driven by effective forces that are determined from the proper and weighed summation and integration of the right hand sides in Eq. (1), extremely light modal structural damping has been added at this stage

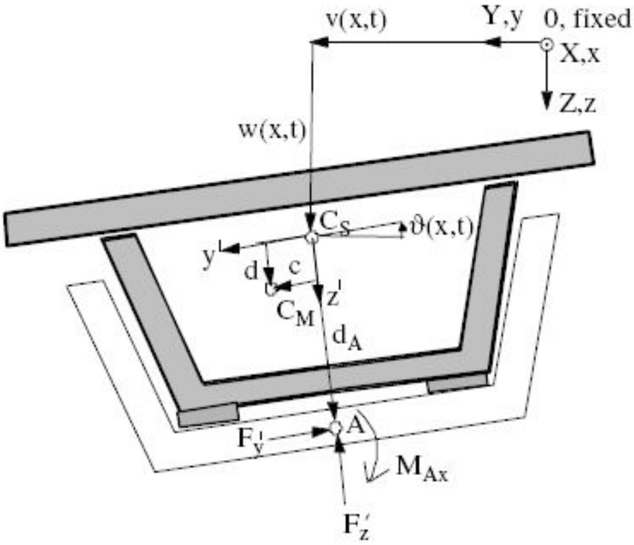


Figure 1. Cross-section of a bridge where the TLCGD is attached. Centre of stiffness C_S and off-symmetry centre of mass C_M indicated. Interaction forces in rotated coordinates primed

$$\ddot{Y}+2\zeta_S\Omega\dot{Y}+\Omega^2 Y=\frac{1}{M}\left[-F_{y'}\left(\chi+\phi\psi\frac{Y}{e}-\frac{d_A}{e}\psi\right)-F_{z'}\left(\phi-\chi\psi\frac{Y}{e}\right)-\psi\frac{M_{Ax}}{e}\right]_{x=\xi}+\frac{F(t)}{M} \quad (3)$$

$$F(t)=\int_0^l\left[\chi(x)p_y(x,t)+\phi(x)p_z(x,t)+\psi(x)\frac{m_x(x,t)}{e}\right]dx$$

Extension to truncated modal analysis of the multiple-degree-of-freedom (MDOF) main structure with several TLCGD attached is done using the Ritz approximation in form of a finite series. Such a three degree-of-freedom rigid frame carrying the single DOF absorber is analyzed computationally and experimentally under severe forcing. Modal tuning with respect to the dominating horizontal displacement and, alternatively, with respect to a dominating rotation, is performed analogous to the classical Den Hartog tuning of a mechanical damper (TMD). Fine-tuning in state space renders the effective damping characteristic of the bridge optimal. Along similar lines the novel VTLCGD is optimized to effectively damp-out vertically dominating flexural vibrations. However, in the course of this paper we restrict Eq. (3) either to the fundamental mode or the first two modes of the bridge. Further, we neglect any torsional deformation by putting $u_T(x,t)=e\vartheta(x,t)=0$ and consequently the moment load per unit of length $m_x\equiv 0$.

2.1 The cantilever method of bridge construction with a single TLCGD attached at the tip

As a background of our study, we reconsider the erection of a "little brother" of the Millau viaduct, the biggest C.E. structure on the A75 motorway in France, consulting engineer and designer *Michel Virlogeux*. It is a multi cable-stayed bridge of eight spans with slender piers and a very light deck. Two end spans of 204m each and six central spans of 342 m each, highest elevation 245 m above the valley ground required quite costly temporary piers mainly due to the expected high wind loads. The steel sections, prefabricated by Eiffel CM, have a cross-sectional profile for the dual carriageway (plus emergency lanes and 1 m shoulders to the central reservation of 4.50 m); rendering a total width of 32 m, that for analysis has been reduced by 10 m in width to a double lane bridge, thereby changing the trapezoidal shape of the cross-section, see Figure 2. Thus, in the construction phase for sake of simplicity of analysis, we consider a cantilevered beam attached to the pier with assigned stiffness, vibrating laterally in its fundamental mode, with a TLCGD attached. The latter provides a horizontally directed control force at the tip. Equation (3) reduces to, see e.g. Blevins [15, p.108]; a uniformly distributed lateral wind load $p_0(t)$ is assumed,

$$\ddot{Y} + 2\zeta_S \Omega_1 \dot{Y} + \Omega_1^2 Y = -\frac{F_y}{M_1}\chi_1\left(x=l\right) + \frac{P_1}{M_1}, P_1 = \int_0^l p_y(x,t)\chi_1(x)dx \to 0.3915 p_0(t) l, \quad (4)$$

$$\Omega_1 = \left(\frac{\lambda_1}{l}\right)^2 \sqrt{\frac{EJ_z}{m}}, \; \lambda_1^2 = 3.516, \; M_1 = \int_0^l m\chi_1^2(x)dx = ml/4, \; \chi_1\left(x=l\right) = 1,$$

$$\chi_1(x) = \frac{1}{2}\left[\cosh\lambda_1\frac{x}{l} - \cos\lambda_1\frac{x}{l} - \sigma_1\left(\sinh\lambda_1\frac{x}{l} - \sin\lambda_1\frac{x}{l}\right)\right], \; \sigma_1 = \frac{\cosh\lambda_1 + \cos\lambda_1}{\sinh\lambda_1 + \sin\lambda_1} = 0.734$$

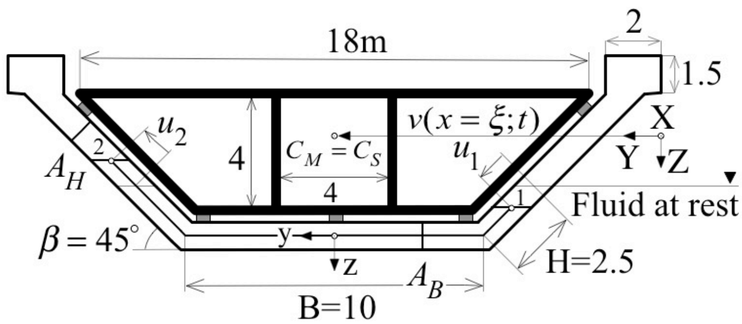


Figure 2. Cross-section of the "little brother" of the Millau steel bridge deck (F-2004), symmetry axis z. TLCGD attached. Note the gas vessels above the fluid in the sealed piping system

At maximum cantilevered length of 170m and with the cross-section of Figure 2 considered, the fundamental bending frequency was estimated $\bar{f}_1 = 0.48$ Hz. Due to the stiffness of the pier, a second natural frequency was estimated, $f_2 = 0.72$ Hz. Dunkerly's approximate lower limit formula, see e.g. [16, p. 452] yields the natural frequency to be inserted in Eq. (4), $f_1 = 0.4$ Hz.

2.2 The tuned liquid column gas damper in horizontal motion

The relative flow of the ideal fluid (just water) in the TLCGD attached at the tip of the cantilevered bridge is considered rendering a generalized Bernoulli type of equation due to the accelerated piping system, $a_A = \ddot{v}(x = l;t)$, for details of derivation see again [16, p. 497] and [12] as well, and consult Figure 2 and, for a piping system that is adapted to a more general bridge cross-section (e.g., of the Millau viaduct), see Figure 3, the fluid stroke is denoted $u_1 = u_2 = u(t)$,

$$\ddot{u} + 2\zeta_A \omega_A \dot{u} + \omega_A^2 u = \kappa \ddot{v}(x = l;t), \kappa = (B_1 + 2B_2 \cos\beta_1 + 2H \cos\beta_2) / L_{eff}, \quad (5)$$

$$f_A = \frac{\omega_A}{2\pi} = \sqrt{\frac{g/\pi^2}{4L_0}}, \; L_{eff} = \frac{A_H}{A_B}(B_1 + 2B_2) + 2H, \; L_0 = (g/\pi^2)/4f_A^2 = L_{eff}/2(\sin\beta_2 + h_0/H_a)$$

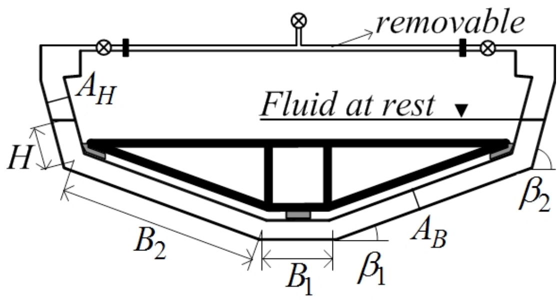


Figure 3. TLCGD (dimensions indicated) fixed to a more general bridge deck (Millau viaduct, thick lines). Piping system sealed. Removable gas pressure supply line sketched

In Eq. (5) the experimentally observed averaged turbulent damping with loss-coefficient δ_L, when equivalently linearized has been added. Consequently, the absorber damping-coefficient becomes depend on the stroke, $\zeta_A = (4\delta_L/3\pi)u_{\max}$. The geometry coefficient κ in Eq. (5) should be maximized within the design limits. The absorber frequency contains the static equilibrium gas pressure p_0 in terms of its head, $h_0 = np_0/\rho g$, the essential new parameter of the optimization process. It results by the linearized quasistatic polytropic gas compression, with the exponent in the range of $1 \le n \le 1.4$. The expansion $p_2 - p_1 = 2n\, p_0 u/H_a + O(u/H_a)^3$ of the gas pressure difference has been considered in the derivation of Eq. (5), see again [10]. The size of the gas volume is defined by $V_a = A_H H_a$. It can be easily shown by applying the control-volume concept, [16, pp. 42, 403], individually for the displaced fluid and the gas volumes that the net control force acting horizontally at the tip of the bridge, $-F_y$, already inserted in Eq. (4), is simply given by the classical conservation of momentum, the y-component of the total acceleration of the center of fluid mass $\mathbf{a}_{C_f}$ in instant location enters,

$$F_y = m_F a_y = m_F \left(\ddot{v}_A + \bar{\kappa}\ddot{u} \right), \; \bar{\kappa} = \kappa L_{eff} / L_1 , \; m_F = \rho A_H L_1 \tag{6}$$

Equation (6) is substituted into Eq. (4) to render together with Eq. (5) an isolated generalized and coupled 2-DOF system that becomes equivalent to the classical one when a TMD of the spring-mass-dashpot type is added, -in that case the geometry coefficients are $\kappa = \bar{\kappa} = 1$ (no dead mass). That isolated system is subjected to a Den Hartog type optimization with formulas of the TMD for both, optimal frequency and damping readily available in the literature, Warburton [17].

2.3 Den Hartog optimization of the V-shaped TLCGD

After choosing the fluid-mass to modal mass ratio $\mu(l) = m_F / M_1$, where $M_1(l) = ml/4$, and the geometry coefficients in the design stage, the mass ratio of the equivalent TMD denoted with a star results by comparing the two 2 DOF systems, see [10] for a detailed derivation and note the altered fundamental frequency of the bridge f_1^* in Eq. (8), caused by the dead fluid mass already apparent in Eq. (7), $(1-\kappa\bar{\kappa})m_F$:

$$\mu^*(l) = \mu(l) \frac{\kappa\bar{\kappa}}{1 + (1-\kappa\bar{\kappa})\mu(l)} < \mu(l) \tag{7}$$

Minimizing the modal deflection of the bridge under (time harmonic) force load requires the TMD parameters to be optimized by the Den Hartog frequency ratio and damping coefficient, see again [17],

$$\delta^*_{opt}(l) = f^*_{Aopt} / f_1^* = 1 / \left(1 + \mu^*\right), \; \zeta^*_{A,opt} = \zeta_{A,opt}(l) = \sqrt{3\mu^*(l) / 8\left(1 + \mu^*(l)\right)} \tag{8}$$

The optimal TLCGD frequency ratio in the first mode of the cantilevered bridge is simply given by a transformation of Eq. (8) with the resulting optimal liquid-absorber frequency,

$$\delta_{opt}(l) = \frac{f_{A,opt}(l)}{f_1(l)} = \frac{\delta^*_{opt}(l)}{\sqrt{1 + (1-\kappa\bar{\kappa})\mu(l)}} < \delta^*_{jopt}(l), \; f_{A,opt}(l) = \frac{f_1(l)}{\left(1 + \mu(l)\right)} \sqrt{1 + (1-\kappa\bar{\kappa})\mu(l)} \tag{9}$$

With the length L_0 of the mathematical pendulum of one and the same linear frequency taken into account, as defined in Eq. (5), the optimal absorber frequency is easily tuned by selecting the proper gas pressure in the design formula,

$$h_0 / H_a = L_{eff} / 2L_0 - \sin\beta , \; h_0 = np_0 / \rho g \tag{10}$$

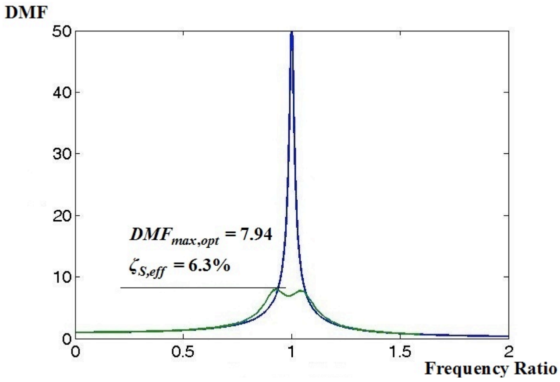
Table 1: Optimal parameter of TLCGD for "little brother's bridge" effective modal damping.

Design parameter		**Parameter optimal, -dynamic**	
Fluid mass m_F, kg	47041	Mass ratio, $\mu_{\min}$	3%
Horiz. length B_1, m	10	Equiv. TMD, $\mu^*_{\min}$	2.4%
Inclined length H, m	2.5	Fund. frequ. f_1, Hz	0.4
Length $L_{eff} = L_1$, m	15	Struct. Damp. ζ_S, %	1
Pipe area $A_H = A_B$,m^2	3.136	δ_{opt}, Eq. (9)	0.974
Incl. angle β_1, rad	$\pi/4$	Absorber $f_{A,opt}$, Hz	0.39
Geom. coeff. $\kappa = \bar{\kappa}$, Eq. (5)	0.902	Absorber $\zeta_{A,opt}$, %	9
Bridge m, kg/m	36895	h_0/H_a, Eqs. (5), (10)	3.87
Cantilever $l_{\max}$, m	170	Gas volume; H_a, m	6.00

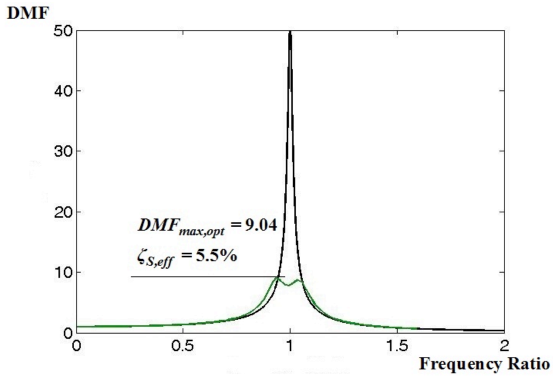
The restrictions $H_a \geq 3u_{\max}$ (to assure linear compressibility) and $30^\circ \leq \beta \leq 90^\circ$ (inclination of the upright pipe section of the V- or U- shaped TLCGD) should be observed in Eq. (10). The gas volume $V_a = H_a A_H = 1.5u_{\max} A_H + V_{a,res}$ must allow a continuous flow of the fluid in its first part and for the remaining portion $V_{a,res}$ the cross-sectional area can be enlarged to save space, Figure 2. In the case of overload, detuning is activated at first by the weak nonlinearity of the gas compression and finally when fluid enters the abruptly changing cross-sectional area of the pipe.

2.4 Numerical results for the "little brother's bridge" in construction and for the full span

The TLCGD at the tip of the cantilevered bridge with mass per unit of length $m = 36895$ kg/m, at its maximum length $l_{\max} = 170$ m, is optimally designed: the modal mass ratio $\mu_F = 3\,\%$ is chosen yielding $m_F = 47041$ kg for the modal mass of the fundamental mode $M_1 = (m/4) l_{\max}$. Equations (7)-(9) render the optimal TLCGD parameters, listed in Table 1 with the essential dimensions, see again Figure 2. The extremely low damping of the steel bridge is effectively increased as shown in Figure 4a by the frequency response functions in the frequency window around the first resonance. For shorter cantilever lengths, the gas pressure can be increased stepwise to keep the absorber frequency tuned despite of the rising fundamental frequencies.



(a)



(b)

Figure 4. Dynamic magnification factors (first mode) of "little brother's bridge without and with optimized TLCGD attached: (a) when cantilevered, TLCGD placed at the tip. (b) when simply supported full span bridge with 3 TLCGD in parallel action placed at mid-span.

Considering the hourly wind speed at Beaufort 12, $v_{wind} = 46$ m/s, in Eq. (4) the static load becomes $p_0 = H_{deck}\rho_{air} v_{wind}^2 / 2 = 5290$ N/m, Figure 2, by simply considering the stagnation pressure. Thus, the static tip-deflection results see again Eq. (4), $v_s(x = l_{\max}) = 0.39\, p_0 / (m/4)\Omega_1^2(l) = 0.0354$ m. Just to convincingly approve the damping of the TLCGD we apply that load time-harmonically at resonance frequency and find the "dramatic" reduction of the modal tip-deflection from 1.775 m to 0.282 m. The fluid stroke reaches 1.210 m.

When uniting the two cantilevered bridge sections, one might keep the two TLCGD in place and add a third one of same design as listed in Table 1. For sake of modelling here, the bridge of length $l = 2l_{\max}$ may be considered simply supported: modal shape function $\bar{\chi}_1 = \sin \pi x / l$, fundamental flexural frequency $\tilde{f}_1 = 0.338$ Hz, modal mass $\tilde{M}_1 = ml/2$, with lateral flexibility of the two piers taken into account, $\tilde{f}_2 = 0.63$ Hz, and thus with the fundamental frequency estimated by Dunkerly's formula, $f_1 = 0.4$ Hz. In a first step, optimization of the three units just considered as one TLCGD with fluid mass $3m_F$ and thus $\mu = 2.25\,\%$ yields the common absorber frequency $f_{A,1-3} = 0.392$ Hz and the absorber damping coefficient $\zeta_{A,1-3} = 8\,\%$. However, the gas pressure must be increased such that $h_0 / H_a = 3.93$, cf. Table 1. From the frequency response function, the effective damping coefficient results, $\zeta_{S,eff} = 5.5\,\%$ at the dynamic magnification of 9.04, Figure 4b. The static deflection at mid-span under the above given wind load is $\bar{v}_s(x = l_{\max}) = 2p_0 / \pi(m/2)\bar{\Omega}_1^2(l)$ $= 0.0289$ m. The modal mid-span amplitude at resonance is reduced from 1.445 m to merely 0.261 m and the fluid stroke is 1.270 m. Fine tuning in state space slightly alters the unit absorber frequencies but drastically lowers the absorber damping and thus renders larger fluid strokes, [9].

2.5 The main span of the Millennium Footbridge reconsidered

Due to the high torsional rigidity any rotational excitation of the TLCGD can be neglected and in oblique bending, Eqs. (1a) and (1b), the horizontal lateral vibrations dominate. Dallard et al [2] presented the bridge dimensions and its dynamic parameters. Those relevant for the main span are: length $l = 144\,\text{m}$, mass per unit of length $m = 1500\,\text{kg/m}$. The normal modes are approximated by sine-shape-functions; consequently the modal mass is constant, $M = ml/2 = 108\times 10^3\,\text{kg}$. Within the first four natural modes in a truncated modal expansion of Eqs. (1a) and (1b), the symmetric fundamental mode with natural frequency $f_{S1} = 0{,}48\,\text{Hz}$ and extremely low linear structural damping coefficient $\zeta_{S1} = 0{,}6\%$ and the neighboring asymmetric mode, $f_{S2} = 0{,}95\,\text{Hz}$, $\zeta_{S2} = 0{,}65\%$ are dominant horizontal and consequently critical for the footbridge. Three TLCGD suffice to properly increase the damping: one attached at mid-span is tuned with respect to the fundamental mode and a pair of TLCGD fixed to the bridge at its quarter points effectively reduce the vibration in the second mode. The fluid-modal mass ratio, $\mu_i = m_{fi}/M \approx 0.01$, yields the water mass constant in all three TLCGD: $m_{f1} = m_{f2} = m_{f3} = 1000\,\text{kg}$. The relevant parameter are listed in Table 2 and adapted to the dimensions of the bridge cross-section.

Table 2: Rigid piping system of three TLCGD within the main span

Design parameter	**TLCGD #1**	**TLCGD #2and #3**
Horizontal length: B,m	2,00	2,00
Inclined length: H, m	2,00	1,50
Cross-sectional area: $A_B = A_H$, m^2	0,17	0,20
Effective length of fluid: $L_{eff} = L_1 = 2H + B$, m	6,00	5,00
Inclination angle: $\beta\,[\text{rad}]$	$\pi/4$	$\pi/4$
Geometry coefficient: $\kappa = \bar{\kappa}$	0,80	0,82

Den Hartog optimization, the relevant modes are considered isolated, Eqs. (7), (8) with Eq. (9) substituted, renders in a first step the absorber parameter $f_{A1} = 0{,}48\,\text{Hz}$, $\zeta_{A1} = 4{,}7\%$ and $f_{A2} = f_{A3} = 0{,}94\,\text{Hz}$, $\zeta_{A2} = \zeta_{A3} = 4{,}8\%$. The excitation by walking pedestrians in a dense population on the bridge is considered by superposition of three time-harmonic functions: (i) symmetric in horizontal and vertical directions and (ii) asymmetric horizontal and symmetric vertically, see again [4] and [12]. The resulting effective modal structural damping coefficients are magnified, in the first mode by the factor 5.2 and in the second critical mode by the factor of 7.0. Consequently, the damping has become sufficiently high to avoid any synchronization effect of irregularly marching pedestrians, Newland [3].

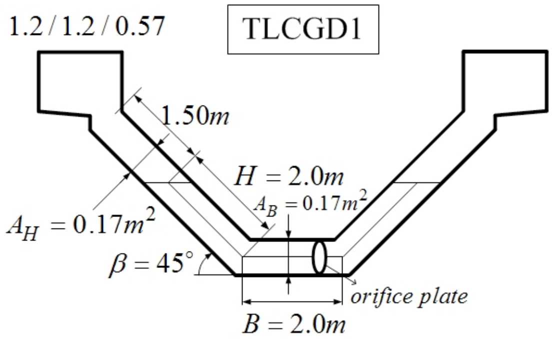


Figure 5. Dimensional sketch of the TLCGD 1 at mid-main span of the Millennium Footbridge

Subsequently performed fine tuning in state space yields the absorber frequencies slightly changed, however, the absorber damping coefficients are dramatically reduced: $f_{A1,opt} = 0{,}47\,\text{Hz}$, $\zeta_{A1,opt} = 3{,}8\%$ and for the pair of TLCGD 2, $f_{A2} = 0{,}98\,\text{Hz}$, $\zeta_{A2,opt} = 3{,}5\%$ and $f_{A3} = 0{,}91\,\text{Hz}$, $\zeta_{A3,opt} = 3{,}1\%$. To realize these absorber natural frequencies when keeping the equilibrium gas pressure equal to the atmospheric pressure, the gas volumes above the fluid columns must be slightly changed (a design with easily adjustable height H_a is required): $V_{a1} = 1{,}061$; $V_{a2} = 0{,}274$; $V_{a3} = 0{,}320\,\text{m}^3$, see Figure 5. More details are collected in [12].

3. THE VERTICALLY ACTING TUNED LIQUID COLUMN GAS DAMPER

We consider a simply supported steel bridge section under vertical forcing. For such dominating vertical vibrations, the absorber setup of the TLCGD is modified in the following way: the length B of the horizontal pipe section is reduced until the pipe sections are close to each other and one of the closed pipe sections is charged with static over-pressure resulting in a static liquid surface displacement. The remaining small asymmetry of the liquid filled, say vertical pipe sections is eliminated by the novel symmetric vertical pipe-in-pipe TLCGD (VTLCGD), as shown in Figure 6. The geometric analogy between the VTLCGD and the equivalent TMD still exists allowing the classical modal Den Hartog tuning in a first design step, before splitting the absorber into smaller units in parallel action or considering neighbouring modes of vibration during subsequent fine tuning in state space. Again, the experimentally observed averaged turbulent damping of the relative fluid flow and the weakly nonlinear gas-spring render the VTLCGD insensitive to overloads and to the parametric forcing of the fluid caused by the vertical motion.

We consider the isolated mode number n of the SS - bridge with modal mass, damping and natural frequency denoted $M_S = ml/2$, ζ_S and $\Omega_S = (n\pi/l)^2 \sqrt{EJ_y/m}$, respectively. Especially for the fundamental mode n=1, a VTLCGD is attached at mid-span supplying the linearized control force $F_z = m_F a_{z,C_f} = m_F \left(\ddot{w}(x = l/2) - \kappa_0 \ddot{u} \right)$, see Figure 6; four pairs of smaller units shown in Figure 7. The modal force excitation is denoted $F(t)$. For the VTLCGD the geometry coefficient is defined by $\kappa_0 = 2H_0 / L_{eff}$, where $L_{eff} = L_1 = L = 2H + B$, Figure 6. Thus, with proper linearization understood, the linear coupled equations of motion of the two DOF isolated system results, ready for optimizing the VTLCGD, see again [10].

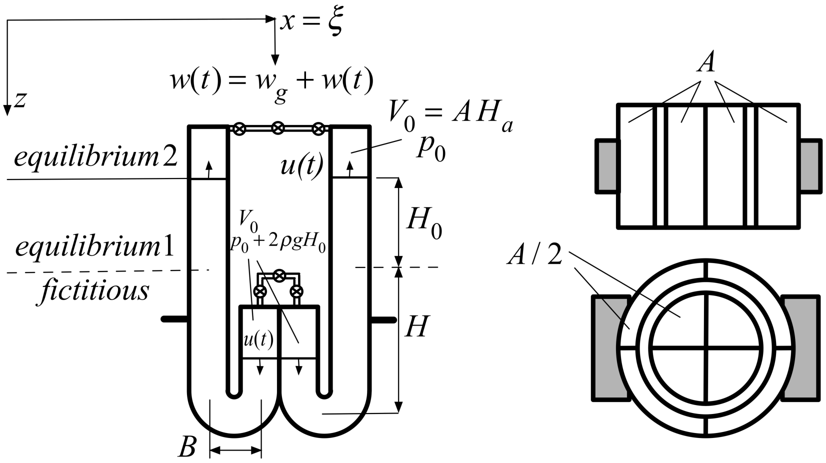


Figure 6. Symmetrical design of a VTLCGD unit, consisting of two (rectangular) or even four (cylindrical) U=shaped TLCGD (width B minimal). The fluid mass in the unit is given by $m_F = 2\rho A L_1$. Design with rectangular and alternatively with circular cross-sections are sketched to reduce vertically dominating bridge vibrations.

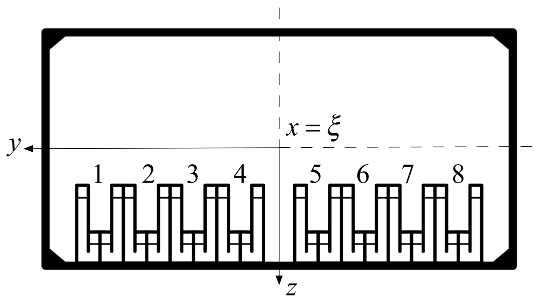


Figure 7. Four pairs of smaller units of VTLCGD sketched and placed symmetrically in the interior of the box-type cross-section of the SS- bridge; for mode 1 at mid-span $\xi = l/2$.

$$\begin{bmatrix} 1+\mu & -\mu\kappa_0 \\ -\kappa_0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{w} \\ \ddot{u} \end{bmatrix} + \begin{bmatrix} 2\zeta_S \Omega_S & 0 \\ 0 & 2\zeta_A \omega_A \end{bmatrix} \begin{bmatrix} \dot{w} \\ \dot{u} \end{bmatrix} + \begin{bmatrix} \Omega_S^2 & 0 \\ 0 & \omega_A^2 \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} \dfrac{F(t)}{M_S} \\ 0 \end{bmatrix} \tag{11}$$

The absorber frequency is given in Eq. (5) with the newly defined length of the equivalent mathematical pendulum substituted,

$$L_0 = \frac{g/\pi^2}{4 f_A^2} = \frac{L_1/2}{1 + \left(h_0 + nH_0\right)/H_a} \tag{12}$$

Thus the design equation for the VTLCGD with respect to gas pressures becomes, analogous to Eq. (10),

$$\left(h_0 + nH_0\right)/H_a = L_1/2L_0 - 1 \tag{13}$$

To avoid any negative influence of the parametric forcing of the fluid, the absorber damping must exceed the cut-off damping coefficient at the most critical double frequency resonance, [18],

$$\zeta_A > \zeta_{A,0}^{(w)} = \max\left|\ddot{w}\left(l/2\right)\right| / \left(L_1/2\right) << 1 \tag{14}$$

The mass ratio of the equivalent TMD is analogously to Eq. (7) defined and consequently, the optimal frequency ratio and damping of the VTLCGD become, Eq. (8) still holds true:

$$\mu^* = \frac{m_A^*}{M_S^*} = \mu \frac{\kappa_0^2}{1 + \mu\left(1 - \kappa_0^2\right)}, \quad \delta_{opt} = \frac{f_{A,opt}}{f_S} = \frac{\delta_{opt}^*}{\sqrt{1 + \left(1 - \kappa_0^2\right)\mu}} = \frac{\sqrt{1 + \mu\left(1 - \kappa_0^2\right)}}{1 + \mu} \tag{15}$$

The dead fluid mass in the VTLCGD is given by $\left(1 - \kappa_0^2\right) m_F$, with a constant cross-sectional area of the piping system understood; see again Figure 6.

Equation (11) takes on a hyper matrix form if the bridge is described by a multiple-degree-of-freedom system (modal expansion truncated and the remaining system preferably described in modal coordinates) with several VTLCGDs attached at properly selected locations, and possibly also converted into smaller units in parallel action at one and the same location. In such a case, fine-tuning in state space becomes necessary.

3.1 Numerical example of an SS-bridge with a VTLCGD attached, first mode

The increase in effective structural damping is demonstrated convincingly by numerical simulations of a simply supported standard (trussed) steel bridge of span $50\,\mathrm{m}$, for sake of simplicity time-harmonically forced at mid-span. The modally tuned VTLCGD with a fluid mass of $m_F = 2000\,\mathrm{kg}$ is designed to suppress the fundamental bridge mode with a modal mass of $M_S = ml/2 = 35720\,\mathrm{kg}$ within the critical frequency window around its fundamental frequency $f_1 = \left(1/2\pi\right)\sqrt{k_1/M_S} = 2.73\,\mathrm{Hz}$. With the pressure in equilibrium state 1 prescribed, $p_0 = 1.2 \times 10^5\,\mathrm{Pa}$, the gauge pressure in action should always be non-negative, and the surplus pressure head $H_0 = 0.70\,\mathrm{m}$ chosen in the equilibrium state 2, $\kappa_0 = 2H_0/L = 0.40$ is inserted in Eq. (15) to obtain the mass ratio of the equivalent TMD,

$\mu^* = 0.9\,\%$, the optimal Den Hartog-parameters of the single VTLCGD become at once, Eqs. (15) and (8), $f_{A1opt} = 2.64\,\text{Hz}$ and $\zeta_{A1opt} = 5.6\,\%$.

Assuming $n = 1.4$, i.e. adiabatic gas compression, yields the height of the gas volume, $H_a = 0.380\,\text{m}$, Eq. (13), and with the cross-sectional area of $A = 0.571\,\text{m}^2$, the principal design of the single VTLCGD is completed. The linearly estimated maximum fluid stroke of $\max|u| = 0.05\,\text{m}$ is compatible within these design dimensions, and the parametric forcing is proven to be fully negligible, $\zeta_{A1opt} = 5.6\,\% > \zeta_{A,0}^{(w)} = 0.08\,\%$. Applying in symmetrical arrangement four pairs of smaller VTLCGD-units with fine tuning in state space, just performed using the tools in the standard program *fminsearch* of [19], with the Den Hartog-parameter serving as the initial values, renders slightly modified absorber frequencies (gas pressures must be adapted) and much lower optimal damping ratios (the maximum fluid stroke is nearly doubled), $f_{A1,8,opt} = 2.80\,\text{Hz}$, $\zeta_{A1,8,opt} = 1.85\,\%$, $f_{A2,7,opt} = 2.51$, $\zeta_{A2,7,opt} = 1.73$, $f_{A3,6,opt} = 2.70$, $\zeta_{A3,6,opt} = 1.73$, $f_{A4,5,opt} = 2.61$, $\zeta_{A4,5,opt} = 1.70$; gas pressures are: $p_{01,8} = 1.42 \times 10^5\,\text{Pa}$, $p_{02,7} = 1.14 \times 10^5$, $p_{03,6} = 1.34 \times 10^5$, $p_{04,5} = 1.23 \times 10^5$. Since the maximum relative fluid speed still remains rather small, namely 1.7 m/s, no problems with respect to the application of the piston theory (intact fluid-gas-interface) are to be expected, [10]. In Figure 8, the dynamic magnification factors are illustrated in the frequency window at the fundamental frequency. Note the more robust damping property of the smaller units.

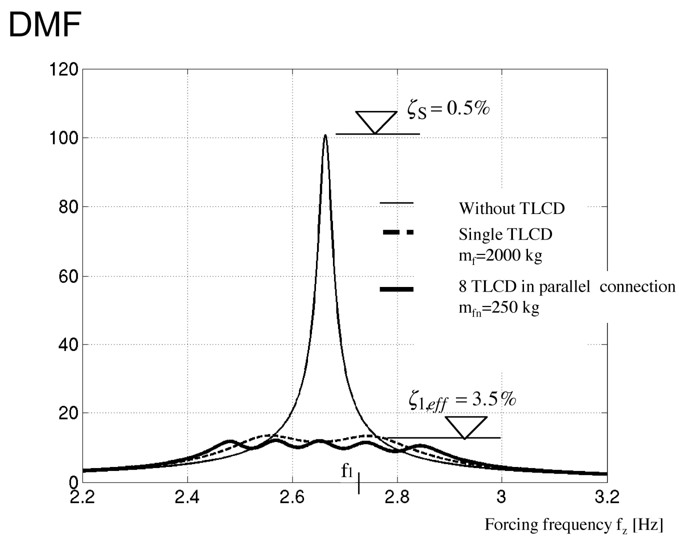


Figure 8. Dynamic magnification factors of the single span steel bridge: VTLCGD optimized with respect to the fundamental mode. More robust damping by fine-tuning of 4 pairs of smaller units

4. CONCLUSION

It is convincingly shown that the Tuned Mechanical Damper, TMD, of the spring-mass-dashpot type can be replaced by the cost-effective Tuned Liquid Column Gas Damper, TLCGD or VTLCGD, to mitigate dominating horizontal (possibly coupled to torsional) or vertical bridge vibrations. The advantages of the liquid absorber are among others no moving mechanical parts, consequently no static friction effects and wear and tear, easy even adjustable frequency tuning by means of the main control parameter: the equilibrium gas pressure in the sealed piping system, the self-controlling property against overload etc. Consequently, in-situ performed frequency tuning takes into account the measured natural frequency of the real bridge structure. Contrary to the Tuned Liquid Damper, TLD relying on the sloshing fluid, the dead fluid-mass can be minimized. Box type bridge cross-sections may allow putting the liquid absorber into the interior, - trussed bridges might allow integrating partly the piping system into truss members. Tuning of the liquid absorber in the design stage just requires a simple transformation of the readily available optimization criteria for the equivalent TMD.

Acknowledgement: Part of the paper has been presented in the Special Session on “Bridges control schemes and devices” at EACS2012, Genova. The first Author gratefully acknowledges a travel grant from the Austrian Center of Competence in Mechatronics, ACCM, 4040 Linz, Austria.

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