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**WORKERS' RECIPROCITY AND THE (IR)RELEVANCE OF WAGE
CYCLICALITY FOR THE VOLATILITY OF JOB CREATION**

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Workers' Reciprocity and the (Ir)Relevance of Wage Cyclicity for the Volatility of Job Creation*

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Abstract

In the last two decades advances in the theory of labour market fluctuations have emphasised the role of *new hires'* wage rigidity—rather than wage rigidity of *existing workers'*—to explain the large volatility of unemployment observed in the data. However, recent evidence suggests that wages paid to newly hired workers are substantially pro-cyclical. By considering the effect that wage changes can have on workers' effort, and therefore on output, this paper provides two novel theoretical results. First, it is shown that the anticipation by firms of the effort response of new hires to wage changes can amplify the magnitude of shocks to the extent that, in contrast with the existing literature, the cyclicity of the hiring wage becomes *irrelevant* for their decision to hire new workers, and hence for the volatility of job creation. Second, it is shown that firms' expectation of existing workers' downward wage rigidity—and the anticipation of their negative reciprocity response to future wage cuts—*does matter* for the expected value of posting a new vacancy, and under certain conditions it may even reduce firms' incentive to hire.

Keywords: reciprocity; wage cyclicity; job creation; unemployment volatility.

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1 Introduction

Workers' and firms' attitudes toward wage setting and the employment relationship have fundamental implications for the cyclical behaviour of the labour market. Grounded in the job flows approach and following the seminal work of [Shimer \(2005\)](#), advances in the theory of labour market fluctuations have placed particular emphasis on the role of rigidities in the wage determination of *newly hired* workers to explain the observed volatility of vacancies and unemployment. This literature has also shifted away from the view that downward wage rigidity in *existing jobs* may be an important driver of large and persistent unemployment fluctuations. Although present, this latter rigidity is irrelevant for the volatility of job creation in existing models ([Pissarides, 2009](#)), and it appears unlikely to be the main driver of the extraordinarily long duration of unemployment ([Elsby, Shin, and Solon, 2016](#)).

However, more recent evidence has shown that wage offers made to newly hired workers are substantially pro-cyclical (e.g. [Haefke, Sonntag, and Rens \(2013\)](#));¹ and that the existing theoretical framework cannot simultaneously accommodate the empirical volatilities of both the relevant hiring wage and the vacancy-unemployment ratio ([Kudlyak, 2014](#)). Hence, although the literature on the subject is particularly developed, it is not yet clear whether the emphasis on new hires' wage cyclicity is well placed, or what impact the wage rigidity of incumbent workers has on the cyclical volatility of job creation.²

This paper contributes to this literature by considering the effects that the anticipation by firms' of their workers' effort responses to wage changes can have on the amplitude and cyclical behaviour of wages and unemployment. In so doing, the paper provides two novel theoretical results. First, in contrast to the existing literature, by appealing to the reciprocity effects induced by wage changes on firms' output, the analysis demonstrates that the cyclicity of the hiring wage is *irrelevant* for the volatility of the vacancy-unemployment ratio. Second, once uncertainty around the evolution of job-match productivity is introduced, it is shown that the expected downward rigidity in the wage of incumbent workers *does matter* for the present value of new employment relationships; and the conditions under which this expectation increases the volatility of job creation and unemployment are derived and discussed.

The main behavioural mechanism underling both these key results is the presence of a pos-

¹See also the recent evidence from [Martins, Solon, and Thomas \(2012\)](#), [Carneiro, Guimarães, and Portugal \(2012\)](#), [Stüber \(2017\)](#) and [Schaefer and Singleton \(2017\)](#).

²Surveys of the existing literature can be found in [Mortensen and Nagypál \(2007\)](#), [Rogerson and Shimer \(2011\)](#), [Elsby, Michaels, and Ratner \(2015\)](#) and [Ljungqvist and Sargent \(2017\)](#).

itive wage-effort relationship, which stems from the optimal effort response of workers to their matched firm's wage setting policy. Whenever the labour market is hit by an exogenous shock and firms anticipate having to adjust their wages, consideration of this relationship enables one to characterise the effect of wage changes on employed workers' effort, which in turn affects output and amplifies the magnitude of the shock. To gain a preliminary intuition of this mechanism, consider the steady-state value of a new match to a firm as given by

$$J = \frac{y(p, e) - w}{1 - \delta(1 - \rho)},$$

which typically determines a firm's vacancy posting decision. Here output y is an increasing function of a productivity shock p and of the employed worker's effort e , which, in accordance to the efficiency wage tradition (e.g. [Akerlof \(1982\)](#), [Akerlof and Yellen \(1990\)](#)) and the literature on reciprocity in labour markets (e.g. [Bewley \(2007\)](#), [Fehr, Goette, and Zehnder \(2009\)](#)), can be written as an increasing function of the wage $e = e(w)$; (δ is the firm's discount factor and ρ an exogenous job destruction rate). For now it is sufficient to suppose that, as a result of optimal wage setting/bargaining, the wage is increasing in p . Next, consider a productivity shock, and let $\varepsilon_{w,p}$ denote the elasticity of wages with respect to productivity. The steady-state elasticity of the value of a new job match with respect to p can therefore be written as

$$\frac{y(p, e(w)) + [y_e(p, e)e'(w) \cdot \varepsilon_{w,p}w] - \varepsilon_{w,p}w}{y(p, e) - w}. \quad (1)$$

The bigger the size of this expression, the larger the effect of the productivity shock on the firm's value of the new match, and hence, the greater the volatility of job creation and unemployment.

In a large body of the existing literature on unemployment volatility the effect of wage changes on effort is not considered: $e'(w) = 0$ and firms' output corresponds to p , implying that the expression above collapses to $\frac{p - \varepsilon_{w,p}w}{p - w}$ (see e.g. [Pissarides \(2009\)](#)). In such a case, the size of (1) crucially depends on the elasticity of the hiring wage $\varepsilon_{w,p}$, and on the difference between the output and the wage, i.e. what is referred to in the literature as the *profit margin* ([Elsby et al., 2015](#)) or the *fundamental surplus* ([Ljungqvist and Sargent, 2017](#)). This simple discussion reflects the insights of [Shimer \(2005\)](#), [Hall \(2005b\)](#), and subsequent theoretical models that emphasised the role of new hires' wage rigidity (i.e. $\varepsilon_{w,p} = 0$), or wage stickiness (i.e. $\varepsilon_{w,p} < 1$), as the main

mechanism to increase the size of (1);³ it also reflects the argument put forward by [Hagedorn and Manovskii \(2008\)](#) and [Pissarides \(2009\)](#) among others, according to which, even if hiring wages were entirely rigid, it is the actual size of the profit margin $p - w$ that matters: only if this margin is sufficiently small will slight changes in productivity generate large fluctuations in the present value of profit, and therefore in vacancy creation.⁴

By considering the effect of wage changes on effort and output—captured by the term $[y_e(p, e)e'(w) \cdot \varepsilon_{w_p} w] > 0$ in (1)—the present paper formally establishes this channel as an additional and important amplification mechanism; and develops a theoretical framework for a transparent analysis of its implications.⁵ In particular, the first contribution of this paper is to show that if firms optimally set the wage to account for their workers' reciprocal response, in contrast with the existing literature concerned with new hires' wage rigidity, the extent of the cyclical volatility of the hiring wage becomes irrelevant for the volatility of vacancies and unemployment. Essentially, the outcome of wage setting is such that the marginal effect of a wage adjustment on profit is optimally balanced by the change in workers' effort at the margin, leaving room for shocks to be fully absorbed by the present value of profit from a new match.⁶ As such, the theoretical framework developed here can generate outcomes that are quantitatively consistent with plausible empirical estimates of the volatilities of both the hiring wage and the vacancy-unemployment ratio (a simple calibration exercise is presented in Appendix A).

Recent empirical evidence on wage and unemployment adjustments in the business cycle has also questioned the role of expected wage rigidity in the wage of incumbent workers for firms' job creation decisions ([Elsby et al., 2016](#)). In his influential paper [Pissarides \(2009\)](#)

³See [Mortensen and Nagypál \(2007\)](#) and [Rogerson and Shimer \(2011\)](#) for surveys. Also, to avoid confusion, the present paper defines wage rigidity as the *acyclical* behaviour of wages, i.e. when wages do not adjust to productivity shocks (downward/upward or both); and wage stickiness as the *less than proportional cyclical* volatility of wages with respect to productivity, i.e. when the wage-productivity elasticity is less than one ([Pissarides, 2009](#)).

⁴See also the discussion in Section 3.1, and [Elsby et al. \(2015\)](#) and [Ljungqvist and Sargent \(2017\)](#) for a complete exposition of these points.

⁵The implications of a positive wage-effort relationship in a search and matching framework have also been analysed by [Wesselbaum \(2013\)](#) and [Kuang and Wang \(2017\)](#). [Wesselbaum \(2013\)](#) considers the effort function proposed by [de la Croix, de Walque, and Wouters \(2009\)](#) and through a calibration exercise shows that, due to cyclical changes in effort, his model quantitatively outperforms a canonical model based on Nash bargaining. However, this result remains sensitive to the value of several parameters that enter the specific effort function considered. As such it is not clear, in the model of [Wesselbaum \(2013\)](#), to what extent the cyclical volatility of effort can amplify shocks. On the other hand, [Kuang and Wang \(2017\)](#) show that gift-exchange considerations by firms generate wage stickiness for new hires which, as the discussion around equation (1) highlighted, contribute to increase unemployment volatility. In fact, as they also point out—and in contrast with the results established in this paper—if hiring wages are instead entirely flexible, their baseline calibration fails to generate the observed volatility of the vacancy-unemployment ratio.

⁶In terms of equation (1), optimal wage setting would imply that $[y_e(p, e)e'(w) \cdot \varepsilon_{w_p} w] - \varepsilon_{w_p} w = 0$. This expression in fact corresponds to the first-order condition characterising the optimal wage in the model of this paper. As such, in this paper, the expression for the steady-state elasticity of market tightness does not feature the elasticity of the hiring wage.

shows that even if the wage of incumbent workers were entirely rigid, firms will be able to internalise these future rigidities in the equilibrium wage negotiated with their new hires at the start of the employment relationship, leaving the volatility of job creation unaffected by the anticipation of wage rigidities in existing jobs. This conclusion has however been recently re-evaluated in two prominent studies: the theoretical model of [Eliaz and Spiegler \(2014\)](#), who show that by generating ex-post inefficient layoffs, existing workers' downward wage rigidity reduces firms' expected duration of new employment relationships, negatively influencing their expected value; and the quantitative analysis of [Bils, Chang, and Kim \(2016\)](#), who show that if firms can contract upon workers' effort (i.e. the employment contract is complete), and the wage of existing workers does not adjust to negative shocks (due to staggered Nash bargaining), firms would require existing workers to be more productive, therefore lowering the relative value of hiring a new worker and reducing job creation.

In light of these findings, the second contribution of this paper is to provide an alternative and complementary perspective to the insights advanced by [Eliaz and Spiegler \(2014\)](#) and [Bils et al. \(2016\)](#), by identifying another channel through which the expectation by firms of downward rigidity in the wage of incumbent workers can affect the volatility of job creation. In particular, even if the employment contract is incomplete (in contrast to [Bils et al. \(2016\)](#)), and even if incumbent workers' wage rigidity does not generate endogenous layoffs (in contrast to [Eliaz and Spiegler \(2014\)](#)), the anticipation by firms of the relatively large cost of implementing wage cuts in the event of a negative shock—that is, the anticipation of stronger negative effort responses from incumbent workers—can negatively influence the expected present value of new employment relationships, dampening hiring incentives and increasing the volatility of job creation and unemployment.

The two novel results of this paper are formally established within a theoretical framework that builds on a growing body of literature exploring the implications of fairness and reciprocity in labour markets ([Fehr et al., 2009](#)).⁷ More specifically, the paper incorporates the model of asymmetric reciprocity and wage setting developed by [Dickson and Fongoni \(2016\)](#) into a canonical search and matching model à la [Pissarides \(1985, 2000\)](#). According to this, wage setting is formalised as a two-stage game where firms (the first movers) make take-it-or-leave-it wage offers to workers (the second movers). Workers evaluate wage contracts with

⁷This literature spans from the efficiency wage models of [Akerlof \(1982\)](#) and [Akerlof and Yellen \(1990\)](#), to more recent applications of fairness and reciprocity in labour markets, such as, [Danthine and Kurmann \(2007, 2010\)](#) and [Eliaz and Spiegler \(2014\)](#).

respect to a reference ‘fair’ wage and are heterogenous on the basis of their employment status and reference wage. New hires arrive at firms with an exogenously-given reference wage; incumbents are characterised by adaptation: their reference wage is endogenously determined by their wage in the previous period. Employed workers’ optimal choice of effort, in light of the wage paid by firms, yields a wage-effort relationship where loss aversion implies a kink at the reference wage, characterising their ‘asymmetric reference-dependent reciprocity’ (Dickson and Fongoni, 2016). Combined with reference wage adaptation in a dynamic environment, asymmetric reciprocity also implies that workers’ optimal effort response to wage cuts is larger than their response to equivalent-sized wage rises, generating downward wage rigidity. Anticipating this, firms setting the optimal wage face an inter-temporal trade-off at the margin between the benefit of a higher wage today, i.e. higher effort, *versus* the cost associated with employing a worker with a higher reference wage in the future, due to reference wage adaptation.⁸ As such, by considering the implications of employed workers’ asymmetric reciprocity for optimal wage setting and job creation, this paper formally establishes that: i) pro-cyclical hiring wages are consistent with large fluctuations in the expected surplus from new matches, since shocks are amplified by the anticipated change in newly hired workers’ effort; and ii) the expected (disproportionate) drop of existing workers’ effort in the event of wage cuts generates downward wage rigidity and can reduce the expected value of new employment relationships, therefore increasing the volatility of job creation.

The remainder of the paper is organised as follows. Section 2 develops the model and characterises firms’ optimal wage setting policy, workers’ optimal effort choice and the steady-state equilibrium. Section 3 studies the role of new hires’ wage and effort cyclicity for the volatility of job creation; while Section 4 introduces uncertainty and studies the effect of expected downward wage rigidity of incumbent workers. Section 5 provides some concluding remarks. Additional material is contained in Appendix A. All proofs are contained in Appendix B.

⁸It can be useful to clarify the conceptual distinction between a worker’s *reservation* wage and a worker’s *reference* wage. The reservation wage is the wage below which a worker would optimally turn down a job offer, stay unemployed and continue to search for jobs. The reference wage instead is a concept that captures a worker’s perception of what is a fair wage, that is, the wage level relative to which the worker evaluates the fairness of an employment contract. While the possibility that the two might coincide should not be ruled out, this paper considers the case where they do not. In doing so the paper shows that, so long as the worker’s reservation wage is not binding—i.e. so long as a wage offer exceeds the reservation wage—firms’ anticipation of how workers evaluate wage contracts relative to a reference ‘fair’ wage generates additional ‘behavioural’ constraints to their wage setting and hiring decisions, with non-trivial consequences for vacancies and unemployment fluctuations.

2 Model

2.1 Labour Market Environment

Consider a labour market with a continuum of infinitely lived identical firms and a continuum of measure one of infinitely lived workers who differ with respect to their employment status. Denote the initial period of an employment relationship by s . At the beginning of each period t a worker can be in one of three states: unemployed and searching for a job, if $t < s$; employed as a new hire, if $t = s$; or employed as an incumbent, if $t > s$ and the worker is not laid off.

Workers' preferences are reference-dependent: they evaluate wage contracts with respect to a reference 'fair' wage $r \in \mathcal{R} \subset \mathbb{R}_+$. It is assumed that a worker's reference wage at the beginning of each t depends on their employment status as follows. In the first employment period $t = s$ newly hired workers, denoted by i , are assumed to be heterogenous with respect to their reference wage r_{it} , which is exogenous. In particular,

A1. r_{it} is the realisation of a random variable on the state space $\mathcal{R} = [0, \bar{r}]$ with density function

$$\gamma_0, \text{ cumulative distribution function } \Gamma_0, \text{ and } \Gamma_0(\bar{r}) = \int_0^{\bar{r}} \gamma_0(r_i) dr_i = 1.$$

On the other hand, from the second employment period onwards $t > s$, incumbent workers, denoted by j , adapt their reference wage to the most recent wage contract $r_{jt} = w_{t-1}$. A such,

A2. the reference wage of employed workers $\{i, j\}$ evolves according to the following adaptation rule:

$$r_{jt+1} = w_t, \quad r_{it} \sim \Gamma_0, \quad \Gamma_0 \text{ given.} \quad (2)$$

Assumptions A1-A2 impose a crucial distinction between newly hired and incumbent workers, entirely captured by their reference wage: a newly hired worker is assumed to have a non-negative, exogenously-given reference wage, while an incumbent's reference wage is assumed to be determined (endogenously) by the wage they were paid in the previous employment period.⁹

⁹This assumption (A2) is consistent with a large body of evidence documented in the labour market literature on reference wage formation as well as by other behavioural science sub-disciplines concerned with reference point formation. The first piece of evidence supporting this idea comes from the seminal experiment of [Kahneman, Knetsch, and Thaler \(1986\)](#). (see also [Kahneman and Thaler \(1991\)](#) and [Baucells and Sarin \(2010\)](#) for a review of the early literature on adaptation, or habituation, in social psychology). Adaptation to past wage contracts is also supported by several anthropological studies (see the survey of [Bewley \(2007\)](#)). In the context of experimental studies, indirect evidence in support of this hypothesis comes from the field experiments of [Gneezy and List \(2006\)](#) and [Mas \(2006\)](#), and the laboratory experiments of [Clark, Masclet, and Villeval \(2010\)](#), [Gächter and Thöni \(2010\)](#) and [Koch \(2016\)](#) among others. Direct evidence of reference wage adaptation has also been documented by the

The unemployment rate is u_t , the employment rate is n_t and the vacancy rate is v_t . The labour force L_t is constant and fixed, and normalised to unity. The number of job matches taking place per unit time is \bar{m}_t , where

A3. \bar{m} is a linearly homogeneous matching function, increasing and concave in both its arguments u and v .

Let the tightness of the labour market be defined by $\theta_t = v_t/u_t$. The probability that a vacant job is matched with a worker is $h(\theta_t) \equiv \bar{m}(u_t, v_t)/v_t$, $h'(\theta_t) < 0$, whilst the probability of an unemployed worker making contact with a vacancy is $f(\theta_t) \equiv \bar{m}(u_t, v_t)/u_t$, $f'(\theta_t) > 0$. The elasticity of the matching function with respect to unemployment is denoted by $\sigma \in (0, 1)$.

The parameters of the model are assumed to be such that every worker-firm match is mutually advantageous: all the unemployed workers that are matched with firms are hired.¹⁰ As such, $f(\theta_t)$ represents the job-finding rate. On the other hand, employed workers move into unemployment at a rate $\rho \in (0, 1)$, the exogenous job-destruction rate. The evolution of mean unemployment can therefore be expressed by the difference between the flows in and out of unemployment:

$$\Delta u_{t+1} = \rho[1 - u_t] - f(\theta_t)u_t, \quad u_0 \text{ given.} \quad (3)$$

2.2 Asymmetric Reciprocity and Optimal Wage Setting

This section presents a model of reciprocity and wage setting following the microeconomic framework developed by [Dickson and Fongoni \(2016\)](#). The model is also extended to an infinite-horizon environment, facilitating the subsequent analysis in the context of a search and matching framework.

field experiment of [Chemin and Kurmann \(2014\)](#) and the laboratory experiment of [Sliwka and Werner \(2017\)](#). Moreover, the idea that ex-ante contracts serve as entitlements for future renegotiations was advanced by [Hart and Moore \(2008\)](#) and further explored in [Herweg and Schmidt \(2012\)](#) in the literature of incomplete contracts. The laboratory experiments of [Fehr, Hart, and Zehnder \(2011, 2014\)](#), [Bartling and Schmidt \(2015\)](#) and [Herz and Taubinsky \(2016\)](#) provide strong support for this hypothesis.

¹⁰By this it is implicitly assumed that firms' zero-profit condition at the time of hiring is always satisfied, and that any wage offer is such that the value to workers of being employed is greater or equal to the value of being unemployed, i.e. the reservation wage is not binding. The former assumption has been widely used in the literature (see for instance [Pissarides \(1987, 2000\)](#)), whilst the latter is a simplification which implies that unemployed workers matched with firms will accept any wage offer (as for instance in [Michaillat \(2012\)](#)). The condition that needs to be satisfied for this latter assumption to hold is spelled out in Section 2.4. Moreover, as shown in Section A.2, Appendix A a calibration of the model reveals that, for conventional values of unemployment income, the workers' reservation wage is never binding. As such notice that in the context of this paper, a worker could accept an employment contract that pays a wage they perceive to be unfair. In fact it is not inconceivable that a worker might prefer to be employed at a wage perceived as unfair, rather than remaining unemployed. Moreover, as long as there exists a wedge between a worker's return from being unemployed and the value of being employed, the results derived hereafter will hold.

Consider a representative worker-firm employment relationship that starts in period s . Information is considered to be complete. At the beginning of each employment period $t \geq s$ the firm learns the match productivity $p \in \mathcal{P} \subset \mathbb{R}_+$ and the worker's reference wage $r \in \mathcal{R} \subset \mathbb{R}_+$, and subsequently decides on the profit-maximising wage contract $w \in \mathcal{W} \subset \mathbb{R}_+$.

After evaluating the firm's wage contract in relation to their reference wage, the worker decides on the utility-maximising level of effort $e \in \mathcal{E} \subset \mathbb{R}_+$ which generates output for the firm according to the production function $y : \mathcal{P} \times \mathcal{E} \rightarrow \mathbb{R}_+$. Payoffs are then realised, the form of which is described next. Therefore, in each employment period $t \geq s$, wage setting is formalised as a two-stage game of complete and perfect information in which the firm makes take-it-or-leave-it wage offers to the worker.¹¹ Since choices are made sequentially and the firm is assumed to be motivated only by profit, the game can be solved by backward induction.¹²

2.2.1 Payoffs

In each employment period $t \geq s$ the instantaneous profit function of an operating firm $\pi : \mathcal{W} \times \mathcal{P} \times \mathcal{E} \rightarrow \mathbb{R}$ takes the following form:¹³

$$\pi(w_t; p_t, e_t) = y(p_t, e_t) - w_t,$$

where

F1. y is strictly increasing and linear on $\mathcal{P} \times \mathcal{E}$, with $y_{ep} > 0$.

Assumption F1 implies a linear production function exhibiting constant returns to effort, where the marginal product of effort is increasing in the match productivity.¹⁴

¹¹For the relative incidence of take-it-or-leave-it wage offers and wage bargaining in employment relationships, see the evidence presented in [Hall and Krueger \(2012\)](#) and [Brenzel, Gartner, and Schnabel \(2014\)](#).

¹²The reader is referred to [Dickson and Fongoni \(2016\)](#) for more details on the validity of backward induction as an equilibrium solution concept in this setting.

¹³All functions considered throughout the analysis are continuously differentiable on their domains unless otherwise specified.

¹⁴Notice that by assuming a concave production function (as in, for instance, [Dickson and Fongoni \(2016\)](#)), i.e. assuming decreasing returns to effort in production, will generate richer out-of-steady-state dynamics. In particular the concavity of the production function with respect to effort will generate (deterministic) endogenous persistence in reciprocity and wage dynamics: for example, in the case of a positive shock, firms will optimally implement a series of wage increases in order to exploit their workers' positive reciprocity in each period. This, combined with workers' adaptation of the reference wage, implies that the marginal gain in effort obtained by firms through wage rises will decrease over time until a new steady state is reached, the properties of which are analogous to those derived at the end of this section. As such, the dynamic implications of a more general production function are not analysed and are left to further research, and notice that this richer dynamics will not affect the predictions of Sections 3 and 4.

The instantaneous utility function of an employed worker $u : \mathcal{E} \times \mathcal{R} \times \mathcal{W} \rightarrow \mathbb{R}$ is additively separable and takes the following form

$$u(e_t; w_t, r_t) = m(w_t) - c(e_t) + M(e_t, w_t, r_t),$$

where $m : \mathcal{W} \rightarrow \mathbb{R}_+$ captures the effect of absolute wage levels on the worker's utility; $c : \mathcal{E} \rightarrow \mathbb{R}_+$ captures the worker's intrinsic psychological and physical *net cost* of productive activity,¹⁵ and the function $M(e, w, r) \equiv en(w|r)$ is the worker's *morale function*.

Morale depends on the worker's evaluation of the wage in relation to the reference wage, which is captured by the function $n : \mathcal{W} \times \mathcal{R} \rightarrow \mathbb{R}$. It is assumed that $n(w|r) \equiv \mu(m(w) - m(r))$ where μ is a gain-loss value function that exhibits loss aversion in the spirit of [Kahneman and Tversky's \(1979\)](#), [Tversky and Kahneman's \(1991\)](#) value function and [Kőszegi and Rabin's \(2006\)](#) universal gain-loss function. Consider the following assumptions:

W1. m is strictly increasing and concave on \mathcal{W} .

W2. c is strictly convex on \mathcal{E} , with $c'(0) < 0$ and $c'''(e) = 0$.

W3. $\mu : \mathcal{X} \rightarrow \mathbb{R}$ is continuous, piecewise linear, and strictly increasing on \mathcal{X} for all $x \neq 0$, with $\mu(0) = 0$. Moreover, for any $x > 0$, $\mu'(-x)/\mu'(x) \equiv \lambda \geq 1$.

Assumption W2 essentially implies that $u(e; w, r)$ is strictly concave on \mathcal{E} , and that the utility-maximising level of effort when $w = r$ is non-negative (which will be referred to as 'normal' effort); while under assumption W3, the gain-loss utility μ takes the following form:

$$\mu(m(w) - m(r)) = \begin{cases} \eta[m(w) - m(r)] & \text{if } w \geq r \\ \lambda\eta[m(w) - m(r)] & \text{if } w < r \end{cases} \quad (4)$$

where $\eta > 0$ is a scaling parameter that represents the importance of gain-loss utility for the worker, and $\lambda \geq 1$ represents the worker's degree of loss aversion.

The morale function captures an additional psychological cost/benefit of productive effort associated with the worker's perception of fairness. If the wage exceeds the reference wage (it is perceived as a gift) the worker gains some additional benefit of productive effort and an

¹⁵This function can be considered as the difference between the worker's physical and psychological costs and the related psychological benefit of the productive activity. A similar assumption is also considered by [Sliwka and Werner \(2017\)](#), [Kaur \(2018\)](#) and [Macera and te Velde \(2018\)](#). The reader is referred to [Dickson and Fongoni \(2016\)](#) for a more thorough discussion.

increase in effort (a gift to the firm) will increase utility. If the wage falls short of the reference wage (it is perceived as unfair) there is a psychological cost of productive effort and a reduction in effort (an ‘unkind’ action towards the firm) increases utility. As such, the morale function implies the worker’s payoff exhibits reciprocity, and since morale is linked to loss aversion, negative reciprocity is stronger than positive reciprocity.

2.2.2 Asymmetric reference-dependent reciprocity

This subsection formally derives an employed worker’s optimal effort response to wage offers in relation to their reference wage. The resulting effort function is identical to the ‘asymmetric reference-dependent reciprocity’ derived by [Dickson and Fongoni \(2016\)](#). Nevertheless, its derivation is repeated here for completeness and clarity of exposition.¹⁶

For any given sequence of wage offers $\{w_t\}_{t=s}^{\infty}$ set by the firm, an employed worker’s $\{i, j\}$ problem consists of choosing a sequence of levels of effort $\{e_t\}_{t=s}^{\infty}$ that maximises their utility, given their evaluation of the wage in relation to their reference wage r_t . In an employment relationship starting in period $t = s$, the employed worker’s problem is:

$$\begin{aligned} \mathcal{W}_s(w_s, r_s) &= \max_{\{e_t\}_{t=s}^{\infty}} \sum_{t=s}^{\infty} \psi^{t-s} u(e_t; w_t, r_t) \\ \text{s.t. } & w_t \text{ given } \forall t \geq s \\ & r_{t+1} = w_t, r_s \text{ given.} \end{aligned} \tag{WP}$$

$\mathcal{W}_s(w_s, r_s)$ is the value function of the employment relationship of a worker hired in period s (hence a newly hired worker); $\psi \in (0, 1)$ is defined as $\psi \equiv \delta(1 - \rho)$, where $\delta \in (0, 1)$ is the discount factor and $\rho \in (0, 1)$ is the exogenous job-destruction probability; w_t and r_t are the two state variables and e_t is the worker’s control variable. Notice that the worker’s choice of effort in each employment period does not affect the evolution of the state variables.

The employed worker’s optimal effort $\tilde{e}_t = \tilde{e}(w_t, r_t, \lambda)$, is therefore characterised by the following first-order condition, which is both necessary and sufficient for an optimum:

$$-c'(e_t) + \mu(m(w_t) - m(r_t)) \leq 0, \tag{5}$$

¹⁶The optimal effort function derived in this section is also related to other asymmetric wage-effort relationships advanced in the microeconomic literature of reciprocity in labour markets (see for instance the models of [Elsby \(2009\)](#), [Sliwka and Werner \(2017\)](#) and [Macera and te Velde \(2018\)](#)).

in which the inequality is replaced with an equality if $e_t > 0$.

Theorem 1. (*Asymmetric Reference-dependent Reciprocity, Dickson and Fongoni (2016)*). For all $t \geq s$, and for any given wage offer w_t relative to their reference wage r_t , the worker's optimal effort function is

$$\tilde{e}_t = \tilde{e}(w_t, r_t, \lambda) = \begin{cases} c'^{-1}(\eta[m(w_t) - m(r_t)]) & \equiv \tilde{e}(w_t, r_t)^+ & \text{if } w_t > r_t \\ c'^{-1}(0) & \equiv \tilde{e}_n & \text{if } w_t = r_t \\ c'^{-1}(\lambda\eta[m(w_t) - m(r_t)]) & \equiv \tilde{e}(w_t, r_t, \lambda)^- & \text{if } w_t < r_t \end{cases} \quad (6)$$

where \tilde{e}_n denotes 'normal' effort; $\tilde{e}(w, r)^+ > \tilde{e}_n$ denotes 'positive reciprocity'; and $\tilde{e}(w, r, \lambda)^- < \tilde{e}_n$ denotes 'negative reciprocity'. Moreover,

- a) For a given r , and for all $w \neq r$, $\tilde{e}(w, r, \lambda)$ is a continuous, increasing and concave function of w ; and

$$\frac{\lim_{\epsilon \rightarrow 0} \tilde{e}_w(r - \epsilon, r, \lambda)^-}{\lim_{\epsilon \rightarrow 0} \tilde{e}_w(r + \epsilon, r)^+} = \lambda,$$

implying that the optimal effort function has a kink at $w = r$ if $\lambda > 1$.¹⁷

- b) For a given w , and for all $w \neq r$, $\tilde{e}(w, r, \lambda)$ is a continuous and decreasing function of r ; and $\tilde{e}_{wr}(w, r, \lambda) = 0$.
- c) For all $w < r$, $\tilde{e}(w, r, \lambda)$ is a continuous and decreasing function of λ ; and $\tilde{e}_{w\lambda}(w, r, \lambda)^- > 0$.

The optimal effort function defined by (6) captures an employed worker's $\{i, j\}$ asymmetric reference-dependent reciprocity. Whenever a worker is paid their reference wage they will exert normal effort, \tilde{e}_n , which is independent of the absolute wage level; while the effect on effort of changes in the wage away from the reference wage is asymmetric for a loss averse worker. This asymmetry has the implication that from an initial wage equal to the reference wage, the effect of negative reciprocity that results from a wage cut will be greater than the effect of positive reciprocity resulting from a wage increase. Moreover, whenever $w_t < r_t$, more loss averse workers will exert less effort, which decreases faster as the wage gets further from the reference wage. Indeed, if a worker is not loss averse ($\lambda = 1$), reciprocity is still reference dependent, but symmetric.

¹⁷Throughout the analysis, where sequences of ϵ are considered over which limits are taken, it is specified that $\{\epsilon_n\}_{n=1}^{\infty} \subset \mathbb{R}_+$, meaning that where the wage is specified to be $r - \epsilon$ and the limit is taken as $\epsilon \rightarrow 0$, it is considered as the wage increasing to the reference wage, and likewise when the wage is specified to be $r + \epsilon$ and the limit is taken as $\epsilon \rightarrow 0$, it is considered as the wage decreasing to the reference wage.

2.2.3 Firms' optimal wage policy

An operating firm's wage setting problem consists of choosing a sequence of wages $\{w_t\}_{t=s}^{\infty}$ that maximises its profit, taking as given their employed worker's $\{i, j\}$ reference wage r_t and their optimal effort responses, defined by the sequence $\{\tilde{e}_t\}_{t=s}^{\infty}$. Since p is parametric and time invariant, for a given initial period s , an operating firm's wage setting problem can be formalised as:

$$\begin{aligned}
 J(r_s) &= \max_{\{w_t\}_{t=s}^{\infty}} \sum_{t=s}^{\infty} \psi^{t-s} \pi(w_t; \tilde{e}_t) \\
 \text{s.t. } \tilde{e}_t &= \tilde{e}(w_t, r_t, \lambda) \\
 r_{t+1} &= w_t, r_s \text{ given.}
 \end{aligned} \tag{FP}$$

$J(r_s)$ is the firm's value function of the employment relationship from period s onwards; \tilde{e}_t is the optimal effort choice of the worker; r_t is the state variable and $r_{t+1} = w_t$ is the control variable. For any given newly hired worker's reference wage $r_s \in \mathcal{R}$ in s , at the beginning of each $t \geq s$ the firm's problem consists of setting the optimal wage w_t , and hence the next period reference wage r_{t+1} , taking as given the current reference wage r_t .

The firm's instantaneous profit can be rewritten as a function of w_t and r_t only, after substituting for the worker's optimal effort function. As such:

Lemma 1. *The firm's profit function π is strictly concave on \mathcal{W} and strictly decreasing on \mathcal{R} , with $\pi_{wr} = 0$. Moreover π is supermodular in (p, e) .*

The relevant functional equation corresponding to the firm's problem in (FP) can be written in recursive form:

$$J(r) = \max_{w \in \mathcal{W}} \{y(p, \tilde{e}(w, r, \lambda)) - w + \psi J(w)\} \tag{7}$$

where r corresponds to the current period worker's reference wage and w corresponds to the current period wage and the following period worker's reference wage.

Denote the optimal wage policy of an operating firm by $\tilde{w} = \tilde{w}(r, p, \lambda)$, which is characterised by the following first-order condition,¹⁸ and which, for convenience, is expressed in

¹⁸This is both necessary and sufficient to characterise a maximum. The technical details which ensures that this is the case, as well as others relevant for the characterisation of the firm's optimal wage policy, are discussed in the Proof of Proposition 2.

terms of the key parameters p and λ (the prime indicates forward values):¹⁹

$$y_e(p, \bar{e})\bar{e}_w(w, r, \lambda) - 1 - \psi y_e(p, \bar{e}') |\bar{e}_r(w', w, \lambda)| = 0 \quad \forall w \neq r \quad (8)$$

The intuition behind condition (8) is the following. Since the worker's effort is increasing in the wage w and decreasing in the reference wage r , the firm will choose the optimal wage such that the *current* marginal benefit in terms of positive reciprocity—or less negative reciprocity—is equalised to the marginal cost of paying a higher wage in the current employment period, net of the additional *expected* marginal cost, in terms of effort, of employing a worker with a higher reference wage in subsequent employment periods.

Proposition 1. *The firm's value function J is strictly concave on \mathcal{W} and strictly decreasing on \mathcal{R} .*

Proposition 2. *For all $t \geq s$ and for any given worker's reference wage r , the time-invariant optimal wage policy of an operating firm employing a worker characterised by asymmetric reference-dependent reciprocity $\bar{e}(w_t, r_t, \lambda)$ with $\lambda > 1$ and adaptation $r_{t+1} = w_t$, is given by*

$$\bar{w}_t = \bar{w}(r_t, p, \lambda) = \begin{cases} \bar{w}(p)^+ > r_t & \text{if } r_t < r_L(p) \\ r_t & \text{if } r_t \in [r_L, r_H] \\ \bar{w}(p, \lambda)^- < r_t & \text{if } r_t > r_H(p, \lambda); \end{cases} \quad (9)$$

where

$$r_L(p) \equiv \{r_t : \lim_{\epsilon \rightarrow 0} y_e(p, \bar{e})\bar{e}_w(r_t + \epsilon, r_t, \lambda) - 1 + \psi y_e(p, \bar{e}')\bar{e}_r(w', r_t + \epsilon, \lambda) = 0\};$$

$$r_H(p, \lambda) \equiv \{r_t : \lim_{\epsilon \rightarrow 0} y_e(p, \bar{e})\bar{e}_w(r_t - \epsilon, r_t, \lambda) - 1 + \psi y_e(p, \bar{e}')\bar{e}_r(w', r_t - \epsilon, \lambda) = 0\}.$$

The optimal wage $\bar{w}(p)^+ (> r)$ is implicitly defined by (8) in which $\bar{e}(w, r, \lambda) = \bar{e}(w, r)^+$; and $\bar{w}(p, \lambda)^- (< r)$ is implicitly defined by (8) in which $\bar{e}(w, r, \lambda) = \bar{e}(w, r, \lambda)^-$. Moreover,

- a) For all $r_t \in \mathcal{R} \setminus [r_L, r_H]$, $\bar{w}(r_t, p, \lambda)$ is increasing in p and independent of r_t ;
- b) For all $r_t \in [r_L, r_H]$, $\bar{w}(r_t, p, \lambda)$ is increasing in r_t and independent of p ;
- c) For all $r_t > r_H(p, \lambda)$, $\bar{w}(p, \lambda)^-$ is increasing in λ .

¹⁹Notice that under assumption F1, y_e is independent of \bar{e} ; and that due the results established in part b) of Theorem 1 (namely that $\bar{e}_{wr} = 0$), \bar{e}_w is independent of its second argument and \bar{e}_r is independent of its first argument. These results and their implications for the characterisation of the optimal wage are rigorously derived in the proof of Proposition 1.

Finally, $r_H(p, \lambda) > r_L(p)$ for all $\lambda > 1$ and if $\lambda = 1$ then $r_H(p, 1) = r_L(p)$ and $\tilde{w}(p)^+ = \tilde{w}(p, 1)^-$.

Proposition 2 shows that the firm's wage setting policy crucially depends on the level of a worker's reference wage. If a worker's reference wage is relatively low, the firm will pay them a relatively low wage, which will then be perceived as a gift $\tilde{w}(p)^+ > r_t$; if a worker's reference wage is relatively high, the firm will pay them a relatively high wage $\tilde{w}_H(p, \lambda)^- < r_t$, which will however be perceived as unfair; while if a worker has a relatively moderate reference wage $r_t \in [r_L, r_H]$, then the firm will pay them their fair wage $\tilde{w}_t = r_t$. This is illustrated in Figure 1, which also shows the optimal wage policy with respect to exogenous changes in p .²⁰

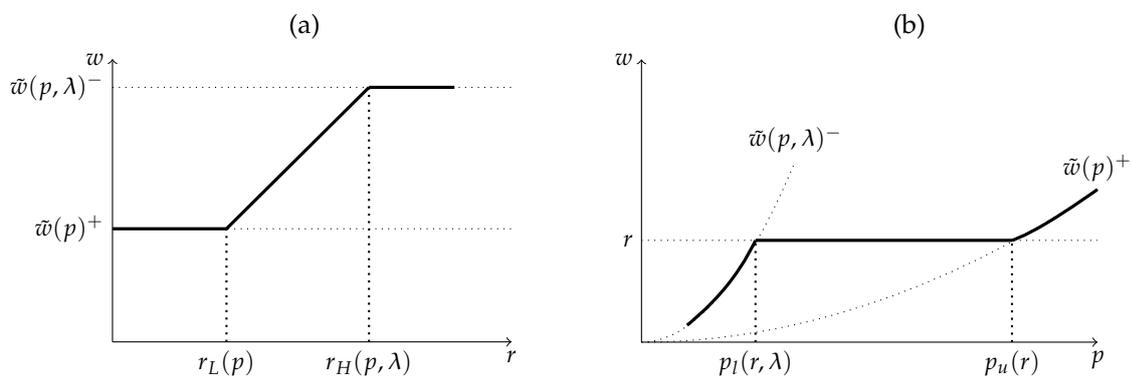


Figure 1:
Optimal Wage Setting Policy

Hence Proposition 2 establishes the existence of a range of worker's reference wages within which it is optimal for the firm to pay them their reference wage. This result hinges crucially on the worker's asymmetric reference dependent reciprocity that stems from their extent of loss aversion $\lambda > 1$. Whenever a firm is facing a worker with a moderate reference wage $r_t \in [r_L, r_H]$, the marginal benefit of setting a lower, but unfair, wage $w_t < r_t$ will not be sufficient to offset the marginal cost generated by the worker's negative reciprocity; similarly, the marginal benefit derived from the worker's positive reciprocity, generated by the wage $w_t > r_t$ being perceived as a gift, will not be enough to offset the marginal cost of paying a higher wage and having to employ a worker with a higher reference wage in the future. In fact, if reciprocity were symmetric, i.e. if $\lambda = 1$, these trade-offs at the margins would disappear.

Finally note that, by the linearity of the firm's production function with respect to effort (i.e. constant returns to effort), whenever the optimal wage is set above or below the worker's

²⁰From the definitions of $r_L(p)$ and $r_H(p, \lambda)$, it is possible to define $p_l(r, \lambda) \equiv r_H^{-1}(r, \lambda)$ and $p_u(r) \equiv r_L^{-1}(r)$.

reference wage, this will be independent of the level of the worker's reference wage.

2.3 Job Creation Condition

The environment described so far is consistent with a labour market in which each period t is characterised by a number of firms searching for unemployed workers; a number of firms employing newly hired workers i with reference wage r_{it} who exert optimal effort \tilde{e}_{it} ; and a number of firms employing incumbent workers j with reference wage $r_{jt} = \tilde{w}_{t-1}$ who exert optimal effort \tilde{e}_{jt} , for all $t > s$.

Let the value of a vacancy to the firm be denoted by V_t ; the value of a job filled by a newly hired worker be denoted by $J(r_{it})$; and the value of a job filled by an incumbent worker by $J(r_{jt})$, in which $r_{jt} = w_{t-1}$. The value of a job filled by a newly hired worker satisfies

$$J(r_{it}) = y(p, \tilde{e}(\tilde{w}_t, r_{it}, \lambda)) - \tilde{w}_t + \delta [(1 - \rho)J(\tilde{w}_t) + \rho V_{t+1}]; \quad r_i \sim \Gamma_0, \Gamma_0 \text{ given.} \quad (10)$$

Let κ be a time-invariant cost of posting a vacancy. The value of a vacancy for a firm, that is facing the probability of matching an unemployed worker in period t to start an employment relationship in the following period, can be expressed as

$$V_t = -\kappa + \delta [h(\theta_t)\mathbb{E}_t [J(r_{it+1})] + (1 - h(\theta_t))V_{t+1}]; \quad r_i \sim \Gamma_0, \Gamma_0 \text{ given,} \quad (11)$$

where $\mathbb{E}_t [J(r_{it+1})]$ denotes the expected value in period t of an employment relationship with a newly hired worker in period $t + 1$ (recall that firms will learn their new hires' reference wage r_i only once they are matched).

To derive a condition governing firms' job creation decisions it is assumed that there is free entry, i.e. $V_t = 0 \forall t$. Hence, the optimal vacancy posting decision of firms can be characterised by the value of θ_t that satisfies the following job creation condition:

$$\frac{\kappa}{h(\theta_t)} = \delta \mathbb{E}_t [J(r_{it+1})] = \delta \int_0^{\bar{r}} J(r_{it+1}) d\Gamma_0(r_i). \quad (\text{JC})$$

2.4 Workers' Reservation Wage

This section explicitly states the condition that needs to be satisfied by the firms' optimal wage offer at the start of an employment relationship, in order for the worker to accept the job and

stop searching.

For any given $r_{it} \sim \Gamma_0$, the value of employment to an incumbent worker j is

$$\mathcal{W}(\tilde{w}_t, r_{jt}) = u(\tilde{e}_t(\tilde{w}_t, r_{jt}, \lambda), \tilde{w}_t, r_{jt}) + \delta \left[\rho \mathcal{U} + [1 - \rho] \mathcal{W}(\tilde{w}_{t+1}, r_{jt+1}) \right]; \quad (12)$$

where $\tilde{w}_t = \tilde{w}(r_{jt}, p, \lambda) = \tilde{w}(r_{it}, p, \lambda)$. The value of employment to a new hire i is:

$$\mathcal{W}(\tilde{w}_t, r_{it}) = u(\tilde{e}_t(\tilde{w}_t, r_{it}, \lambda), \tilde{w}_t, r_{it}) + \delta \left[\rho \mathcal{U} + [1 - \rho] \mathcal{W}(\tilde{w}_{t+1}, r_{it+1}) \right]; \quad (13)$$

where $\tilde{w}_t = \tilde{w}(r_{it}, p, \lambda)$. Finally, the value of unemployment to an unemployed worker is

$$\mathcal{U}_t = u(z) + \delta \left[f(\theta_t) \mathbb{E} [\mathcal{W}(\tilde{w}_{t+1}, r_{it+1})] + [1 - f(\theta_t)] \mathcal{U}_t \right]; \quad (14)$$

where $u(z) = m(z)$, and z represents unemployment income.

As such, define a worker's reservation wage \underline{w}_t as

$$\underline{w}(r_{it}, z, \tilde{e}_n, \theta_t) \equiv \{w_t : \mathcal{W}(w_t, r_{it}) = \mathcal{U}_t\};$$

that is, $\underline{w}_{it} = \underline{w}(r_{it}, z, \tilde{e}_n, \theta_t)$ is the wage offer for which a worker is indifferent between accepting the job or continuing search. So far it has been implicitly assumed that the parameters of the model are such that $\mathcal{W}(w_t, r_{it}) \geq \mathcal{U}_t$. This is equivalent to assume that the parameters of the model are such that the following condition holds:

$$\bar{u}(\tilde{e}_n, \tilde{w}(r_{it}, p, \lambda), r_{it}) \geq \frac{u(z) + \delta f(\theta_t) \int_0^{\bar{r}} \bar{u}(\tilde{e}_n, \tilde{w}(r_{it+1}, p, \lambda), r_{it+1}) d\Gamma_0(r_i)}{1 - \psi + \delta f(\theta_t)}; \quad (15)$$

where the function \bar{u} captures the present discounted value of utility of being employed at the wage $\tilde{w}(r_{it}, p, \lambda)$ from $t = s$ onwards, conditional on job destruction (see Section A.2 of Appendix A for details).²¹ As shown in Appendix A a calibration of the model reveals that, for conventional values of z , condition (15) is always satisfied, that is, a worker with reference wage r_{it} that is matched with a firm offering the wage $\tilde{w}(r_{it}, p, \lambda)$ will always prefer to be hired, rather than continue searching and receive unemployment income z .

²¹ Notice that, in the context of the model derived in this paper, equation (15) is the discrete-time analog of the reservation wage condition derived in Pissarides (2000, p. 150, eq. (6.15)).

2.5 Aggregation and Steady-state Equilibrium

The control variables of the system are: the employed workers' $\{i, j\}$ optimal effort choice \tilde{e}_t , which maximises utility given their matched firm's optimal wage offer \tilde{w}_t evaluated relative to their reference wage r_t ; the optimal wage policy of firms \tilde{w}_t , which maximises their present discounted value of profit taking as given their employed workers' reference wage r_t ; and the level of vacancies, subsumed by labour market tightness θ_t , that satisfies the job creation condition, taking as given the expected present discounted value of a new employment relationship $\mathbb{E}_t [J(r_{it+1})]$ for any given $r_i \sim \Gamma_0$, where the optimal wage paid and the employed workers' reciprocity are endogenously determined by their respective optimisation problems.

The state variables of the system are the employed workers' reference wage r_t and the unemployment rate u_t . The laws of motion describing their evolution are reproduced here for clarity of exposition:

$$r_{t+1} = \tilde{w}(r_t, p, \lambda), r_0 \sim \Gamma_0, \Gamma_0 \text{ given}; \quad (16)$$

$$u_{t+1} = u_t + \rho(1 - u_t) - f(\theta_t)u_t, u_0 \text{ given}. \quad (17)$$

To fully characterise the steady-state equilibrium of the model this section derives first the steady-state levels of wages, reference wages and effort, and subsequently uses these results to derive the steady-state levels of market tightness and unemployment. Note that these can be derived independently as two distinct blocks, since firms' optimal wage policy is independent of labour market tightness and unemployment.²²

Proposition 3. *The steady-state levels of wages, reference wages and effort in the labour market are characterised as follows:*

a) *The steady-state aggregate wage paid to employed (newly hired and incumbent) workers $\{i, j\}$ in*

²²The result that wage setting is independent of labour market tightness in this model is due to the assumption that the distribution of new hires reference wages in the market is exogenous. As such the vacancy rate and the unemployment rate do not affect the optimal effort choice of workers, neither the wage setting behaviour of firms. The same result has also been obtained in the literature by, for instance, [Eliaz and Spiegler \(2014\)](#), who assume that workers have no bargaining power, and [Hall and Milgrom \(2008\)](#), who assume that the workers' threats of quitting to unemployment is not credible. Investigating the consequences of a distribution of new hires' reference wages Γ_0 that is dependent of the state of the labour market is beyond the scope of this paper.

the labour market is given by

$$W^*(\Gamma_0, p, \lambda) = \int_0^{r_L(p)} \tilde{w}^*(p)^+ d\Gamma_0(r_i) + \int_{r_L(p)}^{r_H(p, \lambda)} r_i^* d\Gamma_0(r_i) + \int_{r_H(p, \lambda)}^{\bar{r}} \tilde{w}^*(p, \lambda)^- d\Gamma_0(r_i). \quad (18)$$

b) Newly hired workers' steady-state aggregate level of the reference wage is given by

$$R_i^*(\Gamma_0) = \int_0^{\bar{r}} r_i^* d\Gamma_0(r_i); \quad (19)$$

while their steady-state aggregate level of effort is given by

$$E_i^*(\Gamma_0, p, \lambda) = \int_0^{r_L(p)} \tilde{e}^*(\tilde{w}^*(p)^+, r_i^*)^+ d\Gamma_0(r_i) + \int_{r_L(p)}^{r_H(p, \lambda)} \tilde{e}_n^* d\Gamma_0(r_i) + \int_{r_H(p, \lambda)}^{\bar{r}} \tilde{e}^*(\tilde{w}^*(p, \lambda)^-, r_i^*, \lambda)^- d\Gamma_0(r_i). \quad (20)$$

c) Incumbent workers' steady-state aggregate level of the reference wage is given by

$$R_j^*(\Gamma_0, p, \lambda) = \int_0^{\bar{r}} r_j^* d\Gamma_0(r_i) = \int_0^{\bar{r}} \tilde{w}^*(r_i^*, p, \lambda) d\Gamma_0(r_i) = W^*(\Gamma_0, p, \lambda); \quad (21)$$

while their steady-state aggregate level of effort is given by

$$E_j^* = \int_0^{\bar{r}} \tilde{e}^*(\tilde{w}^*, r_j^*, \lambda) d\Gamma_0(r_i) = \tilde{e}_n^*. \quad (22)$$

Note that these are both steady-state averages and expected values. Proposition 3 establishes a clear distinction between new hires and incumbent workers wages, reference wages, and effort levels in the steady state.

Depending on their initial reference wage, there will be a fraction of new hires that is paid a wage gift $\tilde{w}^* > r_i^*$ and will exert positive reciprocity in their first employment period; a fraction of new hires that is paid a wage perceived as unfair $\tilde{w}^* < r_i^*$, triggering negative reciprocity in their first employment period; and a fraction of new hires that is paid their reference wage $\tilde{w}^* = r_i^*$ and therefore will exert normal effort \tilde{e}_n . On the other hand, independently of the absolute level of the steady-state wage and their initial reference wage, due to reference wage adaptation incumbent workers always perceive their wage as fair, and therefore will exert

normal effort. As such, while firms employing new hires i may expect to experience positive or negative reciprocity in the first employment period, those employing incumbent workers j will always experience normal effort.

Denote by $J(r_t)^+$, $J(r_t)^=$, and $J(r_t)^-$ the value of an employment relationship with a worker $\{i, j\}$ in period t in which a the firm is paying the wage $\bar{w}(p)^+$, r_{it} , and $\bar{w}(p, \lambda)^-$ respectively. Using the results derived thus far it is possible to establish the following:

Lemma 2. *The steady-state expected value of an employment relationship to firms takes the form*

$$\mathbb{E}[J(r_i^*)] = \int_0^{r_L(p)} J(r_i^*)^+ d\Gamma_0(r_i) + \int_{r_L(p)}^{r_H(p, \lambda)} J(r_i^*)^= d\Gamma_0(r_i) + \int_{r_H(p, \lambda)}^{\bar{r}} J(r_i^*)^- d\Gamma_0(r_i); \quad (23)$$

where $J(r_i^*)^+ > J(r_i^*)^= > J(r_i^*)^-$. Moreover, due to assumption A2, the following holds:

$$\begin{aligned} J(r_i^*)^+ &> J(r_j^*)^+ && \text{if } r_i < r_L(p) \\ J(r_i^*)^= &= J(r_j^*)^= && \text{if } r_i \in [r_L, r_H] \\ J(r_i^*)^- &< J(r_j^*)^- && \text{if } r_i > r_H(p, \lambda). \end{aligned}$$

Note that (23) is also the steady-state average value of an employment relationship to firms. Lemma 2 implies that the value of a job filled by a new hire does not always necessarily equal the value of a job filled by an incumbent worker. Notice that in the canonical model there is no such distinction, since wages are re-negotiated at each period and there is no link between one employment period and the next. In fact, even though an employment relationship starts with some degree of positive or negative reciprocity, the value of the job to the firm once a worker becomes incumbent will either shrink or increase due to the dynamic re-normalisation of effort stemming from incumbent workers' reference wage adaptation. As such, the present framework endogenously generates a systematic difference between the output produced by newly formed and existing matches.

Denote the steady-state equilibrium labour market tightness by $\tilde{\theta}^* = \tilde{\theta}^*(\Gamma_0, p, \lambda)$, which is characterised by the solution to (JC); and denote the steady-state unemployment rate by $u^* = u^*(\Gamma_0, p, \lambda)$, which is characterised by the following condition representing the Beveridge Curve:

$$u^* = \frac{\rho}{\rho + f(\tilde{\theta}^*)}. \quad (\text{BC})$$

Proposition 4. *There exists a unique steady-state equilibrium labour market tightness $\tilde{\theta}^*(\Gamma_0, p, \lambda)$*

that satisfies the job creation condition (JC); and a unique steady-state equilibrium unemployment rate $u^*(\Gamma_0, p, \lambda)$ that satisfies condition (BC).

2.5.1 Some Comparative Statics Results

This section investigates some comparative statics properties of the steady state equilibrium. These results will be useful to assess the qualitative relevance of the mechanisms that underlie how the model responds to exogenous shocks.

First, consider the effect of aggregate productivity p .

Proposition 5. $W^*(\Gamma_0, p, \lambda)$, $R_j^*(\Gamma_0, p, \lambda)$, and $E_i^*(\Gamma_0, p, \lambda)$ are increasing in p . Moreover, $\tilde{\theta}^*(\Gamma_0, p, \lambda)$ is increasing in p , implying that $u^*(\Gamma_0, p, \lambda)$ is decreasing in p .

Proposition 5 establishes that, in line with the prediction of the canonical model, wages and labour market tightness are increasing in aggregate productivity, while unemployment is decreasing in p .²³ In addition, it also highlights two distinct predictions with respect to incumbent workers' reference wages and new hires' effort. In particular, the higher steady-state wage implies that: new hires' effort will be higher, since for a given reference wage r_i^* , new hires' effort is increasing in the wage they are paid, which is increasing in p ; and, due to reference wage adaptation, incumbent workers' reference wages will also be higher.

Next, consider the effect of workers' loss aversion λ .

Proposition 6. $W^*(\Gamma_0, p, \lambda)$, $R_j^*(\Gamma_0, p, \lambda)$, are increasing in λ , while the effect of λ on $E_i^*(\Gamma_0, p, \lambda)$ is ambiguous. Moreover, $\tilde{\theta}^*(\Gamma_0, p, \lambda)$ is decreasing in λ , implying that $u^*(\Gamma_0, p, \lambda)$ is increasing in λ .

A higher coefficient of loss aversion implies that workers' negative reciprocity will be stronger, that is, whenever a worker is paid a wage below their reference wage, their optimal effort will be lower. However, the equilibrium outcome of a higher λ on the steady-state level of new hires' effort is ambiguous. This is because, in the anticipation of stronger negative reciprocity, firms' employing newly hired workers with $r_i^* \in (r_H, \bar{r}]$ will set a relatively higher wage in order to stimulate higher effort, partially offsetting the increase in negative reciprocity. Since effort is increasing in the wage but decreasing in the reference wage, whether new hires'

²³As explained by [Pissarides \(2000\)](#), this is not a desirable property of a model in long-run equilibrium, where wages should fully absorb productivity changes and there should exist a balanced-growth equilibrium with constant unemployment. In the canonical model, one way to make the unemployment rate independent of aggregate productivity is to assume that workers' "unemployment income" depends on their "permanent income" (see [Pissarides, 1987](#)). However since the comparative statics results in this section should be considered as approximations of the short-run dynamic adjustment of the model following a shock this issue shall not be addressed here.

effort increases or decreases with λ depends on which of these two aforementioned effect dominates. Note that the latter effect also implies a higher wage paid to incumbent workers for the entire duration of the employment relationship, who, however, will exert normal effort due to their reference wage adaptation. Despite these considerations it is possible to conclude that a higher degree of loss aversion—which essentially captures the anticipation of a greater cost of hiring ‘de-moralised’ workers—unambiguously reduces firms’ expected value of new employment relationships, resulting in fewer vacancies and higher unemployment.

Finally consider the effect of changes in the distribution of new hires’ reference wages Γ_0 .

Proposition 7. *Consider two initial distributions Γ_0 and Γ'_0 , where Γ'_0 is first-order stochastic dominant over Γ_0 . Then, $W^*(\Gamma'_0, p, \lambda) > W^*(\Gamma_0, p, \lambda)$, $R_j^*(\Gamma'_0, p, \lambda) > R_j^*(\Gamma_0, p, \lambda)$, and $E_i^*(\Gamma'_0, p, \lambda) < E_i^*(\Gamma_0, p, \lambda)$. Moreover, $\tilde{\theta}^*(\Gamma'_0, p, \lambda) < \tilde{\theta}^*(\Gamma_0, p, \lambda)$, implying that $u^*(\Gamma'_0, p, \lambda) > u^*(\Gamma_0, p, \lambda)$.*

Proposition 7 establishes that if the reference wage of new hires is expected to be higher, then firms will expect having to pay a higher wage to those workers for which is optimal to pay them their reference wage. This, in turn, raises the steady-state aggregate wage and, due to adaptation, the steady-state aggregate reference wage of incumbent workers. Moreover, a higher expected reference wage from new hires also implies that the expected level of effort, exerted by those new hires for which firms would optimally pay a wage above or below their reference wage, is lower. These considerations enable to conclude that the expectations by firms of employing new hires with, on average, relatively higher reference wages will decrease the expected present value of new employment relationships, reducing job creation and increasing unemployment.

This result emphasises the important role of newly hired workers’ wage entitlements in the market for the determination of unemployment in equilibrium.

3 Unemployment Volatility: New Hires’ Wage Cyclicalilty

This section implements the framework developed in Section 2 to derive and analyse the volatility of labour market tightness with respect to aggregate productivity. A particular emphasis will be placed on the role of new hires’ wage cyclicalilty for the volatility of job creation and unemployment. In so doing, this section contributes to the labour market literature that aims to explain the *amplitude* and co-movement of vacancies and unemployment fluctuations.

3.1 A Concise Discussion of the Relevant Literature

The analysis of the steady-state elasticity of labour market tightness with respect to productivity (referred to as the elasticity of market tightness henceforth) is commonly used as a good approximation of the volatility of vacancies and unemployment when the labour market is hit by exogenous shocks to aggregate productivity (Mortensen and Nagypál, 2007; Elsby et al., 2015). Moreover, as shown by Shimer (2005), this elasticity is particularly important for the assessment of the quantitative implications of the model: a greater elasticity of market tightness implies that job creation is more responsive to exogenous shocks in productivity.

In a highly influential paper Shimer (2005) calibrates a canonical search and matching model and shows that the model cannot quantitatively account for the high volatility of the vacancy-unemployment ratio observed in U.S. data over the period 1951–2003 (see Amaral and Tasci (2016) for a comparable exercise on a set of OECD countries). This *quantitative* failure has been labelled as “the unemployment volatility puzzle” (Pissarides, 2009). Shimer’s insight is that the wage response to productivity shocks predicted by the model is too large, i.e. the elasticity of the wage with respect to productivity is close to unity, offsetting almost all the effect of the productivity shock on job creation. Hence, introducing a degree of wage stickiness will improve the model’s explanatory power. Subsequent to Shimer (2005) the literature attempting to solve the puzzle has flourished, and two main streams of thought have been developed.

On one hand, following the suggestion of Shimer (2005) and starting with the contribution of Hall (2005b), a large body of literature has placed considerable emphasis on the role of the cyclicity of wages by proposing alternative wage determination mechanisms that can generate some form of wage rigidity, i.e. acyclicity, or wage stickiness, i.e. less than proportional cyclicity. For surveys of this literature see, for instance, Mortensen and Nagypál (2007) and Rogerson and Shimer (2011). Given the emphasis on the cyclical behaviour of job creation, these models have stressed the importance of rigidities in *newly hired* workers’ wages, which, as shown by Pissarides (2009), is the relevant wage affecting hiring decisions in the canonical model. However, the more recent empirical literature has challenged the theory underlying these models by providing evidence that wages offered to newly hired workers are instead substantially pro-cyclical (Martins et al., 2012; Carneiro et al., 2012; Haefke et al., 2013; Stüber, 2017; Schaefer and Singleton, 2017). Building on these findings, Kudlyak (2014) has shown that

it is not the hiring wage that is the relevant price of labour for firms, but rather, it is the user cost of labour, i.e. the opportunity cost of delaying hiring decisions. By providing estimates of this measure, and showing that it can be even more pro-cyclical than the hiring wage, [Kudlyak \(2014\)](#) concludes that wage rigidity is not relevant to address the unemployment volatility puzzle.

A different perspective in response to Shimer’s critique has been elucidated by [Mortensen and Nagypál \(2007\)](#), [Hagedorn and Manovskii \(2008\)](#) and [Pissarides \(2009\)](#) among others. These authors have argued that the literature has put too much emphasis on the role of newly hired workers’ wage cyclicality. Even if hiring wages were more sticky, for this to have a substantive effect on the size of the elasticity of market tightness, the present value of the wage would also need to be sufficiently high relative to the firm’s present value of output from a new match ([Elsby et al., 2015](#)). As such, what matters for job creation is the size of the present value of the *profit margin* from a new employment relationship ([Kennan, 2010](#)), i.e. the difference between the present values of output and the wage: only if this margin is small enough will slight changes in productivity generate large fluctuations in the anticipated profits from new matches, and hence in vacancy creation and unemployment. This perspective, which [Ljungqvist and Sargent \(2017\)](#) summarised under the concept of the “fundamental surplus”, downplays the role of new hires’ wage rigidity and shifts the focus to the size of the surplus generated by new employment relationships.

To understand the key insights of these arguments more clearly in the context of this paper, denote the present discounted value of wages and output derived in the present framework as follows:

$$\bar{W}^* \equiv \mathbb{E} \left[\sum_{t=s}^{\infty} \psi^{t-s} \tilde{w}^*(r_t^*, p, \lambda) \right] = \frac{W^*}{1 - \psi}; \quad (24)$$

$$\bar{Y}^*(E^*) \equiv \mathbb{E} \left[\sum_{t=s}^{\infty} \psi^{t-s} y(p, \tilde{e}^*(\tilde{w}^*, r_t^*, \lambda)) \right] = y(p, E_i^*) + \frac{\psi}{1 - \psi} y(p, \tilde{e}_n). \quad (25)$$

As such, the expected present value to firms of new employment relationships in the steady state can be expressed as $\mathbb{E} [J(r_i^*)] = \bar{Y}^*(E^*) - \bar{W}^*$. Next, denote the elasticity of any variable x with respect to productivity p as $\varepsilon_{x_p} = \frac{p}{x} \frac{dx}{dp}$, and consider the following proposition.

Proposition 8. *The elasticity of labour market tightness with respect to aggregate productivity takes*

the form:

$$\varepsilon_{\tilde{\theta}_p^*} = \frac{1}{\sigma} \frac{\bar{Y}^*(E^*) - \Lambda(\varepsilon_{\tilde{w}_p^*}, \bar{W}^*)}{\bar{Y}^*(E^*) - \bar{W}^*} \quad (26)$$

where Λ is a function of the elasticity of wages with respect to productivity $\varepsilon_{\tilde{w}_p^*}$, the present discounted value of wages \bar{W}^* , and other parameters of the model.

Proposition 8 derives an equation for the elasticity of labour market tightness that is directly comparable with the literature (see for instance [Pissarides \(2009\)](#), equation (20), p.1352; or [Elsby et al. \(2015\)](#), equation (11), p.590).²⁴ In the canonical model: $\bar{Y}^*(E^*) = p/(1 - \psi)$; $\bar{W}^* = w/(1 - \psi)$ (where w is the outcome of generalised Nash bargaining solution); and $\Lambda(\varepsilon_{\tilde{w}_p^*}, \bar{W}^*) = \varepsilon_{w_p} w/(1 - \psi)$. Hence, (26) can be expressed as

$$\varepsilon_{\theta_p} = \frac{1}{\sigma} \frac{p - \varepsilon_{w_p} w}{p - w},$$

implying that the size of the elasticity of market tightness hinges crucially on the size of the elasticity of wages ε_{w_p} . Indeed if $\varepsilon_{w_p} = 1$ wages are perfectly proportional to changes in productivity and the elasticity equation collapses to $\varepsilon_{\theta_p} = 1/\sigma$, implying that its size depends crucially on the elasticity of the matching function with respect to unemployment $\sigma \in (0, 1)$. As shown in the literature cited above, for values of $\sigma \in [0.235, 0.72]$ the model fails to generate the target elasticity of $\varepsilon_{\theta_p} = 7.56$ ²⁵ (see [Mortensen and Nagypál \(2007\)](#), [Pissarides \(2009\)](#) and [Kudlyak \(2014\)](#)). This numerical exercise reproduces, in essence, the analysis underlying the insight of [Shimer \(2005\)](#): by implementing a wage setting mechanism that yields an elasticity of wages with respect to productivity lower than unity $\varepsilon_{w_p} < 1$, i.e. by introducing some sort of wage rigidity $\varepsilon_{w_p} = 0$, or stickiness $\varepsilon_{w_p} \in (0, 1)$, the size of ε_{θ_p} will increase, improving the explanatory power of the model.

Ignoring the empirical estimates of ε_{w_p} for the time being, consider the solution proposed by [Hall \(2005b\)](#), in which wages are entirely acyclical, i.e. $\varepsilon_{w_p} = 0$. In such a case the canonical version of equation (26) can be rewritten as

$$\varepsilon_{\theta_p} = \frac{1}{\sigma} \frac{p}{p - w}.$$

It is clear from this expression that, even if wages are entirely rigid, the size of the elasticity

²⁴The discussion that follows draws on [Pissarides \(2009\)](#) and [Elsby et al. \(2015\)](#).

²⁵This figure corresponds to the regression coefficient in a simple regression with labour market tightness as the dependent variable and productivity as the independent variable.

of market tightness depends crucially on the firms' profit margin—the fundamental surplus—from new employment relationships. As pointed out in the previous discussion of the literature, the higher the wage relative to the value of output from a new match the lower the profit margin and therefore the greater the size of the elasticity of market tightness (Elsby et al., 2015).

To conclude this brief *excursus* around the determinants of the size of ε_{θ_p} , notice that the empirical literature has estimated the cyclicalities of hiring wages to be around 1.²⁶ This finding supports the aforementioned conclusions reached by Kudlyak (2014), that the volatility of the hiring wage is not useful to explain the high volatility of vacancies and unemployment observed in the data; and that the canonical model cannot simultaneously accommodate the empirical volatilities of the wage component of the user cost of labour and of the vacancy-unemployment ratio.

How does the framework developed in this paper contribute to the arguments highlighted above? By analysing the qualitative properties of the model with an emphasis on the role of new hires' wage *and* effort cyclicalities, it will be shown that the behavioural mechanisms considered in this paper can provide a novel perspective on the channels through which vacancies and unemployment fluctuations can be amplified.

3.2 Reciprocity and the Irrelevance of New Hires' Wage Cyclicalities

In order to assess the role of new hires' wage cyclicalities for the size of the elasticity of labour market tightness in the model set out in this paper, it is necessary to characterise the function Λ . As derived in detail in the Proof of Proposition 8, $\Lambda(\varepsilon_{\tilde{w}_p^*}, \bar{W}^*)$ takes the following form:

$$\begin{aligned} \Lambda(\varepsilon_{\tilde{w}_p^*}, \bar{W}^*) \equiv & - \left(\int_0^{r_L(p)} \underbrace{\left[y_e \tilde{e}_w^*(\tilde{w}^{*+}, r_i^*)^+ \cdot \varepsilon_{\tilde{w}_p^*} \tilde{w}^{*+} \right]}_{\Delta e \geq 0 \text{ effort response}} - \underbrace{\frac{\varepsilon_{\tilde{w}_p^*} \tilde{w}^{*+}}{1 - \psi}}_{\Delta w \geq 0 \text{ wage adjustment}} d\Gamma_0(r_i) \right. \\ & - \int_{r_L(p)}^{r_H(p, \lambda)} \underbrace{\frac{\varepsilon_{\tilde{w}_p^*} \tilde{w}^{*-}}{1 - \psi}}_{\Delta w = 0 \text{ wage rigidity}} d\Gamma_0(r_i) \\ & \left. + \int_{r_H(p, \lambda)}^{\bar{r}} \underbrace{\left[y_e \tilde{e}_w^*(\tilde{w}^{*-}, r_i^*, \lambda)^- \cdot \varepsilon_{\tilde{w}_p^*} \tilde{w}^{*-} \right]}_{\Delta e \geq 0 \text{ effort response}} - \underbrace{\frac{\varepsilon_{\tilde{w}_p^*} \tilde{w}^{*-}}{1 - \psi}}_{\Delta w \geq 0 \text{ wage adjustment}} d\Gamma_0(r_i) \right). \quad (27) \end{aligned}$$

²⁶For instance, Carneiro et al. (2012) provide an estimate of $\varepsilon_{w_p} = 1.07$; Haefke et al. (2013) an estimate of $\varepsilon_{w_p} = 0.8$; and Schaefer and Singleton (2017) an estimate of $\varepsilon_{w_p} \in [0.82, 0.88]$ for the UK; while using the estimates of Pissarides (2009), Kudlyak (2014) computes a combined elasticity (of the user cost of labour) of $\varepsilon_{w_p} = 1.5$.

First notice that $\Lambda(\varepsilon_{\tilde{w}_p^*}, \bar{W}^*)$ is a function of the elasticity of the wage of newly hired workers $\varepsilon_{\tilde{w}_p^*}$ for any given initial reference wage $r_i \in [0, \bar{r}]$ and, in contrast with other expressions advanced in the literature discussed above, is also a function of the *marginal change in firms' output* induced by a change in new hires' effort, which is triggered by the change in the wage for any given $r_i \in [0, \bar{r}] \setminus [r_L, r_H]$ (see the terms in square brackets).

The interpretation of (27) is as follows. Whenever the labour market is hit by an exogenous shock to productivity p , firms will expect to be in one of two main situations at the start of the employment relationship. On one hand, firms anticipate that if they are matched with workers with moderate reference wages $r_i \in [r_L, r_H]$ they will optimally set a wage equal to their reference wage, which is independent of aggregate productivity. That is, firms anticipate that there is a positive probability of not adjusting the wage for these workers, to avoid the adverse effects of negative reciprocity. As such, $\varepsilon_{\tilde{w}_p^*} = 0$ for all $r_i \in [r_L, r_H]$.²⁷ On the other hand, firms will also anticipate being in a situation in which if they are matched with workers with relatively low $r_i \in [0, r_L)$ or relatively high $r_i \in (r_H, \bar{r}]$ reference wages, then they will adjust the wage to the change in p , as implied by their optimal wage policy (9). However, since as implied by (6) workers' effort optimally responds to wage changes above and below the reference wage, firms also anticipate that any change in the wage of these workers will trigger a change in their effort response, in the form of positive and negative reciprocity. These two marginal effects are in fact optimally balanced by the wage paid to new hires, for all $r_i \in [0, \bar{r}] \setminus [r_L, r_H]$. That is:

$$\begin{aligned} & \left[y_e \tilde{e}_w^*(\tilde{w}^{*+}, r_i^*)^+ \cdot \varepsilon_{\tilde{w}_p^*} \tilde{w}^{*+} \right] - \varepsilon_{\tilde{w}_p^*} \frac{\tilde{w}^{*+}}{1 - \psi} = 0 \quad \forall r_i \in [0, r_L); \text{ and} \\ & \left[y_e \tilde{e}_w^*(\tilde{w}^{*-}, r_i^*, \lambda)^- \cdot \varepsilon_{\tilde{w}_p^*} \tilde{w}^{*-} \right] - \varepsilon_{\tilde{w}_p^*} \frac{\tilde{w}^{*-}}{1 - \psi} = 0 \quad \forall r_i \in (r_H, \bar{r}], \end{aligned}$$

by virtue of the first-order condition (8), characterising the optimal wage setting policy of firms.²⁸ As such, the first and last term in (27) are zero, independently of the size of the elas-

²⁷This particular case reproduces the result generated by the model based on “norms” proposed, and analysed, by Hall (2005b). Moreover, the wage setting model implemented here provides a micro-founded rationale—based on fairness and loss aversion—that endogenously generates rigidity in the wage of newly hired workers for a range of initial reference wages.

²⁸Consider the case $r_i \in [0, r_L)$. Collecting the term $\varepsilon_{\tilde{w}_p^*} \frac{\tilde{w}^{*+}}{1 - \psi}$ as the common factor yields:

$$\varepsilon_{\tilde{w}_p^*} \frac{\tilde{w}^{*+}}{1 - \psi} \{ [1 - \psi] y_e \tilde{e}_w^* - 1 \},$$

in which, as established in the Proof of Proposition 1 (see equation (48)), the term in curly brackets is equivalent to the first-order condition (8) characterising the optimal wage, and is therefore equal to zero. The same argument

ticity of newly hired workers' wages $\varepsilon_{\bar{w}_p^{*+,-}}$.

These considerations underlie the statement of the proposition capturing the first main result of this paper:

Proposition 9. (*Irrelevance Proposition*). *Since $\Lambda(\varepsilon_{\bar{w}_p^*}, \bar{W}^*) = 0$ for any given $\int_0^{\bar{r}} \varepsilon_{\bar{w}_p^*} d\Gamma_0(r_i)$, the elasticity of labour market tightness is:*

$$\varepsilon_{\bar{\theta}_p^*} = \frac{1}{\sigma} \frac{\bar{Y}^*(E^*)}{\bar{Y}^*(E^*) - \bar{W}^*}; \quad (28)$$

which implies that the elasticity of the hiring wage with respect to productivity $\varepsilon_{\bar{w}_p^}$ is irrelevant for the size of the elasticity of labour market tightness $\varepsilon_{\bar{\theta}_p^*}$.*

In contrast with a large body of the existing literature, Proposition 9 establishes that the cyclicity of the hiring wage with respect to productivity is irrelevant for the determination of the size of the elasticity of market tightness. Note that this conclusion holds for any value of the elasticity of the hiring wage, hence irrespectively of the specific value corresponding to its empirical estimate.

For any given change in aggregate productivity firms anticipate either keeping the wage constant at the workers' reference wage, or setting a wage that will optimally balance the intertemporal trade-off between the marginal cost of a higher wage, and the marginal benefit, on output y , generated by the workers' reciprocity (to satisfy the first-order condition (8)). As such, for all $r_i \in [0, \bar{r}] \setminus [r_L, r_H]$ it does not matter how responsive new hires' wages are to productivity, since firms optimally exploit this response by inducing a counteracting response in workers' effort, which positively (or negatively) affects output and leaves room for the impact of the change in productivity to be reflected in the firms' present value of output. Using a terminology familiar with the literature discussed above (e.g. [Haefke et al. \(2013\)](#)), when aggregate productivity increases, firms anticipate being able to turn the additional surplus received by workers, in the form of a higher wage, into an additional surplus that they receive, in the form of higher effort exerted by newly hired workers, which increases their output y . This is the reason why, in the model developed in this paper, the elasticity of the hiring wage with respect to productivity is irrelevant for job creation; and changes in aggregate productivity are fully absorbed by firms' present value of profit.

Note that this conclusion remains qualitatively valid even if reciprocity were symmetric,

applies for the case $r_i \in (r_H, \bar{r}]$.

i.e. if $\lambda = 1$. In this case:

$$\Lambda(\varepsilon_{\tilde{w}_p^*}, \bar{W}^*) = - \left(\int_0^{\bar{r}} \left[y e^{\delta_w^*}(\tilde{w}^*, r_i^*) \cdot \varepsilon_{\tilde{w}_p^*} \tilde{w}^* \right] - \varepsilon_{\tilde{w}_p^*} \frac{\tilde{w}^*}{1 - \psi} d\Gamma_0(r_i) \right),$$

which again is equal to zero for all $r_i \in [0, \bar{r}]$ by virtue of the first-order condition (8). Moreover, at this stage, it is also clear why in the canonical model $\Lambda(\varepsilon_{\tilde{w}_p^*}, \bar{W}^*) = \varepsilon_{\tilde{w}_p^*} \tilde{w}^*/(1 - \psi)$: by neglecting the impact of wage changes on workers' effort (captured by the term in square brackets), the model only considers the marginal cost of a higher wage on firms' profits.

This qualitative result is particularly important for two reasons. First it reinforces the argument summarised in [Elsby et al. \(2015\)](#), that besides the extent of cyclicity of new hires' wages, it is the anticipated present value of the firms' profit margin that matters for the size of the elasticity of labour market tightness. Second, in contrast with a large body of literature that started from the influential contribution of [Shimer \(2005\)](#), it shows that, in the presence of a positive wage-effort relationship in the production function, the extent of new hires' wage cyclicity is irrelevant for the volatility of vacancies and unemployment.

Finally notice that the framework developed here falls into the class of models in which the wage component of the user cost of labour—defined by [Kudlyak \(2014\)](#) as the difference between the expected present value of wages paid to a worker hired in t and the one paid to a worker hired in $t + 1$ —is equal to the wage. As such, this framework will be consistent with any empirical estimate of the cyclicity of the relevant price for labour at the time of hiring, and will be able to generate outcomes that are quantitatively consistent with plausible empirical estimates of the volatilities of both the hiring wage and the vacancy-unemployment ratio. To reinforce these conclusions, a calibration of the model consistent with an elasticity of labour market tightness of 7.56 and an elasticity of the hiring wage of 0.8 ([Haefke et al., 2013](#)) is performed in Section [A.1](#), Appendix [A](#).

4 Unemployment Volatility: Incumbents' Downward Wage Rigidity

The analysis of the previous section has established that the cyclicity of newly hired workers' wages is irrelevant for the volatility of job creation. However, if the wage of incumbent workers is expected to be rigid, what is the role of this expectation for firms' hiring decisions?

In his influential paper [Pissarides \(2009\)](#) addressed this question by showing that even if the wage of incumbent workers were entirely rigid, firms will be able to internalise these future

rigidities in the equilibrium wage negotiated with their new hires at the start of the employment relationship, leaving the volatility of job creation unaffected by the expected rigidity of wages in subsequent employment periods. This theoretical result is general and holds true also under various modifications of the canonical model put forward to address the unemployment volatility puzzle (which, as discussed, have focused on the cyclicity of newly hired workers' wages).

This conclusion has been recently re-evaluated from different perspectives by two prominent studies. Within a framework based on reference dependence, incomplete contracts and fairness, [Eliaz and Spiegel \(2014\)](#) challenge Pissarides' theoretical result with the following *qualitative* insight. In a model where there is uncertainty around the evolution of productivity and wage rigidity of incumbent workers generates ex-post inefficient layoffs, the latter can negatively affect the expected present value of a new employment relationship by reducing its expected duration (essentially working as an additional discount factor). As such, [Eliaz and Spiegel \(2014\)](#) conclude that expected wage rigidity of incumbent workers can negatively influence the volatility of job creation.²⁹ However, [Eliaz and Spiegel \(2014\)](#) are unable to determine whether the extent to which the labour contract is incomplete—which is their relevant source of downward wage rigidity—unambiguously increases the volatility of vacancies and unemployment.

In a different vein, [Bils et al. \(2016\)](#) study a search and matching model with large firms, in which existing workers' wages are rigid (due to a staggered Nash bargaining mechanism), and their effort is observable and contractible (i.e. the employment contract is complete). This framework is used to show *quantitatively* that existing workers' rigidity can negatively affect firms' hiring decisions. According to their model, whenever there is a negative productivity shock and the wage of existing workers does not fall, by the completeness of the labour contract firms would require existing workers to be more productive. This in turn would lower the marginal value of hiring a new worker, reducing job creation and raising unemployment.

The contribution made in this section is to provide an alternative and complementary perspective to the insights advanced by [Eliaz and Spiegel \(2014\)](#) and [Bils et al. \(2016\)](#). To do so the

²⁹The prediction that expected wage rigidity dampens hiring incentives by reducing the expected duration of a match does not appear to be supported by the available evidence (e.g. [Hall \(2005a\)](#) and [Shimer \(2012\)](#)): the observed stability of the unemployment inflow rate during the more recent recessions downplays the role of job duration in determining hiring decisions (see Hall's comments to [Eliaz and Spiegel \(2014\)](#) in the same volume). Moreover, as shown by Moscarini, the volatility of the vacancy-unemployment ratio in the model of [Eliaz and Spiegel \(2014\)](#) is maximised when reference-dependence and wage rigidity do not play any role (see comments from Moscarini in the same volume).

framework developed in this paper is extended by introducing uncertainty around the evolution of an employed worker’s match productivity throughout the employment relationship. In this context it is shown that if the employment contract is incomplete (in contrast to [Bils et al. \(2016\)](#)), and even if incumbent workers’ wage rigidity does not generate endogenous layoffs (as it does in [Eliaz and Spiegel \(2014\)](#)), firms’ expectations of the relatively large cost of implementing wage cuts in the event of a low realisation of future match productivity—that is, the anticipation of stronger negative reciprocity by incumbent workers—can negatively influence the expected present value of new employment relationships, dampening hiring incentives and increasing the volatility of job creation and unemployment.

4.1 Introducing Idiosyncratic Uncertainty

For the purpose of the analysis in this section the model is extended by including a time-variant, idiosyncratic match productivity q_t characterising worker-firm matches. In particular, $q \in \mathcal{Q}$ enters firms’ output function equivalently to the aggregate productivity p as stated in Assumption F1. As such, $y : \mathcal{P} \times \mathcal{Q} \times \mathcal{E} \rightarrow \mathbb{R}_+$ is strictly increasing and linear on $\mathcal{Q} \times \mathcal{E}$, with $y_{eq} > 0$.

U1. The idiosyncratic match productivity q_t of employed workers $\{i, j\}$ evolves according to the following stochastic process:

$$q_{jt+1} = g(q_{it}, \xi), \quad q_{it} \text{ given}; \quad (29)$$

where $\xi \in \mathcal{Z} \subset \mathbb{R}$ is a shock with cumulative distribution function G , and $g : \mathcal{Q} \times \mathcal{Z} \rightarrow \mathcal{Q}$ is a given function.

Assumption U1 implies that employed workers’ match productivity changes stochastically after the first employment period and remain constant for the entire duration of the employment relationship. The dynamics of this process is implied by g and G , and can be represented by a transition probability function Q on \mathcal{Q} , denoted in this particular case by $Q(q_i, dq_j)$. Hence, in contrast with the analysis of Section 3, while new hires’ match productivity is exogenously given at the start of the employment relationship, the match productivity of incumbents is now allowed to change stochastically. The purpose of this assumption is to introduce a form of uncertainty faced by firms at the time of hiring, whom might have to re-adjust wages in the future in the event of exogenous unanticipated changes in per-worker output. The choice to

model uncertainty as a two-period stochastic process is motivated mainly for tractability, and to allow analytical characterisation of the model's steady state and subsequent analysis. Another way to achieve this would have been to impose a two-period employment relationship (as, for instance, in [Dickson and Fongoni \(2016\)](#) or [Eliaz and Spiegler \(2014\)](#)).

U2. The shock ξ follows a log-normal distribution so that $\ln \xi \sim \mathcal{N}(0, \nu^2)$; and workers' normal effort is sufficiently high: $\tilde{e}_n \geq \underline{e}_n(\xi)$, where $\underline{e}_n(\xi) \equiv \max\{\tilde{e}_n : J(r_j, q_j) = 0\}$.

Assumption U2 is instrumental in eliminating the possibility of endogenous job destruction once q_j is realised. In particular, the assumption on the distribution of ξ ensures that q_j is non-negative; while the assumption on \tilde{e}_n ensures that workers' normal effort—which is parametric—is such that the value of a job filled by an incumbent worker is always profitable to the firm, independently of the realisation of q_j . Hence, incumbent workers are never endogenously laid off and additions into unemployment remain determined by the exogenous job destruction rate ρ .

U3. New hires' reference wage is set to zero: $r_{it} = 0$.

Assumption U3 implies that newly hired workers are characterised by 'relatively low' reference wages at the start of the employment relationship.³⁰ The main reason for imposing this assumption is to ensure that firms' optimal wage in the first employment period is perceived as a gift, which implies that new hires always exert positive reciprocity. As a consequence, new hires' wages will be entirely procyclical (since $r_i \in [0, r_L)$ for all r_i). This enables to isolate the analysis of the impact of negative reciprocity (and hence downward wage rigidity) on the expected continuation value of new employment relationships with prospective incumbent workers.

4.2 The Firms' Problem and Wage Setting Behaviour

Given Assumptions U1-U3, the firms' wage setting problem at the start of the employment relationship $t = s$ can be thought as being divided into two separated optimisation problems. Since an incumbent worker's match productivity remains constant after it is revealed, a firm can: i) derive the optimal wage policy for any possible realisation of q_j , taking as given r_j for all $t > s$; then ii) use the obtained 'state-contingent' optimal wage policy to calculate the

³⁰This is analogous to the assumption of "modest aspirations" imposed by [Eliaz and Spiegler \(2014\)](#).

continuation value of an employment relationship with an incumbent worker $J(r_j, q_j)$ for any possible realisation of q_j , the expectation of which is crucial for the characterisation of the optimal wage policy in $t = s$. This two-step structure can be expressed recursively as:

$$J(r_i, q_i) = \max_{w \in \mathcal{W}} \left\{ \pi(r_i, w, q_i) + \psi \int J(w, q_j) Q(q_i, dq_j) \right\}, \quad (30)$$

$$\text{where } J(r_j, q_j) = \max_{w \in \mathcal{W}} \left\{ \pi(r_j, w, q_j) + \psi J(w, q_j) \right\}, \quad (31)$$

Notice that since q_j is constant from $t > s$ onwards, the functional equation (31), corresponding to the wage setting problem faced by a firm employing an incumbent worker, is analogous to the functional equation (7), as analysed in the previous section. The following proposition establishes the solution to (30-31).

Proposition 10. *The optimal wage policy of firms employing workers $\{i, j\}$ which solves the wage setting problem (30-31) is characterised as follows.*

- a) *The optimal wage policy of firms employing incumbent workers j characterised by asymmetric reference-dependent reciprocity $\tilde{e}(w_t, r_{jt}, \lambda)$ with $\lambda > 1$ and adaptation $r_{jt+1} = w_t$ is given by*

$$\tilde{w}_{jt} = \tilde{w}(r_{jt}, p, q_{jt}, \lambda) = \begin{cases} \tilde{w}(p, q_{jt})^+ & \text{if } q_{jt} > q_u(r_{jt}) \\ r_{jt} & \text{if } q_{jt} \in [q_l, q_u] \\ \tilde{w}(p, q_{jt}, \lambda)^- & \text{if } q_{jt} < q_l(r_{jt}, \lambda); \end{cases} \quad (32)$$

where

$$q_u(r_{jt}) \equiv \{q_{jt} : \lim_{\epsilon \rightarrow 0} \pi_w(r_{jt}, r_{jt} + \epsilon, q_{jt}) + \psi J_r(r_{jt} + \epsilon, q_{jt}) = 0\};$$

$$q_l(r_{jt}, \lambda) \equiv \{q_{jt} : \lim_{\epsilon \rightarrow 0} \pi_w(r_{jt}, r_{jt} - \epsilon, q_{jt}) + \psi J_r(r_{jt} - \epsilon, q_{jt}) = 0\}.$$

- b) *The optimal wage policy of firms employing newly hired workers i characterised by asymmetric reference-dependent reciprocity $\tilde{e}(w_t, r_{it}, \lambda)$ with $\lambda > 1$ and $r_{it} = 0$ is given by*

$$\tilde{w}_{it} = \tilde{w}(r_{it}, p, q_{it}, \lambda) = \tilde{w}(p, q_{it}, \lambda)^+ > r_{it}. \quad (33)$$

Part a) of Proposition 10 implies that depending on the realisation of q_j , and for any given

reference wage r_j , once workers become incumbent and their match productivity is revealed, firms' may optimally implement a wage rise, if q_j is sufficiently high ($> q_u$), a wage cut, if q_j is too low ($< q_l$), or a wage freeze, for intermediate values of q_j ($\in [q_l, q_u]$). This characterises the solution to the first step (31). In contrast with the deterministic environment analysed in Section 2, the stochastic change in match productivity and the resulting (potential) wage re-negotiation are such that incumbent workers may exert positive, negative reciprocity or normal effort in the second employment period. These considerations influence the expected continuation value of an employment relationship in $t = s$, and therefore also affect the optimal wage paid to newly hired workers. Although an explicit solution is not provided, as established in part b) of Proposition 10, it can be deduced that the optimal wage paid to newly hired workers is always perceived as a gift and will adjust smoothly to changes in p . This characterises the solution to the second step (30).

4.3 Steady State Characterisation

The results established in the previous sections enable to characterise the steady-state equilibrium of the model under the additional assumptions U1-U3.

In the steady state, new hires are paid the same wage

$$W_i^* \equiv \tilde{w}_i^* \equiv \tilde{w}^*(p, q_i, Q, \lambda)^+ > r_i^*$$

which, as it has been established, is perceived as a gift. Therefore they exert positive reciprocity

$$E_i^* \equiv \tilde{e}_i^* \equiv \tilde{e}^*(\tilde{w}_i^*, r_i^*)^+ > \tilde{e}_n$$

and produce output given by $y(p, q_i, \tilde{e}^*(\tilde{w}_i^*, r_i^*)^+)$. On the other hand, by the same logic explained above, incumbent workers are paid the aggregate steady-state wage

$$W_j^* \equiv W^*(r_j^*, p, Q, \lambda) = \int^{q_l(r_j^*, \lambda)} \tilde{w}^*(p, q_j, \lambda)^- Q(q_i, dq_j) + \int_{q_l(r_j^*, \lambda)}^{q_u(r_j^*)} r_j^* Q(q_i, dq_j) + \int_{q_u(r_j^*)} \tilde{w}^*(p, q_j)^+ Q(q_i, dq_j); \quad (34)$$

and will therefore exert aggregate steady-state effort as given by

$$E_j^* \equiv E^*(r_j^*, p, Q, \lambda) = \int^{q_l(r_j^*, \lambda)} e^*(\bar{w}^*(p, q_j, \lambda)^-, r_j^*, \lambda)^- Q(q_i, dq_j) + \int_{q_l(r_j^*, \lambda)}^{q_u(r_j^*)} \bar{e}_n^* Q(q_i, dq_j) + \int_{q_u(r_j^*)} e^*(\bar{w}^*(p, q_j)^+, r_j^*)^+ Q(q_i, dq_j); \quad (35)$$

and produce aggregate output $\int y(p, q_j, \bar{e}^*(\bar{w}_j^*, r_j^*, \lambda)) Q(q_i, dq_j)$.

The properties of the steady-state equilibrium under a two-period stochastic process for q_t and relatively low wage entitlements by newly hired workers can be summarised as follows. Newly hired workers are homogenous with respect to their match productivity q_i and reference wage r_i . As such, they are paid the same steady-state wage, perceived as a gift, and will exert supra-normal effort in their first employment period. Moreover, in the event of an aggregate shock to productivity p , the results established in Proposition 2 enables to deduce that new hires' wages will be procyclical. On the other hand, incumbent workers are heterogenous with respect to their match productivity q_j , and their reference wage is determined endogenously by the wage they were paid in their initial employment period. As such, as shown by (34) and (35), a fraction of incumbent workers is paid a wage below their reference wage and exert negative reciprocity; a fraction of incumbent workers is paid a wage gift and exert positive reciprocity; and a fraction of incumbent workers are paid the fair wage and therefore exert normal effort. Importantly, in contrast to new hires, there is a fraction of firms employing incumbent workers that will optimally freeze their wage: incumbent workers may experience downward wage rigidity.

Note that these results on wage dynamics are endogenously generated by the model, and stem from the firms' anticipation of their workers' asymmetric reference-dependent reciprocity. As such, despite the focus of the analysis that follows, the theoretical framework developed in this paper is consistent with procyclical hiring wages and a certain degree of downward wage rigidity in the wage of existing jobs, due to firms' optimally anticipating the adverse effects of wage cuts on incumbent workers' morale and reciprocity.

To proceed with the steady-state characterisation, define the steady-state present discounted

values of wages and output from new employment relationships as:

$$\begin{aligned}\bar{W}^*(\lambda) &\equiv \mathbb{E} \left[\sum_{t=s}^{\infty} \psi^{t-s} \bar{w}^*(r_t^*, p, q_t, \lambda) \right] \\ &\equiv \bar{w}^*(p, q_i, Q, \lambda)^+ + \frac{\psi}{1-\psi} \int \bar{w}^*(r_j^*, p, q_j, \lambda) Q(q_i, dq_j)\end{aligned}\quad (36)$$

$$\begin{aligned}\bar{Y}^*(\lambda) &\equiv \mathbb{E} \left[\sum_{t=s}^{\infty} \psi^{t-s} y(p, q_t, \tilde{e}^*(\tilde{w}_t^*, r_t^*, \lambda)) \right] \\ &\equiv y(p, q_i, \tilde{e}^*(\tilde{w}_i^*, r_i^*)^+) + \psi \int y(p, q_j, \tilde{e}^*(\tilde{w}_j^*, r_j^*, \lambda)) + \frac{\psi}{1-\psi} y(p, q_j, \tilde{e}_n^*) Q(q_i, dq_j)\end{aligned}\quad (37)$$

Hence, the expected present value to firms of new employment relationships—that is, the profit margin, or the fundamental surplus—can be expressed as $\mathbb{E} [J(r_i^*, q_i)] = \bar{Y}^*(\lambda) - \bar{W}^*(\lambda)$. This can then be used to determine the unique steady-state equilibrium degree of labour market tightness $\tilde{\theta}^*(\lambda) \equiv \tilde{\theta}^*(p, q_i, Q, \lambda)$ and unemployment rate $u^*(\lambda) \equiv u^*(p, q_i, Q, \lambda)$, which are characterised by the following equilibrium conditions:

$$\frac{\kappa}{h(\tilde{\theta}^*(\lambda))} = \delta [\bar{Y}^*(\lambda) - \bar{W}^*(\lambda)] \quad \text{and} \quad \rho[1 - u^*(\lambda)] = f(\tilde{\theta}^*(\lambda))u^*(\lambda).$$

The remainder of the analysis builds on these results to qualitatively address the following question: what is the effect of expected wage rigidity and anticipated negative reciprocity of incumbent workers on the cyclical behaviour of job creation and on the volatility of vacancies and unemployment?

4.4 Incumbents' Downward Wage Rigidity and the Volatility of Job Creation

To proceed with the analysis, consider first the following proposition, which derives an expression for the elasticity of labour market tightness with respect to productivity characterising the model of this section.

Proposition 11. (*Irrelevance Proposition under Uncertainty*). *The elasticity of labour market tightness with respect to aggregate productivity takes the form:*

$$\varepsilon_{\tilde{\theta}^*} = \frac{1}{\sigma} \frac{\bar{Y}^*(\lambda)}{\bar{Y}^*(\lambda) - \bar{W}^*(\lambda)}.\quad (38)$$

As (38) shows, even in this version of the model the cyclicality of the hiring wage is irrele-

vant for the size of $\varepsilon_{\tilde{\theta}_p^*}$.

Next notice that in the framework developed in this section (in particular due to assumption U3), the loss aversion coefficient λ captures the extent of incumbent workers' negative reciprocity only. Therefore $\lambda > 1$ will imply downward rigidity in the wage of incumbent workers in the second period of employment, once new hires have become incumbents. As such, a comparative statics exercise with respect to λ will enable to qualitatively identify the effect of incumbent workers' downward wage rigidity—expected by firms at the time of hiring—on the elasticity of labour market tightness, i.e. on the volatility of job creation.

Total differentiation of the elasticity of labour market tightness (38) with respect to λ yields

$$\frac{d\varepsilon_{\tilde{\theta}_p^*}}{d\lambda} = \frac{1}{\sigma} \frac{1}{\left[1 - \left[\bar{W}^*(\lambda)/\bar{Y}^*(\lambda)\right]\right]^2 \left[\bar{Y}^*(\lambda)\right]^2} \cdot \left[\frac{d\bar{W}^*(\lambda)}{d\lambda} \bar{Y}^*(\lambda) - \frac{d\bar{Y}^*(\lambda)}{d\lambda} \bar{W}^*(\lambda) \right].$$

The sign of this expression crucially depends on the sign of the second term in square brackets. Further investigation of this term yields the following (details of the derivation of this expression are in the Proof of Proposition 13):

$$\begin{aligned} & \left[\frac{d\bar{W}^*(\lambda)}{d\lambda} \bar{Y}^*(\lambda) - \frac{d\bar{Y}^*(\lambda)}{d\lambda} \bar{W}^*(\lambda) \right] = \\ & \underbrace{\tilde{w}_{i,\lambda}^+ \left[1 + \frac{\psi}{1-\psi} \int_{q_l(r_j^*,\lambda)}^{q_u(r_j^*)} Q(q_i, dq_j) \right]}_{\substack{A < 0 \\ \text{active wage compression}}} \left[\bar{Y}^*(\lambda) - \bar{W}^*(\lambda) \right] \\ & + \underbrace{\frac{\psi}{1-\psi} \int_{q_l(r_j^*,\lambda)}^{q_l(r_j^*,\lambda)} \tilde{w}_{j,\lambda}^- Q(q_i, dq_j) \left[\bar{Y}^*(\lambda) - (1-\psi) \int_{q_l(r_j^*,\lambda)}^{q_l(r_j^*,\lambda)} y_{j,e} \tilde{e}_{j,w}^- Q(q_i, dq_j) \bar{W}^*(\lambda) \right]}_{\substack{B > 0 \\ \text{more muted wage cuts}}} \\ & \underbrace{- \psi \int_{q_l(r_j^*,\lambda)}^{q_l(r_j^*,\lambda)} y_{j,e} \tilde{e}_{j,\lambda}^- Q(q_i, dq_j) \bar{W}^*(\lambda)}_{\substack{C > 0 \\ \text{incumbents' negative reciprocity}}}. \quad (39) \end{aligned}$$

Elaborated as it seems, equation (39) enables to identify precisely the effects of λ on the firms' fundamental surplus at the time of hiring, which, as discussed, is the key component determining the size of $\varepsilon_{\tilde{\theta}_p^*}$.

In particular, there are three major effects at play, labelled here as A, B and C . Before analysing them in detail, consider the following proposition.

Proposition 12. *The steady-state wage paid to newly hired workers $\tilde{w}^*(p, q_i, Q, \lambda)^+$ is decreasing in*

λ ; while the steady-state wage paid to incumbent workers $W^*(r_j^*, p, Q, \lambda)$ is increasing in λ for all $q_j < q_l(r_j^*, \lambda)$ given r_j^* .

The first part of Proposition 12 establishes that the expectation by firms of downward wage rigidity in the second employment period puts downward pressure on the wage paid to newly hired workers (this is captured by the term $\tilde{w}_{i,\lambda}^+ < 0$ in A). This effect captures the “active wage compression” of firms in the presence of downward wage rigidity, which has been analysed in the literature by [Elsby \(2009\)](#), [Benigno and Ricci \(2011\)](#) and [Dickson and Fongoni \(2016\)](#). This result also echoes the argument emphasised by [Pissarides \(2009\)](#), according to which forward looking firms may be able to internalise any potential future negative effect of wage rigidity into the initial wage contract (see the discussion around equation (19), [Pissarides \(2009, p. 1350\)](#))).

The second part of Proposition 12 establishes that the expectation by firms of having to enact a costly wage cut in the event of a low realisation of q_j puts upward pressure on the resulting optimal wage paid to incumbent workers. This result, that wage cuts are “more muted”, derives from firms optimally offsetting their incumbent workers’ negative reciprocity (this is captured by the term $\int^{q_l(r_j^*, \lambda)} \tilde{w}_{j,\lambda}^- Q(q_i, dq_j) > 0$ in B).

Forward-looking firms will anticipate the effects of these considerations on the expected present value of the employment relationships, determining their hiring decisions. As such, the expectation by firms of downward rigidity in the wage of incumbent workers generate the following effects. $A < 0$ captures firms’ anticipation of wage compression in the wage of new hires, which also reduces their employed workers’ wage entitlements once they will become incumbents (as a consequence of reference wage adaptation); this effect partially increases firms’ expected value of a match, since firms expect to pay lower hiring wages. $B > 0$ captures firms’ anticipation of having to set a higher wage in the second period of employment, in order to partially offset incumbent workers’ negative reciprocity response in the event of a wage cut; and $C > 0$ captures firms’ anticipation of an additional decrease in output, due to stronger negative reciprocity by incumbent workers, in the event of a wage cut in the second period of employment (which is captured by the term $\int^{q_l(r_j^*, \lambda)} y_{j,e} \tilde{e}_{j,\lambda}^- Q(q_i, dq_j) > 0$ in C). These two latter effects partially decrease the expected value of a new employment relationship to firms, since they expect to pay higher wages and to endure stronger negative reciprocity in the event of a future wage cut. Notice that this latter channel is absent in [Pissarides \(2009\)](#) and in any other search and matching model developed to date, with the exception of [Eliaz and](#)

Spiegler (2014). However, Eliaz and Spiegler (2014) focus on the incompleteness of the labour contract, and since they implement a reduced-form production function, they are unable to clearly identify, and unambiguously derive conclusions on, the behavioural mechanisms underlying the effect of downward rigidity on the size of $\varepsilon_{\tilde{\theta}_p^*}$ (see Eliaz and Spiegler (2014, p. 174)); As displayed in equation (39), the present analysis enables a transparent discussion of the behavioural incentives underlying these mechanisms.

These considerations lead to conclude the following:

Proposition 13. *The effect of incumbent workers' negative reciprocity and downward wage rigidity on the elasticity of labour market tightness depends on the following condition:*

$$\frac{d\varepsilon_{\tilde{\theta}_p^*}}{d\lambda} \begin{matrix} \geq \\ \leq \end{matrix} 0 \Leftrightarrow |A| \begin{matrix} \leq \\ \geq \end{matrix} B + C.$$

- a) If $|A| < B + C$, then $\varepsilon_{\tilde{\theta}_p^*}$ is increasing in λ ;
- b) If $|A| > B + C$, then $\varepsilon_{\tilde{\theta}_p^*}$ is decreasing in λ ;
- c) If $|A| = B + C$, then changes in λ have no effect on $\varepsilon_{\tilde{\theta}_p^*}$.

Hence, while it is difficult to qualitatively characterise the effect of λ on the volatility of job creation, the statement of Proposition 13 establishes that: if firms expect that the effects of negative reciprocity on output *and* of paying incumbent workers a higher wage are larger than what they can offset through a compression of the hiring wage (i.e. case a)), then the expectation of downward rigidity in the wage of incumbent workers will unambiguously reduce firms' expected present value of new employment relationships, increasing the volatility of job creation and unemployment. On the other hand, if firms manage to sufficiently compress hiring wages—accounting for the expected future costs of negative reciprocity and downward wage rigidity (i.e. case b))—to the extent that the expected profit margin from new employment relationships increases, then the volatility of job creation and unemployment will be lower. Finally, if firms expect that wage compression will be just enough to offset the negative effects captured by B and C (i.e. case c)), then the expectation of incumbent workers' downward wage rigidity is irrelevant for unemployment volatility.

5 Conclusions

Inspired by the literature on efficiency wages (Akerlof, 1982; Akerlof and Yellen, 1990) and reciprocity in labour markets (Fehr et al., 2009), as well as by recent prominent attempts to provide more realistic micro-foundations for the macroeconomic analysis of wage and unemployment fluctuations (e.g. Snell and Thomas (2010), Danthine and Kurmann (2010) and Eliaz and Spiegel (2014)), this paper contributes to the theory of labour market fluctuations by studying the implications of a positive, and asymmetric, wage-effort relationship in a canonical search and matching model à la Pissarides (1985, 2000). This approach allowed a novel and in-depth analysis of the underlying behavioural incentives that determine optimal wage setting and job creation, and shed new light on the relative importance of newly hired and incumbent workers' wage cyclicality for the volatility of job creation.

The contribution of this paper rests on two novel theoretical results. First, in contrast to existing theoretical models, the analysis has shown that, by considering employed workers' optimal effort responses to wage changes, the cyclicality of the hiring wage is irrelevant for the size of the volatility of vacancies and unemployment. Second, after introducing uncertainty around the evolution of a job match productivity, it has been shown that the expectation of downward wage rigidity, and of the relatively large cost of implementing wage cuts—due to employed (incumbent) workers' asymmetric reference-dependent reciprocity—reduces firms' expected surplus from new employment relationships. As such, if firms are unable to offset the impact of these anticipated negative effects by compressing the wage of new hires, expected downward wage rigidity will unambiguously increase the volatility of vacancies and unemployment. This result complements and extends the more recent works of Eliaz and Spiegel (2014) and Bils et al. (2016) by providing a transparent analysis of the behavioural incentives driving firms' optimal wage setting and job creation in long-term employment relationships.

The main behavioural mechanism underling both results is the presence of a positive, and reference-dependent, wage-effort relationship, which stems from the optimal response of employed workers to their firms' wage setting policy. While the existing literature on unemployment volatility seems to have so far ignored the resulting amplification mechanism implied by this channel, the modelling approach adopted here enables a transparent analysis of its implications.

The framework developed in this paper also highlights additional theoretical aspects that

will benefit from further research. First, gaining insight on how wage entitlements at the start of the employment relationship are formed, and whether firms' information about these is complete, can potentially enhance the understanding of the determinants of equilibrium unemployment. For instance, it will be interesting to explore alternative, endogenous or exogenous, reference wage formation processes, and to analyse their related implications in the context of the model developed here (the survey evidence collected under the Wage Dynamics Network could be informative in this respect, see for instance Galuscak, Keeney, Nicolitsas, Smets, Strzelecki, and Vodopivec (2012)). Second, it could be relevant to study whether and how workers' wage entitlements can be systematically 'manipulated' downwards (for instance, during recessions), either by firms, government policies or both. Doing so will not only allow firms to produce more output at the same cost—due to an increase in morale and effort among workers—but will also allow governments to achieve a more desirable equilibrium rate of unemployment.

In conclusion, the approach of this paper has highlighted the existence of behavioural aspects characterising employment relationships—such as workers' asymmetric effort responses and reference wages—that can influence firms' expected surplus from a new match, and has provided a benchmark framework to analyse their implications that is transparent and directly comparable with the existing literature. These behavioural aspects can be exogenous to the specific economic environment, or can be influenced by the labour market institutional and social context. Gaining further insights in this direction can potentially contribute to enhancing our understanding of the cyclical behaviour of labour markets.

A Appendix: Additional Material

A.1 A Simple Calibration Exercise

This section performs a calibration exercise in order to evaluate the quantitative relevance of the behavioural mechanisms considered in the analysis of Section 3, and to show that the theoretical framework developed in this paper can simultaneously accommodate the empirical volatilities of both the hiring wage and of the vacancy-unemployment ratio.

Given the relatively high number of unobservable (exogenous) parameters introduced in this paper, the following calibration strategy and quantitative analysis will be slightly different to the standard calibration approach in the literature. That is, instead of using debatable proxy measures to assign values to unobservable parameters—such as employed workers’ normal effort \tilde{e}_n and the distribution of new hires’ reference wages Γ_0 —and to subsequently calculate the steady-state elasticity of market tightness, the following approach will use the empirical estimate of this volatility measure as a calibration target. More precisely, the analysis will study a combination of the behavioural parameters that can deliver plausible empirical elasticities of both the hiring wage and the vacancy-unemployment ratio with respect to productivity shocks.

A.1.1 A model for calibration

This section presents a parameterised version of the model which enables to characterise closed-form explicit solutions suitable for calibration. As such, consistent with Assumptions A3, F1 and W1–W3 of Section 2, consider the following functional forms: matching function $\bar{m}(u, v) = \hat{m}u^\sigma v^{1-\sigma}$; firms’ per-worker output $y(p, e) = pe$; workers’ utility from the wage $m(w) = \log w$; and their net cost of productive activity $c(e) = e^2/2 - be$, with $b > 0$. To reduce the number of exogenous parameters, the importance of gain-loss utility for workers is set to unity, $\eta = 1$. Finally, the state space characterising the distribution of new hires’ reference wages is discretised as follows:

- C1.** r_i is the realisation of a random variable on the state space $\mathcal{R}_i = \{r^l, r^m, r^h\}$, $\mathcal{R}_i \subset \mathcal{R}$, where $r^l < r_L(p)$, $r^m \in [r_L, r_H]$, $r^h > r_H(p, \lambda)$; and it is distributed according to $\gamma_0 : \mathcal{R}_i \rightarrow \mathbb{R}$ with $\gamma_0(r_i) \geq 0$ for all $r_i \in \mathcal{R}_i$, and $\sum_{r_i \in \mathcal{R}_i} \gamma_0(r_i) = 1$.

Hence, the labour market is populated by a fraction $\gamma_0(r^l)$ of workers with relatively ‘low’ reference wages; a fraction $\gamma_0(r^m)$ of workers with relatively ‘moderate’ reference wages; and a fraction $\gamma_0(r^h)$ of workers with relatively ‘high’ reference wages.³¹

Given these assumptions, the workers’ asymmetric reference-dependent reciprocity $\tilde{e} = \tilde{e}(w, r, \lambda)$ and the corresponding firms’ optimal wage policy $\tilde{w} = \tilde{w}(r, p, \lambda)$ take the following simple forms:

$$\tilde{e} = \begin{cases} \tilde{e}_n + [\log w - \log r] & \text{if } w > r \\ \tilde{e}_n & \text{if } w = r \\ \tilde{e}_n - \lambda[\log r - \log w] & \text{if } w < r \end{cases} \quad \text{and} \quad \tilde{w} = \begin{cases} p[1 - \psi] & \text{if } r < r_L(p) \\ r & \text{if } r \in [r_L, r_H] \\ \lambda p[1 - \psi] & \text{if } r > r_H(p, \lambda); \end{cases}$$

where $\tilde{e}_n \equiv b$; and $r_L(p) = p(1 - \psi)$ and $r_H(p, \lambda) = \lambda p(1 - \psi)$.

These results are sufficient to derive explicit steady-state solutions following the results established in Sections 2 and 3. For clarity of exposition it is useful to show the expression characterising the expected present discounted values of output and wages

$$\bar{Y}^*(E^*) = p \left[\sum_{r_i \in \mathcal{R}_i} \gamma_0(r_i) \tilde{e}(\tilde{w}^*, r_i^*, \lambda) + \frac{\psi}{1 - \psi} \tilde{e}_n \right] \quad \text{and} \quad \bar{W}^* = \sum_{r_i \in \mathcal{R}_i} \gamma_0(r_i) \frac{\tilde{w}(r_i^*, p, \lambda)}{1 - \psi}$$

respectively; which are relevant for the analytical expression of the elasticity of labour market tightness derived in Proposition 8.

Finally, denote the expected average elasticity of the hiring wage by

$$\varepsilon_{W_p^*} \equiv \sum_{r_i \in \mathcal{R}_i} \gamma_0(r_i) \varepsilon_{\tilde{w}_p^*};$$

and notice that due to the assumption that $m(w) = \log w$, the elasticity of the hiring wage with respect to productivity is $\varepsilon_{\tilde{w}_p^*} = 1$ for all $r_i^* = \{r^l, r^h\}$; and indeed $\varepsilon_{\tilde{w}_p^*} = 0$ for all $r_i^* = r^m$. As such it is possible to deduce that the maximum value of $\varepsilon_{W_p^*}$ that can be achieved using this model is bounded above by 1; which can be obtained by any calibration using a reference wage distribution $(\gamma_0(r_i^*))_{r_i^* \in \mathcal{R}_i} \equiv (\gamma_0(r^l), \gamma_0(r^m), \gamma_0(r^h))$ in which $\gamma_0(r^m) = 0$. An alternative assumption for the utility of the wage consistent with Assumption W1 would have been $m(w) = w^\alpha / \alpha$ with $\alpha \in (0, 1)$. In such a case, it can be shown that the maximum value of $\varepsilon_{W_p^*}$

³¹Note that Assumption C1 concerns all workers that can potentially become new hires in the period in which firms post vacancies, and not only those new hires that are successfully matched in their first period of production. Moreover notice that $\sum_{r_i \in \mathcal{R}_i} \gamma_0(r_i) r_i$ represents both the *average* and the *expected* reference wage characterising potential new hires in the labour market.

that can be achieved by the model is bounded above by the factor $\frac{1}{1-\alpha}$, which is greater than 1 for any $\alpha \in (0, 1)$. Since this alternative formulation introduces an additional, unobservable parameter that will have to be calibrated, the analysis that follows will implement the natural logarithm instead.

A.1.2 Calibration strategy

The calibration proceeds as follows: first the conventional parameters of the model are chosen in accordance with the standard approach in the literature; then, the remaining behavioural parameters are calibrated so as to achieve a target elasticity of labour market tightness of $\varepsilon_{\tilde{\theta}_p^*} = 7.56$ and target average elasticity of hiring wages of $\varepsilon_{W_p^*} = 0.8$ (see Haefke et al. (2013)). To enhance comparability with the literature, conventional parameters and targets are chosen following the calibration performed by Pissarides (2009).

Conventional parameters. The time period is given by a quarter. The elasticity of the matching function with respect to unemployment σ is set equal to 0.5 as in Pissarides (2009). This value is at the lower bound of the range of estimates provided by Petrongolo and Pissarides (2001), i.e. $\sigma \in [0.5, 0.7]$, and it is in the middle of the range of values used in the literature, i.e. $\sigma \in [0.235, 0.72]$ (see Kudlyak (2014)). The discount factor $\delta = 0.996$ is set to match a quarterly interest rate of 0.004, and the exogenous job destruction rate ρ is set equal to 0.036 (see Pissarides (2009) and Shimer (2012)). The aggregate match productivity parameter p is normalised to 100 in order to ensure a non-negative wage utility, i.e. so that $m(\tilde{w}^*) = \log \tilde{w}^* \geq 0$. The remaining conventional parameters—namely, the efficiency of matching \hat{m} and the cost of posting a vacancy κ —are calibrated to match an average job finding probability of 0.594, and an average vacancy-unemployment ratio of 0.72 (as in Pissarides (2009)).³² Notice that this calibration yields a steady-state probability that a vacant job is matched with a worker of $h(\theta) = 0.7 \cdot (0.72)^{-0.5} = 0.825$.

Behavioural parameters. The behavioural parameters of the model are: the employed workers' normal effort \tilde{e}_n ; their degree of loss aversion λ , which also affects their extent of negative reciprocity in the event of an unfair wage; their wage entitlements at the start of the employment relationship $r_i^* = \{r^l, r^m, r^h\}$; and the vector of relative frequencies $(\gamma_0(r_i^*))_{r_i^* \in \mathcal{R}_i}$ with which these entitlements are distributed among workers.

³²The relatively high number which results from the calibration of the vacancy cost κ is essentially a product of the non-conventional normalisation of the match productivity p . However note that none of these two parameters are crucial for the determination of the elasticity of market tightness.

The loss aversion parameter λ is set to be equal to 2, which implies that the negative effect of an unfair wage is two times bigger than the positive effect of a wage gift on workers' morale and reciprocity. This parameter value is based on the experimental analysis of [Abdellaoui, Bleichrodt, and Paraschiv \(2007\)](#) and lies below the median of the range of loss aversion parameters $\lambda \in [1.43, 4.8]$ estimated in the literature (see [Abdellaoui et al. \(2007\)](#) for a review).

Table 1: Parameter Values, Quarterly Calibration

Parameter	Value	Description	Source/Reason
<i>Conventional</i>			
σ	0.500	Elasticity of matching	Literature
ρ	0.036	Exogenous job destruction Rate	Literature
δ	0.996	Discount factor	Interest rate = 0.004
\hat{m}	0.700	Efficiency of matching	Job finding probability
κ	45.00	Vacancy cost	v/u ratio
<i>Behavioural</i>			
λ	2.000	Loss aversion parameter	Abdellaoui et al. (2007)
\tilde{e}_n	0.082	Normal effort	1% above e_n
$ \tilde{w}^* - r_i^* /r_i^*$	5%	w-r % deviation $\forall r_i^* = \{r^l, r^h\}$	See Below
$(\gamma_0(r_i^*))_{r_i^* \in \mathcal{R}_i}$	(0.39, 0.20, 0.41)	New hires' reference wage distribution	Calibrated
<i>Normalisations</i>			
p	100.0	Aggregate productivity	$\log \tilde{w}^* > 0$
<i>Targets</i>			
θ	0.720	Average v/u (tightness)	Pissarides (2009)
$\bar{m}\theta^{1-\sigma}$	0.594	Average job finding probability	Pissarides (2009)
$\varepsilon_{\tilde{\theta}_p}$	7.560	Average elasticity of θ w.r.t. p	Literature
$\varepsilon_{W_p^*}$	0.800	Average elasticity of w w.r.t. p	Haefke et al. (2013)

At this stage notice that it is possible to find several combinations of the remaining behavioural parameters that could deliver the two target elasticities of market tightness 7.56 and hiring wages 0.8. This is because the number of unknown parameters is greater than the number of target equations. While this issue could be criticised to be unsatisfactory from a purely quantitative perspective, it also shows that there exists a richer set of potential channels through which the volatility of vacancies and unemployment could be amplified that are consistent with empirical estimates of new hires' wage cyclicity. Nevertheless, it is possible to proceed with the calibration by implementing plausible conditions to pin down these parameters.

Workers' normal effort \tilde{e}_n —which, as established in Section 2, captures the level of effort that minimises the net cost from productive activity whenever a worker is paid their reference wage—is set to be 1% above the minimum effort required to ensure that firms' zero-profit condition when hiring a worker with a relatively high reference wage $r_i^* = r^h$ is always satisfied. That is, $\tilde{e}_n = e_n(r^h, \lambda) \cdot [1 + 1\%]$ where $e_n(r^h, \lambda) \equiv \max\{\tilde{e}_n : J(r^h) = 0\}$. Hence $e_n(r^h, \lambda)$ is

endogenous to the calibration since it depends on the value of r^h , which is unknown. For the baseline calibration, the level of new hires' reference wages are calibrated by simultaneously setting the following conditions: i) the percentage deviations of the relatively low and relatively high reference wages from their respective steady-state wages are assumed to be the same in absolute magnitude, and in particular:³³

$$\frac{|\tilde{w}^*(p)^+ - r^l|}{r^l} = \frac{|\tilde{w}^*(p, \lambda)^- - r^h|}{r^h} = 5\%;$$

and ii) the relatively moderate reference wage is assumed to be the average of the high and low reference wage:

$$r^m = \frac{r^l + r^h}{2}.$$

These conditions yield $\mathcal{R}_i = \{r^l, r^m, r^h\} = \{3.8, 6.1, 8.4\}$ and a value for normal effort of $\tilde{\epsilon}_n \approx 0.081 + 0.001 \approx 0.082$.

Using these values it is now possible to calibrate the frequencies of new hires' reference wages in the market to deliver an elasticity of labour market tightness of 7.56 and an average elasticity of the hiring wage of 0.8. These are pinned down by the following conditions:

$$\gamma_0(r^l) = \{\gamma_0(r^l) : \sum_{r_i \in \mathcal{R}_i} \gamma_0(r_i) = 1\}; \quad (40)$$

$$\gamma_0(r^m) = \{\gamma_0(r^m) : \sum_{r_i \in \mathcal{R}_i} \gamma_0(r_i) \varepsilon_{\tilde{w}_p^*} = 0.8\}; \quad (41)$$

$$\gamma_0(r^h) = \{\gamma_0(r^h) : \varepsilon_{\tilde{\theta}_p^*} = 7.56\}; \quad (42)$$

which yields $(\gamma_0(r_i^*))_{r_i^* \in \mathcal{R}_i} = (0.394, 0.200, 0.406)$. Hence, firms face a probability of $h(\theta) \cdot \gamma_0(r^l) = 0.825 \cdot 0.394 \approx 0.325$ of being matched with a worker for which it is optimal to pay a wage 5% above their reference wage; a probability of $h(\theta) \cdot \gamma_0(r^h) = 0.825 \cdot 0.406 \approx 0.165$ of being matched with a worker for which it is optimal to pay a wage 5% below their reference wage; and a probability of $h(\theta) \cdot \gamma_0(r^m) = 0.825 \cdot 0.200 \approx 0.335$ of being matched with a worker for which it is optimal or to pay them their reference wage.

³³Notice that although this number has been chosen arbitrarily, the calibration remains robust to choices ranging from 1% to 60%. This is because parameters such as $\tilde{\epsilon}_n$ or $(\gamma_0(r_i^*))_{r_i^* \in \mathcal{R}_i}$ are endogenous to the calibration and will adjust accordingly to deliver the required target elasticities of wages and market tightness.

A.1.3 Calibration results and discussion

The calibration results of interest are reported in Tables 2 and 3. The first three rows of each table display the results for a representative worker-firm employment relationship in which the employed worker is characterised by a relatively low, moderate or high reference wage respectively. The last row displays the expected labour market values of the endogenous outcomes in the steady state.

As shown in Table 2, 39% of new hires are characterised by a relatively low reference wage $r^l \approx 3.8$ and are paid the steady-state equilibrium wage $\bar{w}^+ \approx 4.0$. This is perceived as a wage gift of 5%, which triggers an endogenous positive reciprocity response of +25.8% in the first employment period, calculated as the percentage deviation of new hires' supra-normal effort $\tilde{e}^+ \approx 0.103$ from their normal level $\tilde{e}_n = 0.082$. On the contrary, 40.6% of new hires are characterised by a relatively high reference wage $r^h \approx 8.4$ and are paid the steady-state equilibrium wage $\bar{w}^- \approx 8.0$, which however is perceived as unfair. In fact, this wage is 5% below their wage entitlement and therefore triggers an endogenous negative reciprocity response of -54.2%, corresponding to sub-normal effort $\tilde{e}^- \approx 0.038$. Finally, 20% of new hires are paid their reference wage $\bar{w}^m = r^m \approx 6.1$ and therefore exert normal effort $\tilde{e}_n = 0.082$ in their first period of employment. These results imply that firms expect to pay new hires a wage that

Table 2: Newly Hired Workers Statistics

Steady State	Outcomes						
	$\gamma_0(r_i^*)$	W^*	as % of y	R_i^*	w-r gap	E_i^*	% deviation from \tilde{e}_n
Low	0.394	4.0	38.5%	3.8	+5%	0.103	+25.8%
Moderate	0.200	6.1	74.0%	6.1	-	0.082	-
High	0.406	8.0	211.3%	8.4	-5%	0.038	-54.2%
Expected		6.02	83.0%	6.12	-1.56%	0.073	-11.8%

is just -1.56% below the average reference wage in the labour market, but that generates an expected negative reciprocity response of -11.8% in the first employment period.

Table 3 displays the present discounted values of output and wages in the market, the resulting value of a new employment relationship to firms, and related elasticities of new hires' wages and market tightness. From these results it is clear that the expected value of a job filled by a newly hired worker with a high reference wage is very low relative to the one filled by a worker with either a low or a moderate reference wage. The main drivers of this outcome are: the relatively low normal effort—which is 1% above the minimum required for the job to be

profitable; combined with the outcome of optimal wage setting—according to which firms pay a relatively higher (here twice as high) steady-state wage in order to offset the greater cost of new hires’ negative reciprocity in the first employment period. Nevertheless, in expectation, the value of a new employment relationship is ‘reasonably’ large and consistent with a steady-state elasticity of labour market tightness of 7.56 as required.

Table 3: Present Values and Elasticities

Steady State	Outcomes					
	$\gamma_0(r_i^*)$	$\bar{Y}^*(E^*)$	\bar{W}^*	$\mathbb{E}[J(r_i^*)]$	$\varepsilon_{W_p^*}$	$\varepsilon_{\bar{y}_p^*}$
Low	0.394	208.6	100.0	108.6	1.00	3.841
Moderate	0.200	206.5	152.9	53.62	0.00	7.703
High	0.406	202.0	200.0	2.045	1.00	197.6
Expected		205.5	151.2	51.26	0.800	7.560

To conclude, although tackling the unemployment volatility puzzle from an alternative, un-conventional, perspective, the analysis of this section has been useful to show that for plausible values of the behavioural parameters introduced in this paper, the present framework can simultaneously accommodate the empirical volatilities of both the hiring wage and of the vacancy-unemployment ratio, overcoming one of the issues mentioned by [Kudlyak \(2014\)](#). This result enhances the quantitative relevance of the behavioural aspects considered, and suggests a promising route for a richer dynamic stochastic simulation of the model.

A.2 Workers’ Reservation Wage Condition

From reference wage adaptation (Assumption A2) and the results established in Theorem 1 and Proposition 2, the value of employment to an incumbent worker, given by (12), can be rearranged as

$$\mathcal{W}(\tilde{w}(r_i), \tilde{w}(r_i)) = \frac{u(\tilde{e}_n, \tilde{w}(r_i), \tilde{w}(r_i)) + \delta\rho\mathcal{U}}{1 - \psi};$$

since $r_{jt} = \tilde{w}_{t-1}$ and $\tilde{w}_{t+1} = \tilde{w}_t = \tilde{w}_{t-1} = \tilde{w}(r_i)$ (omitting the other functional arguments).

Using this expression, the value of employment to a new hire, given by (13), can be rewritten as

$$\mathcal{W}(\tilde{w}(r_i), r_i) = u(\tilde{e}(\tilde{w}(r_i), r_i, \lambda), \tilde{w}(r_i), r_i) + \frac{\psi}{1 - \psi}u(\tilde{e}_n, \tilde{w}(r_i), \tilde{w}(r_i)) + \frac{\delta\rho}{1 - \psi}\mathcal{U}.$$

Notice that the first two terms on the right-hand side together represent the present discounted value of utility of being employed at the wage $\tilde{w}(r_i)$ from $t = s$ onwards, conditional on the

exogenous job destruction rate. As such, to ease notation, define:

$$\bar{u}(\tilde{\epsilon}_n, \tilde{w}(r_i), r_i) = u(\tilde{\epsilon}(\tilde{w}(r_i), r_i, \lambda), \tilde{w}(r_i), r_i) + \frac{\psi}{1-\psi} u(\tilde{\epsilon}_n, \tilde{w}(r_i), \tilde{w}(r_i)),$$

and re-write the value of employment to a new hire as

$$\mathcal{W}(\tilde{w}(r_i), r_i) = \bar{u}(\tilde{\epsilon}_n, \tilde{w}(r_i), r_i) + \frac{\delta\rho}{1-\psi} \mathcal{U}. \quad (43)$$

Finally, the value of unemployment, given by (14), can be rearranged as

$$\mathcal{U} = \frac{u(z) + \delta f(\theta_t) \mathbb{E} [\mathcal{W}(\tilde{w}(r_i), r_i)]}{1 - \delta[1 - f(\theta_t)]}. \quad (44)$$

Using the definition of the reservation wage $\underline{w}_i \equiv \{w : \mathcal{W}(w, r_i) = \mathcal{U}\}$, by substituting \underline{w}_i into (43), and collecting \mathcal{U} as the common factor yields:

$$\bar{u}(\tilde{\epsilon}_n, \underline{w}_i, r_i) = \mathcal{U} \frac{1-\delta}{1-\psi} \quad (45)$$

[Notice that this is the analog of the one derived by Pissarides 2000, p.150, equation (6.14)].

To write an expression in terms of wages, take the expected value of (43), substitute it into (44), and then rearrange the equation, solving for \mathcal{U} :

$$\mathcal{U} = u(z) \frac{1-\psi}{\check{\psi}(\theta_t)} + \mathbb{E} [\bar{u}(\tilde{\epsilon}_n, \tilde{w}(r_i), r_i)] \frac{\delta f(\theta_t)[1-\psi]}{\check{\psi}(\theta_t)};$$

where $\check{\psi}(\theta_t) \equiv (1-\delta)[1-\psi + \delta f(\theta_t)]$. By using this expression to substitute \mathcal{U} out of (45), and after some algebra, it is possible to establish that a worker's reservation wage is implicitly characterised by the following expression:

$$\bar{u}(\tilde{\epsilon}_n, \underline{w}_i, r_i) = \frac{u(z) + \delta f(\theta_t) \mathbb{E} [\bar{u}(\tilde{\epsilon}_n, \tilde{w}(r_i), r_i)]}{1-\psi + \delta f(\theta_t)}, \quad (46)$$

which is equivalent to (15) in the main body of the paper. [Notice that this is the analog of the one derived in Pissarides 2000, p. 150, eq. (6.15)].

Finally, by using the model and results established in the calibration exercise of Section A.1, it is possible to show that, for conventional values of unemployment income z , the workers' reservation wage condition is also always satisfied, that is $\tilde{w}_i(r_i) \geq \underline{w}_i$ for all $r_i \in \mathcal{R}_i$. This is

done by showing that the following condition always holds

$$\bar{u}(\tilde{e}_n, \tilde{w}^*, r_i^*) \geq \frac{u(z) + \delta f(\theta) \sum_{r_i \in \mathcal{R}_i} \gamma_0(r_i) \bar{u}(\tilde{e}_n, \tilde{w}^*, r_i^*)}{1 - \psi + \delta f(\theta)}; \quad (47)$$

which is the steady-state equivalent of condition (15) of Section 2.4 where $u(z) = \log(z)$ and as in the baseline calibration $\mathcal{R}_i = \{r^l, r^m, r^h\} = \{3.8, 6.1, 8.4\}$. Notice that while the right-hand side (RHS) of (47) depends on the distribution of r_i^* , the left-hand side (LHS) depends on the actual level of r_i^* which can be either ‘low’, ‘moderate’, or ‘high’.

Let unemployment income z be 40% of the expected wage in the market $\sum_{r_i \in \mathcal{R}_i} \gamma_0(r_i) \tilde{w}^*$ (as in Shimer (2005) and Hall (2005b)), that is $z = 6.02 \cdot 40/100 = 2.41$. Using the results obtained from the calibration performed in Section A.1, in equation (47) the RHS = 19.10 is always lower than the LHS = {19.62, 19.78, 19.85} for any $r_i^* = \{3.8, 6.1, 8.4\}$. As such, whenever a worker is matched with a firm and receives a wage offer \tilde{w}^* , the present discounted value of utility from accepting the job and being employed at \tilde{w}^* is always greater than the expected discounted value of utility from continuing searching, being paid unemployment income z and facing the probability $f(\theta)$ of being matched with a firm in the following period.

B Appendix: Proofs

Proof of Theorem 1. Denote the first-order condition characterising the worker’s optimal effort by $\Omega(e; w, r, \lambda)$. Assumptions W1-W3 hold throughout. First notice that when $w < r$, the wage offered may be such that the worker would optimally choose $e < 0$ but cannot due to the constraint that $e \geq 0$. Define

$$\underline{w}(r, \lambda) = \max\{0, w : \Omega(0; w, r, \lambda) = 0\}.$$

Since $\Omega_w(e, w, r, \lambda) = \mu'(m(w) - m(r))m'(w) > 0$ this identifies the threshold wage at which the worker would choose $e = 0$ and below which they would optimally choose $e < 0$ (since $\Omega(e; w, r, \lambda) < 0$ for all $e \geq 0$) but cannot; hence define $\tilde{e}(\underline{w}(r, \lambda), r, \lambda)^- \equiv 0$ for all $w \leq \underline{w}(r, \lambda)$.

So long as $w > \underline{w}(r, \lambda)$ optimal effort is given by the inverse function

$$\tilde{e}(w, r, \lambda) = c'^{-1}(\mu(m(w) - m(r)))$$

which exists since c' is strictly monotonic. Moreover, since c' is a continuous function and μ varies continuously in w and r , $\tilde{e}(w, r, \lambda)$ will be a continuous function of w and r , but it will not be continuously differentiable everywhere as μ has a kink at $w = r$.

When $w = r$, $m(w) = m(r)$ and therefore $\Omega(e; r, r, \lambda) = -c'(e)$. By assumption W2, $\Omega(0; r, r, \lambda) = -c'(0) > 0$ and $\Omega_e(e, w, r, \lambda) = -c''(e) < 0$ which implies $\tilde{e}(r, r, \lambda) = c'^{-1}(0) \equiv \tilde{e}_n > 0$. Recalling the definition of μ in (4), when $w > r$, $\tilde{e}(r, r, \lambda) = c'^{-1}(\eta[m(w) - m(r)]) > \tilde{e}_n$ since $m(w) - m(r) > 0$; whilst when $w < r$, $\tilde{e}(r, r, \lambda) = c'^{-1}(\lambda\eta[m(w) - m(r)]) < \tilde{e}_n$ since $m(w) - m(r) < 0$. As such $\lim_{\epsilon \rightarrow 0} m(r + \epsilon) - m(r) = 0$ implies $\tilde{e}(w, r)^+ \rightarrow \tilde{e}_n$ as $w \rightarrow r$ from above, and $\lim_{\epsilon \rightarrow 0} m(r - \epsilon) - m(r) = 0$ implies $\tilde{e}(w, r)^- \rightarrow \tilde{e}_n$ as $w \rightarrow r$ from below.

When $w \neq r$ and $w > \underline{w}(r, \lambda)$ implicit differentiation of the first-order condition reveals

$$\tilde{e}_w(w, r, \lambda) = -\frac{\Omega_w}{\Omega_e} = \frac{\mu'(m(w) - m(r))m'(w)}{c''(e)} > 0.$$

Further differentiating this expression (and recalling that μ is piecewise linear, by Assumption W3, and that $c'''(e) = 0$, by Assumption W2) yields:

$$\tilde{e}_{ww}(w, r, \lambda) = \frac{\mu'(m(w) - m(r))m''(w)}{c''(e)} < 0.$$

$$\tilde{e}_{wr}(w, r, \lambda) = -\frac{\mu''(m(w) - m(r))m'(w)m'(r)}{c''(e)} = 0.$$

Note from (4) that for $w > r$, $\mu'(x) = \eta$ and when $w < r$, $\mu'(x) = \lambda\eta$. To consider the response of effort to the wage above and below the reference wage, the continuity of $\tilde{e}(w, r, \lambda)$ is used to establish that

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \tilde{e}_w(r - \epsilon, r, \lambda)^- &= -\lim_{\epsilon \rightarrow 0} \frac{\lambda \cdot \eta m'(r - \epsilon)}{b''(\tilde{e}(r - \epsilon, r, \lambda)^-) - c''(\tilde{e}(r - \epsilon, r, \lambda)^-)} \\ &= -\frac{\lambda \cdot \eta m'(r)}{b''(\tilde{e}_n) - c''(\tilde{e}_n)} \\ &= -\lim_{\epsilon \rightarrow 0} \frac{\lambda \cdot \eta m'(r + \epsilon)}{b''(\tilde{e}(r + \epsilon, r)^+) - c''(\tilde{e}(r + \epsilon, r)^+)} \\ &= \lambda \cdot \lim_{\epsilon \rightarrow 0} \tilde{e}_w(r + \epsilon, r)^+. \end{aligned}$$

Since this implies the effort function kinks to a flatter slope as the wage increases, this result combined with the deduction that $\tilde{e}_{ww} < 0$ for all $w \neq r$, implies \tilde{e}_w is everywhere decreasing in w , i.e. the effort function is concave on \mathcal{W} .

The relationship between $\tilde{e}(w, r, \lambda)$ and r is established by implicit differentiation:

$$\tilde{e}_r(w, r, \lambda) = -\frac{\Omega_r}{\Omega_e} = \frac{\mu'(m(w) - m(r))m'(r)}{c''(e)} < 0.$$

Similarly, the effect of the degree of loss aversion on effort when $w < r$ is

$$\tilde{e}_\lambda(w, r, \lambda)^- = -\frac{\Omega_\lambda}{\Omega_e} = \frac{\eta[m(w) - m(r)]}{c''(e)} < 0.$$

Moreover, the effect of λ on $\tilde{e}(w, r, \lambda)$ (for $w < r$) is

$$\tilde{e}_{w\lambda}(w, r, \lambda)^- = \frac{\eta m'(w)}{c''(e)} > 0.$$

■

Proof of Proposition 1. First, consider the following Lemmata (the prime that is not followed by functional arguments denotes forward values):

Lemma 3. $y_e(p, \tilde{e}) = y_e(p, \tilde{e}')$

Proof. From assumption F1, $y_{ee} = 0$ which implies that $y_e(p, \tilde{e})$ is independent of \tilde{e} ; and since p is parametric and time invariant, it follows that $y_e(p, \tilde{e}) = y_e(p, \tilde{e}')$. ■

Lemma 4. $\tilde{e}_r(w', r', \lambda) = -\tilde{e}_w(w, r, \lambda)$ if $r' = w$

Proof. Part b) of Theorem 1 establishes that $\tilde{e}_{wr} = 0$. This implies that $\tilde{e}_r(w', r', \lambda)$ is independent of its first argument w' . and that $\tilde{e}_w(w, r, \lambda)$ is independent of its second argument r . Then, since $r' = w$, from the properties of μ it is possible to deduce that $\mu'(x)m'(r') = -\mu'(x)m'(w)$, which implies that $\tilde{e}_r(w', r', \lambda) = -\tilde{e}_w(w, r, \lambda)$. ■

As such, the first-order condition (8) can be expressed as:

$$\begin{aligned} \Upsilon(w; r, p, \lambda) &\equiv y_e \tilde{e}_w - 1 + \psi y_e' \tilde{e}_r' \\ &= y_e \tilde{e}_w - 1 - \psi y_e \tilde{e}_w \\ &= y_e \tilde{e}_w [1 - \psi] - 1 \end{aligned} \tag{48}$$

where the second line follows from the results established in Lemma 3 and 4. It follows that

$$\Upsilon_w(w; r, p, \lambda) = y_e \tilde{e}_{ww} [1 - \psi] < 0$$

for all $w \neq r$. This implies that $\Upsilon(w; r, p, \lambda)$ is everywhere decreasing in w , establishing concavity of the firm's value function J . This also makes the first-order condition (8) sufficient for the characterisation of a global maximum. ■

Proof of Proposition 2. This proof proceeds as follows. First it will be shown that under assumptions W1-W3, F1, A2 and one additional, though innocuous, restriction on the state and control spaces there exists a unique solution to the functional equation (7). Then, the proof will characterise the properties of the firm's optimal wage policy, following the steps used in the proof of Theorem 2, Dickson and Fongoni (2016).

Preliminaries. By definition, the state and control space \mathcal{R} and \mathcal{W} are both convex subsets of \mathbb{R}_+ . Throughout the proof it is assumed that $\mathcal{W} = \mathcal{R} = [0, \bar{r}]$, where \bar{r} is sufficiently large, in the sense that all the solutions to the firm's maximisation problem are interior (in particular $\bar{r} > r_H(p, \lambda)$, as defined in the statement of the proposition). Notice that since $\pi(w, r)$ is increasing and strictly concave in w and decreasing in r , it is possible to characterise \bar{r} such that it never binds. Moreover notice that since the firm may want to set the wage w_t either above, equal, or below the worker's reference wage r_t in each period t , the values of w that are allowed for any given r is independent of the actual level of r . As such, the set of feasible controls in each t is given by \mathcal{W} , which is nonempty, convex and compact.

Given these premises, it is possible to establish that the instantaneous profit function $\pi(w, r)$ is both bounded and continuous in its domain. This, together with the fact that $\psi \in (0, 1)$, implies that the operator T defined as

$$(TJ)(r) \equiv \max_{w \in \mathcal{W}} \{ \pi(r, w) + \psi J(w) \},$$

which maps the space of continuous and bounded functions into itself, is a contraction with a unique fixed point. Hence there exists a unique solution to the functional equation given by (7); and at least one optimal wage policy exists (see, for instance, Theorem 4.6, p.79 of Stokey and Lucas (1989)).

Hence, the first-order necessary condition to characterise an optimum is

$$\pi_w(r, w) + \psi J'(w) = 0.$$

The related envelope condition is

$$J'(r) = \pi_r(w, r).$$

Combining these two conditions, and denoting the firm's optimal wage policy by $\tilde{w} = \tilde{w}(r)$, yields the following Euler equation

$$\pi_w(r, \tilde{w}(r)) + \psi \pi_r(\tilde{w}(\tilde{w}(r)), \tilde{w}(r)) = 0.$$

Since π is strictly concave in w , and \mathcal{W} is a convex set, to establish uniqueness of the optimal wage policy $\tilde{w}(r)$ it remains to be shown that the firm's value function J is concave (which is not trivial since π is decreasing and convex in r). This is established in Proposition 1, implying that the first-order condition (8) is sufficient for the characterisation of a global maximum. In addition notice that

$$\begin{aligned} \Upsilon_\lambda(w; r, p, \lambda) &= y_e \tilde{e}_{w\lambda} + \psi y'_e \tilde{e}'_{r\lambda} > 0 \quad \text{if } w < r; \\ \Upsilon_\lambda(w; r, p, \lambda) &= y_e \tilde{e}_{w\lambda} + \psi y'_e \tilde{e}'_{r\lambda} = 0 \quad \text{if } w > r. \end{aligned}$$

These results enable to deduce that if $\lambda > 1$, $\Upsilon(w; r, p, \lambda)$ jumps down at the reference wage. Hence it is now possible to proceed with the proof following the same approach implemented in the proof of Theorem 2, [Dickson and Fongoni \(2016\)](#).

Reference wage thresholds. The threshold $r_L(p)$ is the level of a worker's reference wage below which a firm would optimally set a wage above their reference wage, and $r_H(p, \lambda)$ is the level of a worker's reference wage above which a firm would optimally pay the worker a wage below their reference wage. The former, $r_L(p)$, is the value of r below which the value function J is increasing just above the reference wage; the latter, $r_H(p, \lambda)$, is the value of r above which the value function J is decreasing just below the reference wage. These thresholds can be characterised respectively by

$$\lim_{\epsilon \rightarrow 0} \Upsilon(r + \epsilon; r, p, \lambda) = y_e \lim_{\epsilon \rightarrow 0} \tilde{e}_w(r + \epsilon, r)^+ [1 - \psi] - 1 = 0$$

and

$$\lim_{\epsilon \rightarrow 0} \Upsilon(r - \epsilon; r, p, \lambda) = y_e \lim_{\epsilon \rightarrow 0} \tilde{e}_w(r - \epsilon, r, \lambda)^- [1 - \psi] - 1 = 0$$

Using the result established in Theorem 1, part a):

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \Upsilon(r - \epsilon; r, p, \lambda) - \lim_{\epsilon \rightarrow 0} \Upsilon(r + \epsilon; r, p, \lambda) &= \\ &= y_e \lim_{\epsilon \rightarrow 0} \tilde{e}_w(r - \epsilon, r, \lambda)^- [1 - \psi] - 1 - y_e \lim_{\epsilon \rightarrow 0} \tilde{e}_w(r + \epsilon, r)^+ [1 - \psi] + 1 \\ &= y_e \lambda \lim_{\epsilon \rightarrow 0} \tilde{e}(r + \epsilon, r)^+ [1 - \psi] - y_e \lim_{\epsilon \rightarrow 0} \tilde{e}_w(r + \epsilon, r)^+ [1 - \psi] \\ &= y_e \lim_{\epsilon \rightarrow 0} \tilde{e}_w(r + \epsilon, r)^+ [1 - \psi] [\lambda - 1] \geq 0 \end{aligned}$$

with a strict inequality if $\lambda > 1$. Hence, $r_H(p, \lambda)$ is increasing in λ ; $r_H(p, \lambda) > r_L(p)$ for all $\lambda > 1$; and $r_H(p, 1) = r_L(p)$.

Optimal wage setting policy. Consider now the optimal wage setting policy, which depends on the level employed workers' reference wages r in relation to the reference wage thresholds derived above.

If $r \in [r_H(p, \lambda), \bar{r}]$, then from the definition of $r_H(p, \lambda)$, which implies that J is decreasing just below the reference wage, and since $\Upsilon_w < 0$, which implies strict concavity of J on \mathcal{W} , it is possible to deduce that the optimal wage policy must satisfy $w < r$, and will therefore be the solution to

$$y_e(p, \tilde{e}) \tilde{e}_w(w, r, \lambda)^- - 1 + \psi y_e(p, \tilde{e}') \tilde{e}_r(w', w, \lambda)^- = 0 \quad (49)$$

which is denoted by $\tilde{w}(p, \lambda)^-$ (recalling that the expression above is independent of \tilde{e}, r and w' as established in Lemma 3 and 4).

Analogously, if $r \in [0, r_L(p)]$, then from the definition of $r_L(p)$, which implies that J is increasing just above the reference wage, and since J is strictly concave on \mathcal{W} , it is possible to deduce that the optimal wage must satisfy $w > r$, and will therefore be the solution to

$$y_e(p, \tilde{e}) \tilde{e}_w(w, r)^+ - 1 + \psi y_e(p, \tilde{e}') \tilde{e}_r(w', w)^+ = 0 \quad (50)$$

which is denoted by $\tilde{w}(p)^+$ (the expression above being independent of \tilde{e}', r and w'). Since $\Upsilon_\lambda > 0$ it is straightforward to show that $\tilde{w}(p, \lambda)^- > \tilde{w}(p)^+$, and that if $\lambda = 1$ then $\tilde{w}(p, 1)^- = \tilde{w}(p)^+$.

Finally, if $r \in [r_L, r_H]$, from the definition of the reference wage thresholds and the concavity

of the firm's value function, it is possible to deduce that $\Upsilon(w; r, p, \lambda) < 0$ for all $w > r$ and that $\Upsilon(w; r, p, \lambda) > 0$ for all $w < r$. Hence $J(r)$ is maximised if and only if the optimal wage policy is $w = r$.

As such it is straightforward to show that for all $r_t \in \mathcal{R} \setminus [r_L, r_H]$, \tilde{w} is independent of r as noted above and that

$$\tilde{w}_p(r, p, \lambda) = -\frac{\Upsilon_p}{\Upsilon_w} = -\frac{y_{ep}[1-\psi]}{y_e \tilde{e}_{ww}[1-\psi]} > 0$$

which implies that \tilde{w} is increasing in p . Moreover, since $\tilde{w} = r$ for all $r \in [r_L, r_H]$, it is straightforward to see that \tilde{w} is increasing in r and independent of p . Finally, for all $r \in [r_H, \bar{r}]$, implicit differentiation yields:

$$\tilde{w}_\lambda(r, p, \lambda) = -\frac{\Upsilon_\lambda}{\Upsilon_w} = -\frac{y_e \tilde{e}_{w\lambda}[1-\psi]}{y_e \tilde{e}_{ww}[1-\psi]} > 0$$

since $\tilde{e}_{w\lambda} > 0$ as established by Theorem 1, which implies that $\tilde{w}(p, \lambda)^-$ is increasing in λ . ■

Proof of Proposition 3. Consider the employed workers' $\{i, j\}$ reference wage r_t , which evolves according to A2.

New Hires. First of all notice that since γ_0 is given and time-invariant, for all t , any $r_{it} \sim \Gamma_0$ characterises the steady-state reference wage of newly hired workers, and it is denoted by r_i^* . Hence, the steady-state aggregate reference wage of new hires is given by $R_i^*(\Gamma_0) = \int_0^{\bar{r}} r_0 d\Gamma_0(r_i) = \int_0^{\bar{r}} r_i^* d\Gamma_0(r_i)$; As such, following the results established in Theorem 1 and Proposition 2, it is possible to deduce the following. For all $r_i^* \in [0, r_L)$ the steady-state optimal wage paid to new hires is $\tilde{w}^*(r_i^*, p, \lambda) = \tilde{w}^*(p)^+$. and their steady-state optimal effort is $\tilde{e}^*(\tilde{w}^*, r_i^*, \lambda) = \tilde{e}^*(\tilde{w}^*(p)^+, r_i^*)^+$. For all $r_i^* \in (r_H, \bar{r}]$ the steady-state optimal wage paid to new hires is $\tilde{w}^*(r_i^*, p, \lambda) = \tilde{w}^*(p, \lambda)^-$. and their steady-state optimal effort is $\tilde{e}^*(\tilde{w}^*, r_i^*, \lambda) = \tilde{e}^*(\tilde{w}^*(p, \lambda)^-, r_i^*, \lambda)^-$. Finally, for all $r_i^* \in [r_L, r_H]$, the steady-state optimal wage paid to new hires is $\tilde{w}^*(r_i^*, p, \lambda) = r_i^*$ which implies that optimal effort is $\tilde{e}^*(\tilde{w}^*, r_i^*, \lambda) = \tilde{e}^*(r_i^*, r_i^*, \lambda) = \tilde{e}_n^*$.

Incumbents. Any r_j^* that satisfies $r_j^* = \tilde{w}(r_j^*, p, \lambda)$ is a steady state. First, notice that for all $r_0 \in [r_L, r_H]$, any r_0 is already a steady state since, according to the firms' optimal wage policy as established by Proposition 2, $\tilde{w}(r_0, p, \lambda) = r_0$. As such, from the results of Theorem 1, for all $r_j^* \in [r_L, r_H]$, the steady-state optimal wage paid to new hires is $\tilde{w}^*(r_j^*, p, \lambda) = r_j^* (= r_i^*)$, which implies that their optimal effort is $\tilde{e}(r_i^*, r_i^*, \lambda) = \tilde{e}_n^*$.

Next consider the case of $r_0 \in [0, \bar{r}] \setminus [r_L, r_H]$. Existence and uniqueness are straightforward to verify given the results established by Proposition 2. In fact, for all $r_0 \in [0, \bar{r}] \setminus [r_L, r_H]$

the optimal wage policy is independent of r_0 and characterises the steady-state wage paid to incumbents as $\tilde{w}^*(r_0, p, \lambda) = \tilde{w}^*(p)^+$ if $r_0 \in [0, r_L)$; and as $\tilde{w}^*(r_0, p, \lambda) = \tilde{w}^*(p, \lambda)^-$ if $r_0 \in (r_H, \bar{r}]$. As such, from A2 it follows that there exist a unique $r_j^* = \tilde{w}^*(p)^+$ for all $r_0 \in [0, r_L)$, and a unique $r_j^* = \tilde{w}^*(p, \lambda)^-$ for all $r_0 \in (r_H, \bar{r}]$. These hold for all $t > 0$. Moreover, it is possible to verify that

$$\lim_{\epsilon \rightarrow 0} \Upsilon(r + \epsilon; r, p, \lambda) = \Upsilon(w; r, p, \lambda) \Big|_{\tilde{w}^*(p)^+ = r^*} \quad \forall w > r;$$

$$\lim_{\epsilon \rightarrow 0} \Upsilon(r - \epsilon; r, p, \lambda) = \Upsilon(w; r, p, \lambda) \Big|_{\tilde{w}^*(p, \lambda)^- = r^*} \quad \forall w < r.$$

This implies that for all $r_0 \in [0, r_L)$, $r_j^* = r_L(p)$; and that for all $r_0 \in (r_H, \bar{r}]$, $r_j^* = r_H(p, \lambda)$. As such, for all $r_0 \in [0, r_L)$ the steady-state reference wage of incumbents is given by $r_j^* = \tilde{w}^*(r_0, p, \lambda)$, i.e. by $r_L^*(p) = \tilde{w}^*(p)^+$; while for all $r_0 \in (r_H, \bar{r}]$ it is given by $r_j^* = \tilde{w}^*(r_0, p, \lambda)$, i.e. by $r_H^*(p, \lambda) = \tilde{w}^*(p, \lambda)^-$. These results establish that for any given $r_0 \in [0, \bar{r}]$, and for all $t > 0$, $\tilde{w}^* = r_j^*$ which implies that the steady-state optimal effort exerted by incumbents is given by $\tilde{e}(\tilde{w}^*, r^*, \lambda) = \tilde{e}(r_j^*, r_j^*, \lambda) = \tilde{e}_n^*$. ■

Proof of Lemma 2. Following the definitions provided in the text and the results established by Proposition 3, the steady-state values of an employment relationship with an incumbent worker can be expressed as

$$\begin{aligned} J(r_j^*)^+ &= \frac{y(p, \tilde{e}_n^*) - \tilde{w}^*(p)^+}{1 - \psi} && \text{if } r_i < r_L(p); \\ J(r_j^*)^= &= \frac{y(p, \tilde{e}_n^*) - r_i^*}{1 - \psi} && \text{if } r_i \in [r_L, r_H]; \\ J(r_j^*)^- &= \frac{y(p, \tilde{e}_n^*) - \tilde{w}^*(p, \lambda)^-}{1 - \psi} && \text{if } r_i > r_H(p, \lambda). \end{aligned}$$

The corresponding steady-state values of an employment relationship with a new hire can therefore be expressed as

$$\begin{aligned} J(r_i^*)^+ &= y(p, \tilde{e}^*(\tilde{w}^*(p)^+, r_i^*)^+) + \frac{\psi}{1 - \psi} y(p, \tilde{e}_n^*) - \frac{\tilde{w}^*(p)^+}{1 - \psi} && \text{if } r_i < r_L(p); \\ J(r_i^*)^= &= \frac{y(p, \tilde{e}_n^*) - r_i^*}{1 - \psi} && \text{if } r_i \in [r_L, r_H]; \\ J(r_i^*)^- &= y(p, \tilde{e}^*(\tilde{w}^*(p, \lambda)^-, r_i^*)^-) + \frac{\psi}{1 - \psi} y(p, \tilde{e}_n^*) - \frac{\tilde{w}^*(p, \lambda)^-}{1 - \psi} && \text{if } r_i > r_H(p, \lambda). \end{aligned}$$

From these expressions it is now straightforward to conclude that $J(r_i^*)^+ > J(r_i^*)^- > J(r_i^*)^-$, since at the optimum $J(r_i^*)$ is increasing in \tilde{e}^* , and $\tilde{e}^*(\tilde{w}^*(p)^+, r_i^*)^+ > \tilde{e}_n^* > \tilde{e}^*(\tilde{w}^*(p, \lambda)^-, r_i^*, \lambda)^-$. Moreover, the same argument can be used to show that $J(r_i^*)^+ > J(r_j^*)^+, J(r_i^*)^- = J(r_j^*)^-$, and that $J(r_i^*)^- < J(r_j^*)^-$. ■

Proof of Proposition 5. Using the results established in Proposition 2 it is straightforward to show that

$$\frac{dW^*(\Gamma_0, p, \lambda)}{dp} = \int_0^{r_L(p)} \tilde{w}_p^*(p)^+ d\Gamma_0(r_i) + \int_{r_H(p, \lambda)}^{\bar{r}} \tilde{w}_p^*(p, \lambda)^- d\Gamma_0(r_i) > 0,$$

where the derivatives of the limits of the integral with respect to p cancel each other out since \tilde{w} is continuous in p . Moreover since $R_j^*(\Gamma_0, p, \lambda) = W^*(\Gamma_0, p, \lambda)$ it can be deduced that $R_j^*(\Gamma_0, p, \lambda)$ is also increasing in p . This, together with the results established in Theorem 1, enables to deduce that

$$\begin{aligned} \frac{dE_i^*(\Gamma_0, p, \lambda)}{dp} &= \int_0^{r_L(p)} \tilde{e}_w^*(\tilde{w}^*(p)^+, r_i^*)^+ \tilde{w}_p^*(p)^+ d\Gamma_0(r_i) \\ &\quad + \int_{r_H(p, \lambda)}^{\bar{r}} \tilde{e}_w^*(\tilde{w}^*(p, \lambda)^-, r_i^*, \lambda)^- \tilde{w}_p^*(p, \lambda)^- d\Gamma_0(r_i) > 0. \end{aligned}$$

Since $\tilde{\theta}^*(\Gamma_0, p, \lambda)$ is increasing in J and $u^*(\Gamma_0, p, \lambda)$ is decreasing in θ , to prove the statement of the proposition it suffices to prove that $\int_0^{\bar{r}} J(r_i^*) d\Gamma_0(r_i)$ is increasing in p . Using the expressions derived in the proof of Lemma 2 it is possible to show that the derivative of $\int_0^{\bar{r}} J(r_i^*) d\Gamma_0(r_i)$ with respect to p is (some functional arguments are omitted to ease notation)

$$\begin{aligned} \frac{d}{dp} \int_0^{\bar{r}} J(r_i^*) d\Gamma_0(r_i) &= \\ &\int_0^{r_L(p)} y_p(p, \tilde{e}^{*+}) + y_e \tilde{e}_w^{*+} \tilde{w}_p^{*+} + \frac{\psi}{1-\psi} y_p(p, \tilde{e}_n^*) - \frac{1}{1-\psi} \tilde{w}_p^{*+} d\Gamma_0(r_i) \\ &\quad + \int_{r_L(p)}^{r_H(p, \lambda)} \frac{y_p(p, \tilde{e}_n^*)}{1-\psi} d\Gamma_0(r_i) \\ &\quad + \int_{r_H(p, \lambda)}^{\bar{r}} y_p(p, \tilde{e}^{*-}) + y_e \tilde{e}_w^{*-} \tilde{w}_p^{*-} + \frac{\psi}{1-\psi} y_p(p, \tilde{e}_n^*) - \frac{1}{1-\psi} \tilde{w}_p^{*-} d\Gamma_0(r_i) \end{aligned}$$

where the derivatives with respect to the limits cancel each other out since J is continuous in w . Moreover, collecting \tilde{w}_p^{*+} and \tilde{w}_p^{*-} as common factors in the first and second line respectively,

and rearranging, yields:

$$\begin{aligned}
\frac{d}{dp} \int_0^{\bar{r}} J(r_i^*) d\Gamma_0(r_i) = & \\
& \int_0^{r_L(p)} y_p(p, \tilde{e}^{*+}) + \frac{\psi}{1-\psi} y_p(p, \tilde{e}_n^*) + \tilde{w}_p^{*+} \frac{1}{1-\psi} \{ [1-\psi] y_e \tilde{e}_w^{*+} - 1 \} d\Gamma_0(r_i) \\
& + \int_{r_L(p)}^{r_H(p, \lambda)} \frac{y_p(p, \tilde{e}_n^*)}{1-\psi} d\Gamma_0(r_i) \\
& + \int_{r_H(p, \lambda)}^{\bar{r}} y_p(p, \tilde{e}^{*-}) + \frac{\psi}{1-\psi} y_p(p, \tilde{e}_n^*) + \tilde{w}_p^{*-} \frac{1}{1-\psi} \{ [1-\psi] y_e \tilde{e}_w^{*-} - 1 \} d\Gamma_0(r_i) \quad (51)
\end{aligned}$$

where the expressions in the curly brackets are the equivalent of the first-order conditions (50) and (49), as expressed in (48), Proposition 1, for firms' optimal wage setting policy characterising $\tilde{w}^*(p)^+$ and $\tilde{w}^*(p, \lambda)^-$ respectively, and are therefore equal to zero. These deductions yield the following:

$$\begin{aligned}
\frac{d}{dp} \int_0^{\bar{r}} J(r_i^*) d\Gamma_0(r_i) = & \int_0^{r_L(p)} y_p(p, \tilde{e}^{*+}) + \frac{\psi}{1-\psi} y_p(p, \tilde{e}_n^*) d\Gamma_0(r_i) \\
& + \int_{r_L(p)}^{r_H(p, \lambda)} \frac{y_p(p, \tilde{e}_n^*)}{1-\psi} d\Gamma_0(r_i) + \int_{r_H(p, \lambda)}^{\bar{r}} y_p(p, \tilde{e}^{*-}) + \frac{\psi}{1-\psi} y_p(p, \tilde{e}_n^*) d\Gamma_0(r_i) > 0, \quad (52)
\end{aligned}$$

which implies that $\int_0^{\bar{r}} J(r_i^*) d\Gamma_0(r_i)$ is increasing in p . ■

Proof of Proposition 6. Using the results established in Proposition 2 it is straightforward to show that

$$\frac{dW^*(\Gamma_0, p, \lambda)}{d\lambda} = \int_{r_H(p, \lambda)}^{\bar{r}} \tilde{w}_\lambda(p, \lambda)^- d\Gamma_0(r_i) > 0,$$

Moreover since $R_j^*(\Gamma_0, p, \lambda) = W^*(\Gamma_0, p, \lambda)$ it can be deduced that $R_j^*(\Gamma_0, p, \lambda)$ is also increasing in λ . This, together with the results established in Theorem 1, enables to deduce that

$$\frac{dE_i^*(\Gamma_0, p, \lambda)}{d\lambda} = \int_{r_H(p, \lambda)}^{\bar{r}} \tilde{e}_w^*(\tilde{w}^*(p, \lambda)^-, r_i^*, \lambda)^- \tilde{w}_\lambda^*(p, \lambda)^- + \tilde{e}_\lambda^*(\tilde{w}^*(p, \lambda)^-, r_i^*, \lambda)^- d\Gamma_0(r_i),$$

which is ambiguous since the first term is positive but the second term is negative due to $\tilde{e}_\lambda^* < 0$ for all $r_i^* \in [r_H, \bar{r}]$.

Again, since $\tilde{\theta}^*(\Gamma_0, p, \lambda)$ is increasing in J and $u^*(\Gamma_0, p, \lambda)$ is decreasing in θ , to prove the statement of the proposition it suffices to prove that $\int_0^{\bar{r}} J(r_i^*) d\Gamma_0(r_i)$ is decreasing in λ . Using the expressions derived in the proof of Lemma 2 it is possible to show that the derivative of

$\int_0^{\bar{r}} J(r_i^*) d\Gamma_0(r_i)$ with respect to λ is

$$\frac{d}{d\lambda} \int_0^{\bar{r}} J(r_i^*) d\Gamma_0(r_i) = \int_{r_H(p,\lambda)}^{\bar{r}} y_e \tilde{e}_\lambda^{*-} + y_e \tilde{e}_w^{*-} \tilde{w}_\lambda^{*-} - \frac{1}{1-\psi} \tilde{w}_\lambda^{*-} d\Gamma_0(r_i).$$

Collecting \tilde{w}_λ^{*-} as the common factor and rearranging yields:

$$\begin{aligned} \frac{d}{d\lambda} \int_0^{\bar{r}} J(r_i^*) d\Gamma_0(r_i) &= \int_{r_H(p,\lambda)}^{\bar{r}} y_e \tilde{e}_\lambda^{*-} + \tilde{w}_\lambda^{*-} \frac{1}{1-\psi} \{ [1-\psi] y_e \tilde{e}_w^{*-} - 1 \} d\Gamma_0(r_i) \\ &= \int_{r_H(p,\lambda)}^{\bar{r}} y_e \tilde{e}_\lambda^{*-} d\Gamma_0(r_i) < 0; \end{aligned} \quad (53)$$

since the term in curly brackets is equivalent to the first-order condition as expressed in (48), Proposition 1, characterising $\tilde{w}^*(p, \lambda)^-$ and $\tilde{e}_\lambda^* < 0$ for all $r_i^* \in [r_H, \bar{r}]$ as established by Theorem 1. Hence, $\int_0^{\bar{r}} J(r_i^*) d\Gamma_0(r_i)$ is decreasing in λ . ■

Proof of Proposition 7. By the definition of first-order stochastic dominance, to prove the statement of the proposition it suffices to show that $W^*(\Gamma_0, p, \lambda)$ is increasing in r_i^* , and that $E_i^*(\Gamma_0, p, \lambda)$ and $\int_0^{\bar{r}} J(r_i^*) d\Gamma_0(r_i)$ are decreasing in r_i^* . These results are readily established by Proposition 2, Theorem 1 and Proposition 1 respectively. ■

Proof of Proposition 8. Using the expressions for the present discounted value of wages and output, the job creation condition of the model can be re-written as

$$\frac{\kappa}{h(\tilde{\theta}^*)} - \delta \left[\bar{Y}^*(E^*) - \bar{W}^* \right] = 0 \quad (54)$$

Total (implicit) differentiation of (54) with respect to p yields

$$\frac{d\tilde{\theta}^*}{dp} = - \frac{-\delta \left[\frac{d\bar{Y}^*(E^*)}{dp} - \frac{d\bar{W}^*}{dp} \right]}{-\kappa h(\tilde{\theta}^*) - 2h'(\tilde{\theta}^*)}$$

which implies that the elasticity of $\tilde{\theta}^*$ with respect to p is

$$\varepsilon_{\tilde{\theta}^*}^p = \frac{d\tilde{\theta}^*}{dp} \frac{p}{\tilde{\theta}^*} = \frac{\delta \left[\frac{d\bar{Y}^*(E^*)}{dp} - \frac{d\bar{W}^*}{dp} \right] \frac{p}{\tilde{\theta}^*}}{-h'(\tilde{\theta}^*) \frac{1}{h(\tilde{\theta}^*)} \frac{\kappa}{h(\tilde{\theta}^*)}} \quad (55)$$

Consider the denominator first. By appealing to the definition of the elasticity of the matching function with respect to unemployment σ , and by substituting for $\kappa/h(\tilde{\theta}^*)$ using the job creation condition given by (54), the denominator can be expressed as $\sigma\delta \left[\bar{Y}^*(E^*) - \bar{W}^* \right]$. Next consider the numerator, and notice that the term in square brackets is equivalent to $\frac{d}{dp} \int_0^{\bar{r}} J(r_i^*) d\Gamma_0(r_i)$ by definition. Hence,

$$\begin{aligned} \left[\frac{d\bar{Y}^*(E^*)}{dp} - \frac{d\bar{W}^*}{dp} \right] &= \frac{d}{dp} \int_0^{\bar{r}} J(r_i^*) d\Gamma_0(r_i) = \\ &\int_0^{r_L(p)} y_p(p, \tilde{e}^{*+}) + \frac{\psi}{1-\psi} y_p(p, \tilde{e}_n^*) + \frac{d\tilde{w}^{*+}}{dp} \frac{1}{1-\psi} \{ [1-\psi] y_e \tilde{e}_w^{*+} - 1 \} d\Gamma_0(r_i) \\ &\quad + \int_{r_L(p)}^{r_H(p,\lambda)} \frac{y_p(p, \tilde{e}_n^*)}{1-\psi} - \frac{d\tilde{w}^{*+}}{dp} \frac{1}{1-\psi} d\Gamma_0(r_i) \\ &\quad + \int_{r_H(p,\lambda)}^{\bar{r}} y_p(p, \tilde{e}^{*-}) + \frac{\psi}{1-\psi} y_p(p, \tilde{e}_n^*) + \frac{d\tilde{w}^{*-}}{dp} \frac{1}{1-\psi} \{ [1-\psi] y_e \tilde{e}_w^{*-} - 1 \} d\Gamma_0(r_i) \end{aligned}$$

as given in equation (51) and in which $\tilde{w}^{*+} = r_i^*$. Moreover, by the definition of elasticity, substitute $\frac{d\tilde{w}^*}{dp}$ with $\varepsilon_{\tilde{w}_p^*} \frac{\tilde{w}^*}{p}$ for all $r_i^* \in [0, \bar{r}]$. These manipulations lead to the following expression

$$\begin{aligned} \left[\frac{d\bar{Y}^*(E^*)}{dp} - \frac{d\bar{W}^*}{dp} \right] &= \\ &\int_0^{r_L(p)} y_p(p, \tilde{e}^{*+}) + \frac{\psi}{1-\psi} y_p(p, \tilde{e}_n^*) + \varepsilon_{\tilde{w}_p^{*+}} \frac{\tilde{w}^{*+}}{p} \frac{1}{(1-\psi)} \cdot \{ [1-\psi] y_e \tilde{e}_w^{*+} - 1 \} d\Gamma_0(r_i) \\ &\quad + \int_{r_L(p)}^{r_H(p,\lambda)} \frac{y_p(p, \tilde{e}_n^*)}{1-\psi} - \varepsilon_{\tilde{w}_p^{*+}} \frac{\tilde{w}^{*+}}{p} \frac{1}{(1-\psi)} d\Gamma_0(r_i) \\ &\quad + \int_{r_H(p,\lambda)}^{\bar{r}} y_p(p, \tilde{e}^{*-}) + \frac{\psi}{1-\psi} y_p(p, \tilde{e}_n^*) + \varepsilon_{\tilde{w}_p^{*-}} \frac{\tilde{w}^{*-}}{p} \frac{1}{(1-\psi)} \cdot \{ [1-\psi] y_e \tilde{e}_w^{*-} - 1 \} d\Gamma_0(r_i). \end{aligned}$$

Next, multiply $\left[\frac{d\bar{Y}^*(E^*)}{dp} - \frac{d\bar{W}^*}{dp} \right]$ by δp , as required by expression for the numerator of the elasticity of $\tilde{\theta}^*$ with respect to p as derived above (equation (55)), and define

$$\begin{aligned} \Lambda(\varepsilon_{\tilde{w}_p^*}, \bar{W}^*) &\equiv - \left(\int_0^{r_L(p)} \varepsilon_{\tilde{w}_p^{*+}} \frac{\tilde{w}^{*+}}{1-\psi} \cdot \{ [1-\psi] y_e \tilde{e}_w^{*+} - 1 \} d\Gamma_0(r_i) \right. \\ &\quad \left. - \int_{r_L(p)}^{r_H(p,\lambda)} \varepsilon_{\tilde{w}_p^{*+}} \frac{\tilde{w}^{*+}}{1-\psi} d\Gamma_0(r_i) + \int_{r_H(p,\lambda)}^{\bar{r}} \varepsilon_{\tilde{w}_p^{*-}} \frac{\tilde{w}^{*-}}{1-\psi} \cdot \{ [1-\psi] y_e \tilde{e}_w^{*-} - 1 \} d\Gamma_0(r_i) \right). \quad (56) \end{aligned}$$

Finally, using the definitions of the expected present discounted value of output and wages as defined in (24) and (25) respectively (recall that y is linear in p), the numerator takes the

following form

$$\delta p \left[\frac{d\bar{Y}^*(E^*)}{dp} - \frac{d\bar{W}^*}{dp} \right] = \delta \left[\bar{Y}^*(E^*) - \Lambda(\varepsilon_{\bar{w}_p^*}, \bar{W}^*) \right].$$

As such, by substituting the expressions for the numerator and denominator just derived into (55) it yields

$$\varepsilon_{\bar{w}_p^*} = \frac{1}{\sigma} \frac{\bar{Y}^*(E^*) - \Lambda(\varepsilon_{\bar{w}_p^*}, \bar{W}^*)}{\bar{Y}^*(E^*) - \bar{W}^*}.$$

■

Proof of Proposition 9. First notice that since $\bar{w}^{*=} = r_i^*$, for all $r_i^* \in [r_L, r_H]$ the elasticity of the wage with respect to p is zero, that is $\int_{r_L(p)}^{r_H(p,\lambda)} \varepsilon_{\bar{w}_p^=} d\Gamma_0(r_i) = 0$. Then, using expression (56) for $\Lambda(\varepsilon_{\bar{w}_p^*}, \bar{W}^*)$ as derived in the Proof of Proposition 8 above, notice that the terms in curly brackets are equivalent to the first-order conditions characterising the optimal wage policy for all $r_i^* \in [0, r_L]$ and $r_i^* \in [r_H, \bar{r}]$, as expressed by (48), Proposition 1. Hence, they are equal to zero and imply that $\Lambda(\varepsilon_{\bar{w}_p^*}, \bar{W}^*) = 0$ independently of the magnitudes of $\varepsilon_{\bar{w}_p^{*+}}$ and $\varepsilon_{\bar{w}_p^{*-}}$. ■

Proof of Proposition 10. Following the logic of the two-step optimisation problem (30–31), this proof is divided in two key parts. First, the optimal wage setting policy for incumbent workers j will be characterised; then, the results obtained will be used to characterise the optimal wage setting policy for new hires i .

Part 1.

Incumbent workers: preliminaries. Consider the functional equation corresponding to firms' optimisation problem when employing incumbent workers j :

$$J(r_j, q_j) = \max_{w \in \mathcal{W}} \{ \pi(r_j, w, q_j) + \psi J(w, q_j) \}.$$

Since this is analogous to the functional equation (7), the reader is referred to the preliminaries of the Proof of Proposition 2 for technical details regarding existence and uniqueness of solutions. Moreover, it has been established that $J(r_j, q_j)$ is concave. This implies that the following necessary first-order condition are also sufficient for the characterisation of a global maximum:

$$\Upsilon(w; r_j, p, q_j, \lambda) \equiv y_e(p, q_j, \bar{e}) \bar{e}_w(w, r_j, \lambda) - 1 + \psi y_e(p, q_j, \bar{e}') \bar{e}'_r(w', w, \lambda) = 0.$$

Moreover, $\Upsilon(w; r_j, p, q_j, \lambda)$ retains the same properties as the first order condition $\Upsilon(w; r, p, \lambda)$ analysed in the Proof of Proposition 2. That is, $\Upsilon_w < 0, \Upsilon_\lambda > 0$ if $w < r_j$ and $\Upsilon_\lambda = 0$ if $w > r_j$. Moreover, it is straightforward to show that $\Upsilon_q > 0$.

Incumbent workers: productivity thresholds. In contrast to the reference wage thresholds defined in the Proof of Proposition 2, it is useful here to characterise firms' optimal wage setting policy with respect to productivity thresholds on the distribution of match productivity q_j . As such, the threshold $q_l(r_j, \lambda)$ identifies the critical match productivity below which a firm would want to set the wage below the reference wage, and $q_u(r_j)$ is the match productivity above which a firm would want to compensate the worker more than the reference wage. The former is the value of q_j below which the value function is decreasing just below the reference wage; the latter is the value of q_j above which the value function is increasing just above the reference wage:

$$q_u(r_j) \equiv \{q_j : \lim_{\epsilon \rightarrow 0} \Upsilon(r_j + \epsilon; r_j, p, q_j, \lambda) = 0\};$$

$$q_l(r_j, \lambda) \equiv \{q_j : \lim_{\epsilon \rightarrow 0} \Upsilon(r_j - \epsilon; r_j, p, q_j, \lambda) = 0\}.$$

Using the results derived in the Proof of Proposition 2 it can be established that $q_u(r_j) > q_l(r_j, \lambda)$ and that if $\lambda = 1$ then $q_u(r_j) > q_l(r_j, 1)$.

Incumbent workers: optimal wage setting policy. The optimal wage setting policy characterising the wage of incumbent workers depends on the level of match productivity q_j in relation to the thresholds derived above.

If $q_j < q_l(r_j, \lambda)$, the definition of q_l and the concavity of the value function, along with the fact that $\Upsilon_q > 0$, can be used to deduce that the optimal wage must satisfy $w < r_j$, and therefore it will be the solution to

$$y_e(p, q_j, \tilde{e}) \tilde{e}_w(w, r_j, \lambda)^- - 1 + \psi y_e(p, q_j, \tilde{e}') \tilde{e}_r(w', w, \lambda)^- = 0,$$

which is denoted by $\tilde{w}(p, q_j, \lambda)^-$.

If $q_j > q_u(r_j)$, the definition of q_u and the concavity of the value function, along with the fact that $\Upsilon_q > 0$, can be used to deduce that the optimal wage must satisfy $w > r_j$, and therefore it will be the solution to

$$y_e(p, q_j, \tilde{e}) \tilde{e}_w(w, r_j)^+ - 1 + \psi y_e(p, q_j, \tilde{e}') \tilde{e}_r(w', w)^+ = 0,$$

which is denoted by $\tilde{w}(p, q_j)^+$.

Finally, if $q_j \in [q_l, q_u]$ by the same arguments above it is possible to deduce that $\Upsilon < 0$ for all $w > r_j$ and that $\Upsilon > 0$ for all $w < r_j$. Hence $J(r_j, q_j)$ is maximised if and only if the optimal wage policy is $w = r_j$.

Incumbent workers: expected continuation value. It is now possible to use these results to write analytical expressions for the expected continuation value of an employment relationship with an incumbent worker, for any realisation of q_j . By using notation consistent with the rest of the analysis, these can be expressed as follows:

$$\begin{aligned} J(r_j, q_j)^+ &= y(p, q_j, \tilde{e}(\tilde{w}(p, q_j)^+, r_j)^+) + \frac{\psi}{1-\psi} y(p, q_j, \tilde{e}_n) - \frac{\tilde{w}(p, q_j)^+}{1-\psi}; \\ J(r_j, q_j)^= &= \frac{y(p, q_j, \tilde{e}_n) - r_j}{1-\psi}; \\ J(r_j, q_j)^- &= y(p, q_j, \tilde{e}(\tilde{w}(p, q_j, \lambda)^-, r_j, \lambda)^-) + \frac{\psi}{1-\psi} y(p, q_j, \tilde{e}_n) - \frac{\tilde{w}(p, q_j, \lambda)^-}{1-\psi}. \end{aligned}$$

As such, the expected continuation value of an employment relationship with an incumbent worker can be written more compactly as:

$$\begin{aligned} \int J(r_j, q_j) Q(q_i, dq_j) &= \int^{q_l(r_j, \lambda)} J(r_j, q_j)^- Q(q_i, dq_j) \\ &\quad + \int_{q_l(r_j, \lambda)}^{q_u(r_j)} J(r_j, q_j)^= Q(q_i, dq_j) + \int_{q_u(r_j)} J(r_j, q_j)^+ Q(q_i, dq_j). \end{aligned} \quad (57)$$

Part 2.

Newly hired workers: preliminaries. The functional equation corresponding to firms' optimisation problem when employing newly hired workers i is

$$J(r_i, q_i) = \max_{w \in \mathcal{W}} \left\{ \pi(r_i, w, q_i) + \psi \int J(w, q_j) Q(q_i, dq_j) \right\},$$

where $\int J(w, q) Q(q_i, dq_j)$ is the expected continuation value of the employment relationship as expressed in (57), when new hires become incumbents. As such, there exists a unique solution to this functional equation and at least one optimal wage policy exists.

The first order necessary condition to characterise an optimum is

$$\Upsilon(w; r_i, p, q_i, \lambda) \equiv y_e(p, q_i, \tilde{e}) \tilde{e}_w(w, r_i, \lambda) - 1 + \psi \int J_r(w, q_j) Q(q_i, dq_j) = 0;$$

in which, since the value function $J(w, q_j)$ is continuous in all its arguments, the partial derivatives with respect to the limits of integration in $\int J_r(w, q_j) Q(q_i, dq_j)$ cancel out.

The related envelope condition, for any possible realisation of q_j , can be analytically obtained from (57):

$$\begin{aligned} J_r(r_j, q_j)^+ &= y_e(p, q_j, \tilde{e}) \tilde{e}_r(\tilde{w}(p, q_j)^+, r_j)^+ && \text{if } q_j > q_u(r_j); \\ J_r(r_j, q_j)^= &= -\frac{1}{1-\psi} && \text{if } q_j \in [q_l, q_u]; \\ J_r(r_j, q_j)^- &= y_e(p, q_j, \tilde{e}) \tilde{e}_r(\tilde{w}(p, q_j, \lambda)^-, r_j, \lambda)^- && \text{if } q_j < q_l(r_j, \lambda). \end{aligned}$$

To ease the notational burden, denote the optimal wage paid to incumbent workers $\tilde{w}(p, q_j)^+$ and $\tilde{w}(p, q_j, \lambda)^-$ as \tilde{w}_j^+ and \tilde{w}_j^- respectively; and the transition function $Q(q_i, dq_j)$ as dQ . Combining the envelope conditions with the first order condition yields:

$$\begin{aligned} \Upsilon(w; r_i, p, q_i, \lambda) \equiv & \\ & y_e(p, q_i, \tilde{e}) \tilde{e}_w(w, r_i, \lambda) - 1 + \psi \int^{q_l(w, \lambda)} y_e(p, q_j, \tilde{e}) \tilde{e}_r(\tilde{w}_j^-, w, \lambda)^- dQ \\ & - \psi \int_{q_l(w, \lambda)}^{q_u(w)} \frac{1}{1-\psi} dQ + \psi \int_{q_u(w)} y_e(p, q_j, \tilde{e}) \tilde{e}_r(\tilde{w}_j^+, w)^+ dQ = 0; \quad (58) \end{aligned}$$

which is the necessary condition for the characterisation of the optimal wage of newly hired workers. For this condition to be also sufficient, it is left to be shown that the value function $J(r_i, q_i)$ is concave, which is equivalent to show that $\Upsilon_w(w; r_i, p, q_i, \lambda) < 0$.

First notice that by appealing to the results established in the Proof of Theorem 1 and in Lemma 4, it is possible to show that $\tilde{e}_r(\cdot, r_j)^+ = -\tilde{e}_w(w, \cdot)^+$ and that $\tilde{e}_r(\cdot, r_j, \lambda)^- = -\lambda \tilde{e}_w(w, \cdot)^+$ since due to adaptation $r_j = w$ (recall that \tilde{e}_r is independent of its first argument and \tilde{e}_w is independent of its second argument). As such, we can re-write (58) as follows

$$\begin{aligned} y_e(p, q_i, \tilde{e}) \tilde{e}_w - 1 - \lambda \tilde{e}_w^+ \psi \int^{q_l(w, \lambda)} y_e(p, q_j, \tilde{e}) dQ \\ - \frac{\psi}{1-\psi} \int_{q_l(w, \lambda)}^{q_u(w)} dQ - \tilde{e}_w^+ \psi \int_{q_u(w)} y_e(p, q_j, \tilde{e}) dQ = 0. \end{aligned}$$

By collecting \tilde{e}_w^+ as the common factor, and after some algebra, this expression can be re-

arranged in the following useful way:

$$y_e(p, q_i, \tilde{e}) \frac{\tilde{e}_w}{\tilde{e}_w^+} + \frac{1}{\tilde{e}_w^+} \left[1 + \frac{\psi}{1 - \psi} \int_{q_l(w, \lambda)}^{q_u(w)} dQ \right] = \\ = \psi \left[\lambda \int^{q_l(w, \lambda)} y_e(p, q_j, \tilde{e}) dQ + \int_{q_u(w)} y_e(p, q_j, \tilde{e}) dQ \right]. \quad (59)$$

Next, the derivative of the first-order condition $\Upsilon(w; r_i, p, q_i, \lambda)$ with respect to w is

$$y_e(p, q_i, \tilde{e}) \tilde{e}_{ww} - \tilde{e}_{ww}^+ \lambda \psi \int_{q_l(w, \lambda)} y_e(p, q_j, \tilde{e}) dQ - \tilde{e}_{ww}^+ \psi \int_{q_u(w)} y_e(p, q_j, \tilde{e}) dQ$$

where again the derivative with respect to the limits cancel each other out (by the first-order condition characterising \tilde{w}_j). Then, after collecting \tilde{e}_{ww}^+ as the common factor and using the expression derived in (59) to substitute for the term in square brackets, and then again collecting $y_e(p, q_i, \tilde{e})$ as the common factor, this can be written as

$$y_e(p, q_i, \tilde{e}) \left[\tilde{e}_{ww} - \tilde{e}_w \frac{\tilde{e}_{ww}^+}{\tilde{e}_w^+} \right] + \frac{\tilde{e}_{ww}^+}{\tilde{e}_w^+} \left[1 + \frac{\psi}{1 - \psi} \int_{q_l(w, \lambda)}^{q_u(w)} dQ \right].$$

Now, notice that if $\tilde{e} = \tilde{e}^+$ in the first employment period, it implies that $\tilde{e}_w = \tilde{e}_w^+$ and that $\tilde{e}_{ww} = \tilde{e}_{ww}^+$; if $\tilde{e} = \tilde{e}^-$, it implies that $\tilde{e}_w = \lambda \tilde{e}_w^+$ and that $\tilde{e}_{ww} = \lambda \tilde{e}_{ww}^+$; and if $\tilde{e} = \tilde{e}_n$ then $\tilde{e}_w = 0$ and $\tilde{e}_{ww} = 0$. These considerations enable to deduce that the first term in the expression above is zero, and therefore that

$$\Upsilon_w(w; r_i, p, q_i, \lambda) = \frac{\tilde{e}_{ww}^+}{\tilde{e}_w^+} \left[1 + \frac{\psi}{1 - \psi} \int_{q_l(w, \lambda)}^{q_u(w)} dQ \right] < 0;$$

since $\tilde{e}_w^+ > 0$ and $\tilde{e}_{ww}^+ < 0$. This establishes concavity of the value function $J(r_i, q_i)$ and confirms that the first-order condition (58) is also sufficient for the identification of the unique maximum.

Newly hired workers: productivity threshold and optimal wage setting. The productivity thresholds characterising the optimal wage setting policy of firms employing newly hired workers can be defined analogously to those characterising the optimal wage of incumbents. However, note that since $r_i = 0$ by assumption U2, it is straightforward to verify that $\lim_{\epsilon \rightarrow 0} \Upsilon(r_i \pm \epsilon, r_i, p, q_i, \lambda) = 0$ for any given initial q_i , implying that $q_u(0) = q_l(0, \lambda) = 0$.

As such, firms employing newly hired workers would always want to pay them a wage above their reference wage, and the optimal wage must satisfy the first-order condition (58)

for $w > r_i$, and it is therefore denoted by $\tilde{w}(p, q_i, \lambda)^+ > r_i$. ■

Proof of Proposition 11. This proof follows the same steps and logic of the Proof of Proposition 8. Total(implicit) differentiation of the job creation condition

$$\frac{\kappa}{h(\tilde{\theta}^*(\lambda))} - \delta [\bar{Y}^*(\lambda) - \bar{W}^*(\lambda)] = 0$$

with respect to p yields

$$\frac{d\tilde{\theta}^*}{dp} = -\frac{-\delta \left[\frac{d\bar{Y}^*(\lambda)}{dp} - \frac{d\bar{W}^*(\lambda)}{dp} \right]}{-\kappa h(\tilde{\theta}^*)^{-2} h'(\tilde{\theta}^*)};$$

which implies that the elasticity of $\tilde{\theta}^*(\lambda)$ with respect to p is

$$\varepsilon_{\tilde{\theta}^*} = \frac{d\tilde{\theta}^*}{dp} \frac{p}{\tilde{\theta}^*} = \frac{\delta \left[\frac{d\bar{Y}^*(\lambda)}{dp} - \frac{d\bar{W}^*(\lambda)}{dp} \right]}{-h'(\tilde{\theta}^*)} \frac{1}{h(\tilde{\theta}^*)} \frac{\kappa}{h(\tilde{\theta}^*)} \frac{p}{\tilde{\theta}^*}. \quad (60)$$

By appealing to the definition of the elasticity of the matching function with respect to unemployment σ , and by substituting for $\kappa/h(\tilde{\theta}^*(\lambda))$ using the job creation condition above, the denominator can be expressed as $\sigma \delta [\bar{Y}^*(\lambda) - \bar{W}^*(\lambda)]$. Next consider the numerator

$$\delta p \left[\frac{d\bar{Y}^*(\lambda)}{dp} - \frac{d\bar{W}^*(\lambda)}{dp} \right],$$

and in particular the two terms in square brackets. From the definitions of $\bar{Y}^*(\lambda)$ and $\bar{W}^*(\lambda)$ as given by (37) and (36) respectively (functional arguments are omitted and notation will be slightly abused to reduce the notational burden):

$$\begin{aligned} \frac{d\bar{Y}^*(\lambda)}{dp} &= y_{i,p} + y_{i,e} \tilde{e}_{i,w}^{*+} \tilde{w}_{i,p}^{*+} + \psi \int^{q_l} y_{j,p} + y_{j,e} \tilde{e}_{j,w}^{*-} \tilde{w}_{j,p}^{*-} + y_{j,e} \tilde{e}_{j,r}^{*-} \tilde{w}_{i,p}^{*+} + \frac{\psi}{1-\psi} y_{j,p} dQ \\ &\quad + \psi \int_{q_l}^{q_u} \frac{1}{1-\psi} y_{j,p} dQ \\ &\quad + \psi \int_{q_u} y_{j,p} + y_{j,e} \tilde{e}_{j,w}^{*+} \tilde{w}_{j,p}^{*+} + y_{j,e} \tilde{e}_{j,r}^{*+} \tilde{w}_{i,p}^{*+} + \frac{\psi}{1-\psi} y_{j,p} dQ; \end{aligned}$$

$$\frac{d\bar{W}^*(\lambda)}{dp} = \tilde{w}_{i,p}^{*+} + \frac{\psi}{1-\psi} \int^{q_l} \tilde{w}_{j,p}^{*-} dQ + \frac{\psi}{1-\psi} \int_{q_l}^{q_u} \tilde{w}_{i,p}^{*+} dQ + \frac{\psi}{1-\psi} \int_{q_u} \tilde{w}_{j,p}^{*+} dQ.$$

Notice that in both expressions the derivatives with respect to the limits cancel each other out, by the continuity of $\bar{Y}^*(\lambda)$ and $\bar{W}^*(\lambda)$ with respect to q . Combining these two expressions together as in the numerator, and collecting the terms $\frac{\psi}{1-\psi}\tilde{w}_{j,p}^{*-}$, $\frac{\psi}{1-\psi}\tilde{w}_{j,p}^{*+}$ and $\tilde{w}_{i,p}^{*+}$ as common factors yields:

$$\begin{aligned} \frac{d\bar{Y}^*(\lambda)}{dp} - \frac{d\bar{W}^*(\lambda)}{dp} &= y_{i,p} + \int y_{j,p} + \frac{\psi}{1-\psi}y_{j,p} dQ \\ &\quad \tilde{w}_{i,p}^{*+} \left[y_{i,e}\tilde{e}_{i,w}^{*+} - 1 + \psi \int^{q_l} y_{j,e}\tilde{e}_{j,r}^{*-} dQ - \psi \int_{q_l}^{q_u} \frac{1}{1-\psi} dQ + \psi \int_{q_u} y_{j,e}\tilde{e}_{j,r}^{*+} dQ \right] \\ &\quad + \frac{\psi}{1-\psi}\tilde{w}_{j,p}^{*-} \int^{q_l} \left\{ [1-\psi]y_{j,e}\tilde{e}_{j,w}^{*-} - 1 \right\} dQ + \frac{\psi}{1-\psi}\tilde{w}_{j,p}^{*+} \int_{q_u} \left\{ [1-\psi]y_{j,e}\tilde{e}_{j,w}^{*+} - 1 \right\} dQ. \end{aligned}$$

It can now be noticed that the expression in square brackets in the second line is equivalent to the first-order condition characterising the optimal wage paid to new hires i , as given by (58), as established in the proof of Proposition 10; while the expressions in curly brackets in the third line are equivalent to the first-order conditions characterising the wage of incumbent workers as established in the proof of Proposition 10. As such, both the second and third line are equal to zero and the numerator in (60) is:

$$\delta p \left[\frac{d\bar{Y}^*(\lambda)}{dp} - \frac{d\bar{W}^*(\lambda)}{dp} \right] = \delta p \left[y_{i,p} + \int y_{j,p} + \frac{\psi}{1-\psi}y_p dQ \right] = \delta \bar{Y}^*(\lambda)$$

due to the linearity of y with respect to p . Hence, the elasticity expression (60) collapses to

$$\varepsilon_{\tilde{\theta}_p^*} = \frac{1}{\sigma} \frac{\bar{Y}^*(\lambda)}{\bar{Y}^*(\lambda) - \bar{W}^*(\lambda)}.$$

■

Proof of Proposition 12. Consider the steady-state wage paid to newly hired workers $\tilde{w}(p, q_i, \lambda)^+ > r_i$. Using the results established in the Proof of Proposition 10, implicit (partial) differentiation with respect to λ yields:

$$\tilde{w}_\lambda^*(p, q_i, \lambda)^+ = -\frac{\Upsilon_\lambda(w; r_i, p, q_i, \lambda)}{\Upsilon_w(w; r_i, p, q_i, \lambda)} = \frac{(\tilde{e}_w^+)^2 \psi \int^{q_l(w, \lambda)} y_e dQ}{\tilde{e}_{w\lambda}^+ \left[1 + \frac{\psi}{1-\psi} \int_{q_l(w, \lambda)}^{q_u(w)} dQ \right]} < 0,$$

since $\tilde{e}_{w\lambda}^+ < 0$ due to the concavity of the optimal effort function on \mathcal{W} .

Next consider the steady-state wage paid to incumbent workers $W^*(r_j^*, p, Q, \lambda)$. For any given r_j^* , using the results established in the Proof of Proposition 2, it is straightforward to conclude that

$$W_\lambda^*(r_j^*, p, Q, \lambda) = \int^{q_l(w, \lambda)} \tilde{w}_\lambda^*(p, q_j, \lambda)^- dQ > 0. \quad \blacksquare$$

Proof of Proposition 13. Total derivative of the elasticity of labour market tightness (38) with respect to λ yields

$$\frac{d\varepsilon_{\tilde{w}_p^*}}{d\lambda} = \frac{1}{\sigma} \frac{1}{\left[1 - \left[\overline{W}^*(\lambda)/\overline{Y}^*(\lambda)\right]\right]^2 \left[\overline{Y}^*(\lambda)\right]^2} \cdot \left[\frac{d\overline{W}^*(\lambda)}{d\lambda} \overline{Y}^*(\lambda) - \frac{d\overline{Y}^*(\lambda)}{d\lambda} \overline{W}^*(\lambda) \right].$$

The sign of this expression crucially depends on the sign of the second term in square brackets. From the results established in the Proof of Proposition 12 and from the expressions characterising the expected present values of wages and output, as given by (36) and (37) respectively, it is possible to establish that

$$\begin{aligned} \frac{d\overline{W}^*(\lambda)}{d\lambda} &= \tilde{w}_{i,\lambda}^{*+} + \frac{\psi}{1-\psi} \left[\int^{q_l(w, \lambda)} \tilde{w}_{j,\lambda}^{*-} - dQ + \int_{q_l(w, \lambda)}^{q_u(w)} \tilde{w}_{i,\lambda}^{*+} dQ \right] \\ &= \tilde{w}_{i,\lambda}^{*+} \left[1 - \frac{\psi}{1-\psi} \int_{q_l(w, \lambda)}^{q_u(w)} dQ \right] + \frac{\psi}{1-\psi} \int^{q_l(w, \lambda)} \tilde{w}_{j,\lambda}^{*-} dQ, \end{aligned}$$

where the second line follows from collecting $\tilde{w}_{i,\lambda}^{*+}$ as the common factor; and that

$$\begin{aligned} \frac{d\overline{Y}^*(\lambda)}{d\lambda} &= y_{i,e} \tilde{e}_{i,w}^{*+} \tilde{w}_{i,\lambda}^{*+} + \psi \int^{q_l} y_{j,e} \tilde{e}_{j,w}^{*+} \tilde{w}_{j,\lambda}^{*-} + y_{j,e} \tilde{e}_{j,r}^{*-} \tilde{w}_{i,\lambda}^{*+} + y_{j,e} \tilde{e}_{j,\lambda}^{*-} dQ + \psi \int_{q_u} y_{j,e} \tilde{e}_{j,r}^{*+} \tilde{w}_{i,\lambda}^{*+} dQ \\ &= \tilde{e}_{i,w}^{*+} \tilde{w}_{i,\lambda}^{*+} \left\{ y_{i,e} - \psi \left[\lambda \int^{q_l} y_{j,e} dQ + \int_{q_u} y_{j,e} dQ \right] \right\} + \psi \int^{q_l} y_{j,e} \tilde{e}_{j,w}^{*+} \tilde{w}_{j,\lambda}^{*-} + y_{j,e} \tilde{e}_{j,\lambda}^{*-} dQ \\ &= \tilde{w}_{i,\lambda}^{*+} \left[1 - \frac{\psi}{1-\psi} \int_{q_l(w, \lambda)}^{q_u(w)} dQ \right] + \psi \int^{q_l} y_{j,e} \tilde{e}_{j,w}^{*+} \tilde{w}_{j,\lambda}^{*-} dQ + \psi \int^{q_l} y_{j,e} \tilde{e}_{j,\lambda}^{*-} dQ, \end{aligned}$$

where the second line follows from the fact that $\tilde{e}_r^- = -\lambda \tilde{e}_w^+$, after collecting $\tilde{e}_{i,w}^{*+} \tilde{w}_{i,\lambda}^{*+}$ as the common factor, and the third line uses an algebraic manipulation of equation (59).

By combining these two expressions as required by the term in square brackets and collecting $\tilde{w}_{i,\lambda}^{*+} \left[1 - \frac{\psi}{1-\psi} \int_{q_l(w, \lambda)}^{q_u(w)} dQ \right]$ and $\frac{\psi}{1-\psi} \int^{q_l(w, \lambda)} \tilde{w}_{j,\lambda}^{*-} dQ$ as common factors yields expression (39) in the body of the paper. The statement of the proposition is then readily verified given the results established by Proposition 12. \blacksquare

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