Structure of High Frequency Green's Function in Non-Axisymmetric (Chevron-type) Transversely Sheared Mean Flows using a Ray Tracing Solver within the Generalized Acoustic Analogy Formulation

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Introduction

The chevron nozzle continues to remain a popular approach to reducing jet noise, which works by breaking up the turbulence structures at all scales. As indicated in Figure 1, high frequency sound waves are produced by the fine-scale turbulence structures and are emitted at a larger angle from the jet axis than low frequency waves meaning they are the greatest risk to health for airfield employees.

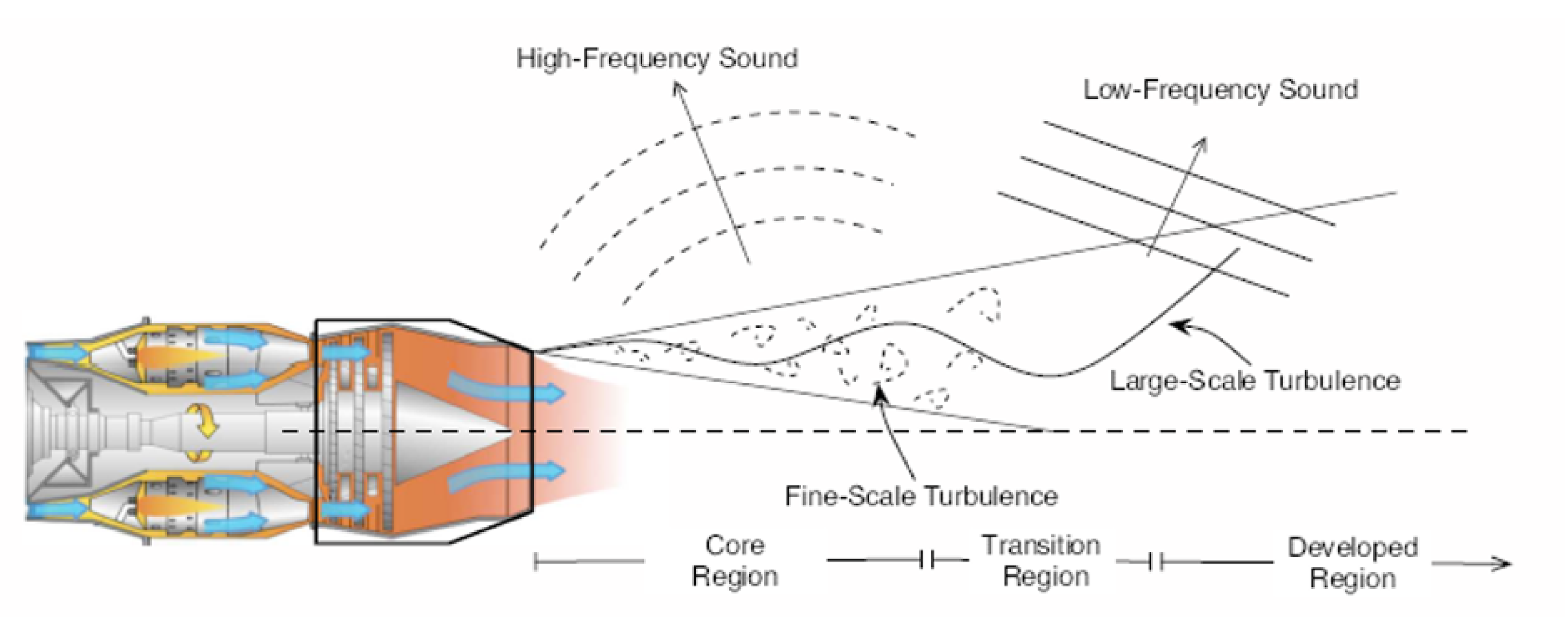

Figure 1: Regions of the jet and turbulence structures (Gudmundsson, 2010)

High frequency noise can be studied using ray theory, and here a previously developed high frequency ray tracing approach was applied to the case of a chevron with multiple lobes. The aim was to numerically evaluate the scaled Green's function to see how it was affected by increasing the number of lobes, and see how this could reduce jet noise.

Method

Goldstein's generalised acoustic analogy shows that the acoustic pressure at the far field limit is given by the convolution product of a rank 2 tensor propagator and the fluctuating Reynolds Stress:

$$p(\boldsymbol{x},\omega) = \int_{V_\infty(\boldsymbol{y})} \hat{G}_{ij}(\boldsymbol{y}|\boldsymbol{x};\omega)\hat{T}_{ij}(\boldsymbol{y},\omega)d^3y$$

where $\hat{G}_{ij}$ (the propagator tensor) is related to the vector adjoint Green's function of the linearised Euler operator. The "analogy" assumes that $\hat{T}_{ij}(\boldsymbol{y},\omega)$ is a known localised radiator of sound through its autocovariance. The propagator tensor is linearly related to the Rayleigh equation:

$$L_R\hat{G}_0(\boldsymbol{y}_T|\boldsymbol{x}_T;\omega) = \delta(\boldsymbol{y}_T - \boldsymbol{x}_T)$$

The temporal-streamwise Fourier transform of which, reduces to Sturm-Louville equation (Goldstein, 2003):

$$\left[\frac{\partial}{\partial \boldsymbol{y}_T}\Big(A(\boldsymbol{y}_T|\boldsymbol{x}_T;\omega)\frac{\partial}{\partial \boldsymbol{y}_T}\Big) + k_\infty^2 q^2(\boldsymbol{y}_T|\boldsymbol{x}_T;\omega)\right]\tilde{G}_0 = \delta(\boldsymbol{y}_T - \boldsymbol{x}_T)$$

We can use a ray theory ansatz (in which far-field wavenumber, $k_\infty >> O(1)$) to determine the solution of the Green's function problem. This is done using a previously developed code sourced from (Leib & Goldstein, 2001). The code calculates the ray trajectories by solving a set of differential equations ((Leib & Goldstein, 2001) equations 10,11,12) using the Runge-Kutta (RK4) method. It then calculates the scaled Green's function.

Results

Our results show that the chevron jet introduces a richer structure in the scaled Green's function (i.e. more non-periodic modulation). Figure 2 shows the mean flow used for several number of lobes (2n). Figure 3 shows the resulting scaled Green's function and compares to that for a round jet (which would be produced by a straight nozzle) where $n = 0$.

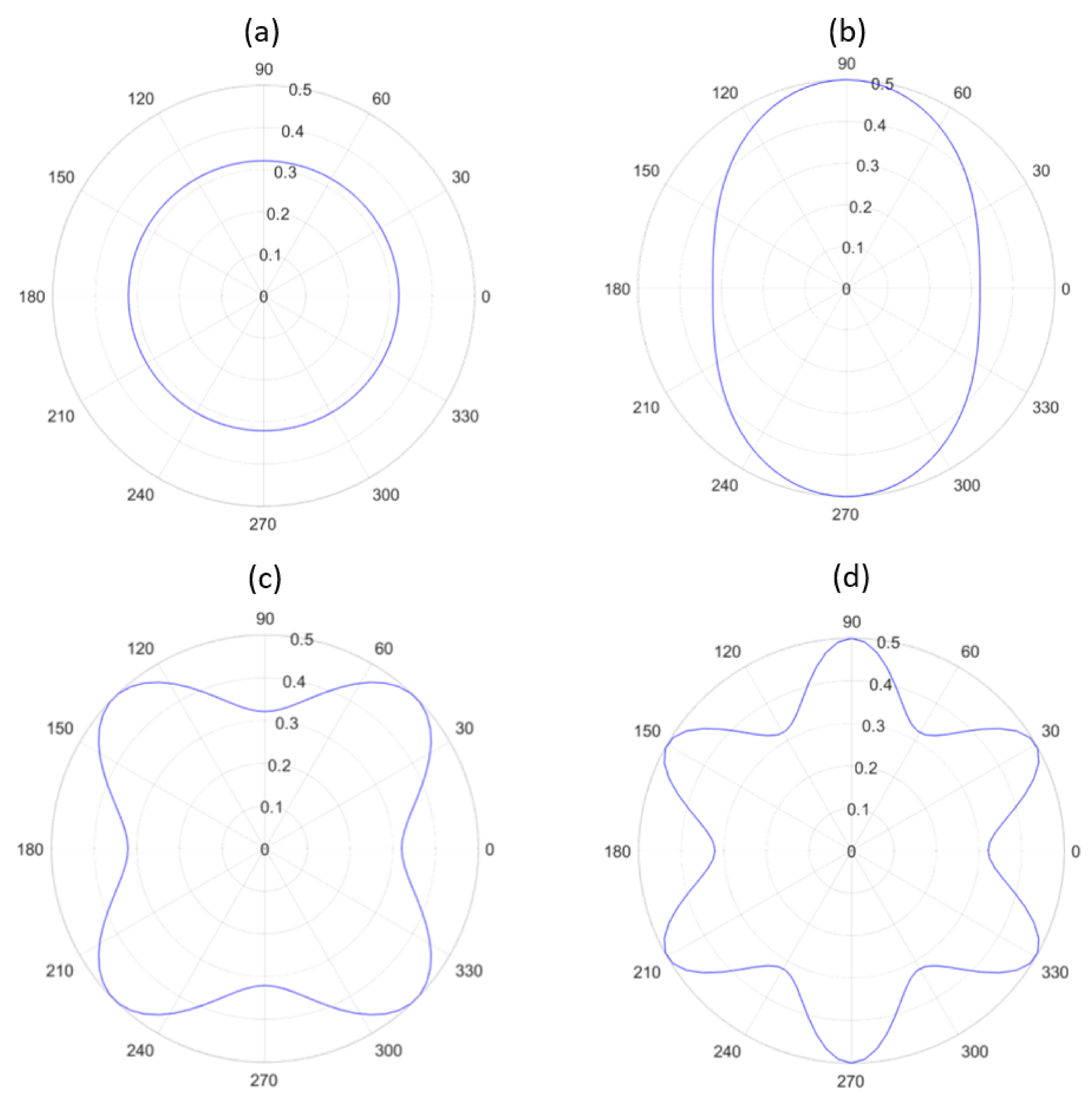

Figure 1: Meanflow profile with constant r=0.8 and varying the number of chevron lobes (a) $n = 0.0$ (b) $n = 1.0$ (c) $n = 2.0$ (d) $n = 3.0$

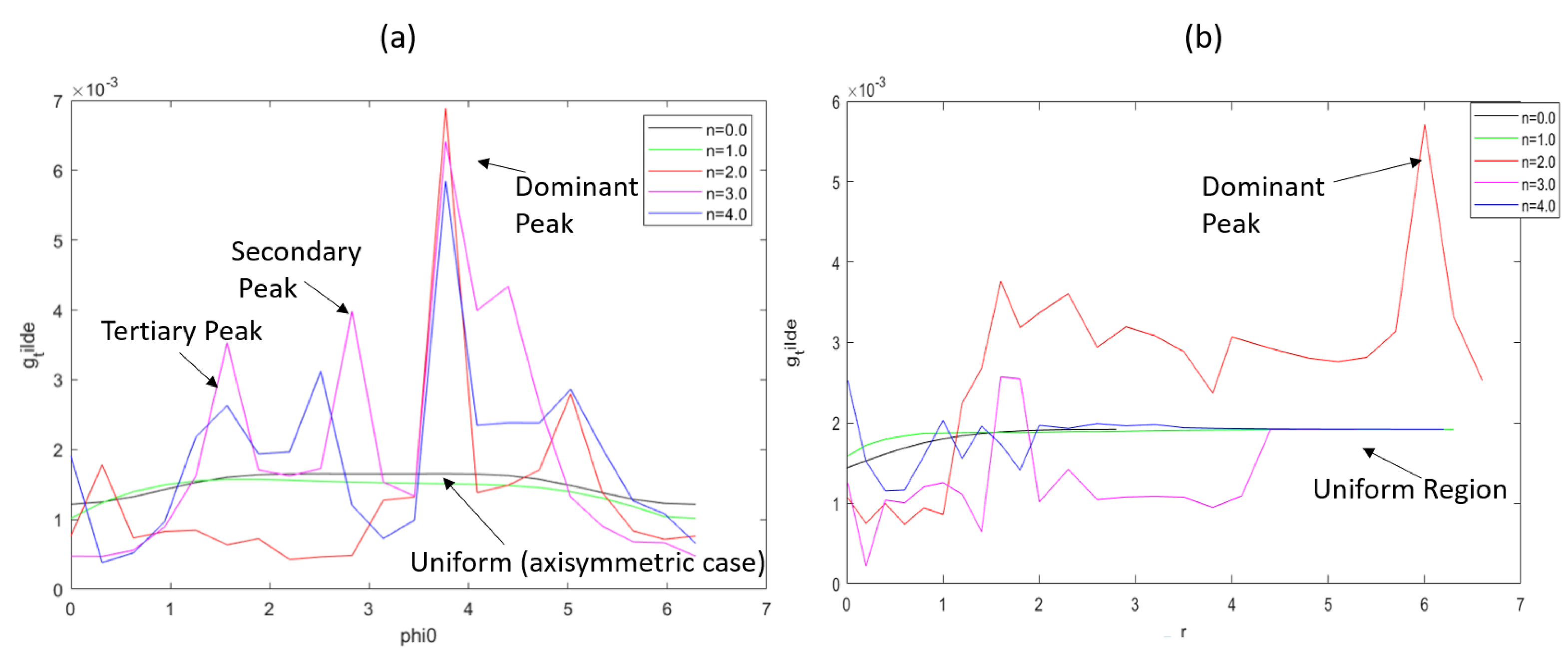

Figure 2: Results for the scaled Green's function varying the number of chevron lobes (2n) with (a) ϕ and (b) r

Conclusions

- The chevron jet resulted in a jagged scaled Green's function with several peaks and troughs
- If the peak turbulence is moved to the troughs (via an actuator, synthetic jet blowing etc.) there would be a decrease in jet noise.
- It is not possible to determine which chevron jet could result in the largest noise reduction as the scaled Green's function varies a lot with different source locations (r, ϕ). Different number of lobe are better in different areas.

References

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