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**CO-MOVEMENTS IN REAL EFFECTIVE EXCHANGE RATES:  
EVIDENCE FROM THE DYNAMIC HIERARCHICAL FACTOR  
MODEL**

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# Co-movements in Real Effective Exchange Rates: Evidence from the Dynamic Hierarchical Factor Model

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## Abstract

We analyze and quantify co-movements in real effective exchange rates while considering the regional location of countries. More specifically, using the dynamic hierarchical factor model (Moench et al. (2011)), we decompose exchange rate movements into several latent components; worldwide and two regional factors as well as country-specific elements. Then, we provide evidence that the worldwide common factor is closely related to monetary policies in large advanced countries while regional common factors tend to be captured by those in the rest of the countries in a region. However, a substantial proportion of the variation in the real exchange rates is reported to be country-specific; even in Europe country-specific movements exceed worldwide and regional common factors.

**JEL classification:** F31

**Keywords:** Real effective exchange rates, dynamic hierarchical factor model, variance decomposition, Bayesian model averaging

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# 1 Introduction

In the past, some studies have been carried out on co-movements in exchange rates. The co-movements, which can be measured by the sensitivity of one currency to another in regression analysis or the simple correlation coefficient, are important since changes in one currency indeed often affect the currency of other countries (e.g., McKinnon and Schnabl (2003)) particularly for those implementing a flexible exchange rate regime. Furthermore, currency interdependence has been examined in the context of inferring actual exchange rate regimes which may be deviating from officially announced ones (e.g., Frankel and Wei (2008)).

The co-movements of exchange rates are also underlined during financial crisis periods; deterioration in one's currency value almost simultaneously affecting others by, for example, speculative attacks (e.g., Gerlach and Smets (1995), Masson (1998)). Such an effect is often called contagion in academic literature, and has been increasingly prominent over recent years when a series of financial crises have affected the world economy. Recent examples include the 1997 Asian crisis which erupted in Thailand, the Lehman Shock (2008) in the US, and the European sovereign debt crisis which started in Greece (2009). Each lead not only its own economy but also its regional and/or the world economy into recession.

Generally, co-movements in financial asset prices and returns are shown to be time-varying and increase during financial crises (see the next section), and as theoretical explanations, two categories of propagation channels affecting other countries have been identified (see Forbes and Rigobon (2001)). The first group is closely linked with economic fundamentals and tranquil periods, and considers international trade, common economic policies, learning behaviors of investors and global shocks as relevant propagation channels. The second group of channels is more closely associated with crisis periods and is characterized by multiple equilibria in investors' expectations, endogenous liquidity shocks and political influence over other countries.

However, previous studies have been carried out with rather different approaches and focuses. Such heterogeneity in research arises from different definitions of co-movements and different approaches to measuring them.<sup>1</sup> Indeed, there are many terminologies similar to co-movements such as interdependence, cross country linkages, spillovers and contagion (see Forbes and Rigobon (2001), Pesaran and Pick (2007)): while contagion is specific to financial crises and the interdependence to tranquil periods, the rest are often used to refer to cross country move-

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<sup>1</sup>See for example Forbes and Rigobon (2001) regarding the definitions of financial contagion.

ments during any time period.<sup>2</sup> Similarly, there are several approaches to analyze co-movements, e.g., by looking at the correlation between exchange rates, the probability of speculative attacks and the propagation of volatility.

In addition, given that a country normally has multiple trading partners, this effect should be analyzed in a multi-country setting. However, a bilateral nominal exchange rate, often vis-à-vis the US dollar, dominates the exchange rate literature (e.g., Macdonald and Taylor (1992) and the next section), rather than a real effective exchange rate which is also important, e.g., as an indicator of the international competitiveness of a country.

Against this background, this paper analyzes and quantifies co-movements in real effective exchange rates, mainly for advanced countries, while making use of information on the regional location of countries. Unlike previous contagion studies which focused solely on spillovers during crises, this paper will calculate and evaluate the size and evolution of spillovers, without making a clear distinction between crisis and non-crisis periods.<sup>3</sup> Therefore, here co-movements are synonymous with cross country linkages and spillovers, and in this context, we attempt to estimate worldwide and regional common factors by using an advanced statistical method and identify countries which are influential over other exchange rates.

Thus the distinguishing features of this paper are: 1) decomposition of the real effective exchange rates of 30 countries into several factors (hierarchies) by applying the recently developed statistical method, the dynamic hierarchical factor model (Moench et al. (2011)), 2) quantification of the contribution of each hierarchy to the total variation in exchange rates, and 3) identification of countries who are influential over other countries' exchange rates. However, unlike contagion studies, this paper does not emphasize the direction of causality from one country to another.

The paper consists of 5 sections. In the next section, we summarize previous studies focusing on statistical methods which can be used to estimate co-movements in exchange rates. Section 3 describes the dynamic hierarchical factor model which have recently been proposed by Moench et al. (2011)). Section 4 describes the data and presents our estimates of co-movements in exchange rates from the dynamic hierarchical factor model; furthermore, this section investigates the driving forces of co-movements by means of the Bayesian Model Averaging (BMA) method.

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<sup>2</sup>For example, Pesaran and Pick (2007) make a clear distinction between contagion and interdependence; the latter referring to spillover effects during the tranquil period.

<sup>3</sup>This circumvents making a priori assumptions about crisis periods which are often required in contagion studies.

This paper ends with our conclusion in Section 5.

## 2 Review of Measuring the Co-movements

Co-movements in financial asset prices have frequently been studied in the context of the evaluation of financial crises and the international capital market integration, and equity returns rather than exchange rates have been the main focus in previous studies. Therefore, these studies have profound implications for building a financial portfolio, e.g., see the capital asset pricing model. However, the same concept of co-movements and statistical approach can be applied to exchange rate analysis; therefore, we shall review previous literature by focusing on the statistical methodologies used to measure them for a variety of financial assets. Such academic literature seems to have employed one of the following approaches.

The traditional and probably most popular approach is to use correlation measures between international stock returns, and increased correlation is regarded as evidence of increased cross-country linkages. This can be carried out either by simply calculating correlation coefficients among stock returns or estimating the stock return equation of one country with other countries' stock returns as explanatory variables. Classic studies include Makridakis and Wheelwright (1974) that showed unstable interrelationships in the major stock exchanges of the world. King and Wadhvani (1990) reported increased correlation at the time of the October 1987 crash. A similar result has been reported for more recent crises by a number of researchers (Longin and Solnik (1995), Liu et al. (1998), Reinhart and Calvo (1996), Bayoumi et al. (2007)). However, there are potential problems. Obviously, the regression based approach requires an exogeneity assumption about explanatory variables. But it may be difficult to justify this assumption using volatile financial asset data. Furthermore, Forbes and Rigobon (2002) argue that the regression analysis should take into account market volatility which differs during crisis and non-crisis periods, and they conclude that once this effect is considered, no evidence of contagion is found in Black Monday (1987), the Mexican currency crisis (1994) or the Asian crisis (1997).

Second, co-movements can be estimated using the factor model or the principal components approach. The factor model is often used to distinguish between worldwide and country-specific elements, and according to this approach, increases in the proportion of the worldwide factor become evidence of higher cross-country linkages (Koedijk and Schotman (1989), Dungey (1999), Cayen et al. (2010)). The commonality in data can also be estimated by the principal components

approach. For example, Nellis (1982) analyzed financial market integration using corporate and government bonds with the expectation that their yields will be dominated by common factors in a highly integrated financial market. Similarly, Volosovych (2013) studied financial market integration utilizing government bond yields from 1875 to 2009 and provided evidence of a higher integration from the data through the end of the 20th century. These traditional approaches often hinge upon the assumption of the stationarity of data. In this regard, Eickmeier (2009) and Byrne and Nagayasu (2012) used a nonstationary dynamic factor model (Bai and Ng (2004)) to highlight homogeneity in European financial markets.

However, the number of countries under investigation from these methods seems to be rather limited, and there is a strong tendency for research to focus on advanced countries. Using the factor model, Koedijk and Schotman (1989) studied the exchange rates of 15 industrialized countries, Dungey (1999) analyzed 6 Pacific Rim currencies and Cayen et al. (2010) dealt with 6 countries against the US dollar. Using the principal components method, Nellis (1982) employed the bond yields of 5 advanced countries, and Volosovych (2012) studied the government bond yields of 15 industrialized countries. Using a nonstationary factor model, Eickmeier (2009) considered 7 core Euro area countries and Byrne and Nagayasu (2012) focused on 11 emerging European countries.

The limited coverage of countries seems to be due to several reasons. Obviously, data availability is one issue for deciding the number of countries under investigation, at least in the past. However, more countries now exist in the world; especially, the number of countries has increased after the breakdown of socialist countries, and today there are a number of countries who have disseminated a reasonably long history of data. Second, the economic interpretation of commonality may become more difficult with a large data set because the number of common factors often increases along with an expansion in country coverage. This also may have become a reason for the analysis of small data sets.

However, over the last decade much progress has been made in estimating multiple commonality in large data sets, especially in the area of studies on business cycles and general commodity (not financial asset) inflation. Using a modified factor model, Forni et al. (2000), Kose et al. (2003) and Foerster et al. (2011) analyzed international or domestic business cycles for a large number of countries or economic sectors. The recent relevant studies on inflation includes Bernanke et al. (2005), Canova and Ciccarelli (2009) and Mumtaz and Surico (2012).

Due to the increased complexity of the model, it has become standard to estimate the model using the Bayesian estimation technique.

This study follows the second strand of the literature (i.e., the factor model) by extending it in a number of ways. In particular, rather than a two level classification, we consider four level hierarchies in our estimation. This is an essential modification in order to study many countries with a number of economically interpretable common factors. The increased complexity of the model cannot be estimated by conventional statistical methods and thus will be executed by the Bayesian method.

### 3 Econometric Method: The Factor Model

Correlation between economic variables has been of interest to economists since economic events are often highly correlated with one another. As reviewed in previous studies, classic studies used the factor model to extract commonality among a panel of stationary variables. One recent extension is to decompose data into common and idiosyncratic factors (i.e., a two level decomposition) in a nonstationary environment (Bai and Ng (2004)). Another extension is Moench et al. (2011) that has proposed the four level hierarchical factor model for a panel of stationary variables. The latter is also attractive in the presence of multiple common factors. By imposing extra information when grouping countries into hierarchies, this model facilitates researchers in identifying and interpreting each factor. Furthermore, they argue that this extra information results in reduction in computational burdens compared with Kose et al. (2003) who also proposed the Bayesian method to decompose a large data set. Here we make use of geographical information since adjacent countries tend to possess similar cultures and economic structures, to be very important trade partners, and to share common trading hours through which investors receive information simultaneously (see e.g., Goldstein (1998))

Following closely the notation used in Moench et al. (2011), this four level decomposition of a vector  $Z_{bsnt}$  can be written, from low to high level, as:

$$Z_{bsnt} = \Lambda_{Hbsn}H_{bst} + e_{Zbsnt} \quad (1)$$

$$H_{bst} = \Lambda_{Gbs}G_{bt} + e_{Hbst} \quad (2)$$

$$G_{bt} = \Lambda_{Fb}F_t + e_{Gbt} \quad (3)$$

where the Greek letters are loadings, and  $t$  is time ( $t = 1, \dots, T$ ), and components which cannot be explained by the factors ( $F$ ,  $G$  and  $H$ ) are treated as residuals ( $e$ ). Thus, we assume that the universal factors affect more regional factors; the worldwide factor ( $F$ ) affects regional factors ( $G$ ), and  $G$  influences  $H$  which is more region specific.

In Eqs 1 to 3, the lowest (i.e., individual) level classification is characterized as  $Z_{bsnt}$  containing individually the exchange rates of all countries (Eq. 1). Each exchange rate movement is decomposed to a regional level common factor  $H_{bst}$  and country-specific elements  $e_{Zbsnt}$ . Since the latter do not have a common factor, country-specific elements are assumed to be independent across countries.

In our analysis, there are two levels of regional classification (Levels 2 and 3, Table 1). Level 2 consists of 4 regional factors; namely  $H_{11t}$  for the Euro area,  $H_{12t}$  for the non-Euro European area,  $H_{21t}$  for the Asia-Pacific area, and  $H_{22t}$  for the American area, (i.e.,  $b = 1, 2$  and  $s = 1, 2$ ). This classification is based largely on geographical information. But a distinction is made between Euro zone members and non-members, since within the single currency area nominal exchange rates are identical among member countries, and thus a high level of correlation is expected among them.<sup>4</sup>

Another regional classification (Level 3) is based on whether or not countries are in Europe; European factors are shown as  $G_{1t}$ , and non-Europeans as  $G_{2t}$  ( $b = 1, 2$ ). These factors affect the second level regional factors. In turn, former regional factors ( $G_{1t}$  and  $G_{2t}$ ) are influenced by a worldwide commonality ( $F$ ) (Eq. 3). Here, one common factor is assumed to exist in each group (i.e.,  $G_1, G_2, H_{11}, H_{12}, H_{21}, H_{22}$  and  $F$ ) in our model.<sup>5</sup>

For the estimation, each factor is assumed to be stationary and is in the form of the first-order autoregression.

$$\begin{aligned}
 F_t &= \rho_F F_{t-1} + \varepsilon_{Ft}, \varepsilon_{Ft} \sim N(0, \sigma_F^2) \\
 e_{Gbt} &= \rho_{Gb} e_{Gbt-1} + \varepsilon_{Gbt}, \varepsilon_{Gbt} \sim N(0, \sigma_{Gb}^2) \\
 e_{Hbst} &= \rho_{Hbs} e_{Hbst-1} + \varepsilon_{Hbst}, \varepsilon_{Hbst} \sim N(0, \sigma_{Hbs}^2) \\
 e_{Zbsnt} &= \rho_{Zbsn} e_{Zbsnt-1} + \varepsilon_{Zbsnt}, \varepsilon_{Zbsnt} \sim N(0, \sigma_{Zbsn}^2)
 \end{aligned}$$

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<sup>4</sup>Our classification method may be ad hoc but is largely consistent with Koedijk and Schotman (1989).

<sup>5</sup>While not reported here, assumption of the presence of common factors is consistent with evidence from the statistical tests (e.g., Bai and Ng (2007)).



The abovementioned model is estimated using the Markov Chain Monte Carlo (MCMC) method. We follow the assumption about the prior distribution used in Moench et al. (2011); for example, the prior distribution of parameters are assumed to be standard normal,  $\Lambda, \rho \sim N(0, 1)$ , and that of variance an inverse of the scaled  $\chi^2$  distribution.<sup>6</sup> The initial values for latent variables are estimated by the principle components approach. In order to obtain reliable estimates, the first 50,000 out of our 100,000 draws are discarded, and every 50th observation from the remaining 50,000 draws is used for the analysis. This leaves us 1,000 draws for each parameter at each point in time. Furthermore, convergence diagnostics are calculated based on the Geweke test, but are not reported here for the sake of brevity.

## 4 Empirical results

### 4.1 Estimation of Common Factors

Real effective exchange rate data are obtained from the International Financial Statistics (IFS) of the International Monetary Fund. They (IFS code..REUZF, 2005=100) are constructed using the consumer price index and weights determined by the size of trade (unit values) to each trading partner, and cover the sample period from 1980Q1 to 2012Q2 for 30 countries, most of which are advanced ones (see Table 1).<sup>7</sup> These countries are classified as Group 1 (12 Euro countries), Group 2 (10 non-Euro European countries), Group 3 (4 Asia-Pacific countries) and Group 4 (4 American countries). In the subsequent analysis, we analyze exchange rate growth, i.e., the first difference of log exchange rates ( $\text{Log}(S_t/S_{t-1}) \times 100$ ), in order to be congruent with a priori assumption of the data required for the factor model.

The basic statistics are summarized in Table 2, and the conventional correlations (the highest- and lowest-40) of exchange rate pairs are listed in Table 3. The sign of the average exchange rate suggests that the direction of exchange rate movements is quite diversified; half of the countries have experienced an exchange rate increase (Table 2). Furthermore, among them, the

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<sup>6</sup>Our estimation is based on the Matlab codes of Professor Ng which are disseminated on her homepage. Details of assumptions about the prior distributions are stated there. The scale factor for the scaled inverse  $\chi^2$  is set as 0.01.

<sup>7</sup>Mexico and Costa Rica are not categorized as advanced countries according to the IMF classification (as of this writing). They are included for analysis because Mexico is part of the North American Free Trade Agreement (NAFTA). Inclusion of Costa Rica is for computational reasons. We failed to obtain results from the dynamic hierarchical factor model using only NAFTA countries.

Polish Zloty rate has experienced high volatility. The volatility can be measured by the standard deviation and is closely associated with acceleration in domestic inflation from the late 1980s to the early 1990s. Otherwise, other exchange rates appear quite comparable.

In addition, as expected, high correlation is obtained from a pair of Euro member countries, particularly Germany and the Netherlands (Table 3). In contrast, the lowest correlation is obtained between Greece and Singapore, countries from different geographical groups. As in the classic principle component approach, all the data are standardized to have zero mean and variance equal to one before we implement the dynamic hierarchical factor model.

Figures 1 to 3 plot estimates of the common factors from the dynamic hierarchical factor models, where all 1,000 observations are shown for each time period. They suggest that these factors are stationary regardless of the level of hierarchies, and the non-European common factor ( $G_2$ ) which includes the US, exhibits sharper changes around 2008/09 than the European factor ( $G_1$ ) in response to adverse effects from the Lehman Shock. Furthermore, the European factor ( $G_1$ ) is more tightly distributed than the non-European factor, suggesting less uncertainty about the estimates for Europe and homogeneity of their exchange rates. Apart from these points, it is very difficult to draw any concrete conclusions from these graphs.

Therefore, in order to understand better these common factors, we carry out the variance decomposition analysis using the common factors which are equal to the average values in Figures 1 to 3 at each point in time. The total variance of exchange rates  $Var(Z_{bsn})$  can be decomposed to:

$$Var(Z_{bsn}) = \gamma_F(Var(F)) + \gamma_G(Var(e_{Gb})) + \gamma_H(Var(e_{Hbs})) + \gamma_G(Var(e_{Zbsn})) \quad (4)$$

where  $\gamma$  is a composite of parameters,  $\Lambda$  in Eq. (1) to (3), and the following ratios are presented in Table 4

$$\begin{aligned} & \gamma_F(Var(F))/Var(Z_{bsn}) \\ & \gamma_G(Var(e_{Gb}))/Var(Z_{bsn}) \\ & \gamma_H(Var(e_{Hbs}))/Var(Z_{bsn}) \\ & \gamma_G(Var(e_{Zbsn}))/Var(Z_{bsn}) \end{aligned} \quad (5)$$

From this table, we can report several findings. First, there is a clear difference between country groups in the contribution of each factor to the total variation of exchange rates, but a

country-specific element is generally most significant (71% on average, Table 4). This element is more important for a group which contains countries with a heterogeneous background. In this regard, the country-specific variation is least important among the Euro members (Group 1) but still accounts for nearly 60% of the total variation.

There may be two reasons for this outcome. One obvious reason is that our observations include the pre-Euro period. Prior to 1999, there is heterogeneity in nominal exchange rates of euro candidate countries although they had been making efforts to meet the convergence criteria. Second, given that their nominal rates are identical after the introduction of the Euro, heterogeneity in prices and trading partners seem to be significantly different among Euro member countries. This implies the presence of a diversified external competitiveness within the Euro area.

With respect to other regions, the non-Euro European and Asia-Pacific groups have exhibited a similar proportion of country-specific effects (over 80%). This outcome for non-Euro European countries may be due to their heterogeneities among countries; this group consists of member and non-member states of the European Union (EU) as well as countries which have recently joined the Euro zone (Table 1).

Table 4 also shows that country-specific effects are relatively low in American countries (Group 4) although they are slightly higher than the level of Group 1. These results likely reflect the fact that 3 countries (the USA, Canada and Mexico) in this group form NAFTA, leading their markets to become more homogeneous.

Finally, given the fact that the Euro was introduced in January 1999, we have repeated the same analysis this time for the Euro Group. The results are also reported in Table 4 and suggests a slight increase (drop) in the importance of country-specific (worldwide) components from the data prior to 1999. It implies that, while the relationship with regional factors remains relatively similar to the pre-Euro period, real exchange rates in the Euro members have been more synchronized with and have become more dominant over the worldwide factor after 1999.

## **4.2 Countries Influential over the Common Factors**

Then what are the driving forces of these common factors? Economic theory suggests that real exchange rate would be determined by the real interest rate differential or the productivity differential in non-tradeable sectors (known as the Balassa-Samuelson theorem), among others.

Here we use real interest rates which seem to be available for more countries, and shall summarize below their theoretical link following Obstfeld and Kenneth (1996). Their derivation of the model is more attractive than the conventional one using only the purchasing power theorem and the covered interest rate parity condition (see MacDonald and Nagayasu (2000)) in the sense that sticky prices are considered in the model.

Let's consider domestic inflation which can be explain by the Dornbusch type inflation specification for an open economy.

$$\Delta p_{t+1} = \gamma(y_t^d - \bar{y}_t) + \Delta s_{t+1} + \Delta \tilde{p}_{t+1}^* \quad (6)$$

where  $y_t^d$  is the demand for home country output,  $s$  is the nominal exchange rate and  $p$  is the price. Because all variables are in log form, and  $\Delta$  represents the differenced variable; therefore,  $\Delta p_{t+1} = p_{t+1} - p_t$  becomes inflation. A variable with a bar indicates a natural level, and a foreign variable is denoted with an asterisk. In the presence of multiple partner countries, the latter can be thought of as a weighted average of foreign variables; thus the tilde suggests the weighted average of prices in partner countries. The  $\gamma > 0$  implies that home inflation increases due to excessive demand for home products, exchange rate depreciation, and increases in foreign inflation. Otherwise, there is no market clearance, i.e.,  $\Delta p_{t+1} \neq 0$ .

Further, the demand for home products ( $y_t^d$ ) is assumed to be expressed as:

$$y_t^d = \bar{y}_t + \delta(s_t - p_t + \tilde{p}_t^* - \bar{q}) \quad (7)$$

where  $\delta > 0$ . For simplicity, the long-run (or equilibrium) real exchange rate is assumed to be fixed here. According to this equation, the demand for domestic goods exceeds its natural level to an extent proportional to the level of currency misalignment.

Using the definition of the real exchange rate ( $q_t \equiv s_t - p_t + \tilde{p}_t^*$ ) and the uncovered interest parity condition ( $\Delta s_{t+1} = i_t - \tilde{i}_t^*$ , where  $i$  is the nominal interest rate), Eqs. 6 and 7 yield:

$$\Delta p_{t+1} = \gamma\delta(q_t - \bar{q}) + i_t - \tilde{i}_t^* + \Delta \tilde{p}_{t+1}^* \quad (8)$$

In addition, using the Fisher parity condition ( $i_t = r_t + \Delta p_{t+1}$ ) and rearranging Eq 8 in term of

the real exchange rate, we can obtain the following relationship:

$$q_t = \bar{q} - \frac{1}{\gamma\delta}(r_t - \tilde{r}_t^*) \quad (9)$$

Since all parameters are theoretically positive, this equation asserts that there would be home currency depreciation when the real interest rate falls at home. Given that many advanced countries have recently been experiencing a minimal inflation level, the nominal interest rate guided by monetary policy has a close link with the exchange rate.

Indeed there are many previous studies investigating the relationship between real exchange rates and real interest rates. Early studies tend to cast doubt on the credibility of this relationship (Edison and Pauls (1993), Edison and Melick (1999)); however, stronger evidence is reported by more recent studies (MacDonald and Nagayasu (2000), Byrne and Nagayasu (2010)) when taking into account the possibility of nonstationary elements in exchange rates and interest rates in the panel data context. Since the latter studies utilize cross country information and thus yield more reliable statistical results, we could base our analysis on this theoretical framework.

For the estimation, we consider the equation of exchange rate changes which is consistent with our derivation of common factors. Furthermore, since there are several components in real effective exchange rates, we shall estimate each factor using real interest rate data, based on the following liner equation.

$$\Delta q_t^{Com} = c + \beta_1 \Delta r_t + \beta_2 \Delta \tilde{r}_t^* + u_t \quad (10)$$

where  $q_t^{Com}$  is the common factor in the real effective exchange rates, and can be the worldwide factor ( $Com = F$ ) or regional common factors ( $Com = G_1, G_2, H_{11}, H_{12}, H_{21}$  or  $H_{22}$ ). The  $u_t$  is a residual, and the theoretical sign for this equation is  $\beta_1 < 0$  and  $\beta_2 > 0$  as suggested by Eq. 9. For the estimation, the real interest rate of each country is directly introduced in the specification rather than assigning a weight to foreign variables prior to the estimation as in Eq. 10 .

$$\Delta q_t^{Com} = c + \beta_1 \Delta r_{1t} + \beta_2 \Delta r_{2t}^* + \beta_3 \Delta r_{3t}^* + \dots + \beta_j \Delta r_{jt}^* + u_t \quad (11)$$

We shall apply the Bayesian Model Averaging (BMA) technique to this theoretical relation-

ship in order to clarify the relevance of countries to the common factors. The distinguishing feature of the BMA is to extend the standard Bayesian statistical method which addresses parameter uncertainty to also considering model uncertainty. This approach has drawn considerable interests from researchers over recent decades, and has been applied to identifying relevant variables for explaining economic growth, which potentially has a number of candidate explanatory variables (Fernandez et al. (2001)).

The concept of the BMA can be summarized very concisely using the following equation. (See Appendix about more detailed explanations about the BMA.) For a normal linear equation with  $k$  real interest rates, a parameter from the BMA can be summarized as:

$$\beta_{BMA} = \sum_{j=1}^k w_j \beta_j \quad (12)$$

where  $w_j$  is a weight attached to parameters  $\beta_j$  for model specification,  $j$ ; each model has a different combination of explanatory variables. Parameters  $\beta_j$  are estimated by the conventional Bayesian method and thus parameter uncertainty will be addressed here. Consideration of weight,  $w_j$ , captures model uncertainty. With the intercept term always in the specification, the BMA evaluates the importance of  $2^k$  models which will be estimated by the Markov Chain Monte Carlo model composition ( $MC^3$ )(Madigan et al. (1995)).

Here, most data are based on money market nominal rates (IFS code: 60B..ZF) but in their absence, deposit rates (60L..ZF) are used as alternatives.<sup>8</sup> We obtain real interest rates by subtracting inflation rates from the nominal interest rates, and calculate inflation rates using the consumer price index (CPI) (IFS code: 64...ZF). Since the CPI is not available from the IFS for Germany or the UK, their inflation rates are obtained from the OECD Main Indicators and the Office of National Statistics, UK, respectively. Due to data availability, our data set does not cover the real interest rates of all but 16 countries.<sup>9</sup> This leaves us analysis of  $2^{16}$  (= 65, 536) models.

Table 5 reports BMA results for each common factor. Model uncertainty is shown as the posterior inclusion probability (PIP) which is the sum of posterior model probabilities (PMP) from all models (see Appendix). The PIP shows the likelihood of appearance in the common

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<sup>8</sup>Deposit rates are used for France, Hungary and Mexico.

<sup>9</sup>IMF and OECD data are obtained from the UK Data Service. See Table 5 about a list of countries whose real interest rates are used in the analysis.

factor equations, and its high value can be considered as more relevant to explaining common factors. Against this background, advanced countries; notably, the US, France and Japan, are reported to be most relevant to the worldwide common factor. Their PIP exceeds more than 90 percent, which confirms that large economies become more influential over the worldwide variation in exchange rates. Because there is a substantial difference in the PIP among countries, our result is also consistent with the asymmetric effects of monetary policy which have been identified in the past. For example, Bansal and Dahlquist (2000) and Byrne and Nagayasu (2012) discuss that US monetary policy is very influential over emerging markets and the US influence remains higher than domestic ones even for other advanced economies (Nagayasu (2003)).

As we move on to regional factors, some other countries which are not regarded as most relevant to the worldwide common factor, are found to be closely related to regional factors. For example, the interest rates of Italy and Sweden are reported to be closely associated with  $G_1$ , and Australia with  $G_2$ . Similarly, Italy and Ireland exhibit a close association with  $H_{11}$  while France and Germany are less relevant to this level of the common factor. In  $H_{22}$ , Mexico is found to be more relevant than the US and Canada which are reported to be closely linked with the worldwide factor. Obviously, this is a general observation, but is consistent with the conventional belief.

Furthermore, the different magnitude of our estimates (PIP) in different common factors implies that the worldwide common factor is more closely associated with interest rates compared with regional factors; its PIP is much higher than that found from regional factors. Thus this finding implies that the determinants of regional factors are more complex and this underlines the potential importance of other economic and financial factors. The increase in this complexity can be understood by Eqs 1 to 3 where lower level hierarchical factors are driven by more region specific and heterogeneous information.

Needless to say, Table 5 does not give much information about country specific factors, but as discussed, our study (Table 4) shows that more than half of variation in real effective exchange rates are attributable to country specific factors. The results remain valid even for the Euro member countries whose nominal exchange rates are fixed within member countries.

## 5 Conclusion

Given that exchange rates are believed to be highly correlated among countries, we calculate co-movements in real effective exchange rates using the recently developed data decomposition method (Moench et al. (2011)). Then, our results suggest that the evolution of real effective exchange rate changes is rather country-specific. As expected, the country-specific elements are less significant within the Euro area compared with other regions, and their importance has dropped after the introduction of the Euro. However, these components still account for around 60% of the total variation, implying heterogeneous inflation and external competitiveness among member countries. This is in sharp contrast to the expectation that real exchange rate movements are dominated by common factors in a single currency union, and underlines the importance of both idiosyncratic and 3<sup>rd</sup> country effects when understanding exchange rate dynamics. Therefore, a single country analysis remains important and should deliver valuable information even when assessing the economic conditions of a single currency area.

Our further analysis has identified countries which are influential over common movements in real effective exchange rates using the BMA. In this paper, the empirical analysis is carried out within the theoretical framework of real exchange rates and interest rates, and thus our study is closely related to the influence of monetary policy over exchange rates. Then we report that the worldwide common factor is often associated with interest rates in large advanced countries, notably the USA. Therefore, US monetary policy is confirmed to be very influential over world exchange rates. By contrast, regional factors are to a larger extent affected by economic and financial developments in relatively smaller countries.



## Appendix. Bayesian Model Averaging

This appendix summarizes the concept and specific assumptions used for the Bayesian Model Averaging (BMA) in this paper. (See also Koop (2003) regarding the general concept of the BMA.) Maintaining the mathematical notation used in the main text, the BMA is summarized below.

With  $k$  explanatory variables, the posterior for the parameters for  $M_j$  can be written using the Bayesian rule as:

$$p(\beta_j|y, X) = \sum_{j=1}^{2^k} p(\beta|M_j, y, X)p(M_j|y, X) \quad (13)$$

It states that the posterior for the parameters of interest consists of the weighted average of the posterior distribution of the parameters from all models and the posterior model probabilities (PMP). Furthermore, the PMP can be expressed as:

$$p(M_j|y, X) = \frac{p(y|M_j, X)p(M_j)}{p(y|X)} \quad (14)$$

Since the denominator is constant over the models, we could focus only on the numerator;  $p(M_j)$  is assumed to follow a uniform prior probability for each model,  $p(M_j) \propto 1$ , and  $p(M_j) = 1/2^k$ . Then dropping this term which is constant over models, the PMP can be expressed as:

$$p(M_j|y, X) = \frac{p(y|M_j, X)}{\sum_{i=1}^{2^k} p(y|M_i, X)} \quad (15)$$

The posterior inclusion probably (PIP) is the sum of PMP from all models including variable  $x_h$ :

$$p(x_h|y, X) = \sum_{i:x_h \in M_j} p(M_i|y, X) \quad (16)$$

For more specific assumptions, we follow Zellner's  $g$  prior. In other word, based on a Bayesian linear model, we consider

$$\beta_j|g \sim N(0, \sigma^2 g(X_j'X_j)^{-1}) \quad (17)$$

where  $g$  measures the level of uncertainty and is set as  $g = \max(T, K^2)$ ; uncertainty increases along with rises in  $g$ . Then the marginal likelihood for  $M_j$  is:

$$p(y|M_j, X) \propto \left(\frac{g}{g+1}\right)^{\frac{k_j}{2}} \left[ \frac{1}{g+1} y' P_{x_j} y + \frac{g}{g+1} (y - \bar{y})' (y - \bar{y}) \right]^{-\frac{T-1}{2}} \quad (18)$$

where  $P_{X_j} = I_T - X_j(X_j'X_j)^{-1}X_j'$ .

As the size of the model is large ( $2^{16} = 65,536$ ), our results are based on the Markov Chain Monte Carlo model composition (known as  $MC^3$ , with the Metropolis-Hastings algorithm (see Madigan et al 1995)) with 10,000 iterations and the burn-in equal to 5,000.

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Table 1: List of Countries and Hierarchical Classification

Level 4	All countries ( $F$ )			
Level 3	Europe ( $G_1$ )		Non-Europe ( $G_2$ )	
Level 2	Group 1 ( $H_{11}$ )	Group 2 ( $H_{12}$ )	Group 3 ( $H_{21}$ )	Group 4 ( $H_{22}$ )
	Euro	Non-Euro Europe	Asia-Pacific	America
Level 1 [ $Z$ ]	Austria	UK	Japan	US
	Belgium	Denmark	Australia	Canada
	France	Sweden	NZ	Costa Rica
	Germany	Hungary	Singapore	Mexico
	Italy	Poland		
	Luxembourg	Malta		
	Netherlands	Cyprus		
	Finland	Norway		
	Greece	Switzerland		
	Ireland	Iceland		
	Portugal			
	Spain			

**Notes:** Malta and Cyprus are Euro member countries, but are treated here as Non-Euro European countries since their entry date is very recent (2008). Common factors are denoted as  $F$ ,  $G$ , and  $H$ , and  $Z$  are country vectors.

Table 2: Basic Statistics for Changes in Exchange Rates

Country	Mean	Std Dev	Min	Max
Austria (AUS)	0.011	0.968	-2.468	2.239
Belgium (BEL)	-0.113	1.383	-5.284	2.346
France (FRA)	-0.142	1.410	-6.667	2.742
Germany (GER)	-0.158	1.584	-4.580	3.392
Italy (ITA)	0.031	2.044	-11.460	6.390
Luxembourg (LUX)	-0.070	0.864	-4.012	1.399
Netherlands (NET)	-0.082	1.413	-3.727	4.274
Finland (FIN)	-0.101	2.136	-9.539	5.635
Greece (GRE)	0.130	2.274	-13.343	5.811
Ireland (IRE)	0.120	2.136	-5.890	6.286
Portugal (POR)	0.217	1.688	-8.933	4.924
Spain (SPA)	0.015	1.868	-7.960	4.238
UK	-0.025	3.244	-13.232	9.723
Denmark (DEN)	0.042	1.413	-4.740	3.320
Sweden (SWE)	-0.255	2.875	-13.828	7.354
Hungary (HUN)	0.437	3.226	-10.961	9.379
Poland (POL)	-1.484	22.652	-245.141	24.027
Malta (MAL)	-0.070	1.493	-4.150	4.465
Cyprus (CYP)	-0.086	1.449	-4.299	6.025
Norway (NOR)	0.058	2.029	-10.624	5.062
Switzerland (SWI)	0.172	2.244	-7.592	8.085
Iceland (ICE)	-0.313	4.268	-24.550	11.286
Japan (JAP)	0.308	4.507	-10.541	19.259
Australia (AUST)	0.126	4.192	-20.560	10.403
NZ	0.174	3.768	-15.554	10.787
Singapore (SIN)	0.084	1.692	-7.160	4.489
US	-0.059	2.778	-5.879	8.731
Canada (CAN)	0.072	2.629	-11.283	6.583
Costa Rica (COS)	-0.170	6.976	-48.710	17.013
Mexico (MEX)	-0.143	6.826	-43.974	14.401

**Notes:** Full sample (1980Q1-2012Q2).

Table 3: Correlations Between Exchange Rate Changes

Lowest 40 Pairs		Corr	Highest 40 Pairs		Corr
GRE	SIN	0.001	GER	SWE	0.470
POR	POL	0.001	FRA	SIN	-0.473
SWE	CYP	0.002	GRE	SWE	0.473
UK	COS	-0.002	BEL	SWE	0.476
IRE	SPA	-0.003	FRA	SWE	0.478
HUN	NZ	-0.003	FRA	LUX	0.481
FIN	USA	-0.003	FRA	POR	0.483
IRE	UK	-0.004	BEL	SIN	-0.494
UK	SWE	0.005	NET	POL	0.500
BEL	AUST	-0.006	GRE	SIN	-0.501
FIN	GRE	-0.006	NET	SWE	0.509
ITA	USA	-0.007	NET	SIN	-0.531
ITA	GRE	-0.007	GER	SIN	-0.544
POR	UK	-0.007	AUS	LUX	0.574
IRE	USA	-0.008	GER	LUX	0.594
LUX	JAP	0.008	JAP	USA	0.599
FRA	POR	0.008	LUX	NET	0.600
UK	USA	-0.009	FRA	IRE	0.626
FRA	CYP	0.009	BEL	FRA	0.630
ITA	DEN	-0.010	GER	IRE	0.650
FIN	CYP	-0.011	IRE	GRE	0.654
AUS	AUST	0.012	FRA	GER	0.654
POL	NZ	-0.014	FRA	NET	0.658
SPA	CAN	-0.014	BEL	IRE	0.661
BEL	JAP	-0.015	LUX	GRE	0.662
SPA	SWE	-0.015	AUS	IRE	0.664
ITA	UK	0.016	NET	IRE	0.672
FIN	UK	-0.016	FRA	GRE	0.692
GRE	UK	-0.016	AUS	FRA	0.703
UK	POL	-0.016	AUS	BEL	0.742
UK	HUN	-0.016	BEL	NET	0.756
HUN	COS	0.018	GER	GRE	0.762
GER	AUST	-0.019	NET	GRE	0.764
POR	USA	-0.019	BEL	GER	0.766
LUX	USA	-0.019	AUS	GRE	0.771
ITA	UK	-0.019	BEL	GRE	0.806
GER	UK	-0.020	BEL	LUX	0.856
SWE	JAP	0.020	AUS	NET	0.887
FRA	SPA	-0.020	AUS	GER	0.898
GRE	IRE	0.023	GER	NET	0.916

**Notes:** See Table 2 for abbreviation of country names. Full sample.

Table 4: Variance Decomposition

Country	Worldwide Factor: $Var(F)$	Regional factor I: $Var(e_{Gb})$	Regional factor II: $Var(e_{Hbs})$	Country specific factor: $Var(e_{Zbsn})$
Euro	0.197	0.087	0.087	0.630
Euro (until 1998)	0.128	0.086	0.089	0.697
Non-Euro Europe	0.084	0.037	0.058	0.822
Asia-Pacific	0.028	0.063	0.069	0.839
America	0.010	0.112	0.217	0.661
Average (Full sample)	0.108	0.082	0.104	0.706

**Notes:** The table shows the proportion of each factor to the total variation in the real effective exchange rate using the dynamic hierarchical factor model. Results are full sample unless otherwise stated.

Table 5: Bayesian Model Averaging: Common Factors and Real Interest Rates

$F$	$G_1$	$G_2$	$H_{11}$	$H_{12}$	$H_{21}$	$H_{22}$							
US	0.998	ITA	0.441	AUST	0.218	ITA	0.271	SWE	0.157	AUST	0.260	MEX	0.107
FRA	0.929	SWE	0.395	JAP	0.152	IRE	0.250	UK	0.097	JAP	0.162	US	0.092
JAP	0.921	IRE	0.335	MEX	0.135	FIN	0.237	DEN	0.093	SIN	0.098	Can	0.089
CAN	0.745	FRA	0.202	CAN	0.124	FRA	0.163	SWI	0.084				
AUST	0.611	FIN	0.194	US	0.119	GER	0.104	HUN	0.076				
GER	0.485	SWI	0.115	SIN	0.109								
UK	0.234	Hun	0.106										
ITA	0.232	GER	0.095										
FIN	0.207	UK	0.095										
SWI	0.205	DEN	0.082										
HUN	0.190												
MEX	0.177												
IRE	0.166												
SWE	0.109												
SIN	0.075												
DEN	0.051												

**Notes:** Full sample and based on Eq. 11. The PIP is reported, and its higher value indicates the relevance of the real interest rate of that country in explaining common factors. Standard errors in parentheses.



Figure 1: The Worldwide Common Factor ( $F$ )

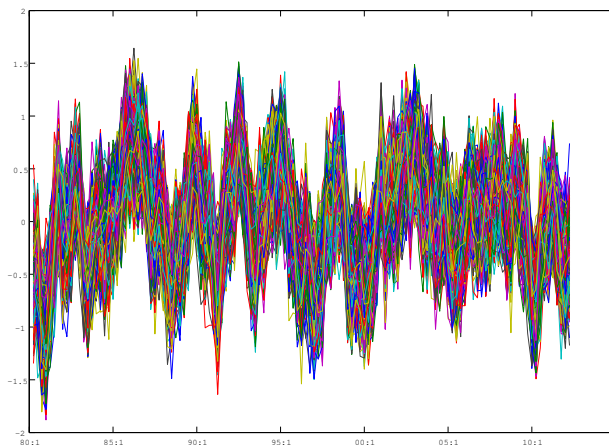


Figure 2: The Regional Factor ( $G_1$ )

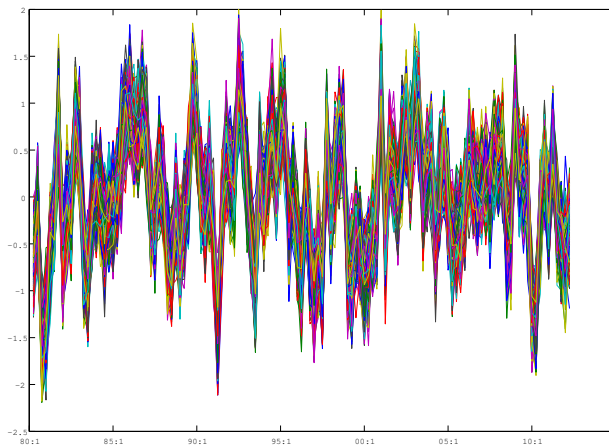


Figure 3: The Regional Common Factor ( $G_2$ )

