Anandan *et al.* Reply: We agree with Bhandari [1] that our mixed state phase $\phi = \arg\mathrm{Tr}(U_i\rho_0)$ is undefined in the special cases,

$$\mathrm{Tr}(U_i\rho_0) = 0. \tag{1}$$

However, for the example in our paper [2] that Bhandari criticizes $\mathrm{Tr}(U_i\rho_0) = -1 \neq 0$. In this example of interferometry with unpolarized neutrons, where one beam is given a rotation of 2π radians, our mixed state phase shift is π (modulo 2π), in agreement with the experiments. But Bhandari claims that this phase shift is "indeterminate" because it could be π or $-\pi$; but these two phases differ by 2π. So, the only difference between Bhandari's viewpoint and ours is that our phase is defined modulo 2π, whereas Bhandari argues that two phases that differ by $2\pi n$, n integer, may be distinguished experimentally in a history-dependent manner.

Bhandari's singularities are defined by (1) in relation to the input state ρ_0, and his nonmodular phase is associated with the evolution path that originates at ρ_0. This phase has the disadvantage that it becomes undefined even at points of the parameter space for which $\mathrm{Tr}(U_i\rho_0) \neq 0$ if the path has passed through a "singularity" (see Fig. 1 of [1]). But our phase modulo 2π is well defined at all such points, as in the above example, because it does not depend on the path. For the special case of spin $1/2$ or qubit pure state $\rho_0 = |\psi\rangle\langle\psi|$, the singularity is the point opposite to ρ_0 in the Bloch sphere or the Poincaré sphere. The Pancharatnam phase is undefined for this pair of orthogonal states, which is not a problem for this phase, and similarly (1) is not a problem for our phase. The interesting fringe shift in the interference pattern that Bhandari obtains in his experiments (Refs. [3–5] of [1]) when the path goes around a singularity, but not around any other point, may be explained by the change in $e^{i\phi}$, in which the phase is defined modulo 2π, instead of using his nonmodular phase ϕ. Also, for arbitrary quantum systems in pure or mixed states, these singularities may be detected, without the use of the nonmodular phase, by the vanishing of the visibility.

For arbitrary spin also Bhandari's approach does not give any additional information as implied at the end of his Comment. The geometric phases in this case may be obtained by parallel transporting around the circuit C traced by the direction of the evolving spin quantization axis on the sphere SU(2)/U(1). This holonomy transformation gives [3] the geometric phases for the states with spin quantum numbers j as

$$\beta_j = j\alpha (\mathrm{mod}\, 2\pi), \qquad j = -J, -J+1, \ldots, J, \tag{2}$$

where α is the solid angle of either of the complementary surfaces S_1 and S_2 on this sphere spanned by C, and the spin J is an integer or half integer. The freedom to choose either S_1 or S_2 *requires that the phase should be defined modulo* 2π, because their solid angles add to 4π. This is an interesting aspect of the Dirac monopole geometry which gives rise to the geometric phases. The mixed state geometric phase is then $\beta = \arg\{\sum_j \lambda_j \exp(i\beta_j)\}$, where λ_j are nondegenerate eigenvalues of the density matrix ρ_0 ($\lambda_j \geq 0, \sum_j \lambda_j = 1$). Now, (2) is equivalent to $\beta_j = j\alpha + 2\pi n$, where n is a particular integer. Suppose C is infinitesimal. Since β_j is obtained in any experiment from $e^{i\beta_j}$, we may instead regard its values corresponding to all possible values of n to be equivalent. In particular, both α and β_j may be chosen to be infinitesimal, which corresponds to $n = 0$. Then the spin quantum number $j = \beta_j/\alpha$ is obtained from the known values of β_j and α, without having to go around a "singularity."

In the geometrically analogous magnetic monopole case, this corresponds to determining the magnetic charge by simply measuring the field strength at the infinitesimal (i.e., $\ll 2\pi$) circuit. Bhandari's history-dependent, nonmodular phase implicitly chooses a gauge that has the analog of a Dirac string whose intersection with the sphere is his "singularity." His phase then is defined using the solid angle of one of the two surfaces S_1, S_2 that has no singularity. But this is contained as a special case, with appropriate choice of n, of the above more general treatment that is valid in all gauges.

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