SQA qualifications in Mathematics and English (1986-2018): a statistical overview

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Executive summary

For Scottish learners, teachers, employers and policymakers, a clear understanding of SQA qualifications is important. Unfortunately, misunderstandings about attainment in these qualifications are common. This report addresses some of those misunderstandings by examining in detail the patterns of attainment in SQA qualifications in Mathematics and English over a period of more than three decades, including the recent reform of National Qualifications.

Our principal messages for stakeholders are the following.

1. Patterns of attainment in Mathematics and English are different.

Most notably:

(a) The national pass rate at National 5 is higher in English (85%) than in Mathematics (65%).
(b) The uptake of Higher English is almost twice as high as the uptake of Higher Mathematics.
(c) A higher fraction of entries are awarded grade A in Higher Mathematics than in Higher English.
(d) The relative difficulty of Mathematics qualifications as measured by the National Ratings is greater than that of English qualifications.

Most of these differences pre-date the reform of National Qualifications but some may have been accentuated by it.

2. Patterns of attainment are not constant in either Mathematics or English.

Reforms affect qualifications both directly through changes to the specifications and indirectly through changes to the cohorts taking them, and both changes are reflected in attainment statistics. Against a continually changing educational, social and demographic background, changes in attainment statistics alone cannot provide reliable evidence of a change in "standards". Small year-to-year changes, in particular, are essentially meaningless.

In addition to these changes over time, attainment at the level of individual schools is likely to reflect local conditions. We therefore caution against the use of national data as a baseline with which to compare school performance.

3. Attainment in Mathematics is more sensitive to prior qualifications than attainment in English.

It is plausible that this reflects the strongly structured nature of mathematical knowledge, and it has obvious consequences for decisions about subject choice and presentation. It might be helpful for some learners to take more flexible routes through SCQF5 and SCQF6 rather than to follow the default pathway of National 5 in S4 followed by Higher in S5.

Although there are some systematic differences between attainment in different subjects, we do not recommend that learners give this undue weight when choosing subjects. Learners' aptitude for and interest in different subjects will generally make far more difference to their attainment.

4. Attainment figures considered in isolation are potentially misleading.

The SQA invest considerable effort in monitoring attainment with the aims of maintaining the standards of their qualifications over time and ensuring that qualifications at the same level are broadly comparable. Although these aims cannot be defined in a purely statistical sense, at present the vagueness surrounding them means that stakeholders are liable to impose their own partial interpretations on the evidence provided. No single measure is adequate to capture the full picture, and different statistics can present apparently contradictory pictures, which do not necessarily agree with more qualitative studies.

With these considerations in mind, we suggest that it would be valuable for the SQA to work with stakeholders to encourage greater understanding of assessment standards and of the various evidence that captures aspects of learners' attainment.

1. Introduction and scope

This report presents statistical and other information on Mathematics and English qualifications in Scotland over the last three decades. It aims to help teachers, learners and other stakeholders set more recent figures in context and to inform any subsequent discussions of these qualifications.

This work was prompted by the recent reform of National Qualifications, particularly at SCQF levels 5 and 6. The Scottish Mathematical Council have been concerned by reports that some learners have had an inaccurate understanding of the new qualifications, leading to candidates being presented at inappropriate levels. We have also been concerned by reports that some school leadership teams have had unrealistic expectations of attainment, leading to unreasonable pressure on teachers to replicate changes in uptake or pass rates seen in other subjects.

With these concerns in mind, we aim to provide insight into the following questions.

- What have been the patterns of attainment in Mathematics and English qualifications over the study period?
- How have these patterns been affected by the recent qualifications reform?
- How do these patterns, and the changes within them, differ between these subjects?

We consider Mathematics and English in parallel because they have consistently been the two largest subjects in terms of entry numbers, and because they have analogous roles as "gateway" qualifications for many subsequent qualifications and careers. Despite these analogous roles, the cohorts taking these subjects are different and the qualifications have different purposes. We therefore do not assume that Mathematics and English are, or ought to be, directly equivalent in any sense; indeed, the evidence presented here suggests how problematic it would be to define any such direct equivalence.

The focus of this report is on Higher, National 5 (N5) and Standard Grade (SG). We do not consider in detail other pathways such as Lifeskills / Applications of Mathematics, Intermediate 1 and 2, or the older O Grade qualifications. We also do not consider more advanced or specialised qualifications such as CSYS or Advanced Higher.

We begin in Section 2 by giving an overview of the various evidence that we will present. In Section 3 we present and describe this evidence chronologically, before briefly discussing some features and the key messages that emerge (Section 4). The references and data sources are listed in the Appendix.

A summary spreadsheet containing the data used to generate the figures is available from the SMC website.[1]

[1] http://www.scottishmathematicalcouncil.org/wp1/sqa-exam-data/

2. Sources and types of evidence and comments on interpretation

2.1 Attainment statistics and demographic data

The number of entries and the grade distribution for every subject, from 1986-2018, are available via the SQA Statistics Archive[2]. When two versions of the same qualification were offered in the same year we have combined the figures. N5 Lifeskills Mathematics / Applications of Mathematics, with typical entry numbers in the range 2000-3000, is not included in the N5 Mathematics figures. English assessed by alternative means has not been included; the numbers involved are in any case typically very small.

The numbers of 16- and 17-year-olds in Scotland have changed substantially over the study period: the population of 17-year-olds was nearly 85 000 in 1986 and a little under 56 000 in 2018. This means that raw entry or attainment numbers are potentially misleading. We have therefore normalised numbers throughout by expressing them as fractions of the population of 16- and 17-year-olds, using the data provided by the National Records of Scotland.[3] Although not all candidates who take SCQF5 or SCQF6 qualifications do so at these ages, normalising by these population figures allows us to correct roughly for demographic changes.[4] Except where otherwise stated in Section 3.1, when we refer to entry numbers this will mean the population-normalised entry numbers.

Comments

Entry numbers and attainment figures are the primary data that stakeholders expect the SQA to produce in its role as an awarding body. Of these, the pass rates for each subject receive the most public attention, and are possibly also the most open to misinterpretation. Each qualification is taken by a different cohort of students: some qualifications may attract relatively small numbers of students, all of whom are specialists in (or at least enthusiasts for) that subject; others are taken by much larger numbers of students, who are on average significantly less well motivated or prepared. The nature of the cohort taking a given qualification may also change over time. Attainment statistics must therefore be viewed in the context of demographic data and entry numbers, and treated with caution.

A further caveat is that although Standard Grade results were classified as belonging to a particular SCQF level, candidates typically attempted two levels (e.g. Credit and General) and so entries were not classified as belonging to a particular SCQF level. This means that entry numbers at SG may not be directly comparable with those at N5 and Intermediate.

2.2 National Ratings (NRs)

National Ratings for all qualifications are produced annually by the SQA but are not published on the SQA website; they are available by request from the SQA Statistics Team, who supplied us with the NRs from 2001 onward.

Comments

The NRs are intended to provide a statistical measure of the "difficulty" of a qualification relative to other qualifications at the same SCQF level.[5] The NRs were originally produced alongside "relative ratings" for individual schools, in order to provide a baseline against which schools could gauge their performance in

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[2] https://www.sqa.org.uk/sqa/57518.html. Scott (2018) quotes slightly different figures for the numbers of entries at SCQF5; as the difference in methodology is not clear, we have not made use of Scott's analysis here.
[3] These data were extracted from the *Population Pyramids of Scotland, 1981-2041*. Estimates may have been revised subsequently but are unlikely to have changed substantially.
[4] SQA (2018) uses school roll numbers as a reference for the eligible population. Although these numbers are resolved by school year they have two disadvantages for our purposes: they only include state-funded schools (excluding independent schools and colleges), and the available data only cover 1995-2017.
[5] Although our focus is on statistical information, we note that relative difficulty can be defined in many ways, and that the use of purely statistical definitions is highly contestable. See, e.g., Baird *et al.* (2000); Wiliam (2001); Jones *et al.* (2011) for discussion of this point.

individual subjects. These relative ratings, which were not statistically robust (Sparkes 2000), have since been superseded by a more sophisticated benchmarking tool (Insight[6]).

The description of the methodology in the National Ratings Explained section of the NR spreadsheet is quite strongly simplified.[7] The methodology was introduced by Alison Kelly in the mid 1970s, and is well described by Coe *et al*. (2008):

Kelly's method begins by comparing the grades achieved by candidates in one subject with their average grades in all their other subjects, and so estimating the difficulty of that subject. This is done for each subject under consideration, using the grades achieved by all candidates who have taken it with at least one other in the set. These 'difficulty estimates' are then used to apply a correction factor to the grades achieved in that subject. So, for example, if chemistry is found to be half a grade more difficult than the average, that half grade is added to the achieved grade for all chemistry examinees. The whole process is then repeated using the 'difficulty corrected' grades in each subject instead of the actual achieved grades, to produce a new estimate of the relative difficulty of these subjects with corrected grades. After a small number of iterations, the corrections shrink to zero and so the estimates of difficulty of each subject converge.

The purpose of this iterative process[8] is to eliminate the effect of subject clustering, since students tend to take groups of related subjects at Higher and so the baselines against which corrections are calculated at the first iteration may not be the same.

The reported NRs are negative versions of Kelly's correction terms: a positive NR means that students typically perform better in that qualification than in the others they take. For SG, an NR of 1 corresponded to a correction of one grade on the seven-grade scale; for N5, an NR of 1 corresponds to a correction of two bands on the nine-band scale, or approximately one grade. Although the NRs are not given much weight in the setting of grade boundaries, they provide the only quantitative definition of comparability employed by the SQA, whose aim is that most NRs should lie between -0.5 and 0.5.[9] (The SQA also consider qualitative means of defining comparability: see SQA (2017a).)

Although the NRs are designed to compensate for cohort effects, they do so only under the assumption that a candidate's performance in one qualification can be expected to correlate with their performance in another (i.e. that all qualifications reflect a single underlying measure of ability). There are cases in which this may not apply, such as "heritage languages" which many candidates learn and use outwith the classroom, and subjects such as PE that emphasise practical skills. If one qualification effectively acts as a gatekeeper to another qualification with which it is poorly correlated, this may bias the NR for that qualification.[10] Even if two qualifications do depend on the same underlying ability[11], the NRs and the raw attainment statistics may suggest rather differ-

[6] See https://insight.scotxed.net.
[7] This was confirmed by Dr Noel Thomson (SQA statistician) in evidence to the RSE Learned Societies' Group, 16 October 2018.
[8] In practice the fixed point of the iteration is located by solving a system of linear equations rather than by iteration; this distinction is not important from the point of view of understanding the process.
[9] Dr Gill Stewart (Director of Qualifications Development, SQA), giving evidence to the RSE's Learned Societies' Group, 16 October 2018.
[10] This is easiest to illustrate with a thought experiment. Consider two subjects, say chess and tiddlywinks. Higher Chess is widely regarded as an essential qualification which all school-leavers ought to have. Higher Tiddlywinks is widely regarded as nice but inessential, so only learners who are considered certain to perform well in Higher Chess are encouraged to take Higher Tiddlywinks. If performance in tiddlywinks is uncorrelated with that in chess, this will mean that Higher Tiddlywinks will have a negative NR even if it has a reasonable grade distribution, because almost all Higher Tiddlywinks candidates will have obtained an A in Higher Chess and their Higher Tiddlywinks grade will on average be poorer than this.
[11] It may be worth noting that Coe *et al.* (2008: p. 123) report a moderately strong correlation (r=0.77) between candidates' performance in GCSE Mathematics and GCSE English. Of course, the NRs for Mathematics and English are influenced by performance in many subjects, not just in each other.

ent conclusions, and so it is important to consider them together.[12]

A point not reflected in Coe *et al.*'s description is that the NRs only take into account qualifications taken in the same year.[13] This is likely to have a particularly strong effect on the NRs for Highers that are taken predominantly in S6.

A further technical caveat concerns the use of bands rather than raw marks to calculate NRs. This means that the NRs are affected by changes in the number of bands or grades (e.g. the change from seven grades at SG to nine bands at N5) and by changes to the width of bands (e.g. the change between 2017 and 2018 from setting the lower boundary of N5 grade D at 45% to setting it at 40%[14]). It may also tend to mask the effect if very high performances are more common in one subject than in another: with typical N5 grade boundaries, a mark of 100% and a mark of 85% would both be classified as band A1 and treated as identical.

Some other weaknesses of the NRs are discussed by Sparkes (2000), who points out that a statistic based on candidates' average grades conceals significant variations between individual candidates and between sub-cohorts of candidates (e.g. male/female; "arts-focused"/ "science-focused"; etc.) Sparkes's analysis of the NRs for Higher English and Higher Mathematics found that there were indeed differences between sub-cohorts, although it also found that the NR for Higher English remained neutral or slightly positive, whereas that for Higher Mathematics remained negative, for all sub-cohorts.

Notwithstanding Sparkes's criticisms and these other caveats, the analysis of A-Level and GCSE data by Coe *et al*. (2008; chapter 5) found that Kelly's method agreed with other statistical measures of relative difficulty. Although more sophisticated methods are available, such as average marks scaling or the Rasch model (Coe *et al*. 2008; sections 2.1.2-2.1.3), the robustness of this method suggests that insofar as relative difficulty is a measurable concept, the NRs provide a reasonable measure of it.

2.3 Progression tables (PTs)

Progression tables for all subjects taken at Higher in 2013 through 2017, along with PTs for other progressions, are available via the SQA Statistics Archive.[15] PTs for 2018 are not yet available.[16]

Comments

Each PT considers the students who took a given qualification in a given year having taken a given prior qualification in the previous year. It categorises those students according to their grade in the prior qualification, and within each category it lists the percentage who achieved each grade in the later qualification. For example, the PT for Higher Mathematics in 2016 categorises the students into those who had previously achieved A, B, C, D or no award in N5 Mathematics in 2015, and for each category states the percentage of those achieving A, B, C, D or no award in Higher Mathematics in 2016.

There are some important caveats. First, because the PTs only include those students who take a qualification in a given year having taken the prior qualification in the previous year, they

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[12] A notorious case is A-Level Further Mathematics (see Coe *et al.* 2008, pp. 86-88). A similar case may be Advanced Higher Mathematics of Mechanics, which in 2017 had a very high percentage of grade As (53.7%; the highest of any STEM subject) and a high percentage of grades A-C (74.5%; second only to Chemistry among STEM subjects), but an NR of -0.58 (the lowest of any subject at AH that year). We may surmise that this qualification attracted mathematically strong students who nevertheless found it more challenging than their other AH subjects.

[13] For example, if a candidate completed five Highers in S5 and one Higher in S6 then their Higher results in S5 would contribute to the NRs for the five subjects completed, but their Higher result in S6 would not contribute to any NRs that year as the candidate would have no other Higher performances to compare it with. I am grateful to Dr Sue Pope and Dr Noel Thomson (SQA) for elucidating this point.

[14] See the Grade Boundaries spreadsheets available via the SQA Information page, https://www.sqa.org.uk/sqa/64718.8314.html, and the Information Archive, https://www.sqa.org.uk/sqa/57520.8315.html.

[15] https://www.sqa.org.uk/sqa/84433.html.

[16] Publication is expected in April 2019; see https://www.sqa.org.uk/sqa/48513.8316.html.

do not include, for example, students who take Higher in S6 having taken N5 in S4.[17] Second, the PTs do not indicate how many students who get a given grade in N5 then go on not to take a Higher in that subject. (Consequently they do not give, for example, the probability that a student will go on to achieve a B in Higher Mathematics given that this student has achieved a C in N5 Mathematics.) Nevertheless, the PTs provide valuable information about progression pathways in each subject.

The PTs do not reflect "difficulty" in the same sense as the NRs, because they look at progress within a subject rather than comparing different subjects. They may, however, reflect differences in the difficulty of progressing, which in turn may reflect both intrinsic properties of a subject and the perceptions of difficulty that drive student choices. For example, it would not be surprising if attainment in Higher Mathematics were to depend more strongly on prior knowledge from N5 Mathematics than attainment in Higher English depended on prior knowledge from N5 English, because Mathematics is a more strongly structured discipline based on a more hierarchically arranged body of knowledge. Equally, if students are aware of this then one might expect that students who had performed poorly in N5 Mathematics would be more likely to drop it at Higher than students who performed poorly in N5 English would be to drop English. This would mean that the cohorts taking Higher Mathematics and Higher English were "filtered" differently, with implications for the comparability of these qualifications.

The PTs bear a family resemblance to the approach of "Comparative Progression Analysis" (CPA) which has recently been discussed in England (Ofqual 2017), although the presentation of the data is somewhat different. A useful critique of CPA, with accompanying thought experiments, has been presented by Benton & Bramley (2017). In particular, Benton & Bramley point out that "even if two subjects are equivalent according to CPA for hypothetically defined populations… non-random choice of A-level subjects and different GCSE A-level correlations in different subjects can create the appearance of different CPA for the groups that actually take the A levels in different subjects". The converse holds as well: subject-choice effects might conceal differences in progression and attainment between subjects with apparently similar CPA results.

2.4 Monitoring Standards Over Time (MSOT) reports

A final source of evidence should be mentioned even although it is not principally statistical in nature. On an occasional basis the SQA carry out Monitoring Standards Over Time exercises, in which two sittings of a qualification (typically four years or more apart) are compared in detail, complementing the three-year horizon considered in the annual Awarding Meetings.[18] These exercises consider attainment statistics but focus on the content of the assessments, looking both at the assessment instruments and at specimen scripts awarded various grades; any changes to the course specifications are also considered.

MSOT reports from 2011 and from 2013-2017 are available from the SQA website.[19] Higher Mathematics and Higher English were both considered in 2011, being compared with the 2004 and 2005 versions respectively. Higher English was also considered in 2015, when it was compared with the 2011 version. These reports will be referred to as appropriate in Section 3.

[17] Similarly, they cannot include candidates who "crash" a Higher without a prior qualification in that subject, who have a prior qualification such as GCSE from outwith the SQA portfolio, or who do not use a consistent Scottish Candidate Number.

[18] Dr Gill Stewart, evidence to RSE Learned Societies' Group, 16 October 2018.

[19] See https://www.sqa.org.uk/sqa/74496.8276.html. The subjects considered in 2012 are listed but the report is not available from this web page.

3. Chronological description of the evidence

3.1 Entry numbers

Figures 1 and 2 show the number of entries to Mathematics and English at O Grade / Standard Grade / Intermediate 2 / National 5 (Figure 1) and at Higher (Figure 2) from 1986 until 2018, as a fraction of the total population of 16-year-olds (Figure 1) or 17-year-olds (Figure 2) in Scotland. Figure 2a also shows the absolute entry numbers and the population of 17-year-olds for comparison.

Several changes to the qualifications took place over this period, the most notable of which were the introduction of Standard Grades (1986) and Revised Highers (1987), the "Higher Still" reform in 1999-2000, and the introduction of the new National Qualifications in 2014-2015 following the introduction of Curriculum for Excellence.

The last O Grade sittings were in 1992. Over the period 1986-1992, SGs gradually replaced OGs (Figure 1a,b); the replacement began more rapidly for Mathematics than for English, but by about 1989 entries to the two subjects were nearly equal. Between 1992 and 2000 the pattern of entries remained stable, with nearly all learners in the appropriate age group sitting both SG Mathematics and SG English. The introduction of the new qualifications in 1999-2000 saw a divergence from this pattern: entries to both SG Mathematics and SG English dropped off (Figure 1a), while entries to the Intermediate qualifications rose (Figure 1b). The decline at SG was more marked for Mathematics than for English, so by the final SG sitting in 2013 there were roughly 20% more entries to SG English than to SG Mathematics.

The introduction of NQs saw a step change: N5 entries in 2014 were anomalously low, presumably reflecting deferred entries to the new system and students still taking Intermediate qualifications (Figure 1b), but since then entry numbers for both Mathematics and English have recovered to levels comparable with mid-2000s SG. The uptake of National 5 Lifeskills Mathematics[20] has remained low. Entry numbers to N5 English remain a little higher than to N5 Mathematics.

Changes to the entry numbers for Higher Mathematics and Higher English have been less dramatic but still substantial (Figure 2a,b). Over the period 1987-1994, when both "old" and Revised Highers were offered, both subjects saw entry numbers rise relative to the population (Figure 2b), as they remained fairly stable in absolute terms despite a substantial fall in the population of 17-year-olds (Figure 2a). Entries to Higher English were consistently about 60% to 80% higher than entries to Higher Mathematics. Over the period 1994-1999, as the population grew and then fell slightly, the Higher Mathematics entries remained fairly stable and there was a slight decline in Higher English entries. New Highers were introduced in 2000 and Revised Highers were offered for the last time in 2001; this period saw a further drop in entries to Higher English while Higher Mathematics entries remained stable. Between 2001 and 2015 there was an extended period during which Higher English entries were consistently about 50% higher than Higher Mathematics entries (Figure 2c), even as both subjects saw a slight decline from 2001 to 2009 followed by an increase relative to the population from 2009 to 2015 (Figure 2b).

The new Highers were introduced in 2015 and the last sittings of the old Highers were in 2016; this period saw a small fall in Higher Mathematics entries but a sharp increase in Higher English entries. Since 2016 both subjects have seen modest increases in entries relative to the population[21], with the result that entries to Higher English now stand at about 65% of the eligible population, their highest level ever; entries to Higher Mathematics stand at about 33% of the eligible population, slightly short of their peak of 35% in 2014.[22] There are currently

[20] This qualification was renamed Applications of Mathematics in 2018.

[21] Higher English has in fact increased in absolute numbers while Higher Mathematics has fallen (Figure 2a).

[22] These entry numbers should be seen against the background of evidence that suggests a general narrowing of course choice in schools with the introduction of N5s (Shapira 2018). This evidence suggests that learners typically take fewer subjects at N5 than was usual at SG, and that there is a tendency for schools to focus more on English,

over 90% more entries to Higher English than to Higher Mathematics (Figure 2c).

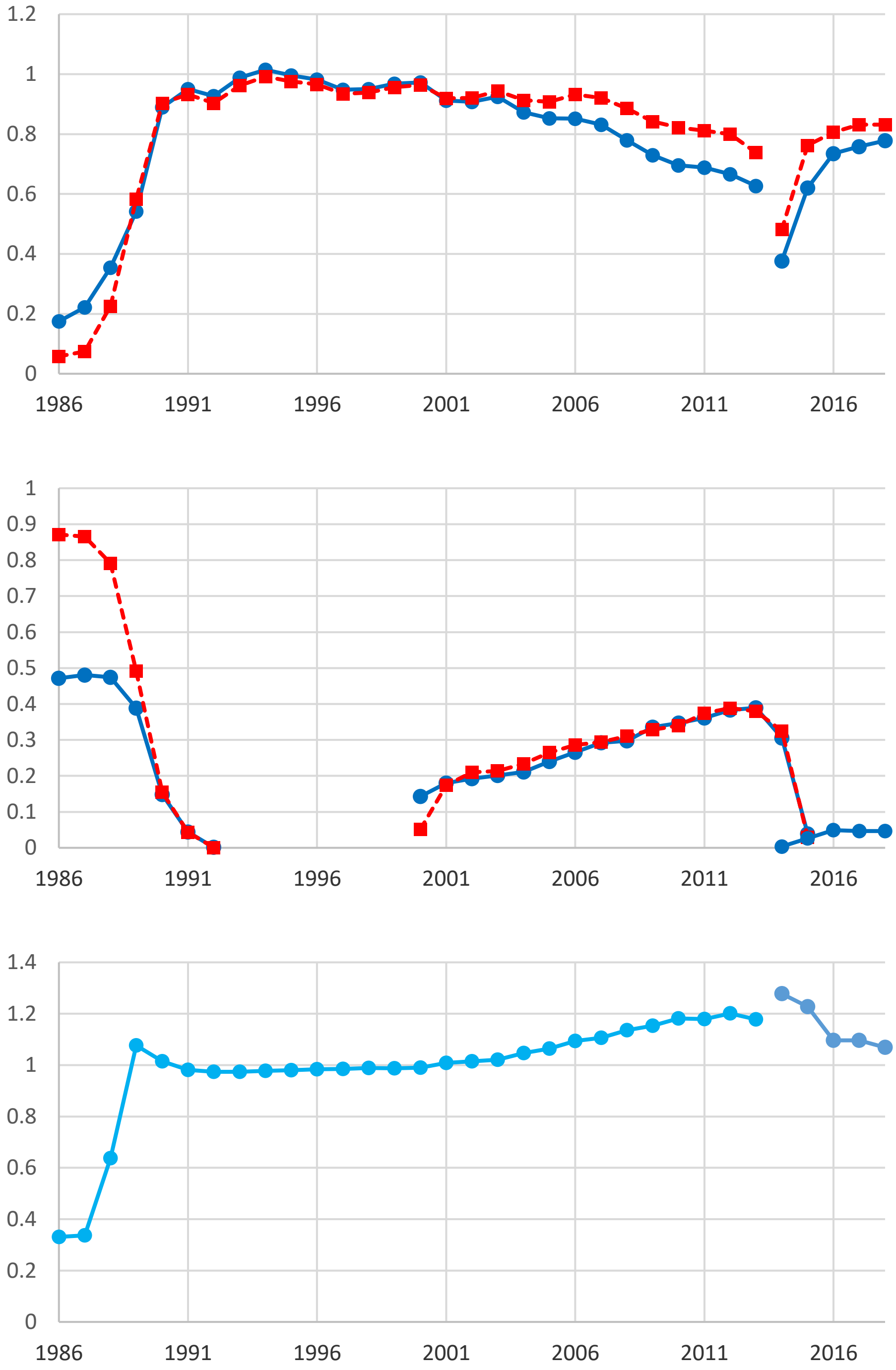

Figure 1. (a) Entries to Standard Grade (1986-2013) or National 5 (2014-2018) Mathematics (blue, solid; circles) and English (red, dashed; squares) as fractions of the population of 16-year-olds. (b) Entries to O Grade (1986-1992), Intermediate 2 (2000-2015), and N5 Lifeskills Mathematics (2014-2018) (blue, solid; circles) and O Grade/ Intermediate 2 English (red, dashed; squares) as fractions of the population of 16-year-olds. (c) The ratio of Standard Grade or National 5 English to Standard Grade or National 5 Mathematics entries.

Mathematics and the sciences. A narrowed choice at N5 is likely to restrict learners' options at Higher, possibly bolstering the uptake of the subjects that remain strongly supported at N5.

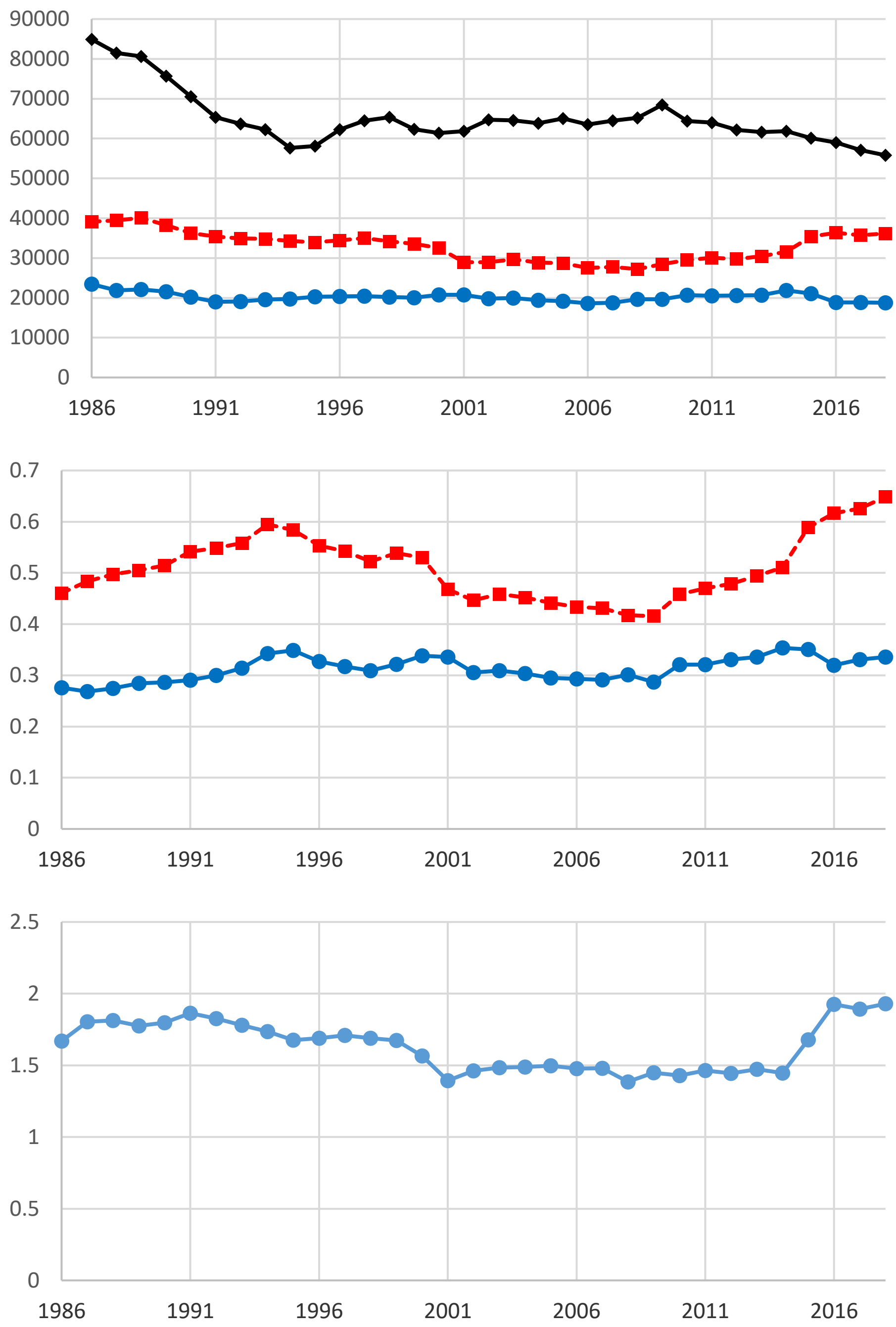

Figure 2. (a) Entries to Higher Mathematics (blue, solid; circles) and Higher English (red, dashed; squares), with population of 17-year-olds (black). (b) Entries to Higher Mathematics and Higher English as fractions of the population of 17-year-olds. (c) The ratio of Higher English to Higher Mathematics entries.

3.2 Attainment statistics

Figure 3 shows the fractions of entries attaining grade A or A-C (upper lines) in English and Mathematics at N5 (Figure 3a, 2014-2018), or attaining grades 1, 1-2, and 1-3 at SG (Figure 3a, 1986-2013), and those attaining grades A or A-C at Higher (Figure 3b); we will refer to these data as attainment relative to the cohort, or cohort-referenced attainment. In contrast, Figure 4 shows the same data, but this time expressed as fractions of the total eligible population; we will refer to these data as attainment relative to the population, or population-referenced attainment. It is important to note that there is not a

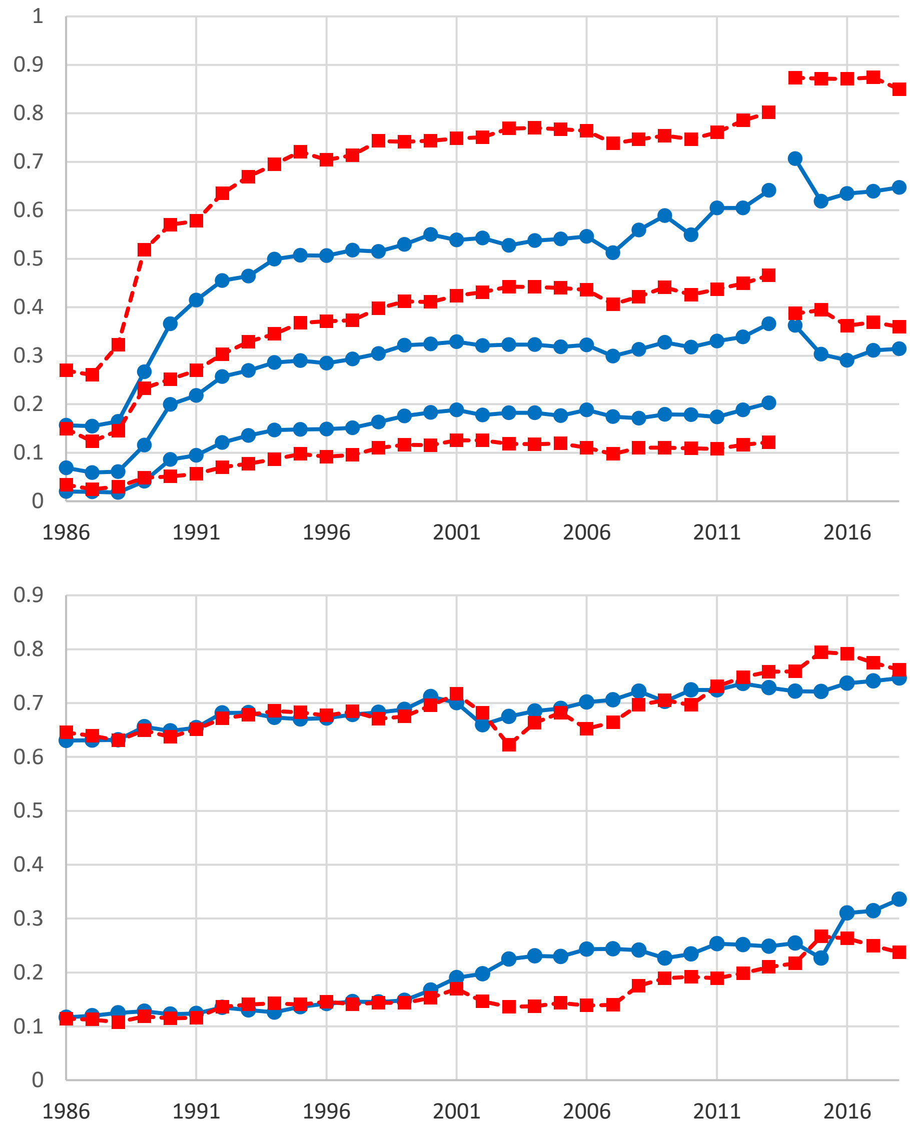

Figure 3. (a) Fractions of entries awarded grades 1, 1-2, 1-3 in English and in Mathematics at Standard Grade (1986-2013) and awarded grades A or A-C in English and Mathematics at National 5 (2014-2018). (b) Fractions of entries awarded grade A or A-C in English and in Mathematics at Higher. In both plots, blue solid lines with circles denote Mathematics and red dashed lines with squares denote English.

direct equivalence between the N5 and SG grades plotted.

The "start-up" period of SG from 1986-1992 saw rapidly increasing cohort-referenced attainment (Figure 3a), suggesting that many schools continued to enter higher-attaining students for O Grade until SG had become firmly established. The period from 1993-2013 saw very gradually increasing attainment relative to the cohort in both subjects and at all grades, but overall the pattern of cohort-referenced attainment was stable. More entries to SG Mathematics attained the top grade than in SG English, but this pattern reversed for grade 2 and below; "pass" rates (grades 1-3) were typically 50% to 60% for SG Mathematics and 70% to 80% for SG English.[23] Attainment relative to the population (Figure 4a) looks slightly different because of the different trends in entry numbers from 2000

[23] It is known that in other contexts, particularly higher education, very different mark distributions are seen in different disciplines. Mathematics tends to have a wide spread of marks on a percentage scale, while English typically has a much narrower distribution (Bridges *et al.* 1999; Simpson 2016).

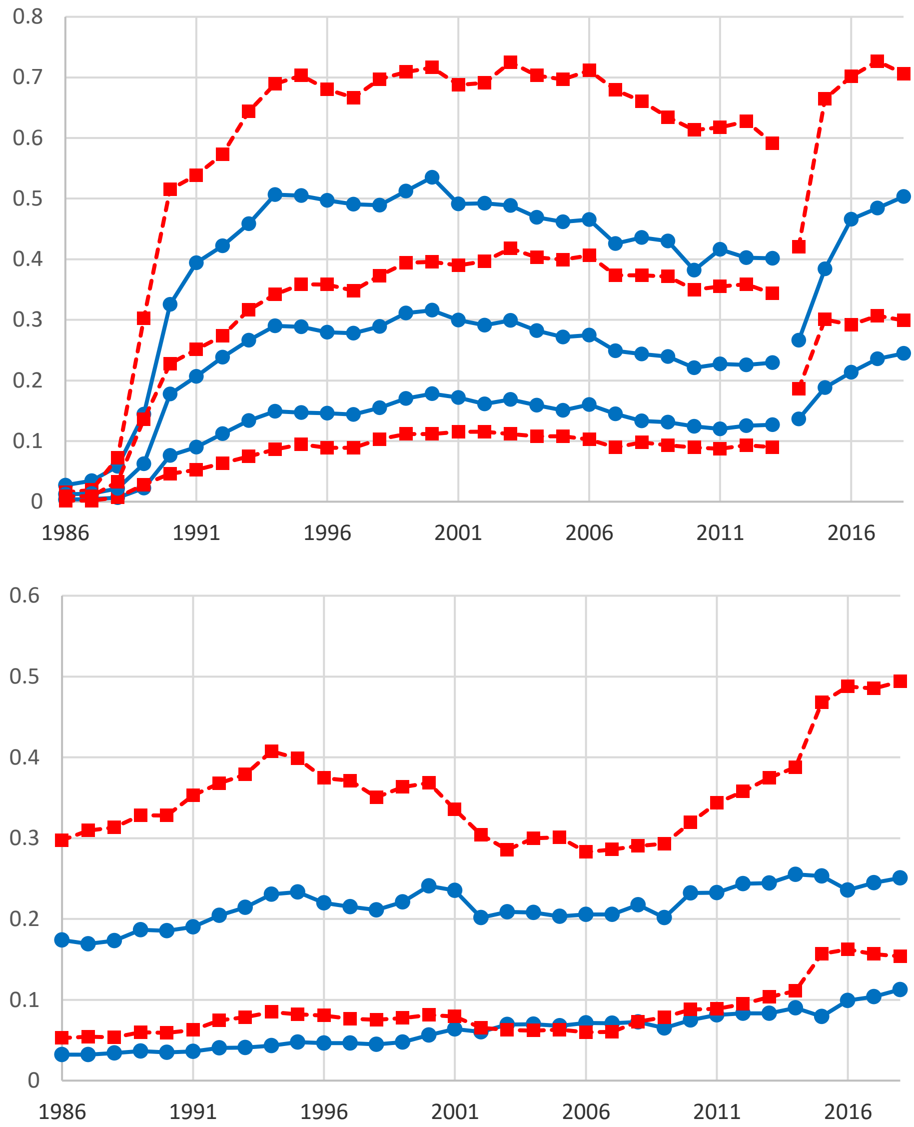

Figure 4. (a) Fractions of the 16-year-old population awarded grades 1, 1-2 or 1-3 in English and in Mathematics at Standard Grade (1986-2013) and fractions awarded grades A or A-C in English and Mathematics at National 5 (2014-2018). (b) Fractions of the 17-year-old population awarded grade A or A-C in English and in Mathematics at Higher. In both plots, blue solid lines with circles denote Mathematics and red dashed lines with squares denote English.

onward (Figure 1a): the fraction of the eligible population attaining a "pass" in SG English fell from around 70% in 2001 to just under 60% in 2013, while it fell from about 50% to 40% for SG Mathematics. Attainment at grade 1 in Mathematics fell only slightly relative to the population while in English it remained stable (Figure 4a), suggesting that it was generally the lower-attaining students who were taking alternative qualifications.

The first year of N5 was marked by high cohort-referenced attainment in N5 Mathematics (Figure 3a), followed by a downward "correction" the following year and by a very gradual increase thereafter; N5 English has remained fairly consistent. It is noticeable that although the "pass" rate in N5 English remains substantially higher than in N5 Mathematics (85% to 65% in 2018), the replacement of the old grade 1 with the broader grade A (roughly equivalent to grades 1-2 at SG) means that the trend for SG Mathematics to have more top grades than SG English has now reversed; in 2018 the figures were 31% for N5 Mathematics and 36% for N5 English.

Again, the population-referenced data (Figure 4a) look slightly different; low entry numbers mean that 2014 was clearly anomalous, and since then there has been a pronounced increase in population-referenced attainment in N5 Mathematics while N5 English has remained fairly stable. Very roughly speaking, the fraction of the population who attained grade A in N5 Mathematics was similar in 2014 to the fraction who attained grade 1 in SG Mathematics in 2013 and is now similar to the fraction who attained grades 1-2 at SG in 2013; meanwhile the fraction who attained grades A-C in N5 Mathematics in 2014 was similar in 2014 to the fraction who attained grades 1-2 in SG Mathematics in 2013 and is now similar to the fraction who attained grades 1-3 at the peak of SG. This contrasts with English, where the fraction of the population who attain grade A in N5 is somewhat lower than the fraction who attained grades 1-2 in SG, but the fraction who attain grades A-C in N5 is again comparable with those who attained grades 1-3 in SG.

Given these rapid changes to the cohort and the difference between cohort-referenced and population-referenced attainment, it is not surprising that there has been confusion over the appropriate level of presentation for some candidates, exacerbated by a sense that N4 is not valued by parents/carers and employers.[24] Patterns of presentation may still be adjusting, although the number of entries (Figure 1a) appears to be gradually rising, rather than falling as one would expect if the number of "aspirational presentations" had started to decline.

Two distinct periods are evident in the cohort-referenced attainment data for Higher Mathematics and Higher English (Figure 3b). From 1986 to 2001 there was a gradual increase in attainment, and the two subjects remained very close at both grade A and at grades A-C. Following the qualification reform in 2000-2001 the subjects diverged significantly; the fraction of entries attaining an A in Higher Mathematics increased more rapidly than in Higher English, although by 2014 Higher English had nearly caught up. The pass rate for Higher Mathematics remained stable during this period, while that for Higher English fell in the early years of the revised qualifications and then gradually increased, eventually overtaking Higher Mathematics.

The Monitoring Standards Over Time reports for Higher English (2005/2011 and 2011/2015) both found that the overall demand of the qualifications was broadly the same, attributing the higher attainment to candidates' and teachers' increasingly clear understanding of the required standards and (in 2015) to better "dovetailing" with N5. (The pass rate in 2005 was 68%; in 2011 it was 73%; and in 2015 it was 79%.) In contrast, the MSOT report for Higher Mathematics (2004/2011) found consistency at the lower grade C boundary (note that the pass rate in 2004 was 68% and the pass rate in 2011 was 72%) but expressed some concern that the quality of scripts at the A/B boundary had fallen; it recommended that this boundary be scrutinised carefully in 2013. (The grade-A rate in 2004 was 23%; in 2011 it was 25%; and in 2013 it was 25%.)

The introduction of the new Highers in 2015-16 was associated with a small fluctuation in cohort-referenced attainment. Overall attainment for the controversial Higher Mathematics[25] in 2015 was similar to that in the previous year; this was followed by an increase in the fraction of entries attaining grade A, so that in 2018 this stood at 34%, its highest ever. Meanwhile there has been a slight decline in the fractions of entries attaining A or A-C in Higher English.

Again, the population-referenced attainment data (Figure 4b) present a slightly different picture. Between 1986 and 2000 there were substantial fluctuations in the fraction of the population attaining a pass at Higher in both subjects, substantially driven by changes in the number of entries (Figure 2b), and the higher entry numbers in English meant that a higher fraction of the population were attaining both passes and grade As in Higher English than in Higher Mathematics. After 2001, the increase in the fraction of Higher Mathematics entries awarded grade A, and the drop in the fraction of Higher English entries awarded grade A, meant that until 2014 comparable fractions of the

[24] See, for example, section 3.4 of the SQA report on the National Course fieldwork visits (SQA 2017b).
[25] See, e.g., the BBC News coverage of this story at https://www.bbc.co.uk/news/uk-scotland-33760350.

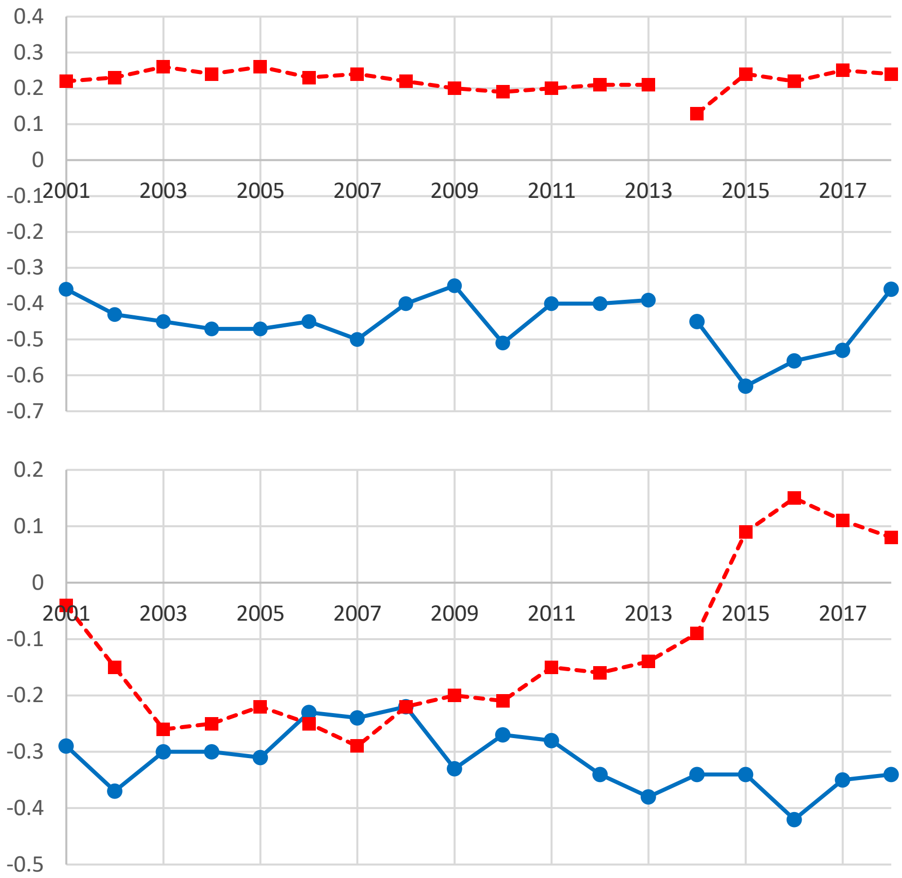

Figure 5. National Ratings for (a) Standard Grade or National 5 English and Standard Grade or National 5 Mathematics; (b) Higher English and Higher Mathematics. In both plots, blue solid lines with circles denote Mathematics and red dashed lines with squares denote English.

population were attaining grade A in each subject; however, Higher English showed a more rapid increase over this period than Higher Mathematics. During this period the large increase in the number of Higher English passes, and the more moderate increase in the number of Higher Mathematics passes, was largely driven by changes in entry numbers (Figure 2b). The large increase in the cohort-referenced grade-A rate in Higher Mathematics after 2015 (Figure 3b) is more moderate in population-referenced terms (Figure 4b) because it is offset by a slight decrease in the number of entries. Meanwhile the large increase in entries to Higher English has led to significantly higher fractions of the population attaining both passes and grade As since 2015; at 49% and 15% respectively, both are now comfortably above their previous maxima.

3.3 National Ratings

Figure 5 shows the NRs for Mathematics and English at Standard Grade and N5 (Figure 5a) and at Higher (Figure 5b), over the period 2001 to 2018, i.e. since the "Higher Still" reforms. It is important to recall that although we show only the NRs for Mathematics and English, the NRs are affected by results across every subject taken together with them, so fluctuations in the NRs for either may reflect changes in other qualifications as well.

The NRs for SG English were fairly stable, between 0.19 and 0.26, from 2001 to 2013 (Figure 5a). The NRs for SG Mathematics fluctuated somewhat more, varying between -0.51 and -0.35, indicating that in the sense measured by the NRs, SG Mathematics was consistently more "difficult" than SG English, but usually remained within the ±0.5 bounds regarded as acceptable by the SQA. The introduction of N5 saw the NRs for both subjects fall slightly.[26] N5 English promptly recovered, and its NRs since 2015 have been similar to those for SG English (despite the slightly higher pass rate (Figure 3a)). N5 Mathematics was distinctly out of line in 2016, with an NR of -0.61. Since then it appears to have recovered — although this appearance depends strongly on the upturn in 2018 which may prove to be a one-off — and in 2018 its NR was -0.36.[27]

The NRs for Higher Mathematics and Higher English (Figure 5b) appear more volatile. From 2001 to 2014, the NRs for Higher Mathematics varied between -0.37 and -0.22, with no systematic trend apparent; the NRs for Higher English varied rather more, becoming briefly comparable to Higher Mathematics in the mid-2000s and then seeing a sustained increase from -0.29 in 2007 to -0.09 in 2014, which coincided with an increase in entry numbers (Figure 2b) and cohort-referenced attainment (Figure 3b). As noted above (Section 3.2), the MSOT reports for 2005/2011 and 2011/2015 concluded that standards in Higher English had been broadly maintained over this period.

The NRs for Higher Mathematics have not changed much since the introduction of the new qualifications in 2015, despite the drop in the fraction of students taking the qualification; the 2016 rating was slightly out of line at -0.42 but the NRs appear to have stabilised since then. Higher English, in contrast, became relatively easier between 2014 and 2016, when its NR peaked at 0.15; this coincided with an increase in entry numbers (Figure 2b) and an increase in cohort-referenced attainment over this period (Figure 3b). There has been a slight fall since then, but the NR for Higher English remains slightly positive while that for Higher Mathematics remains negative.

3.4 Progression Tables

The Progression Tables from SG or Intermediate 2 in 2013 to Higher in 2014 are shown in Table 1; those from N5 in 2015 to Higher in 2016 and from N5 in 2016 to Higher in 2017 are shown in Table 2. The figures for 2012-13 are very similar to those for 2013-14, while those for 2014-15 are complicated by the multiple versions of Higher that were available in 2015 and so are omitted here.

Standard Grade and Intermediate 2 to Higher (Table 1)

There were 31 589 entries for Higher English in 2014 (cf. 13 677 learners progressing from SG and 11 064 learners progressing from Intermediate 2); there were 21 851 entries for Higher Mathematics in 2014 (cf. 10 133 learners progressing from SG and 5753 learners progressing from Intermediate 2). Thus, of the candidates for Higher English in 2014, 43% had taken SG in 2013, 35% had taken Intermediate 2 in 2013, and the remaining 22% had presumably taken qualifications in an earlier sitting or elsewhere. Meanwhile, of the candidates for Higher Mathematics in 2014, 46% had taken SG in 2013, 26% had taken Intermediate 2 in 2013, and 28% had taken qualifications in an earlier sitting.

We can also view these figures in terms of the proportion of candidates progressing from each prior qualification. Of the 46 656 candidates for SG English in 2013, 29% proceeded to Higher English in 2014, while of the 23 465 candidates for Intermediate 2 English in 2013, 47% proceeded to Higher English in 2014. Meanwhile, of the 38 685 candidates for SG Mathematics in 2013, 25% proceeded to Higher Mathematics in 2014, while of the 24 064 candidates for Intermediate 2 Mathematics in 2013, 24% proceeded to Higher Mathematics in 2014. It is clear that the route from Intermediate 2 to Higher was rather more strongly established in English than in Mathematics.

[26] The shift from the seven-grade scale at SG to the nine-band scale at N5 would be expected, all other things being equal, to increase the magnitude of NRs proportionately without changing their sign.
[27] Following the widening of grade D between 2017 and 2018, grade D awards increased from 5% to 9.1% of entries in N5 English, and from 7.2% to 13.8% of entries in N5 Mathematics. It is not clear how much this change in band for about 5% of students affected the NRs, but it seems unlikely that it was the main factor behind the increase in the NR for N5 Mathematics in 2018; no comparable change occurred for N5 English.

		Mathematics: Percentage of Learners Gaining Higher 2014					
	Result	**A**	**B**	**C**	**D**	**No Award**	**Total**
Standard Grade 2013	**1**	44%	29%	19%	5%	4%	100%
	2	3%	16%	30%	18%	33%	100%
	3	8%	15%	22%	17%	38%	100%

Total Learners Progressing	10,133

		English: Percentage of Learners Gaining Higher 2014					
	Result	**A**	**B**	**C**	**D**	**No Award**	**Total**
Standard Grade 2013	**1**	51%	29%	16%	2%	2%	100%
	2	13%	26%	35%	12%	14%	100%
	3	3%	9%	33%	19%	36%	100%

Total Learners Progressing	13,677

		Mathematics: Percentage of Learners Gaining Higher 2014					
	Result	**A**	**B**	**C**	**D**	**No Award**	**Total**
Intermediate 2 2013	**A**	26%	20%	23%	10%	21%	100%
	B	1%	5%	17%	15%	62%	100%
	C	0%	5%	12%	11%	71%	100%
	D	0%	5%	15%	15%	65%	100%
	No Award	0%	4%	21%	13%	63%	100%

Total Learners Progressing	5,753

		English: Percentage of Learners Gaining Higher 2014					
	Result	**A**	**B**	**C**	**D**	**No Award**	**Total**
Intermediate 2 2013	**A**	37%	29%	23%	5%	5%	100%
	B	5%	18%	36%	16%	25%	100%
	C	2%	6%	27%	18%	47%	100%
	D	0%	1%	18%	24%	56%	100%
	No Award	4%	5%	12%	14%	65%	100%

Total Learners Progressing	11,064

Table 1. Progression Tables from Standard Grade or Intermediate 2 to Higher in 2014. In the Standard Grade tables, all entries for grades 4 are blank (fewer than 20 students) so they have been omitted.

This tendency is confirmed by the outcomes in the Progression Tables, which show distinctly higher attainment for those progressing from Intermediate 2 to Higher in English than in Mathematics (Table 1c,d): for any grade other than an A in Intermediate 2 Mathematics, someone who progressed to Higher Mathematics the following year was likely to receive no award, whereas just over half of those progressing from Intermediate 2 English to Higher English even with a grade C received an award at Higher.

		Mathematics: Percentage of Learners Gaining Higher 2017					
	Result	A	B	C	D	No Award	Total
National 5 2016	A	49%	26%	15%	4%	6%	100%
	B	6%	17%	28%	15%	34%	100%
	C	2%	8%	20%	14%	57%	100%
	D	2%	5%	18%	10%	65%	100%
	No Award	3%	14%	8%	3%	72%	100%

Total Learners Progressing	15,072

		English: Percentage of Learners Gaining Higher 2017					
	Result	A	B	C	D	No Award	Total
National 5 2016	A	47%	32%	16%	3%	2%	100%
	B	9%	26%	35%	15%	15%	100%
	C	2%	10%	28%	20%	40%	100%
	D	0%	5%	21%	15%	59%	100%
	No Award	15%	12%	23%	19%	31%	100%

Total Learners Progressing	29,331

		Mathematics: Percentage of Learners Gaining Higher 2016					
	Result	A	B	C	D	No Award	Total
National 5 2015	A	49%	26%	15%	4%	6%	100%
	B	5%	18%	30%	14%	33%	100%
	C	1%	9%	21%	15%	53%	100%
	D	0%	6%	23%	15%	56%	100%
	No Award	2%	2%	22%	22%	51%	100%

Total Learners Progressing	14,315

		English: Percentage of Learners Gaining Higher 2016					
	Result	A	B	C	D	No Award	Total
National 5 2015	A	47%	30%	18%	3%	2%	100%
	B	9%	24%	37%	15%	15%	100%
	C	2%	10%	31%	22%	35%	100%
	D	0%	8%	22%	14%	55%	100%
	No Award	2%	7%	18%	15%	58%	100%

Total Learners Progressing	29,130

Table 2. Progression Tables from National 5 to Higher in 2016 and in 2017.

A similar pattern can be seen in the PTs for those progressing from SG to Higher (Table 1a,b). Those progressing with a grade 1 in English performed slightly better than those progressing with a grade 1 in Mathematics, but the difference is much stronger for those progressing with a grade 2.[28] Taken together with the outcomes for learners progressing from Intermediate 2, this suggests that prior knowledge has a stronger effect on attainment in Mathematics than in English.

National 5 to Higher (Table 2)

There were 35 716 entries for Higher English in 2017 (cf. 29 331 learners progressing from SG in 2016), and there were 18 861 entries for Higher Mathematics (cf. 15 072); thus around 18% of Higher English entries and 20% of Higher Mathematics entries had not taken N5 the previous year. There were 45 856 entries for N5 English in 2016, and there were 41 780 entries for N5 Mathematics in 2016 (plus 2796 for Lifeskills Mathematics, of whom only 5 progressed to Higher Mathematics in 2017); thus around 64% of learners who took N5 English progressed to Higher English in the following year while 36% of learners who took N5 Mathematics progressed to Higher Mathematics in the following year. The proportions for the previous year are similar.[29] Thus, both N5 English and N5 Mathematics have a higher proportion of learners progressing immediately to Higher than either Standard Grade or Intermediate 2 did, but the increase since the introduction of N5s is rather more marked in English than in Mathematics.

The pattern of progression represented by the PTs has been fairly stable over the two years of progression from N5 to Higher. As before, the greatest variation in outcomes is for learners who progressed to Higher with poor grades (D or NA at N5), and we can reasonably assume that this represents a fairly small number of candidates. The pattern of achievement at Higher for learners who progress with an A at N5 is rather similar in English and Mathematics. Learners progressing with a B or a C tend to do worse in Mathematics than in English, which again plausibly represents the greater importance of prior knowledge. Bearing in mind that an A at N5 represents a similar fraction of the population to a grade 1 or 2 at SG (Figures 3a and 4a), the attainment of learners progressing with grades at this level has somewhat improved in both subjects since the introduction of N5: compare in particular the "no award" figures for grade A in Tables 2a,b with those for grades 1 and 2 in Tables 1a,b. In each subject, the fraction of entries with a B at N5 receiving no award at Higher is close to the fraction of entries with a 2 at SG receiving no award at Higher.

What the patterns of progression do not reflect is the much stronger "filtering" between N5 and Higher represented by the fractions of learners progressing; this filtering presumably means that the cohort taking Higher Mathematics is significantly more specialised (and considerably more strongly weighted towards the higher performers at N5) than the cohort taking Higher English.[30] It is not possible to quantify the effect that this "filtering" has on the grade distribution, because the PTs only list percentages of the (unknown) total numbers of candidates in each row of the PT rather than absolute numbers.[31]

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[28] The results for those with grade 3 are more similar, but it is likely that the number progressing with a grade 3 was small. Note that the number who attained grades 1-2 in each subject in 2013 was larger than the total number of learners progressing.

[29] In 2016 around 20% of Higher English entries and 24% of Higher Mathematics entries had not taken N5 the previous year, while around 65% proceeded from N5 English in 2015 to Higher English in 2016, compared to 39% who proceeded from N5 to Higher Mathematics. We omit the raw figures here for brevity.

[30] Although extreme caution is required when drawing parallels between the Scottish and English education systems, it is suggestive that Ofqual (2017: p. 20) found "strong evidence that schools and colleges tend to operate different progression requirements across subject areas: a 'minimum C' policy for English and the humanities; versus a 'minimum B' policy for the sciences and languages".

[31] It is impossible to reconstruct the absolute numbers without knowing either the distribution by N5 grade of those not progressing or the grade distribution at Higher of those who did not take N5 Mathematics in the previous year.

4. Conclusions

4.1 Summary and comments

Scottish school education has been in a state of continual change over the last three decades, and attainment in Mathematics and English reflects this change. There is no evidence that in this period there has ever been a "golden era of consistency" in which entry numbers, attainment rates and measures of relative difficulty all remained constant. Rather, there has been small-scale volatility from year to year; there have been changes associated with every qualification reform; and there has been a long-term trend of increased attainment, especially when considered in population-referenced terms.

At a national level, pass rates and the fraction of entries awarded particular grades have historically varied by as much as a few percentage points from year to year, so any annual changes smaller than this cannot safely be separated from random noise. Bearing in mind that Mathematics and English are the largest subjects nationally, we might reasonably expect to see greater volatility than this both in smaller subjects and in the same subjects considered at regional or local level.

There has been a persistent trend for Higher English to be more popular than Higher Mathematics, but the extent to which this is the case has varied considerably. Currently the ratio of Higher English entries to Higher Mathematics entries is at its highest level during the period considered (Figure 2c). This is principally due to the significantly higher uptake of Higher English since the qualifications reform in 2015 associated with the advent of Curriculum for Excellence. Higher Mathematics has not received a similar boost from CfE.

These differences in uptake naturally invite the argument that Higher English is significantly "easier" than Higher Mathematics, but the evidence for this is mixed. The National Ratings do suggest that English has become slightly easier than other Higher subjects in recent years, while Mathematics remains slightly harder: the difference between the NRs for Higher Mathematics and Higher English is currently around 0.5, corresponding to half a grade. From about 2003 the pass rate in Higher English rose more rapidly than in Higher Mathematics but from a lower baseline, overtaking it around 2010. This rise was amplified by the higher proportion of the population taking Higher English than Higher Mathematics, and coincided with the increase in the NR of Higher English. In contrast, however, since 2001 there has been a fairly consistent trend for a greater fraction of entries to receive an A in Higher Mathematics than in Higher English; indeed, the grade-A rate in Higher Mathematics reached a record 34% in 2018.

The changing size of the cohorts taking each subject makes long-term comparability, even in a statistical sense, difficult to appraise. It might appear reasonable to argue that for the sake of consistency the pass rate for a given qualification should remain constant over time. It might appear equally reasonable to argue that the fraction of the eligible population passing that qualification should remain constant over time. These criteria are not the same even for subjects the size of Mathematics and English, as the period between 2001 and 2014 illustrates (cf. Figures 3b and 4b). Moreover, the qualitative analysis of assessment demand conducted as part of the Monitoring Standards Over Time exercise is sometimes at odds with what the attainment statistics, considered in isolation, might suggest (see the comments in Section 3.2).

These considerations for Higher also apply to N5 and its precursor Standard Grade. Except perhaps during the period from 1992 to 2000, the cohorts taking these qualifications have changed considerably over the years, making trends difficult to discern. There has been a consistent gap between the National Ratings for SG/N5 Mathematics and English, with English typically appearing "easier" than Mathematics by around 0.6 to 0.7; this gap widened to 0.87 immediately following the introduction of N5s, and now appears to be narrowing (Figure 5b). The other change associated with the introduction of N5s is that Standard Grade 1 has no directly equivalent grade at N5; this has reversed the tendency for a higher fraction of entries to receive top grades in Mathematics than in English. Meanwhile the pass rate in

English remains significantly higher (85% versus 65% in 2018; see Figure 3a). The Progression Tables (Tables 1 and 2) suggest that success in Higher Mathematics depends more strongly on a good grade in N5 Mathematics (and, previously, in Standard Grade Mathematics) than success in Higher English does on a good grade in N5 English; thus, the lower attainment in N5 Mathematics may act as a stronger filter restricting progress to Higher Mathematics.

The qualification reforms in 2014-15 inevitably affected patterns of both attainment and progression. Most notably, they appear to have accelerated the existing trend of increasing attainment in Higher English. Less obviously but importantly, the reforms disrupted the pathways of progression to Higher, both because there is no direct equivalence between Standard Grade and National 5 grades and because of the removal of the Intermediate 2 pathway. Under the circumstances, it is not surprising that there has been confusion over the appropriate level of presentation for some candidates, with consequences both for attainment and for the perception of the qualifications. It is important that all discussions of the reform and of current attainment recognise both this disruption and the achievement of the Mathematics teaching profession in maintaining, and even improving, learners' attainment through this highly challenging period.

4.2 Some key messages for stakeholders

1. Patterns of attainment in Mathematics and English are different. Most notably: the national pass rate at National 5 is higher in English (85%) than in Mathematics (65%); the uptake of Higher English is almost twice as high as the uptake of Higher Mathematics; the fraction of entries awarded grade A in Higher Mathematics is substantially higher than the fraction in Higher English; the relative difficulty of Mathematics qualifications as measured by the National Ratings is greater than the relative difficulty of English qualifications. Most of these differences pre-date the recent qualification reform but some may have been accentuated by it. As a consequence of these differences at national level, it is not meaningful to compare attainment in Mathematics directly with attainment in English at the level of a school or a local authority. Such a comparison would tend to misrepresent the achievements of learners and teachers alike.

Comparing patterns of attainment in other subjects lies beyond the remit of this report. Nevertheless, given that these differences exist between the two largest subjects in Scotland, which have analogous roles as gateways to further study, one might reasonably expect to find differences of at least comparable magnitude between smaller and more specialised subjects with more widely differing cohorts of candidates.

It also lies beyond the remit of this report to consider the reasons why patterns of attainment should differ between different subjects, and whether these differences are justifiable. Such questions require consideration of the purpose of the qualifications and the process of assessment, and cannot be addressed solely on the basis of statistical evidence.

2. Patterns of attainment are not constant in either Mathematics or English. Policy-makers and those commenting on educational policy should be aware of the historical scale of year-to-year variation and of longer-term patterns. Focusing on small details of annual variation is essentially meaningless. They should also be aware that reforms affect qualifications both directly through changes to the specifications and indirectly through changes to the cohorts taking them, and that both changes are reflected in attainment statistics. Against a continually changing educational, social and demographic background, changes in attainment statistics — especially taken in isolation — cannot provide reliable evidence of a change in "standards".

In addition to these changes over time, attainment at the level of individual schools is likely to reflect local conditions. For these reasons, we caution against the use of national data as a baseline with which to compare school performance; more sophisticated benchmarking tools are required for this purpose.

3. Attainment in Mathematics is more sensitive to prior qualifications than attainment in English. It is plausible that this reflects the strongly structured nature of mathematical knowledge, and it has obvious consequences for decisions about subject choice and presentation. It might be helpful for some learners to take more flexible routes through SCQF5 and SCQF6 rather than to follow the default pathway of taking National 5 in S4 followed by Higher in S5.

As noted above, there are also some systematic differences between attainment patterns in Mathematics and in English. However, it should be recognised by learners and those advising them that these systematic differences are likely to be smaller than the differences between individuals' interest in and aptitude for these subjects. Interest and aptitude, along with the educational and career options opened by a given qualification, provide a much better guide to appropriate subject choice than difficulty or perceived difficulty overall.

4. Attainment figures considered in isolation are potentially misleading. The SQA invest considerable effort in monitoring attainment with the aim of maintaining the standards of their qualifications over time. No single set of figures is adequate to capture the full picture, and different figures (for example pass rates and National Ratings) can present apparently contradictory pictures.

The data collated in this report indicate the thoroughness of the SQA's monitoring processes, and in general — with the exception of the period 2015-2017 — the SQA have been successful in keeping the National Ratings for Mathematics and English within the intended tolerance. However, the NRs do not provide a full picture of the nature and demands of qualifications. The Monitoring Standards Over Time process is designed to scrutinise changes in assessments over longer timescales and, like the annual Awarding Meetings, considers both statistical evidence and expert appraisal of the assessments. The last MSOT exercise for Higher English was in 2015; taken together with the 2011 exercise it indicated that despite rapid improvements in attainment the demand of this qualification had remained broadly the same since 2005. No MSOT report for Higher Mathematics has been published since 2011. In the light of the recent changes to these important qualifications, it may be an appropriate time to revisit this process.

We also suggest that at present it is easy for stakeholders to misinterpret the available data and the SQA's objectives in setting standards. In particular:

(a) The National Ratings, while not published on the SQA website, circulate among schools and teachers and are not always thoroughly understood. It is unfortunate that the description of the methodology in the official spreadsheet is so strongly simplified.
(b) The SQA's stated aim (SQA 2017a) is to ensure that qualifications in different subjects at a given level are "broadly" comparable. Although this aim cannot be defined in a purely statistical sense, at present the vagueness surrounding it means that stakeholders are liable to impose their own partial interpretations on the processes and on the information provided. This leads to frustration and fails to do justice either to the achievements of learners or to the SQA's own processes.

With these considerations in mind, we suggest that it would be valuable for the SQA to work with stakeholders to encourage greater understanding of assessment standards and of the various statistics that capture aspects of learners' attainment.

To conclude, the debate surrounding qualifications in Scotland in recent years has often suffered from misunderstandings between stakeholders, including teachers, learners and policymakers, and misinterpretations of the evidence available. We hope that this report will help to clear up some of these misunderstandings and to provide those involved in Scottish mathematical education and assessment with a basis for constructive discussions in the future.

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Appendix

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Data sources

Population figures are taken from the spreadsheet *Population Pyramids of Scotland, 1981-2041*, published by the National Records of Scotland and available online from
https://www.nrscotland.gov.uk/statistics-and-data/statistics/statistics-by-theme/population/population-projections/population-projections-scotland/population-pyramids-of-scotland.

Attainment statistics are tabulated in spreadsheets variously titled *Attainment Statistics (December)* (2014-2018); *Post-Appeals National Qualification Results* (2006-2013) and other names pre-2006; post-appeals data have been taken throughout. All these are available online via the SQA Statistics Archive,
https://www.sqa.org.uk/sqa/57518.8313.html.

Grade boundaries are available via the SQA Information Archive,
https://www.sqa.org.uk/sqa/57520.html.

National Ratings are not published online but are available from the SQA on request. NRs for all qualifications are tabulated in the spreadsheets titled *National Ratings December* (2014-2018) or *Post Appeal National Ratings* (2001-2013).

Progression tables for the years 2013 to 2017 are in the spreadsheets available online from
https://www.sqa.org.uk/sqa/84433.html.

Monitoring Standards Over Time reports are available online from
https://www.sqa.org.uk/sqa/74496.8276.html.

The numbers of pupils in state-funded schools from 1995-2017 are published by the Scottish Government, and are available online from
https://www2.gov.scot/Topics/Statistics/Browse/School-Education/HistoricDatasets/PupilTeacherHistoric.