PROBABILISTIC STUDY ON HYDRAULIC CONDUCTIVITY OF CONCRETE AT MESO-SCALE

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ABSTRACT

Hydraulic conductivity of concrete can be used as a key indicator in assessment of service life of concrete structures. In this paper, a probabilistic investigation on hydraulic conductivity of concrete is conducted, allowing for variation in hydraulic properties of concrete constituents. Concrete is modeled as a three-phase composite at meso-scale, consisting of mortar, aggregates and the Interfacial Transition Zone (ITZ). A Finite Element (FE) method is developed to calculate the hydraulic conductivity of concrete, which is then verified using available experimental results. Based on a large pool of samples generated from Monte Carlo simulation, a conceptual model relating hydraulic conductivity of concrete to aggregate volume fraction ratio and hydraulic conductivity of mortar and the ITZ is proposed. It is shown from the probabilistic-based sensitivity analysis that hydraulic conductivity and thickness of the ITZ are amongst the most influential factors affecting the bulk hydraulic conductivity of concrete. It is also shown that for high aggregate volume fraction ratios, due to increasing volume of the ITZ, the coefficient of variation of hydraulic conductivity can be as high as 0.36.

Keywords: Concrete; Hydraulic Conductivity; Meso-scale; Probabilistic

INTRODUCTION

In most of the concrete deterioration processes, e.g., corrosion of reinforcing steel, water is either the principal cause of deterioration or the principal medium by which aggressive agents such as chloride or sulphate ions are transported into the concrete. It has been reported that water ingress is the main cause of deterioration in tunnel linings (ITA Working Group on Maintenance 1991; Russell and Gilmore 1997) leading to considerable cost of remediation and strengthening. Therefore, the long-term durability of concrete material is essentially related to its moisture transport properties. Service life of concrete structures, as a measure of their durability, can be directly quantified using moisture transport properties of concrete (Ho and Chirgwin 1996; Murata et al. 2004). Water ingress in concrete can be classified into two types: pressurized seepage flow through saturated concrete and the strongly nonlinear capillary-driven flow through unsaturated concrete. Water flow through unsaturated concrete follows the Fickian diffusion (Hall 1994; Leech et al. 2003), in which the hydraulic diffusivity coefficient is the most important moisture transport property. In this type of flow, the water penetration depth can be used as a service life measure (Lockington et al. 2002). On the other hand, flow of water in saturated media including concrete follows the Darcy's law with the hydraulic conductivity as the main water transport property. In order to make an adequately accurate service life prediction, it is vital that the water transport properties of concrete and factors influencing them are correctly characterized.

As concrete is a heterogeneous composite comprising of cement paste and randomly distributed fine and coarse aggregates, water transport properties of concrete significantly depend on its pore structure, e.g., the pore diameter and distribution, and the pore continuity and tortuosity, and to some extent micro cracking (Li et al. 2005). In porous media, transport properties are defined according to porosity, which is a function of water cement ratio and degree of hydration (Young and Hansen 1987). Relationships linking the diffusivity and conductivity to porosity can be found in the literature (Garboczi and Bentz 1996; Halamickova et al. 1995). The Interfacial Transition Zone (ITZ), which is formed due to the presence of aggregates, is highly porous. It is believed that the

porosity and as such conductivity of the ITZ is considerably higher than that of the surrounding matrix, i.e., the cement paste (Winslow et al. 1994), which can have a considerable effect on the transport performance of concrete. Nonetheless, there is little information about size and porosity of this zone (Zheng et al. 2005). A wide range of 10 μm (3.94×10^{-7} in)-300 μm (1.18×10^{-5} in) has been reported for the size of ITZ (Nemati and Gardoni 2005; Ollivier et al. 1995). Breton et al. (1992) have reported that the effective diffusivity of this zone is 6-12 greater than that of bulk cement paste. Shane et al. (2000) reported that the mortar conductivity is predicted to be about 10 times higher than that of a bulk mortar. It has also been shown that porosity of this zone is highest at the contact with aggregates and decreases with distance from aggregates (Bentz and Garboczi 1991; Scrivener et al. 2004; Wu et al. 2016).

On the other hand, since most aggregates used in concrete are dense compared to the cement paste, it is usually assumed that they have zero conductivity or diffusivity (Li et al. 2016; Wang and Ueda 2013). The inclusion of these nonconductive aggregates has an inverse effect on the diffusivity and conductivity of the concrete composite. Various expressions relating the conductivity and diffusivity of composites to volume fraction of aggregates are available in the literature (Hall et al. 1993; Hasselman and Johnson 1987; Neale and Nader 1973; Pietrak and Wisniewski 2015). Volume fraction of aggregates and size and porosity of the ITZ have contradicting effects on the transport properties of concrete. In a statistical study on conductivity of concrete, Zhou and Li (2010) showed that, the decreasing effect of aggregate volume fraction ratio can be completely cancelled out by high conductivity of the ITZ.

In order to study water movement in concrete, multi-scaled modeling is well accepted (Maekawa et al. 2009). In this approach, water movement in concrete is modeled using the so-called meso-scale method, in which concrete is modeled as a three-phase composite material with aggregates embedded in cement paste matrix and the interfacial transition zone (ITZ) on the interface between the aggregates and the surrounding cement paste. Numerical meso-scale modeling of concrete

material allows detailed investigation of the effect of ITZ and aggregate size and distribution on the water transport properties of concrete. This approach can be very computationally demanding. However, with the significant advance in computer technology, this is now possible. Several meso-scale numerical studies by means of either the Finite Element (FE) method (Li et al. 2017; Li et al. 2016; Zhou and Li 2010) or lattice-type network models based on Voronoi tessellation (Dehghanpoor Abyaneh et al. 2013; Wang and Ueda 2011; Wang and Ueda 2013) have been established to model the water ingress in concrete material at meso-scale level.

From the above discussion, it can be inferred that transport properties of concrete as a composite material depends on volume fraction ratio of aggregates, transport properties of cement paste matrix and size and transport properties of the ITZ, most of which, especially the properties of ITZ, are highly variable (Winslow et al. 1994). Therefore, investigation of transport properties of concrete requires a probabilistic approach. Although Zhou and Li (2010), using a two-dimensional FE model, conducted a sensitivity analysis on the effect of ITZ properties on hydraulic conductivity of concrete, their analysis was not probabilistic. In this paper, in order to study the effect of the above-mentioned factors on conductivity of concrete, a three-dimensional FE analysis, at meso-scale level, embedded in a probabilistic procedure is employed. The formulation of the FE model is presented and the model is verified with available experimental results. Based on rational probability density functions for the conductivity of mortar matrix and the ITZ, a probabilistic sensitivity analysis is conducted. Finally, an expression quantifying effect of aggregated volume fraction ratio and conductivity of mortar and ITZ on overall water conductivity of concrete is presented.

RESEARCH SIGNIFICANCE

As a major water transport property, which can be used as an indicator to assess durability of concrete, hydraulic conductivity depends on highly variable hydraulic conductivity of its constituents: mortar, aggregate and the ITZ. Despite many studies that have been carried out on the influence of properties of concrete constituents on its conductivity, due to highly variable nature of

these properties, the published results are still not consistence. In order to investigate the extent of effect of variability of concrete constituents on its hydraulic conductivity, a probabilistic analysis is needed. It is in this regard that the present paper proposes a probabilistic methodology for investigating relation of hydraulic conductivity of concrete to properties of its constituents.

PRINCIPAL MECHANISM OF WATER FLOW IN CONCRETE

Water flow in saturated porous media such as concrete can be mathematically described using Darcy's law. Analogues to other simple linear transport laws in electricity (Ohm's law), diffusion (Fick's law) and the heat transfer (Fourier's law), Darcy's law establishes a relationship between flow and energy gradient, which is hydraulic potential in the case of water flow, through a transport property referred to as the conductivity. A unified theoretical framework can be applied to heat, diffusion and electrical current and water flow (Hall and Hoff 2011). Conduction can be defined as a flow process of water through a saturated medium, here concrete, under a hydraulic gradient. The validity of Darcy's law will mathematically be acknowledged as,

$$q = -K\nabla h \tag{1}$$

where K is the conductivity coefficient, q is water flow, ∇ is the gradient operator and h is the hydraulic potential. For steady state conduction, continuity requires that,

$$\nabla q = 0 \tag{2}$$

Therefore, given that K is constant, Equations (1) and (2) reduce to,

$$\nabla^2 h = 0 \tag{3}$$

where ∇^2 is the Laplace operator, and the resulted differential equation is the Laplace equation. By having the boundary conditions, variation of hydraulic potential and flow can be determined. There are two types of boundary conditions for the general water transport process, which can be formulated as follows,

$$h = f(x, y, z) \text{ for x, y and z on } \Gamma_1 \text{ domain, [Dirichlet boundary condition]} \quad (4a)$$

$$q = -K \frac{\partial h}{\partial n} \text{ for x, y and z on } \Gamma_2 \text{ domain, [Neumann boundary condition]} \quad (4b)$$

where q is the outward flux normal to the boundary (in direction n); Γ_1 and Γ_2 are the boundary domains with prescribed potential and normal flux, respectively.

The unsaturated moisture transport in porous media on the other hand can be simulated using Richards' equation (Hall 1994). In the unsaturated water flow, which is strongly driven by the capillary forces, the conductivity is highly dependent on the water content, and the solution will be time dependent. In the unsaturated water flow, generally the diffusivity coefficient, which is related to conductivity, is used instead of the conductivity coefficient.

NUMERICAL MODELING

A cylindrical shape with length L and cross-section of A is used for investigating the effective hydraulic conductivity of concrete, K_e, at the meso-scale level. By employing the numerical analysis using the FE method, the steady-state water flow, Q, over the whole cross section of cylinder, A, can be calculated. Then, the effective hydraulic conductivity of the cylindrical specimens considering the particle inclusion and the ITZ can be calculated as follows,

$$K_e = \frac{Q}{A(\Delta h / L)} \quad (5)$$

where $\Delta h/L$ is the hydraulic gradient applied on the cylinder. The ratio of effective hydraulic conductivity to that of mortar, K_e/K_m, is a measure that can show the effect of the aggregate inclusion and the ITZ properties.

Meso-scale Structure of Concrete

In a meso-scale level, concrete comprises of mortar matrix, large aggregates and the ITZ. The total volume of aggregates generally is about 60–80% of the whole volume of the concrete, and the volume fraction of coarse aggregates is between 0.40 and 0.50 (Wriggers and Moftah 2006). The

size distribution of aggregates is determined by given grading curve from sieve analysis. In the numerical analysis of concrete at the meso-scale level, it is common to use an optimal gradation initially proposed by the Fuller for the aggregate size distribution (Li et al. 2016; Ma et al. 2016; Wang et al. 2015).

$$P(d) = 100\left(\frac{d}{d_{\max}}\right)^{\gamma} \tag{6}$$

where d is the size of the aggregate, d_{max} is the maximum size of aggregates and $P(d)$ is the percentage of aggregates having a size smaller than d. γ is a constant varying from 0.45 to 0.70. In this study, a value of 0.50 is used for the gradation curve. Furthermore, for simplicity, only coarse aggregates larger than 3.00 mm (0.118 in) are modeled, and the maximum aggregate size is 20 mm (0.787 in). The large number of fine aggregates together with the cement matrix is treated as mortar, for which homogenous water transport properties, conductivity in this study, are used.

For random generation of aggregates, a standard procedure from the available literature (Ma et al. 2016; Wang et al. 2015) is devised. The basic idea is to generate and place aggregates in a repeated manner and with no overlap and boundary wall collision, until the target volume fraction is achieved. Spherical particles are used to model the coarse aggregates for convenience.

Finite Element (FE) Model

In order to solve the differential equation governing hydraulic conduction in concrete at meso-scale level, shown in Equation (3), different methods such as the finite difference method or the FE can be used. In the numerical methods that use the finite difference technique, discretization based on lattice-type network is commonly employed (Dehghanpoor Abyaneh et al. 2013). The FE method, which is based on meshing the body into small elements, has also successfully been employed for solving the water transport (Li et al. 2017; Li et al. 2016) as well as mechanical (Wang et al. 2015; Xu and Chen 2016) problems at meso-scale level. Furthermore, the FE method has also been used

in the macro-scale modeling of water movement in concrete (Rahal et al. 2016; Van Belleghem et al. 2016; West and Holmes 2005).

Mesh generation of complex three-dimensional meso-scale models in presence of the ITZ, which is extremely thin compared to the normal size of coarse aggregates, is still a challenging task. Xu and Chen (2016) concluded that using solid elements for the ITZ requires very fine meshing and as such it is computationally demanding. They suggested the interface element as an alternative for modeling this zone. The zero-thickness interface element has been used by Li et al. (2016) for modeling the ITZ. The interface element is inserted between the solid elements representing the aggregates and the mortar matrix. When used in the transport problems, this element conducts water between the aggregates and the mortar.

In this paper, using the commercially available FE program, ANSYS (2016), a different approach in modeling the ITZ is followed. As **Fig. 1** shows, the ITZ is modeled using shell elements, and solid element is used for modeling the mortar matrix. Furthermore, as conductivity of aggregates is negligible (compared with that of mortar matrix) the aggregates are not modeled.

Fig. 1

Following the previous discussion of the analogy between governing equation in the heat transfer and water transport problems, in this study, capabilities of ANSYS program in heat transfer are used to model water flow concrete. A three-dimensional conductive element SOLID70 with three-dimensional conduction, shown in **Fig. 2(a)** is used for modeling the mortar matrix. The element has eight nodes (six nodes in case of tetrahedral element) with a single degree of freedom, temperature, at each node.

Fig. 2

The element SHELL57 is a three-dimensional shell element capable of modeling in-plane conduction, shown in **Fig. 2(b)**, is used to model conductivity of the ITZ. The element has four

nodes (or three nodes in case of triangular element) capable of having temperature degrees of freedom, hydraulic potential in this study, at each node. As **Fig. 1** shows, using zero-flux boundary condition, see Equation (4b), water flow normal to aggregate surface is set to zero.

The FE model proposed in this study eliminates the need for sophisticated meshing required in cases of modelling the ITZ using three-dimensional solid element. By introducing the shell element for modelling the ITZ, there is no need to use interface element, which adds extra nodes in the FE model. The number of nodes required for modelling the ITZ using the interface element are twice those required when shell element is used. Furthermore, by replacing the non-conductive aggregate elements with appropriate flux boundary conditions, the number of solid element is considerably reduced. The proposed FE method is robust and can be used for evaluation of hydraulic conductivity of concrete with different aggregate volume fractions. Nonetheless, given the very fine mesh required in cases with high aggregate volume fraction, the meshing process is still time consuming and requires large storage space. Any improvement, which leads to reduction in meshing time and increases the meshing quality, would enhance the proposed FE method.

Validation

In order to validate the FE model described in the previous section for the analysis of hydraulic conductivity of concrete at meso-scale level, samples from Li et al. (2016) study are considered in this section. The laboratory specimens have cylindrical shape with 61.8 μ (2.43 in) diameter and 40.0 mm (1.57 in) height. For coarse aggregate, non-conductive equal-sized spherical Glass particles with 6 mm (0.236 in) and 12 mm (0.472 in) were used. Typical random aggregate generation for specimens with 0.10 to 0.40 aggregate Volume Fraction Ratios (VFR), denoted as φ in this paper, can be seen in **Fig. 3**. For the specimen with VFR = 0.5, number of particles with 6 mm (0.236 in) and 12 mm (0.472 in) size are 292 and 30, respectively. In order to generate the flow gradient, hydraulic potentials of 1300 mm (51.18 in) and 0 mm (0 in) were applied to the top and bottom surface of the cylindrical specimen, respectively.

Fig. 3

In the FE analysis, the aggregate configuration is kept the same with experimental specimens as depicted in **Fig. 3**. The Glass particles are considered as voids with zero flux inside the mortar body as is indicated in **Fig. 4**. The mortar body is meshed using tetrahedral solid elements with maximum size of 2.5 mm (0.10 in). A sensitivity analysis on mesh size showed that using this size lead to adequately accurate results. In the boundary of internal voids and the mortar triangular shell elements representing the ITZ are used. In **Fig. 4**, meshed body with the applied boundary conditions for specimen with VFR of 0.30 is shown. In the experimental program of Li et al. (2016), the outer face of cylinder was sealed. Thus, on the cylinder wall, a zero flux boundary condition is applied.

Fig. 4

Results of the experimental program by Li et al. (2016) showed that the mean hydraulic conductivity of mortar, K_m, is 0.0195 mm/sec (7.67×10^{-4} in/sec). As there is no direct method for measurement of ITZ thickness, t_i, and conductivity, K_i, Li et al. (2016) used sensitivity analysis to find the appropriate properties of this zone. They suggested that a thickness of 20 μm (7.87×10^{-7} in) and a mean conductivity equal to 10 times that of mortar be used. For consistency, in the current study, these values are used for validation of the proposed FE model.

Comparison between FE results obtained in this study and those of experimental program, as shown in **Fig. 5**, shows that there is good agreement between the numerical and test results. For each specimen, five readings have been reported by Li et al. (2016). The numerical FE result in **Fig. 5** is based on average of five random particle generations. For high VFR values, the agreement between the numerical and experimental results seems to be less. This could be attributed to higher surface area between aggregates and the mortar, and as such larger ITZ volume, for high volume fraction ratios. As properties of the ITZ are highly variable, variability of the effective conductivity would be more for specimens with higher VFR values. It also worth noting that, in their numerical

analysis, Li et al. (2016) also attributed the difference between numerical and experimental results to the fact that the property of ITZ is determined by assumptions and the sensitive analyses not from experimental measurement.

Fig. 5

There are various simplified analytical and empirical expressions for effect of inclusion (no ITZ effect) on the effective hydraulic conductivity of porous media with random dispersion of spherical particles. A comparison between results of these models, the FE analysis used in this study and the experimental results is shown in **Table 1**. The VFR of inclusion in this table is denoted as φ. It should be noted that, for the experimental results in **Table 1**, effect of ITZ properties is included.

Table 1

Comparison of the results in **Table 1** shows that the conceptual models (Bruggeman 1935; Jeffrey 1973; Neale and Nader 1973) predict the effective conductivity very well. Furthermore, by comparing the result of FE with no ITZ with the experimental results (mean of five reported readings shown in **Fig. 5**), it can be concluded that the effect of ITZ is more pronounced for high VFR values.

It is also worth noting that considering the hydraulic conductivity of 0.0195 mm/sec for mortar, for the aggregate volume fraction ratios of 0.4 and 0.5, the lower bound k_e/k_m ratios that are obtained by ignoring hydraulic conductivity of the particles and the ITZ are 0.5 and 0.4, respectively (see Table 1). These values are higher than those of Li et al.'s (2016) tests. This suggests that the hydraulic conductivity of mortar in the mixes prepared for higher particle volume fractions deviate from that of mortar with no particle.

PROBABILISTIC ANALYSIS

Hydraulic conductivity of concrete is a function of geometry and conductivity of its three phases, i.e., mortar, aggregates and the ITZ. As all of these properties are highly uncertain, hydraulic

conductivity of concrete should be treated as a random variable. VFR and size distribution of coarse aggregates is random and varies from place to place depending on the homogeneity of the concrete mix. In this study, conductivity of a volume body of concrete is of interest, for which the VFR is fixed. Nonetheless, the coarse aggregates that follow a predefined size distribution, shown in Equation (6), are placed randomly in the volume cell. Five different VFR values of 0.10, 0.20, 0.30, 0.40 and 0.50 are considered.

Hydraulic conductivity of mortar and the thickness and conductivity of the ITZ are treated as random variables. As the conductivity of adequately large volume cell is independent of its size, in this study, the same concrete cylinder used in the verification section [from Li et al. (2016)] is considered in the probabilistic analysis. The element used in the FE analysis and the mesh size is also similar to that discussed in the verification section. For mortar conductivity, the mean and coefficient of variation (COV) obtained by Li et al. (2016) are used, and it is assumed that this variable follows the lognormal distribution. Statistical measurements for properties of the ITZ are rare in the current literature. Only range of the ITZ size has been reported (Nemati and Gardoni 2005; Ollivier et al. 1995). On the other hand, there is little information about the statistical distribution of the hydraulic conductivity of the ITZ (Bentz and Garboczi 1991; Wang et al. 2015). In the absence of probabilistic models for the size and conductivity of the ITZ, the Uniform distribution with the ranges available from the current literature can be used for modeling these variables, as shown in **Table 2**. In the absence of information about frequency distribution of a random variable, the Uniform distribution is a logical choice (Benjamin and Cornell 1975). Off course, availability of more reliable probabilistic model will be a boost to the methodology proposed in this study.

Table 2

Given the statistics for each random variable, the Monte Carlo technique is employed to find the cumulative density function of the hydraulic conductivity of concrete as the independent response.

Trial simulations were conducted by Monte Carlo technique so that the effect of the sample size on the results could be investigated. For this purpose, various sample sizes ranging between 100 and 1,500 for randomly generated values of the basic random variables were used. **Fig. 6** shows the mean effective concrete conductivity (normalized with respect to conductivity of the mortar) obtained with various sample sizes for the case with VFR of 0.40.

Fig. 6

As can be seen from **Fig. 6**, the mean response become stable when the sample size is 1000 and do not change significantly for larger sample sizes. Thus, statistical assessment was performed for the sample size of 1000 for the remaining part of the study. Furthermore, for each VFR, five samples with different random aggregate distributions are used.

RESULTS AND DISCUSSION

Having the geometry, aggregate size distribution and material properties, the hydraulic conductivity of concrete composite can be predicted by means of FE analysis according to Equation (5). As the main variables are random in nature, the conductivity of concrete is probabilistic and its distributions can be simulated by means of the Monte Carlo technique. Statistical measures of concrete conductivity are investigated in this section. The Monte Carlo technique will also be used to conduct sensitivity analysis and derive a relationship between the concrete conductivity and the main random variables, i.e., hydraulic conductivity of mortar and properties of the ITZ.

Uncertainty in hydraulic conductivity of concrete

A typical result of simulated K_e/K_m ratios, based on one the randomly distributed aggregate cases, is shown in **Fig. 7**. As is expected, by increasing the VFR, the K_e/K_m ratio decreases. It is clear that the scatter of the K_e/K_m ratio increases with increasing aggregate VFR, and the range of the simulate K_e/K_m ratio becomes wider. This can be attributed to increasing effect of ITZ properties, which have more variability than that of conductivity of mortar. For higher VFR, the aggregate surface

available for the formation of ITZ is bigger, and as such, contribution of this zone in the final conductivity would be higher. As the results of **Fig. 7** show, the coefficient of variation (COV), is nearly proportional to the aggregate volume fraction ratio. It is worth noting that for consistency, the same bin number has been used in plotting the histogram of K_e/K_m ratio for each of the VFR in **Fig. 7**.

Fig. 7

For other cases of randomly distributed aggregates, similar results are obtained. In **Table 3**, for the volume fraction ratio of 0.40, statistics of K_e/K_m ratio for all the considered cases (cases 1-5) are shown. As it can be seen, for all of these cases, same trend of statistics is observed indicating that the statistical measures obtained from each of these cases are consistent. This shows that the random position of aggregates does not have considerable effect on statistics of the concrete conductivity. Therefore, for the next subsections, results of simulation for all of the considered cases (for each VFR) are combined together, i.e., for each VFR, 5000 samples are used.

Table 3

The minimum and maximum values in **Table 3** correspond to the lower and upper bound values of the ITZ properties shown in **Table 2**. If the model proposed by Neale and Nader (1973) is used for predicting the effective hydraulic conductivity of concrete, a value of 0.50 is obtained for the K_e/K_m ratio (see **Table 1**). This is consistent with the minimum value shown in **Table 3**. By comparing the maximum and minimum values, it is interesting to note that inclusion of the ITZ properties could results in an effective hydraulic conductivity three times that of without the inclusion. In the subsequent sections, this will be investigated in more details.

Sensitivity analysis

In order to find the most important random variables affecting conductivity of concrete, that are the hydraulic conductivity of mortar and conductivity and thickness of the ITZ, a probabilistic

sensitivity analysis based on Pearson's correlation coefficient, was conducted. In the probabilistic sensitivities (based on sample simulated by the Monte Carlo technique), any interaction among the input random variables will be correctly reflected in the probabilistic sensitivities. The bar chart in **Fig. 8** presents the main random variables, as well as the correlation coefficients associated with each of these random variables. It is worth noting the simulated values of all the five cases are combined in the results shown in **Fig. 8**. This means that a total number of 5000 samples are used in derivation of each correlation coefficient.

Fig. 8

Results of the sensitivity analysis shown in **Fig. 8** clearly indicate that sensitivity coefficients, represented by Pearson's correlation coefficient, ρ, vary with the aggregate VFR. For low VFR, the ITZ properties, i.e., the hydraulic conductivity K_i and thickness, t_i, do not have big effect, while for high aggregate VFR, the effect of these properties is considerable. Correlation of effective hydraulic conductivity, K_e, with conductivity and thickness of the ITZ is the same. This is natural, as passage of water though this zone can be logically resembled as movement of water within a water channel, in which thickness of ITZ represents the channel width and ITZ conductivity represents the water speed. The total quantity of water passing through the channel would be proportional to the product of these quantities. Therefore, the unified term $K_i \times t_i$ can be used to relate ITZ properties to conductivity of concrete as a composite material. For the reference, correlation of effective hydraulic conductivity of concrete to this term is added in **Fig. 8**. There is high correlation between this product term and the concrete conductivity. It can be concluded that except for the aggregate volume fraction of 0.10, for all other cases, the term $K_i \times t_i$ has a dominant impact on the concrete conductivity. This highlights the importance of the ITZ and its effect on the overall hydraulic conductivity of concrete as a composite material.

Model fitting

According to Neale and Nader (1973), using a rational function, effective hydraulic conductivity of composite material with randomly dispersed spherical dispersion can be related to volume fraction ratio of inclusion, here the aggregate VFR (see **Table 1**). For low VFR (up to 0.50), with adequate accuracy, this relationship can be linearized as follows,

$$\frac{K_e}{K_m} = 1 - \alpha\varphi \tag{7}$$

The parameter α is a reduction factor, which accounts for effect of aggregate VFR. This relationship does not account for the effect of ITZ properties. Results of the sensitivity factors in the previous section showed that properties of the ITZ represented by the product term $K_i \times t_i$ have the biggest impact on the effective hydraulic conductivity of concrete, K_e. Normalizing the effective hydraulic conductivity, K_e, with respect to hydraulic conductivity of mortar, K_m, results in the $K_i \times t_i / K_m$ term. If the original relationship between normalised conductivity, K_e/K_m, proposed by Neale and Nader (1973) is used as a basis for a new relationship considering effect of ITZ properties, the following general model can be advised,

$$\frac{K_e}{K_m} = 1 - \left(\alpha - \beta \frac{K_i t_i}{K_m d_{avg}} \right) \varphi \tag{8}$$

where α and β are the fitting parameters. The variable d_{avg} is the average size of aggregate that can be easily determined using the aggregate size distribution. This variable is introduced to define a new dimensionless quantity, $K_i \times t_i / (K_m \times D_{avg})$, which facilitate fitting of a new model relating the normalized concrete conductivity, K_e/K_m, to two dimensionless quantities that are φ and $K_i \times t_i / (K_m \times d_{avg})$. The general model shown in Equation 8 physically predicts consistent values for extreme values of the dimensionless variables: as $\varphi \rightarrow 0$, $K_e/K_m \rightarrow 0$, and as $K_i \times t_i \rightarrow 0$, $K_e/K_m \rightarrow 1 - \alpha\varphi$. Considering the assumptions made for the aggregate size distribution in this study [d_{min} = 3 mm (0.118 in), d_{max} = 20 mm (0.787 in) and γ = 0.50 for Fuller curve], the average aggregate size can be determined as follows,

$$d_{avg} = \int_{d_{\min}}^{d_{\max}} x.f_D(x)dx \tag{9a}$$

$$d_{avg} = \int_{d_{\min}}^{d_{\max}} x.\frac{\gamma}{d_{\max}} x^{\gamma-1}dx = \frac{\gamma}{d_{\max}} \int_{d_{\min}}^{d_{\max}} x^{\gamma}dx = \frac{\gamma}{\gamma+1}\left[1-\left(\frac{d_{\max}}{d_{\min}}\right)^{\gamma+1}\right] d_{\max} = 6.28\ mm(0.247\,in) \tag{9b}$$

where $f_D(x)$ is the probability density function of the aggregate size, which is determined by taking the first derivative of the aggregate distribution size function given in Equation (5). Fitting the results obtained from 5000 simulations to Equation (8) results in the following equation relating the normalized concrete conductivity to the two-abovementioned dimensionless parameters.

$$\frac{K_e}{K_m} = 1.0 - \left(1.1 - 3.0\frac{K_i t_i}{K_m d_{avg}}\right)\varphi \tag{10}$$

For convenience, the fitting parameters α and β are rounded. A comparison between results of simulation for concrete conductivity with those obtained from Equation (10) shows that the coefficient of determination, R^2, is about 0.98. This shows that the proposed expression well predicts the concrete conductivity considering the most important variables affecting the concrete conductivity. In **Fig. 9**, the relationship between the independent dimensionless variables and the effective hydraulic conductivity, K_e/K_m, is graphically shown.

Fig. 9

Equation (10) can be used to find the critical ITZ properties that cancel out the adverse effect of aggregate VFR. According to this equation, two extreme values lead to a K_e/K_m = 1.0. The first solution is φ = 1.0, which is a trivial solution. The other solution relates the critical properties of ITZ to properties of matrix (mortar here) and size distribution of aggregates represented by d_{avg}. It shows that when the product $K_i \times t_i$ exceeds 0.367($K_m \times d_{avg}$), the adverse effect of aggregate volume fraction on concrete conductivity is cancelled out. In **Fig. 9**, these two solutions are shown. It is worth noting that if the values used in the model verification are substituted to find the $K_i \times t_i$ / ($K_m \times d_{avg}$) ratio, a value of 0.032 results, which is considerably lower that the critical value, i.e.,

0.367. On the other hand, using the extreme value shown in **Table 2** for the ITZ properties [K_i/K_m = 50 and t_i = 0.10 mm (0.0039 in)] results in 0.796 for this ratio. The available information about properties of the ITZ is limited to make a solid conclusion on whether this critical ratio is met or not. Nonetheless, the proposed expression in Equation (10) shows the significance of the ITZ and its effect on conductivity of concrete as a composite material. More research on identification of properties of ITZ using experimental data is required.

CONCLUSION

In this paper, with the aid of the Monte Carlo technique, a probabilistic study on variability of the hydraulic conductivity of concrete and its relation to conductivity of concrete constituents, i.e., aggregate, mortar and the ITZ, at the meso-scale level is conducted. Within this probabilistic procedure, a numerical analysis using the FE method, which was verified using experimental results, is proposed. The proposed numerical model takes advantage of analogy between heat transfer and water flow. It was shown that the properties of the ITZ, i.e., its thickness and hydraulic conductivity have high correlation with the hydraulic conductivity of concrete. By taking advantage of some of the available analytical models relating conductivity of composite material to volume fraction of non-conductive particles, a new conceptual model relating effective hydraulic conductivity of concrete to aggregate volume fraction, thickness of the ITZ and hydraulic conductivities of mortar and the ITZ is proposed. Comparing the results predicted based on this model and those obtained from the FE method, showed that this model can predict hydraulic conductivity of concrete with very good accuracy. The model can be a useful tool for further parametric studies on the role of the ITZ and mortar properties on overall conductivity of concrete as a composite material.

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TABLES AND FIGURES

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Table 1–Results of K_e/K_m ratio with no ITZ (except test results)

				Neale and Nader (1973)	Jeffery (1973)	Bruggeman (1935)
No.	VFR	Test	FE	$\frac{2(1-\varphi)}{2+\varphi}$	$1-1.5\varphi+0.588\varphi^2$	$(1-\varphi)^{3/2}$
1	0.1	0.93	0.83	0.86	0.86	0.85
2	0.2	0.75	0.71	0.73	0.72	0.72
3	0.3	0.70	0.60	0.61	0.60	0.59
4	0.4	0.32	0.51	0.50	0.49	0.46
5	0.5	0.18	0.42	0.40	0.40	0.35

Table 2–Statistical models for the basic random variables

Var.	Parameters	Distribution
K_m	Mean = 0.0195 mm/sec (7.67×10^{-4} in/sec), COV = 0.15	Lognormal
K_i	Min = 0, Max = 50×0.0195 mm/sec ($50 \times 7.67 \times 10^{-4}$ in/sec)	Uniform
t_i	Min = 0 , Max = 100 μm (0.0039 in)	Uniform

Table 3–Statistics of the hydraulic conductivity of concrete expressed as K_e/K_m ratio (φ = 0.40)

Case	Min	Max	Mean	SD[1]	COV[2]
1	0.51	1.47	0.81	0.234	0.288
2	0.51	1.64	0.82	0.235	0.287
3	0.52	1.44	0.82	0.223	0.273
4	0.52	1.51	0.82	0.219	0.268
5	0.51	1.58	0.83	0.240	0.290
All	0.51	1.64	0.82	0.230	0.280

[1]SD = Standard Deviation

[2]COV = Coefficient of Variation

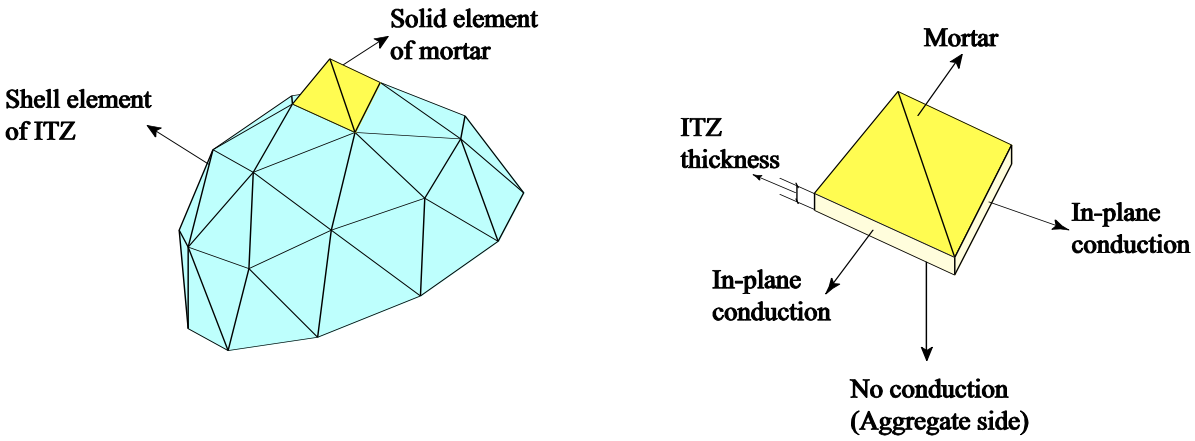
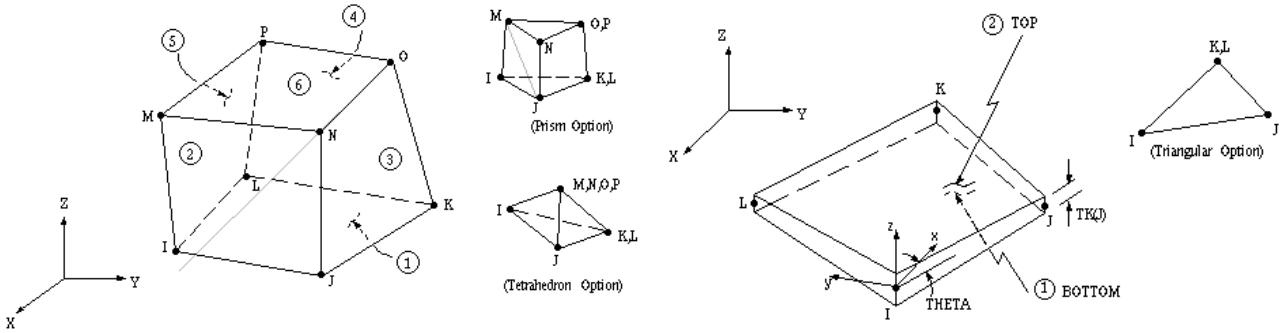
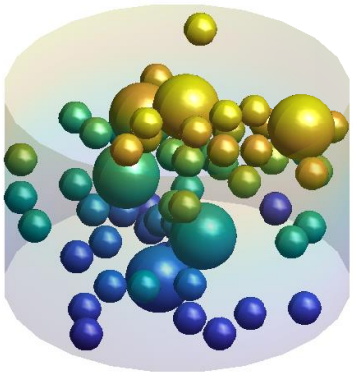


Fig. 1–FE modeling of the ITZ



(a) SOLID70 element (b) SHELL57 element

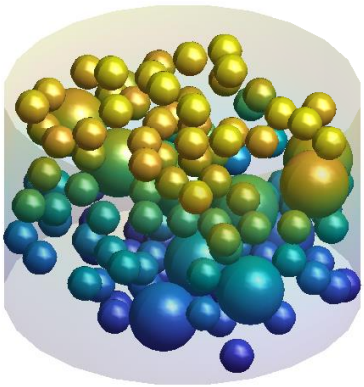
Fig. 2–Finite elements used in the numerical analysis (ANSYS 2016)



Number of 6 mm (0.236 in) = 58

Number of 12 mm (0.471 in) size = 6

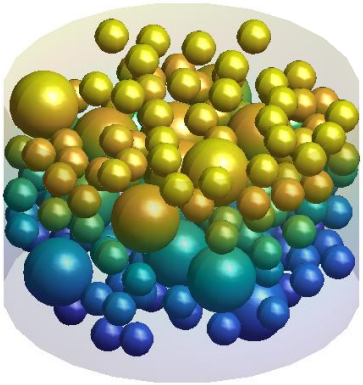
(a) φ = 0.10



Number of 6 mm (0.236 in) = 117

Number of 12 mm (0.471 in) size = 12

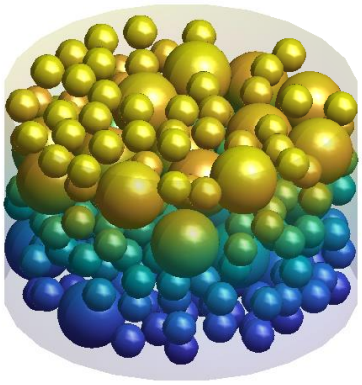
(b) φ = 0.20



Number of 6 mm (0.236 in) = 175

Number of 12 mm (0.471 in) size = 18

(c) φ = 0.50



Number of 6 mm (0.236 in) = 234

Number of 12 mm (0.471 in) size = 24

(d) φ = 0.40

Fig. 3–Typical samples with aggregate different volume fraction ratios

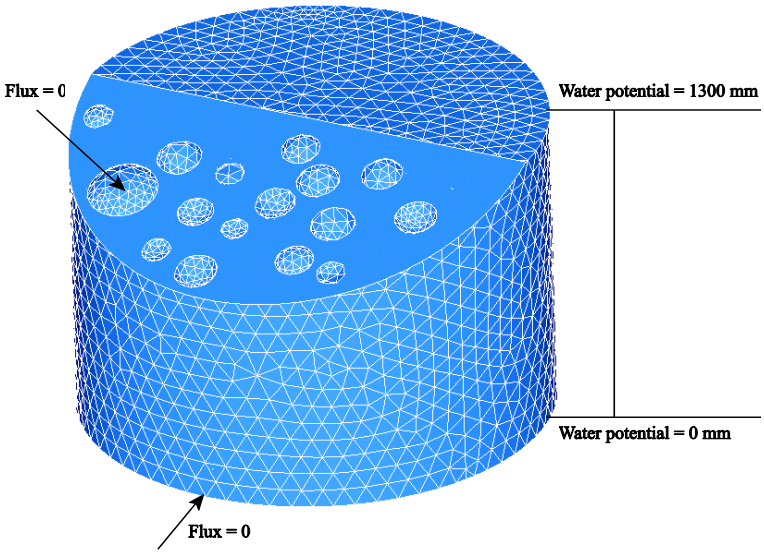


Fig. 4–FE mesh and boundary conditions

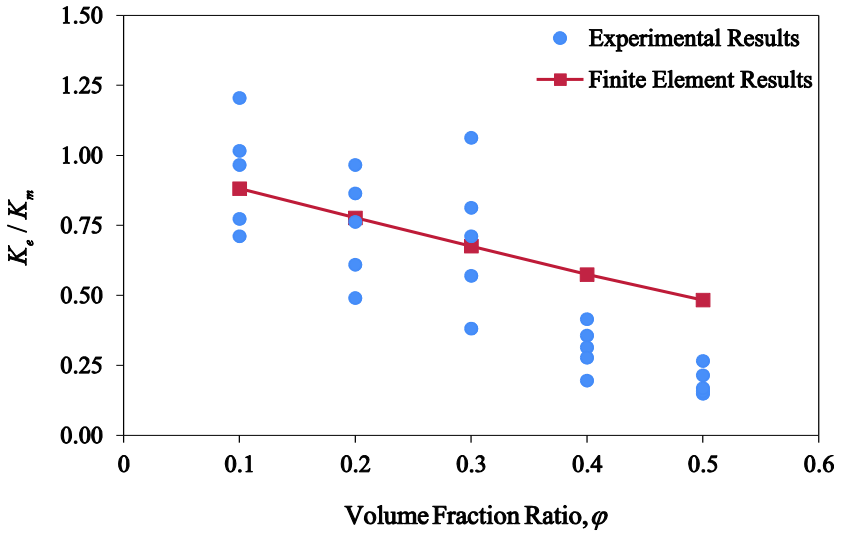


Fig. 5–FE versus experimental results from Li et al. (2016)

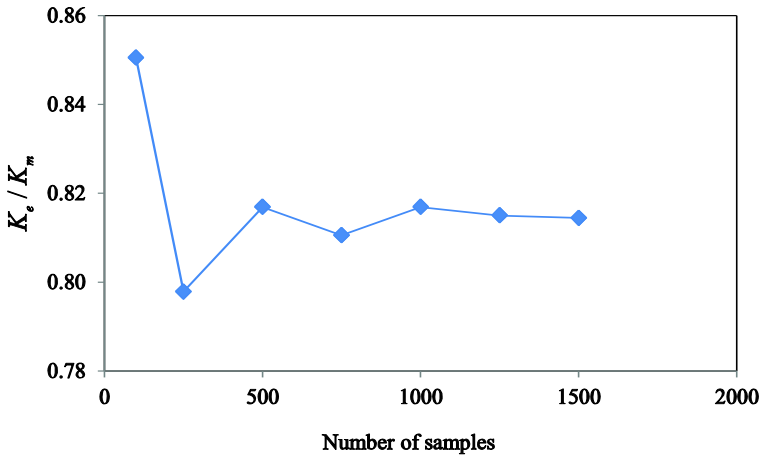


Fig. 6–Effect of sample size on concrete effective conductivity (φ = 0.40)

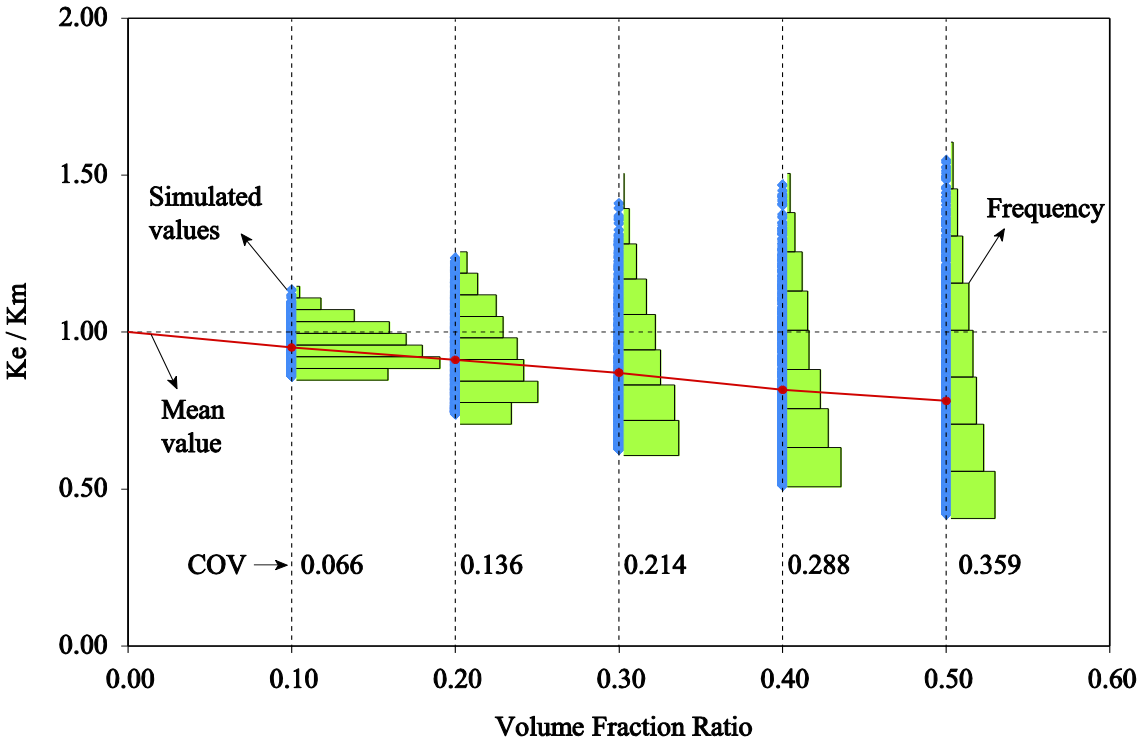


Fig. 7–Typical probability density functions of concrete conductivity

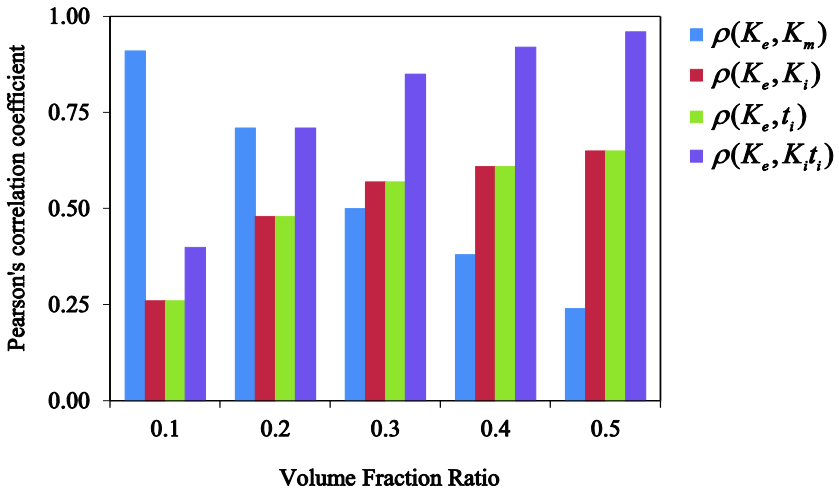


Fig. 8–Sensitivity of the hydraulic conductivity of concrete to the basic random variables

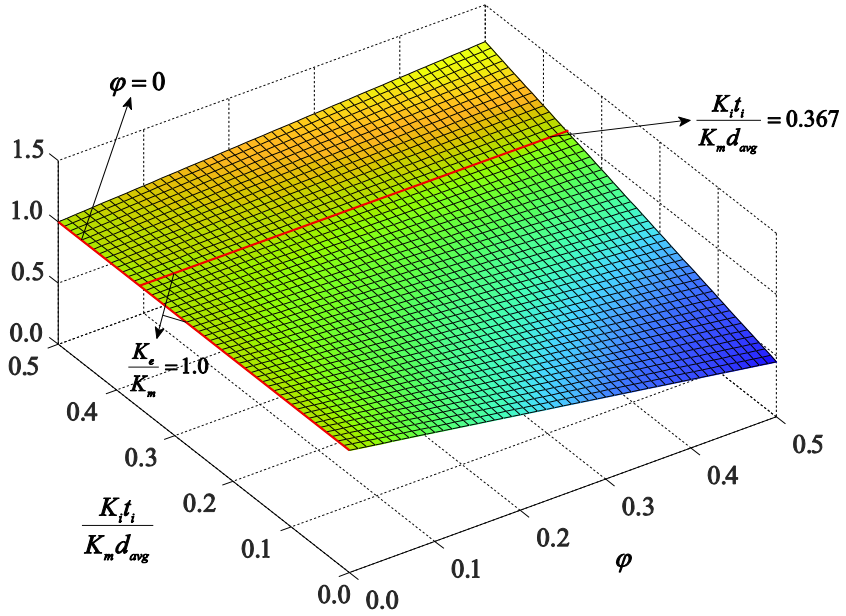


Fig. 9–Simulation versus predicted hydraulic conductivity of concrete