Second generation calculation method for use in the inclining experiment

Kristian Bertheussen Karolius, *Maritime Safety Research Centre* kristian.karolius@strath.ac.uk

Dracos Vassalos, *Maritime Safety Research Centre* d.vassalos@strath.ac.uk

ABSTRACT

The vertical centre of gravity (VCG) is paramount in assessing intact and damage stability being the baseline for any condition of loading. It is well known that the *Classical* method used to calculate the VCG is limited by assuming an unchanged metacentre position and may produce error prone results. An alternative method, namely the *Polar* method, will be presented in this paper. Possible implications inherent in the *Classical* method on stability performance and safety will be assessed utilising the attained index A. The study clearly highlights the accuracy and flexibility of the *Polar* method and demonstrates the importance of correct VCG calculation as even minor errors in the order of millimetres may translate into extensive weights and moments compromising stability and safety.

Keywords: Stability, Safety, Inclining experiment, Lightweight, VCG, Calculation method

1. INTRODUCTION

The centre of gravity of a vessel and specifically its vertical centre of gravity (VCG) is paramount for assessing intact and damage stability performance, being the baseline for any condition of loading. It is also affecting other important aspects such as vessel motion behaviour through the rolling period, hence linked to the new second generation intact stability criteria (IMO, 2016). The VCG is utilised in most intact and damage stability legislation through enforcing requirements to the GZ righting curve. It can therefore also be regarded as a safety baseline.

It is a well-known fact that the so called *Classical* method, in which the VCG is calculated following inclining experiments, has its limitations on performance in terms of applied heel angle magnitude, applied loading condition and accuracy for certain hull forms. This is due to the assumption made of unchanged metacentre position when the vessel is heeled. As a result of the limiting assumptions

in the *Classical* calculation method, more accurate and flexible calculation methods have been proposed. A detailed study on such methods has been presented by Karolius & Vassalos (2018a), highlighting possible limitations inherent in the *Classical* method whilst demonstrating due flexibility and higher accuracy through the use of the new methods.

A second study by Karolius & Vassalos (2008b) has also been presented, but with higher focus on design implications in terms of stability and cargo carrying capacity. In this paper, focus will be on the *Polar* method. The method derivation will be outlined and its accuracy and superiority over the *Classical* method will be highlighted through a technical inclining experiment utilising a completely box-shaped vessel, enabling first principle calculations. The paper will further assess possible implications on stability performance and subsequent safety resulting from incorrect

VCG calculations using the *Classical* method based on the two vessels showing highest errors from the initial study by Karolius & Vassalos (2018a).

Assessment of possible stability implications is achieved by identifying the ensuing false safety resulting from calculating incorrect VCG using the *Classical* method, hence impact on the attained index A as set out in SOLAS Reg. II-1/7-8 (IMO, 2009). By performing calculations for both actual lightweight VCG and calculated lightweight VCG from the technical inclining experiment, the false safety can be identified, highlighting the importance in achieving a correct VCG value following the inclining experiment for a safe vessel design.

2. BACKGROUND

2.1 The inclining experiment

Before vessel stability in any condition of loading can be assessed, the initial lightweight condition needs to be identified using the inclining experiment. All other loading conditions are created using the lightweight condition as a basis, applying loads in terms of cargo, crew, consumables and other equipment, and checked against given stability criteria. As such, it can be considered to be the main stability reference and measure of loading capacity for a vessel. Any errors in determining the lightweight particulars will be a consequential error on all other loading conditions that are to be assessed against relevant intact and damage stability criteria.

SOLAS Reg. II-1/5 (IMO, 2009) requires every passenger ship, regardless of size, and every cargo ship above 24 meters in length, to be inclined upon its completion or following any design alterations affecting stability. High-speed and light-craft have similar requirements found in the HSC Code Reg. II/2.7 (IMO, 2000), and in Torremolinos Reg. III/9 (IMO, 1977 as amended), for fishing vessels. Even smaller recreational craft above 6 meters in length have equivalent requirements in ISO standard 12217-2 (ISO, 2013).

Passenger vessels are further required by SOLAS to be inclined every 5 years if lightweight surveys identify a weight change above a given threshold limit. Evolution of the lightweight is very common, as most vessels are refurbished and converted through their operating-life. The inclining experiment report is subject for approval by flag administration and class and comprise approval of the vessel's lightweight particulars which is needed for the purpose of stability approval and its control. The approval establishes the stability baseline setting limiting constraints on the vessels loading conditions, thus ensuring safe operations.

2.2 Classical method assumptions

The validity of the *Classical* method is based on the assumption of unchanged position of the metacentre when the vessel is heeled. This is illustrated in Figure 1.

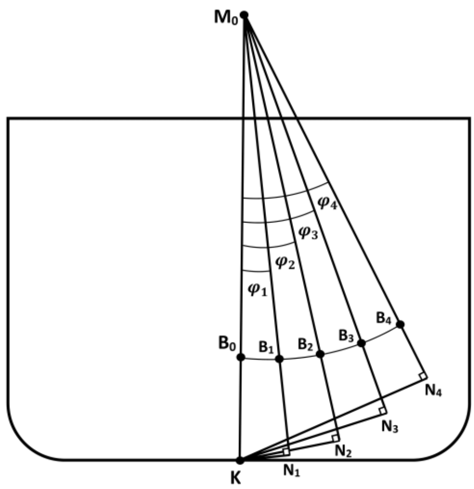

Figure 1. Assumption of unchanged metacentre position.

The position of the metacentre can be represented by the metacentre-radius (BM) given by (1):

$$BM = \frac{I_{XX}}{\nabla} \tag{1}$$

were I_{XX} = second moment of the waterplane area, and ∇ = displaced volume.

The vessel displaced volume is constant during the incline and the change in the position of the metacentre is, therefore, proportional to the change in the second moment of the waterplane area and, consequently, the waterplane area itself. A more realistic movement of the metacentre with increased waterplane area is illustrated in Figure 2. The assumption in the *Classical* method, however, relates to smaller heel angles, and may hold to an acceptable level for more traditional hull forms. In an attempt to ensure the correct application of the *Classical* method, various requirements have been set out in the 2008 IS Code Part B Ch. 8 and Annex I (IMO, 2008).

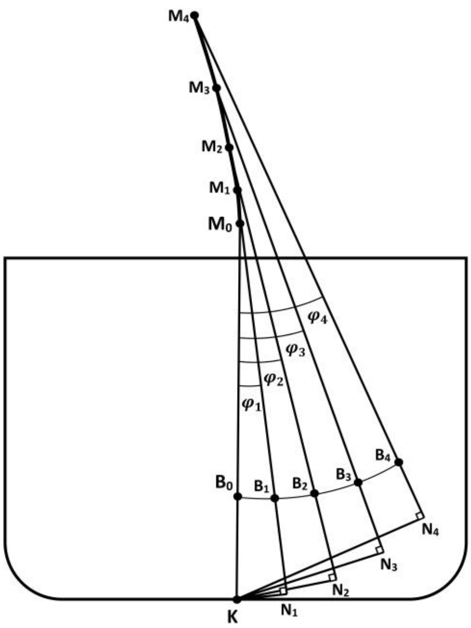

Figure 2. More realistic movement of metacentre position.

Most vessels have today various design features that may result in higher change in the waterplane area than should be accepted even for smaller heel angles. This is the main reason for the *Classical* method being subjected to scrutiny and debate. Such design features may include:

- Chine lines and knuckles
- Large fore- and aft flare
- Misc. appendages
- Large change in trim during heel
- Other unconventional hull forms

The reason for the assumption in the *Classical* method is to utilise a simplified trigonometric relationship as illustrated in Figure 3, which facilitates a formula for VCG to be derived using (2-7). The assumption further enables the use of upright hydrostatics in the calculation of GM for every weight shift.

$$\tan(\varphi) = \frac{G_0 G_\varphi}{G_0 M_0} \quad (2)$$

$$G_0 M_0 = \frac{G_0 G_\varphi}{\tan(\varphi)} \quad (3)$$

$$G_0 G_\varphi = \frac{wd}{\Delta} \quad (4)$$

$$GM = G_0 M_0 = \frac{wd}{\Delta \tan(\varphi)} \quad (5)$$

$$\tan(\varphi) = \frac{r}{L} \quad (6)$$

$$VCG = KM - GM \quad (7)$$

were w = inclining weight, d = movement distance, Δ = displacement, r = pendulum reading, and L = pendulum length. Remaining parameters are explained using figure 3.

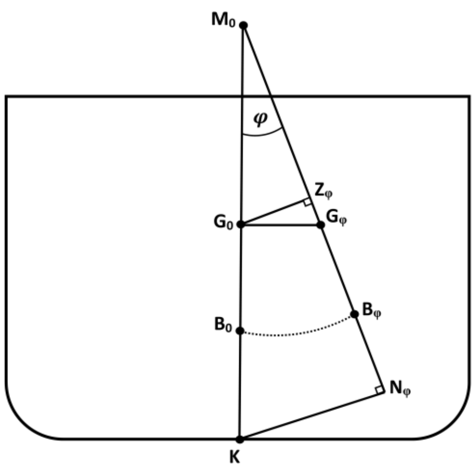

Figure 3. Simplified trigonometric relationship for deriving the Classical formula for GM.

2.3 Implications on stability

The waterplane area may increase or decrease depending on heel magnitude and which specific design features are emerged or submerged during the incline. This can be addressed as two specific cases:

Case 1: Increase in waterplane area:

$$BM_0 < BM_\varphi \quad (8)$$

$$GM_0 < GM_\varphi \quad (9)$$

$$VCG_0 > VCG_\varphi \quad (10)$$

Case 2: Decrease in waterplane area:

$$BM_0 > BM_\varphi \quad (11)$$

$$GM_0 > GM_\varphi \quad (12)$$

$$VCG_0 < VCG_\varphi \quad (13)$$

By using the *Classical* method, Case 1 will overestimate vessel stability, thus producing a lower VCG value than is the actual case, while Case 2 will underestimate the vessel stability leading to a higher VCG than is the actual case. Moreover, the VCG is utilised in most intact and damage stability legislation through enforcing requirements on the GZ righting curve. The GZ curve is represented by (14) and it is clear that any error in VCG will lead to subsequent errors in the GZ curve and hence incorrect assessment against relevant stability criteria.

$$GZ(\varphi) = KN(\varphi) - VCG\sin(\varphi) \quad (14)$$

Another way to illustrate possible implications on stability, a traditional VCG stability limit curve can be used as shown in Figure 4.

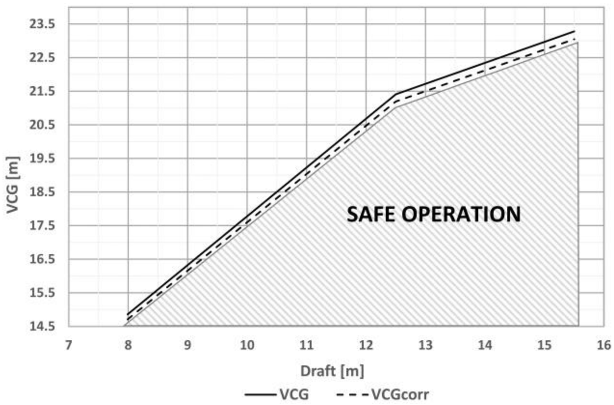

Figure 4. Stability limit curve.

The limit curve serves as the safe operational envelope for a vessel and represents the operational conditions for which the relevant intact and damage stability requirements are fulfilled. The black curve represents the limit curve prepared using the lightweight VCG, as obtained from the *Classical* method. For the sake of argument, an error of 1% underestimation in VCG is assumed. The actual curve is then represented by the stapled line as the underestimation of the lightweight VCG has resulted in a more lenient operational limit. This clearly shows that a vessel may be operating in an unsafe area if the lightweight VCG is underestimated, presenting false safety to the operators.

3. THE POLAR METHOD

The *Polar* method was presented in the study by Karolius & Vassalos (2018a), and is derived utilising the line through point (x, y) represented in polar coordinates, i.e. polar line (PL), illustrated in Figure 5. The representation in polar coordinates is seen in (17) and is derived using (15) and (16).

$$a = y + \frac{x}{\tan(\varphi)} \quad (15)$$

$$z = a \cdot \sin(\varphi) \quad (16)$$

$$\therefore \qquad z = \left(y + \frac{x}{\tan(\varphi)}\right)\sin(\varphi)$$

$$= y\sin(\varphi) + \frac{x\sin(\varphi)}{\tan(\varphi)}$$

$$= x\cos(\varphi) + y\sin(\varphi)$$

$$z = x\cos(\varphi) + y\sin(\varphi) \quad (17)$$

If corrected for actual KN and HZ values for each weight shift using (18), the line will pass through the point (x, y) for any arbitrary weight shift from the neutral position, i.e. Inclining weights in initial position. Further, knowing that the x-coordinate is equal to TCG, and the y-coordinate equal to VCG as represented by (19) and (20) respectively, equation (21) is obtained.

$$z = KN - HZ \quad (18)$$

$$x = TCG \quad (19)$$

$$y = VCG \quad (20)$$

$$KN - HZ = TCG\cos(\varphi) + VCG\sin(\varphi) \quad (21)$$

Using equation (21) and solving for VCG and TCG, we get equations (22) and (23).

$$VCG = \frac{TCG\cos(\varphi) - KN + HZ}{\sin(\varphi)} \quad (22)$$

$$TCG = \frac{-VCG\sin(\varphi) + KN - HZ}{\cos(\varphi)} \quad (23)$$

The method takes advantage of the fact that both VCG and TCG need to be located on the PL line in the initial condition and to be kept constant in this position for each individual weight shift, i.e. the initial VCG_0 and TCG_0 are kept constant on this line, while the overall system TCG is shifted a distance G_0G_i for each shift i as represented by (24) and (25).

$$TCG_0 = TCG_i \quad (24)$$

$$VCG_0 = VCG_i \quad (25)$$

There are, as a result, two equations to derive the two unknown parameters and by using (22) and (23), and following some deduction, (24) results in a solution for VCG given by (26) and (25) in a solution for TCG given by (27) in their most general form:

$$VCG = \frac{(KN_i - HZ_i)\cos(\varphi_0) - (KN_0 - HZ_0)\cos(\varphi_i)}{\cos(\varphi_0)\sin(\varphi_i) - \sin(\varphi_0)\cos(\varphi_i)}$$
(26)

$$TCG = \frac{(KN_i - HZ_i)\sin(\varphi_0) - (KN_0 - HZ_0)\sin(\varphi_i)}{\cos(\varphi_i)\sin(\varphi_0) - \sin(\varphi_i)\cos(\varphi_0)}$$
(27)

The equations can further be simplified using the trigonometric relations in (28) and (29) and knowing that the heeling arm in the neutral position HZ_0 needs to be zero, this results in (30) and (31).

$$\cos(\varphi_0)\sin(\varphi_i) - \sin(\varphi_0)\cos(\varphi_i) = \sin(\varphi_i - \varphi_0) \quad (28)$$

$$\cos(\varphi_i)\sin(\varphi_0) - \sin(\varphi_i)\cos(\varphi_0) = \sin(\varphi_0 - \varphi_i) \quad (29)$$

$$VCG = \frac{(KN_i - HZ_i)\cos(\varphi_0) - (KN_0)\cos(\varphi_i)}{\sin(\varphi_i - \varphi_0)}$$
(30)

$$TCG = \frac{(KN_i - HZ_i)\sin(\varphi_0) - (KN_0)\sin(\varphi_i)}{\sin(\varphi_0 - \varphi_i)}$$
(31)

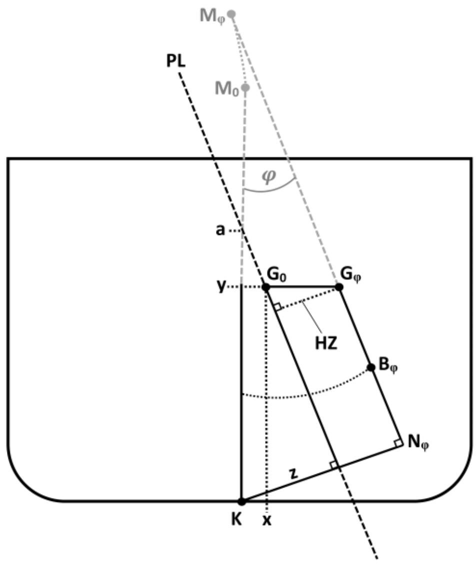

Figure 5. Main parameters of the Polar method.

The *Polar* method can in theory be used to calculate VCG directly for any arbitrary shift from the neutral position, but to account for other sources of errors, it is recommended to utilise a least square linear regression similar to that of the *Classical* method, by plotting the denominator against the numerator and calculating the regression slope.

4. VALIDITY ASSESSMENT

4.1 Approach

For assessing the mathematical validity and accuracy of the *Classical* and *Polar* calculation methods, a technical inclining experiment has been performed using a completely box shaped vessel, enabling first principles calculations. The main particulars of the box shaped vessel are seen in table 1, and the lightweight particulars of the vessel are seen in table 2.

Table 1. Main particulars of box shaped vessel.

Vessel type	L_{BP} [m]	B [m]	D [m]	T [m]
Box-shape	100	40	40	10

Table 2. Lightweight particulars of box shaped vessel.

Para.	Δ [tonnes]	LCG [m]	TCG [m]	VCG [m]
Val.	392400	50.00	0.00	12.00

The technical inclining experiment has been performed using small heel angles of 4° in line with the IMO requirements and larger heel angles of 10° to clearly show the limitation of the *Classical* method when it comes to larger heel angles. Both calculation methods have been applied using a least squares linear regression, and 8 weight shifts.

4.2 Result

The result from the technical inclining experiment is presented in table 3. It is clear that the *Classical* method is highly dependent on the heel angle magnitude, and will produce result with increasing errors with increasing heel angle. This finding is in line with the earlier studies by Karolius & Vassalos (2008a-b). It is further shown that the *Polar* method produce exact results with no errors compared to the actual VCG value.

Table 3. Validity assessment results.

	Classical		Polar	
Heel [°]	4	10	4	10
$VCG_{calculated}$ [m]	11.97	11.84	12.00	12.00
VCG_{actual} [m]	12.00	12.00	12.00	12.00
Error [%]	0.21	1.34	0.00	0.00
Error [mm]	25.36	161.29	0.00	0.00

5. STABILITY AND SAFETY ASSESMENT

5.1 Approach

For most vessels, it is the damage stability requirements that is governing and limits the operational envelope. For the sake of illustrating possible implications on stability, the probabilistic damage stability requirements in accordance with SOLAS Reg. II-1/7-8 (IMO, 2009) have been utilised. The attained index A for the operational VCG and the corrected VCG have been calculated, making it possible to gauge the impact of incorrect VCG calculation using (32).

$$Risk = 1 - A \quad (32)$$

Knowing the ensuing risk, this allows obtaining a measure of false safety inherent in the vessel as a result of the inaccurate VCG calculated using the *Classical* method

5.2 Test vessels

The test vessels used in the following for assessing possible implications on stability performance comprise one *RoPax* and one *Container* vessel. These are the two vessels identified in the study by Karolius & Vassalos (2018a) to have highest underestimated VCG values corresponding to Case 1 explained in Section 2.3 above. The vessels main particulars are presented in table 4, and the corrected VCG values are presented in table 5.

Table 4. Test vessels utilised in the study.

Vessel type	L_{BP} [m]	B [m]	D [m]	C_B [m]
RoPax	195.3	25.8	14.8	0.79
Cont. vessel	320.00	48.20	27.20	0.76

Table 5. Corrected VCG values.

Vessel	VCG [m]	Correction [mm]	VCGcorr [m]
RoPax	13.171	41.478	13.213
Cont. vessel	17.228	60.813	17.288

5.3 Results

Table 6 presents the false safety inherent in the vessel as a result of the inaccurate VCG calculated using the *Classical* method. For both vessels, an error in safety estimation of around 3% is seen due to the error in VCG. The table further presents the difference in number of capsize cases and it is seen that the *RoPax* vessel has 6 additional capsize cases not accounted for due to the error in VCG. This corresponds to a 7.5% error in estimated capsize cases. The *Container* vessel has 37 additional capsize cases, corresponding to a 4% error in estimation.

Table 6. Underestimated VCG translated to overestimated probabilistic damage stability performance, i.e. false safety.

Vessel	LW case	A	Risk = 1-A	Capsize cases
RoPax	VCG [m]	0.725	0.275	74
	VCG_{corr} [m]	0.717	0.283	80
	Difference [%]	1.11	2.83	7.50
Cont. Vessel	VCG [m]	0.711	0.289	896
	VCG_{corr} [m]	0.702	0.298	933
	Difference [%]	1.28	3.02	3.96

6. CONCLUDING REMARKS

As can be seen from the technical inclining experiment, the *Classical* method is highly dependent on heel angle magnitude and may produce unacceptable errors that could affect stability performance. The additional measures imposed by IMO are unnecessary when applying the *Polar* method, as no reference is made to the metacentre in the equation. The *Polar* method produce accurate results for any floating position, in terms of draught, heel magnitude and initial heel as they utilise actual KN values corresponding to each floating position. This reduces the possibility of making mistakes and can therefore be considered more reliable and flexible than the *Classical* method.

As the Classical calculation method was developed in the late 17th century (Hoste, 1693) when detailed software models were not available, the limiting assumptions makes sense as it enables upright hydrostatics to be utilised. Today, however, the strife is towards higher accuracy and there exists a range of tools for this purpose, making such simplifications and requisite assumptions obsolete.

The results further highlight the importance of achieving correct VCG value following the inclining experiment for a safe vessel design, as even minor errors in the order of millimetres may translate into extensive weights and moments compromising safety. The most common argument for maintaining the *Classical* method is that the errors are small and insignificant in comparison with other sources of errors, but the validity of this argument can be questioned.

Considering the results from this study, the industry should be more critical when applying the *Classical* method and it may even be time to replace it with better and more flexible calculation methods. It is at least important for the industry to know that there are other more reliable alternatives to the *Classical* method and should be accounted for in the regulations and guidelines in use today.

7. REFERENCES

Hoste, Paul., 1697, "Théorie de la Construction des Vais-seaux" ("Theory of the Construction of Vessels"), Arisson & Posule, Lyon.

International Maritime Organization, 2016, SDC 3/WP.5, Development of Second Generation Intact Stability Criteria. Report of the Working Group, London, UK.

International Maritime Organization, 2009, "Reg. II-1/5 of SOLAS Consolidated Edition 2009", as adopted in IMO Res. MSC 216(82)), 2006.

International Maritime Organization, 2008, "Part B Annex I of the International Code on Intact Stability 2008", as adopted in IMO Res. MSC.267(85), 2008.

International Maritime Organization, 2000, "Reg. II/2.7 of the International Code of Safety for High- Speed Craft Code, 2008 Edition", as adopted in IMO Res. MSC.97(73)

International Maritime Organization, 1977, "Reg. III/9 of the Torremolinos International Convention for the Safety of Fishing Vessels, as modified by the 1993 Protocol (SFV/Torremolinos Convention)".

International Organization for Standardization, 2013, "ISO 12217-2:2013 – Small craft – Stability and buoyancy assessment and categorization".

Karolius, Kristian and Vassalos, Dracos, 2018a, "Tearing down the wall – The inclining experiment", Ocean Engineering Vol. 148, pp 442-475.

Karolius, Kristian and Vassalos, Dracos, 2018b, "Weight and buoyancy is the foundation in design: Get it right", IMDC 2018: 13th International Marine Design Conference.