Microwave undulator using a helically corrugated waveguide

Liang Zhang, Wenlong He, Jim Clarke, Kevin Ronald, Alan D.R. Phelps and Adrian W. Cross

***Abstract*—Microwave undulators (MU) can be an alternative to the permanent magnet undulators in a free electron laser (FEL). MUs normally employ a cavity-type structure that supports the standing wave to boost the electromagnetic field strength. In this paper, a waveguide-type MU based on a helically corrugated waveguide (HCW) that supports a traveling wave is studied. The use of the HCW allows partial conversion of a traveling wave mode into a backward traveling wave mode, hence increasing the effective interaction length. When operating with a TE_{11} mode at 30.3 GHz, the MU was predicted to realize a field strength of ~0.3 T and an undulator period of 4.95 mm when driven by 1 GW of microwave power.**

***Index Terms*— microwave undulator, free electron laser, helically corrugated waveguide, traveling wave mode.**

I. Introduction

In a free electron laser (FEL) [1, 2], the relativistic electron beam passes through a transverse periodic magnetic field to generate short-wavelength radiation. An undulator that is able to create such a magnetic field is important for FEL operation. The use of conventional permanent magnet undulators (PMUs) in FELs is dominant.

The periodic magnetic field can also be generated by an electromagnetic wave. Such types of undulators are known as RF undulators, or microwave undulators (MU), depending on the wavelength. The concept was proposed in 1982 and the experiment was demonstrated a year later [3, 4]. It is advantageous that the polarization and the field strength in an MU can be easily controlled through adjustment of the input microwave signal. However, for Permanent Magnet Undulators (PMUs), it is hard to achieve polarization control as mechanical adjustment is required. In addition, MUs are capable of achieving a shorter period by operating at higher frequency. It is difficult for PMUs to scale to millimeter undulator periods due to the constraint of the physical dimension of the magnets.

A MU is essentially a metallic cavity that can support the desired operating mode. Progress has been challenging since the first experiment because of two major difficulties in achieving a high equivalent magnetic field. One is a high Q cavity is needed to maintain a high field inside the cavity. The other is that a high power microwave source is required.

The first MU experiment was carried out with a ridged rectangular cavity achieving a quality factor of 7100. When driven by a 300 kW, 2.856 GHz microwave source, an equivalent magnetic field B_u of 0.045 T and an undulator period of 54.9 mm were achieved [3]. This structure was limited by the large electric field at the cavity wall which can cause microwave breakdown in the MU cavity when operating at a higher microwave power. A significant improvement was achieved using a low loss HE_{11} mode existing in a cavity composed of a corrugated waveguide, in which the microwave power is mostly concentrated in the central region of the cavity and the electric field at the cavity wall is much smaller. A MU of the HE_{11} type operating in X-band achieved a quality factor of 91000 [5]. When driven by a 50 MW klystron at 12GHz, an equivalent B_u of 0.65 T and a period of 13.9 mm were achieved in the experiment. Such performance is close to a state-of-the-art PMU, for example, 0.85 T with 15 mm period used in the Swiss-FEL [6].

An ideal undulator should have a shorter period to generate shorter wavelength radiation from the FEL. Further improvements on the MU including higher field and longer cavity section are dependent on access to microwave sources capable of both high power (tens of megawatts) and long duration (a few microseconds) at a higher frequency (Ka-band).

However, generation of short pulse duration (nanosecond range), GW level microwave radiation is feasible and such a source can even be compact because of the low average power when operating in pulsed mode. For example, at X band, a peak output power of 3 GW can be achieved with a relativistic backward wave oscillator [7]. At Ka-band, the GW-level output power can be achieved by a Cherenkov maser [8], or a superradiant Cherenkov source [9, 10].

A "flying" MU, which uses a waveguide structure that supports a traveling wave instead of a standing electromagnetic wave, was proposed by Kuzikov *et al* [11]. It can potentially make use of the high-power, short-pulse microwave sources to drive the MUs. The present paper systemically studies the helically corrugated waveguide (HCW) which is able to couple two arbitrary modes for use as a waveguide-type microwave undulator. Both the circular polarized and linearly polarized operating modes were investigated. The operating mode chosen

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was the one that had the biggest field strength in the waveguide center. It also allows a large beam aperture. The analysis is also relevant to a range of possible operating modes.

The paper is organized as follows. Section II introduces the principle of a MU operating in the traveling wave mode. Section III describes the selection of the operating mode, as well as the unique properties of a HCW. Section IV presents the design of a MU operating at Ka-band, with numercal predictions of the behaviour of such an undulator.

II. The Principle of the Flying Microwave Undulator

The principle of the MU can be found in [3, 11-13]. The high-frequency FEL radiation is generated from the stimulated Compton counter scattering of the backward traveling wave by the relativistic electron bunch, which has a high Doppler up-shift frequency [14]. The wavelength of the radiated wave propagating at the same direction of the electron beam can be expressed as

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}(1+\frac{K^2}{2}) \quad (1)$$

where K is the undulator parameter, γ is the Lorentz factor, and λ_u is the period of the undulator. FELs with a radiation wavelength of ~1 nm usually have electron beam energy at the GeV level, while the K value of a microwave undulator operating in the millimeter wave range with a backward traveling wave of power 1 GW is less than 1. Therefore the wavelength of the radiation wave is about 6-orders of magnitude smaller than the period of the MU. Because the dimensions of the MU is 6-orders of magnitude larger than the wavelength of the radiation wave, the high-frequency FEL radiation wave will not be affected by the wall of the MU. The excitation, amplification of the radiated wave occurs at the axial direction, which is the same as the relativistic electron beam. The principle of the FEL in the periodic permanent magnet therefore equates to the case when a microwave undulator is used.

In a MU cavity operating at a TE-mode and with a mode pattern defined by $E_x \neq 0, E_y = 0$, the relativistic electrons see both the electric field $E_x = E_0 \sin(2\pi z/\lambda_g) \cdot \sin(\omega t)$ and magnetic field$B_y = B_0 \cos(2\pi z/\lambda_g) \cdot \cos(\omega t)$. The electron's motion along the *x* axis can be written as $\frac{d\boldsymbol{p}_x}{dt} = -e(E_x - v_z B_y)$. It can be further rewritten in the form of

$$\frac{d\boldsymbol{p}_x}{dt} = \frac{eE_0}{2}\left(\frac{\varsigma}{Z_w}+1\right)\cos\left(\omega t+\frac{2\pi z}{\lambda_g}\right)+\frac{eE_0}{2}\left(\frac{\varsigma}{Z_w}-1\right)\cos(\omega t-\frac{2\pi z}{\lambda_g}) \quad (2)$$

where E_0, B_0 are respectively the peak electric and magnetic field strengths in the MU cavity. Z_w and ς are the wave impedances in the cavity and in free space, respectively.

In a PMU that only has the magnetic field, the electron's motion is

$$d\boldsymbol{p}_x/dt = eB_u cos(2\pi z/\lambda_u) \quad (3)$$

Comparing with Eq. 2 and 3, the force in an MU contains two terms. The first term denotes the force from a backward traveling (counter propagating) wave and the second term is the force of a forward traveling (co-propagating) wave. It indicates the electrons are modulated by the standing wave in the cavity composed of the forward and backward waves.

The transverse motion of the electron bunch in the backward traveling wave was analytically studied in reference [13], under the assumption of a paraxial wave condition. The ratio between the transverse and axial velocities was found to be on the order of K/γ, which is also a small value.

The co-propagating wave will also modulate the electron bunch to generate low-frequency motion. From Eq. 2, its impact can be minimized if the impedance of the MU is close to the impedance of free space. Recent study on the electron motion in reference [13] showed the effect of the co-propagating wave could not be ignored and it could cause spectrum degradation at larger K values. In this paper, only the electromagnetic properties were considered when designing the microwave undulator. The electron beam dynamics will be analyzed in separate work.

It is possible that the electron beam interacts with the backward traveling wave in a waveguide instead of a cavity structure, as shown in Fig. 1(a). In this counter-propagation mode, the effective interaction time is $\tau = L/(v_e + v_g)$, where L is the waveguide length, and v_e, v_g are the velocity and group velocity of the electron beam and the microwaves, respectively. However the effective ineraction length is only $L' = v_g\tau$. In a FEL, the relativistic electron has $v_e \approx c$, therefore L' will be less than $L/2$. On the other hand, as shown in Fig. 1(b), if the EM wave co-propagates with the electron bunch, and generates a proportion of backward traveling wave, or reflected wave, at the same time, the effective interaction time can be much longer and becomes $\tau = L/(v_e - v_g)$. The effective interaction length in this case is

$$L' = v_g L/(v_e - v_g) \quad (4)$$

To achieve the same interaction length, the waveguide length in the counter-propagation mode will need to be 3 times longer compared with the co-propagation mode when $v_g = 0.5v_e$. The ratio will be even greater as the value of v_g increases.

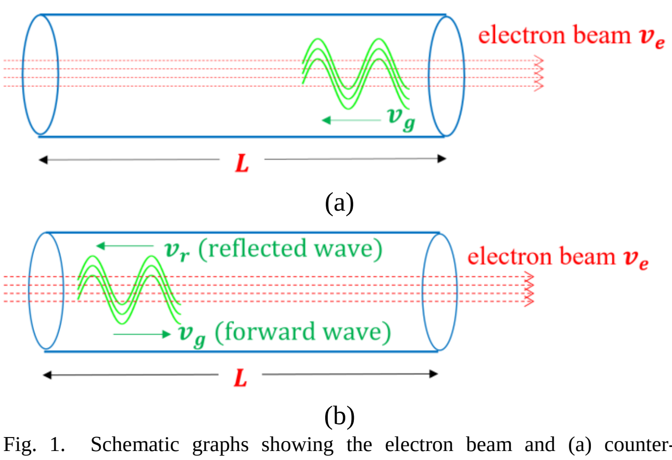


Fig. 1. Schematic graphs showing the electron beam and (a) counter-propagation and (b) co-propagation microwave signals.

III. HCW as Microwave Undulator

In order to achieve the required microwave behavior, a HCW that has both azimuthal and axial periodicities was studied. It has interesting features as it can couple two operating modes in the circular waveguide to generate new eigenmodes. It has attracted considerable interest and found its applications in gyrotron backward wave oscillators (gyro-BWOs) [15, 16], gyrotron traveling wave amplifiers (gyro-TWAs) [17-19], mode converters [20, 21], microwave pulse compressors [22, 23], and as a microwave undulator [11]. The inner surface of an HCW can be expressed in cylindrical coordinates as

$$r(\theta, z) = R_0 + R_1 cos(m_B\theta + 2\pi z/d) \quad (5)$$

where R_0 is the mean radius of the circular waveguide, R_1 is the corrugation depth, m_B is the fold number, and d is the axial period of the corrugation. A typical dispersion diagram for a HCW is shown in Fig. 2.

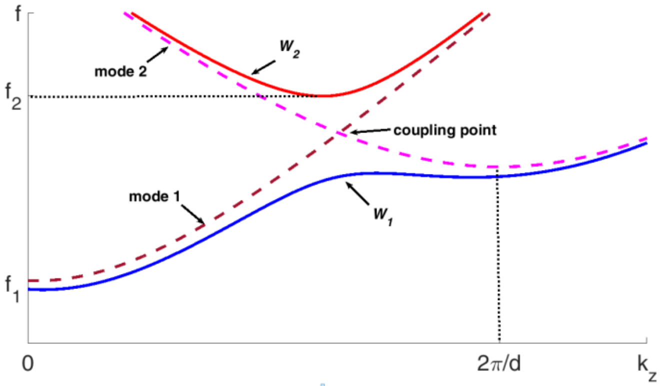


Fig. 2. Mode coupling and dispersion curves of a HCW.

When $R_1=0$, the dispersion curves become the uncoupled TE/TM modes in the circular waveguide, labeled as mode 1 and 2. When $R_1\neq 0$, the two modes can couple which generates eigenmodes W_1 and W_2 if the synchronism conditions are satisfied. Being hybrid modes, W_1 and W_2 have characteristics of both modes 1 and 2. However, the mode percentages depend on the coupling strength and operating frequency. The strongest coupling happens at the crossing point, where the coupling modes 1 and 2 have opposite directions of group velocities. Thus it satisfies the requirement of the MU operating with the backward traveling wave.

Eq. 4 indicates that a high group velocity of the backward traveling wave component is preferred to achieve a short waveguide length. Therefore mode 2 will not operate close to cutoff frequency. Also, it should be noted that the group velocity at the coupling point is usually smaller than its neighbors. Therefore, the desired operating frequency will have an offset to the coupling point.

The operating mode in the HCW is a left-handed circularly polarized wave, the major high power microwave sources with GW level output capability are slow wave devices, where the output mode is linear polarized. A linear polarization version of the HCW can be obtained by adding the right-handed circularly polarization component to its inner surface, which can be written as

$$r(\theta, z) = R_0 + R_1\left[cos\left(m_B\theta + \frac{2\pi z}{d}\right) + cos\left(m_B\theta - \frac{2\pi z}{d}\right)\right]$$
$$= R_0 + R_2 cos(m_B\theta) cos\left(\frac{2\pi z}{d}\right) \quad (6)$$

where R_2 is the corrugation depth of the linearly polarized HCW. To allow a large electron beam aperture, it was assumed that the electron beam tunnel is in the waveguide center. When using an HCW as a MU, it should meet the following criteria. (1) The axial electric field at the waveguide center should be small so that the electron beam will not be modulated in the axial direction. (2) Achieve maximum transverse electric field at the waveguide center, which results in a bigger equivalent magnetic field. (3) The Ohmic loss should be as small as possible, because the undulator is long and operates at high power.

Most of the TE/TM modes satisfy the criteria (1) except the TM_{0n} modes. Criteria (2) means lower-order modes are preferred. Operating at a higher-order mode will result in a larger waveguide radius at a specific operating frequency, which will lead to smaller field strength. A TE_{11} mode was chosen as the operating mode due to the strong equivalent transverse magnetic field it excited in the waveguide axis, as well as the large flat field region in the waveguide center, although it has the disadvantage of large Ohmic loss. The TE_{01} mode has the lowest Ohmic loss and can be a good candidate, as it will be the ideal mode if an annular beam can be used in the microwave undulator or a small diameter beam placed at the peak of the electric field of the TE_{01} mode in the undulator [24].

IV. Design of the HCW

Following the appropriate choice of the suitable modes, a MU operating at Ka-band was designed. The design of the HCW geometry can start from perturbation theory, where a small corrugation depth was assumed. It is suitable for both the linearly and circularly polarized corrugation profiles. The coupling coefficient between two TE modes (mode m and mode j) can be rewritten as [25]

$$S_{jm} = \frac{R_1[\xi_{mq}^2\xi_{jp}^2 + (h_{jp}h_{mp} + k^2)jmR_0^2]}{h_{jp}(h_{jp} - h_{mq})R_0^3\sqrt{\xi_{mq}^2 - m^2}\sqrt{\xi_{jp}^2 - j^2}} \quad (7)$$

where h_{jp}, h_{mq} are the axial wave numbers of the coupled modes and k is the free-space wave number. ξ_{mq} is the q^{th} zero of the derivative of J_m and J is the first kind of Bessel function. ξ_{jp} is the p^{th} root of J_n. In this study, the coupling modes 1 and 2 are TE_{11} and its spatial harmonic. $m= j=1$ and $\xi_{mq}= \xi_{jp} =1.841$. At the coupling point, the equation can be further simplified as,

$$S = [0.315(d/R_1)^2 + 0.419] \cdot (R_1/R_0) \quad (8)$$

when applying the synchronism conditions $m_B = 2$ and $h_{mq} = -h_{jp} = \pi/d$. The coupling coefficient is proportional to the corrugation depth (R_1 or R_2, respectively for circularly or linearly polarized cases). The coupling coefficient is also controlled by the ratio between the period and average radius. For the MU application, a HCW should not have an excessive coupling, because that will cause a small group velocity. A small corrugation depth is preferred in this application as the power in the waveguide is large. The corrugation depth can be set at a fixed value and the system tuned by varying the period to optimize the waveguide for the required application.

It should be noted that for the linearly polarized coupling modes, $m_B = m \pm j$ both satisfy the synchronism conditions. Therefore when the coupling modes are chosen to have the same azimuthal index numbers, where $m = j$, then $m_B = 0$ can be used. In this case, the HCW can be further simplified as an axisymmetric waveguide with steps. Such a TE_{11} mode reflector has been studied using a mode-matching method [26]. In this study the general case that $m_B \neq 0$ was chosen to provide a systematic study of the HCW dimensions based on the selected operating modes and the conclusions are also valid for the particular case of $m_B = 0$.

The mean radius R_0 dictates the cutoff frequency which mainly controls the operating frequency. When operating at 30 GHz, the minimum waveguide radius is 2.93 mm to ensure a traveling TE_{11} mode can propagate. To obtain a large group velocity, a relatively larger waveguide radius of 5.8 mm was chosen. A small corrugation depth, $R_1 = R_2 = 0.3$ mm was chosen for circularly or linearly polarized cases.

The parameter range of the corrugation period can be determined from the dispersion curve of the HCW. Fig. 3 shows the dispersion curves (i.e. phase advance over one period of the structure) for different corrugation periods. There is a stop band between the coupled eigenmodes W_1 and W_2 due to the mode coupling. Microwaves with frequency in the stop band will be totally reflected. The group velocity when the phase shift $k_z d = 180°$ is zero. The group velocity increases and the value is similar to the group velocity of the TE_{11} mode if the phase shift has an offset beyond 15 degrees. The operating point can therefore be chosen in this range. It also can be seen that the frequency at the coupling point increases with a decrease of the corrugation period. The tuning of the operating frequency can be realized by varying the waveguide mean radius or the period.

More accurate dispersion curves can be calculated by the 3D finite-difference time-domain (FDTD) method or eigenmode solver using the finite element method (FEM) [27]. The corrugation period for the microwave undulator operating at 30 GHz was chosen to be 5.6 mm. The geometry, the dispersion curves of the coupled modes and the group velocity simulated by CST Microwave Studio and by the analytical method are shown in Fig. 4. The results only represent the linearly polarized corrugation profile, the circularly polarized HCW has similar results. Fig. 4(c) shows the group velocities at a single frequency have opposite signs which means the waveguide supports both forward traveling and backward traveling waves. At 30 GHz, the group velocity is as high as 0.8c, which is preferred in the application as a microwave undulator. At 30.3 GHz, the group velocity decreases however to about 0.6c.

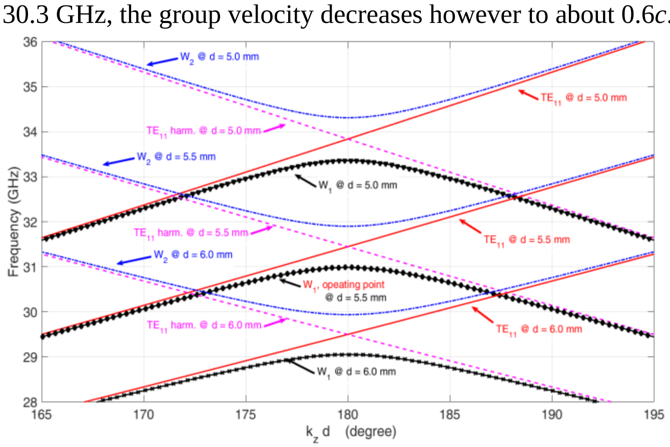
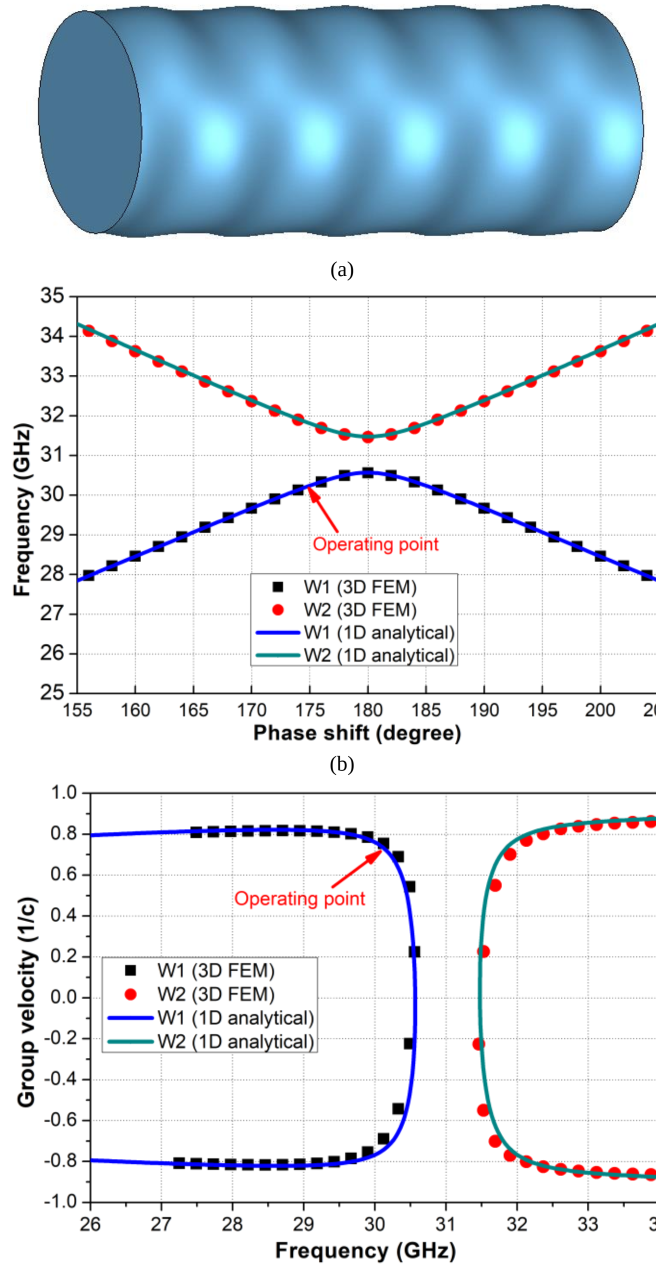


Fig. 3. Dispersion curves at different corrugation periods.



(c)

Fig. 4. (a) The geometry, (b) dispersion curves and (c) group velocities of the designed HCW.

To examine the field pattern inside the HCW, the 3D Finite-Difference Time-Domain method (FDTD) in CST Microwave Studio was used to simulate the scattering parameters. It should be noted that for this periodic structure, the scattering parameters may be a function of the number of periods. The microwave undulator usually requires long length, on a scale of a few meters, therefore HCWs with a different number of periods were simulated until the results converged. The simulation results are shown in Fig. 5. The reflection has a reduced variation after 150 periods. The operating frequency was adjusted to be 30.3 GHz, because it offered a larger amplitude of the backward traveling wave than at 30 GHz. The ratio for the amplitude of the reflected wave to the input wave was about 0.6. Compared with the cavity type MU, the advantage of the HCW is it allows a certain bandwidth. This helps to reduce the difficulty to match the frequency of the high power microwave source.

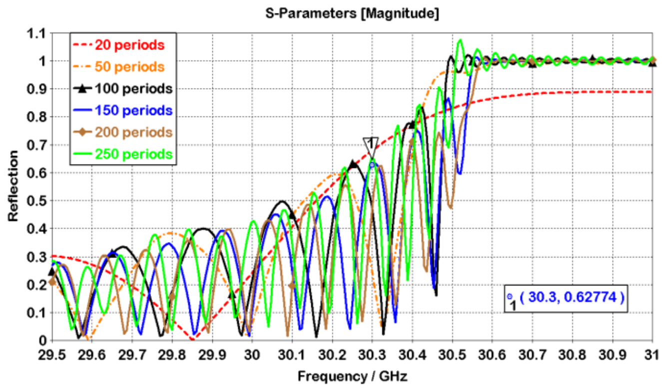


Fig. 5. Scattering parameters of the simulated structure.

The performance of the HCW at steady state was also simulated. In this case, a continuous wave of 30.3 GHz with 1 GW power was excited at one end of the waveguide. The electric fields on the axis at different positions over the simulation time were observed. The results shown in Fig. 6 are the electric fields on the axis at waveguide positions of 250 mm and 300 mm. They have similar trends and the maximum field strengths are about 150 MV/m. With this field strength, the equivalent B_u is about 0.3 T. As the undulator period is 4.95 mm, the undulator parameter *K* is about 0.14.

Two electric fields at probes with position difference of $\Delta z =$ 1.0 mm were exported. The electric fields were fitted by the sum of sine functions $F_i(t, z_i) = \sum_N A_{i,N} \sin(\omega_{i,N} t - \varphi_{i,N})$, individually. Where *i* =1, 2 denotes the probe number. The best fit happens when *N* = 4, which indicates 4 waves inside the waveguide. Two waves have large amplitudes but different signs. They are the forward and backward traveling waves, respectively. Another two waves have much smaller amplitudes but with close $\omega_{i,N}$ values. They are the forward and backward traveling waves with different polarizations caused by the mode conversion inside the waveguide. The group velocities of the wave components can be estimated by $(\omega_{1,N} - \omega_{2,N})\Delta z/(\varphi_{1,N} - \varphi_{2,N})$. They were found to be -0.62*c* and 0.65*c*, respectively. This also proves the existence of the backward traveling wave inside the HCW.

The Ohmic loss inside the HCW waveguide was also considered. In the simulations, a 1.12-meter waveguide with 2/3 of the conductivity of pure copper was used. The loss was found to be about -11 dB, corresponding to 8% of the input power. However, the effect of the field drop-off due to the Ohmic loss on the radiation of the FEL still requires further investigation.

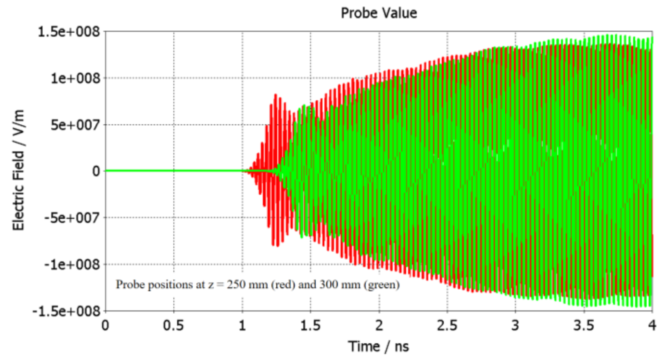


Fig. 6. Electric fields at two different positions over the simulation time.

V. Discussion and Conclusion

This paper presented the design of a waveguide-type microwave undulator by using a helically corrugated waveguide (HCW) operating at 30.3 GHz. When driven by a high power microwave source with an output power of 1 GW and pulse duration of 15 ns, the equivalent magnetic field strength B_{ub} was about 0.3T, and the undulator period λ_u was 4.95 mm which results in an undulator parameter *K* value of 0.14.

This paper has concentrated on study of the electromagnetic field and waveguide structure of the microwave undulator. The motion of the relativistic electrons in the MU is very important as well as the electron bunch dynamics. Both of these topics will be part of future work to enable deeper understanding of the microwave undulator that has been designed.

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