

# 1 Maximization of energy absorption for a wave energy converter using the 2 deep machine learning

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## 7 Abstract

8 A controller is usually used to maximize the energy absorption of wave energy converter. Despite the  
9 development of various control strategies, the practical implementation of wave energy control is still  
10 difficult since the control inputs are the future wave forces. In this work, the artificial intelligence  
11 technique is adopted to tackle this problem. A multi-layer artificial neural network is developed and  
12 trained by the deep machine learning algorithm to forecast the short-term wave forces. The model  
13 predictive control strategy is used to implement real-time latching control action to a heaving point-  
14 absorber. Simulation results show that the average energy absorption is increased substantially with  
15 the controller. Since the future wave forces are predicted, the controller is applicable to a full-scale  
16 wave energy converter in practice. Further analysis indicates that the prediction error has a negative  
17 effect on the control performance, leading to the reduction of energy absorption.

18 *Keywords:* wave energy converter; wave energy control; energy absorption; neural network; deep  
19 machine learning; wave force prediction.

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## 20 1. Introduction

21 To keep up with the growth of global energy demand, various energy systems have been  
22 developed to extract power from marine energy sources (offshore wind, ocean waves, tide, etc) [1-3].  
23 Compared with other marine energy resources, wave energy is a kind of resource with high power  
24 density and all-day availability. Owing to these advantages, wave energy is regarded as a prospective  
25 solution to the sustainable generation of power. The device used to harvest energy from ocean waves  
26 is called the wave energy converter (WEC). Li et al. [4] showed the power output of an oscillating-  
27 body WEC installed on a spar-type floating wind turbine. He et al. [5] utilized a floater breakwater to  
28 harvest energy from the waves. Experimental study of the concept was performed. Falcao and  
29 Henriques [6] presented a review on the oscillating-water-column WEC. Stansby et al. [7] examined  
30 the dynamics of multi-float WEC concept M4.

31 Although a set of WEC concepts have been developed, the energy harvesting efficiency is still not  
32 satisfactory, especially in the off-resonance state. One of the solutions is the usage of a non-linear  
33 power take-off (PTO) system. Zhang and Yang [8] showed that a PTO system with nonlinear spring  
34 could harvest more energy in random waves. Xiao et al. [9] investigated the power capture of an

35 oscillating-body WEC with three different PTO systems. They showed that the nonlinear behavior of  
36 the PTO was beneficial to the power capture. An more widely accepted approach is to regulate the  
37 WEC dynamics with a controller. Babarit et al. [10] studied how the declutching control influenced  
38 the energy absorption of a WEC in regular and irregular waves. Tom et al. [11] optimized the power  
39 capture of an oscillating surge WEC using the pseudo-spectral control method. The latching control  
40 was firstly introduced by Budal and Falnes [12]. They found that one condition for maximizing  
41 energy absorption was to keep the velocity in phase with the wave excitation force. Inspired by their  
42 pioneering work, many researchers begin to adopt the latching control to enhance wave energy  
43 efficiency. Babarit and Clement [13] assessed the benefits produced by the latching control. Based on  
44 the pre-generated wave elevations, the optimal command theory was applied to derive the control  
45 command. Henriques et al. [14] applied the latching control to an oscillating-water-column WEC.  
46 Until now, the WEC control studies mainly concentrate on the development of control strategy  
47 whereas the practical application of control is seldom reported. It is mainly because the  
48 implementation of the controll to a realistic WEC requires the prediction of future wave forces.

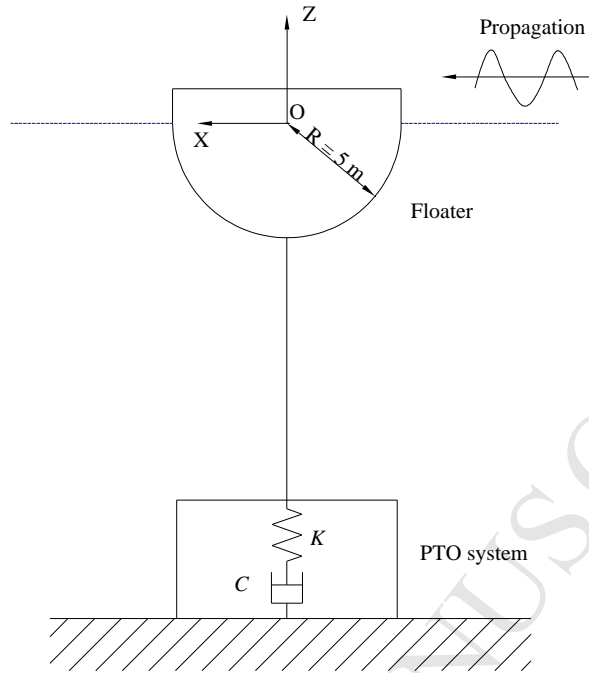
49 Given the explosive growth of the artificial intelligence, the deep machine learning algorithm  
50 based on the artificial neural network has been widely used for regression and classification. The  
51 artificial neural network was firstly proposed by Mcculloch and Pitts [15]. At that time, the structure  
52 of the neural network was very simple since the inference between densely connected nets with many  
53 hidden layers is rather difficult. In 2006, Hinton et al. [16] proposed a fast, greedy algorithm for the  
54 multi-layers network. Their work marked the era of ‘deep learning’. Although the deep machine  
55 learning is basically employed in the recognition and interaction of signal, it is also powerful for  
56 prediction in many fields. Lv et al. [17] used the deep learning approach to predict the traffic flow.  
57 Islam and Morimoto [18] forecasted the inside air temperature of a pillar cooler with the neural  
58 network. Recently, the machine learning was introduced to marine hydrodynamic prediction.  
59 Pourzangbar et al. [19] predicted scour of breakwaters using the genetic programming and the  
60 artificial neural network, respectively. Ebtehaj et al. [20] developed an integrated framework of  
61 learning machines to predict scour at pile groups.

62 The present study is aimed at developing a real-time controller applicable in practice by  
63 considering the short-term wave force forecasting. An artificial neural network is developed for the  
64 wave force prediction. The neural network is trained with the deep machine learning algorithm to  
65 learn the underlying relationship between wave forces in the past and future wave forces. The smart  
66 controller is implemented to a heaving point-absorber to maximize the energy absorption. The  
67 advantage of the neural network against traditional prediction method will be discussed.

## 68 2. Numerical model

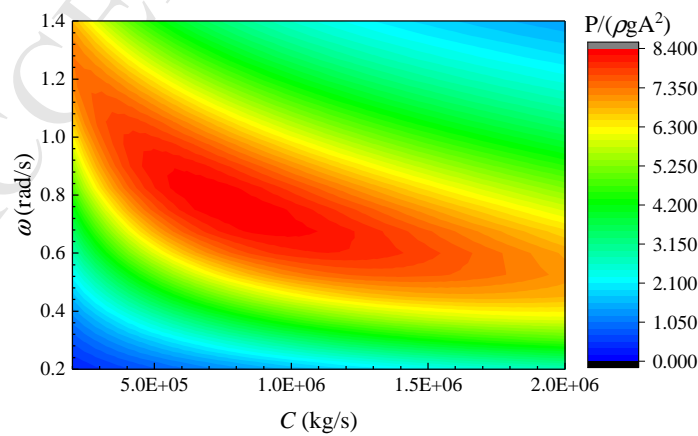
69 Fig. 1 gives the sketch of the heaving point-absorber WEC studied in this work. The point-  
70 absorber consists of a floater harvesting wave energy and a PTO system conversing wave energy into

71 electrical power. The floater, a hemisphere with a radius of 5 m, is rigidly connected to the PTO  
 72 system fixed at the seabed. Only heave motion of the floater is allowed.



73  
74 Fig. 1. Wave energy converter.

75 The generator force is modeled with a linear damping coefficient  $C$  and stiffness  $K$ . According to  
 76 Vicente et al. [21], the stiffness of a PTO system is typically around ten percent of the hydrostatic  
 77 coefficient. Therefore,  $K = 0.1\rho g\pi R^2$  is adopted.  $\rho$  is the water density and  $g$  is the acceleration of  
 78 gravity. Fig. 2 illustrates the sensitivity of the PTO system to wave frequency  $\omega$  and damping  
 79 coefficient  $C$ . To harvest as much energy as possible,  $C = 8.14 \times 10^5$  kg/s is used. The controller  
 80 regulates the WEC response by locking and releasing the floater alternately following a certain rule. A  
 81 very large but finite damping coefficient  $c$  is used to lock the floater. This kind of control is widely  
 82 known as the latching control.



83  
84 Fig. 2. The sensitivity of energy absorption to wave frequency and damping coefficient  $C$  in regular waves. Wave amplitude  
 85  $A = 1$  m.

86 The linear potential flow theory is adopted to address the wave-structure interaction. The viscous  
 87 effect is not considered. It is worth noting that the linear dynamic model is invalid in extreme sea state,  
 88 where the free surface condition involves strong nonlinearity. Since a full-scale WEC just works in  
 89 moderate sea state, the linear dynamic model is still applicable. A right-handed coordinate system  
 90 fixed to the earth is used (see Fig. 1). The center of the coordinate system is fixed at the mean sea  
 91 surface.  $Z$  axial is positive upward.  $X$  axial is along the propagation direction of the sea waves.

92 Based on the impulse response theory [22], the time-domain motion equation of the floater is given  
 93 by

$$94 \quad (M + m) \ddot{z}(t) + \int_0^t H(t - \tau) \dot{z}(\tau) d\tau + \rho g \pi R^2 z(t) = F_{wave}(t) - C\dot{z}(t) - Kz(t) - \beta(t)c\dot{z}(t) \quad (1)$$

95 where  $M$  is the mass of the floater and  $m$  is the added mass corresponding to infinite frequency.  $z$ ,  $\dot{z}$   
 96 and  $\ddot{z}$  are the displacement, velocity and acceleration.  $F_{wave}$  is the wave excitation force.  $\beta(t)$  is the  
 97 binary control sequence. When  $\beta = 1$ , the floater is locked; when  $\beta = 0$ , it is free to oscillate. The  
 98 point-absorber switches abruptly between two states ( $\beta = 0, 1$ ) so that the latching control is a bang-  
 99 bang control.  $H$  is the so-called retardation kernel function which represents the memory effect of  
 100 radiation force. It can be obtained either from the added mass  $a(\omega)$  or the potential damping  $b(\omega)$

$$101 \quad H(t) = \frac{2}{\pi} \int_0^\infty \frac{b(\omega)}{\omega} \sin(\omega t) d\omega = \frac{2}{\pi} \int_0^\infty a(\omega) \cos(\omega t) d\omega \quad (2)$$

102 Although Eq. (1) is widely used to simulate the wave-structure interaction, such form makes it  
 103 inconvenient to implement the control strategy. An alternative model is thus developed to simulate the  
 104 dynamics of the point-absorber in random waves, in which the convolution term is replaced by a state-  
 105 space representation.

$$106 \quad \int_0^t H(t - \tau) \dot{z}(\tau) d\tau = \bar{C}u(t) \quad (3)$$

$$\dot{u}(t) = \bar{A}u(t) + \bar{B}\dot{z}(t)$$

107 where  $\bar{A}$ ,  $\bar{B}$  and  $\bar{C}$  are constant matrices identifying the system with dimensions  $n \times n$ ,  $n \times 1$  and  $1 \times n$ .  $n$   
 108 is the order of the system. These matrices can be derived from the hydrodynamic coefficients of the  
 109 point-absorber by system identification.  $u$  is an intermediate vector with dimension  $n \times 1$ . Define a  
 110 state vector  $x = [z, \dot{z}, u^T]^T$  with dimension  $(n+2) \times 1$ . Then Eq. (1) can be re-expressed as

$$111 \quad \dot{x} = \gamma \cdot x + \eta$$

$$\gamma = \begin{bmatrix} 0 & 1 & \mathbf{0} \\ -\frac{\rho g \pi R^2 + K}{M + m} & -\frac{C + \beta c}{M + m} & -\frac{\bar{C}}{M + m} \\ \mathbf{0} & \bar{B} & \bar{A} \end{bmatrix}, \eta = \begin{bmatrix} 0 \\ \frac{F_{wave}}{M + m} \\ \mathbf{0} \end{bmatrix} \quad (4)$$

112 Eq. (4) is a first-order, one-variable differential formula, which is easier to handle. Given the initial  
 113 condition  $x(0) = \mathbf{0}$ , it becomes a classical initial-value problem and the time series of floater

114 movement can be obtained by the 4<sup>th</sup> Runge-Kutta method. Then, the average energy absorption  
 115 during simulation interval  $[0, T]$  is given by

$$116 \quad P = \frac{1}{T} \int_0^T C \cdot \dot{z}(t, \beta)^2 dt \quad (5)$$

117 The random sea waves can be efficiently approximated by a set of regular wave components with  
 118 various frequencies and random phases

$$119 \quad \zeta(t) = \text{Re} \left[ \sum_{j=1}^N A_j e^{i(\omega_j t + \varepsilon_j)} \right] \quad (6)$$

$$A_j = \sqrt{2S(\omega_j)} \omega$$

120 where  $A_j$ ,  $\omega_j$ , and  $\varepsilon_j$  are the amplitude, frequency and random phase of the regular wave component  $j$ .  
 121  $S(\omega)$  is the wave spectrum adopted to describe the statistical feature of the random waves.  $N$  is the  
 122 number of regular wave components in the wave spectrum. If  $\omega_j$  is uniformly distributed over the  
 123 wave frequency range, the stochastic wave elevations will start to repeat after a certain duration [23].  
 124 To address this issue, the correction technique in Ref [24] is adopted here. The wave frequency range  
 125 is first uniformly divided into  $N$  segments and  $\omega_j$  is afterward randomly distributed within segment  $j$ .  
 126 Given the time series of random wave elevations, the linear wave forces are obtained with the first-  
 127 order transfer function  $\Psi$ .

$$128 \quad F_{\text{wave}}(t) = \text{Re} \left[ \sum_{j=1}^N \psi(\omega_j) A_j e^{i(\omega_j t + \varepsilon_j)} \right] \quad (7)$$

### 129 3. Control strategy

#### 130 3.1. Optimal command theory

131 The objective of latching control is to maximize the average energy absorption through the binary  
 132 control sequence  $\beta(t)$

$$133 \quad \max P = \frac{1}{T} \int_0^T C \cdot \dot{z}(t, \beta)^2 dt \quad (8)$$

134 From a mathematical point of view, it is required to find the maximum of  $P$  subject to constraint  
 135 Eq. (4). If the incident wave is regular, it becomes an impedance matching problem and can be solved  
 136 analytically [13]. Otherwise, the solution is non-causal [25]. Regardless of the incident waves, define  
 137 a Hamiltonian  $H$ :

$$138 \quad H = C\dot{z}^2 + \lambda(\gamma \cdot x + \eta) \quad (9)$$

139  $\lambda$  is a state vector with dimension  $1 \times (n+2)$ , which can be regarded as the Lagrange multipliers.  $\gamma$  and  $\eta$   
 140 have the same definitions in Eq. (4).

141 According to the Pontryagin's maximum principle, the optimal  $\beta$  is the one maximizing the  
 142 Hamiltonian at every time instant throughout  $[0, T]$ . The Hamiltonian is a linear function of  $\beta$  so that  $\beta$

143 must be the extremal values (0 or 1) to maximize the Hamiltonian. It is easy to find that the  
 144 Hamiltonian reaches the maximum value on condition that

$$145 \quad \beta = \begin{cases} 1 & \lambda_2 c \dot{z} < 0 \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

146 Assume that the random waves within the interval  $[0, T]$  are already known, the time series of  
 147 floater movement can be calculated. Subsequently, it is required to calculate  $\lambda_2$  at each time instant  
 148 and apply the latching control based on the binary sequence. Please note that the state vector satisfies  
 149 the following relationships.

$$150 \quad \begin{aligned} \dot{\lambda}_i &= -\frac{\partial H}{\partial x_i}(t, \mathbf{x}, \boldsymbol{\beta}), i = 1, 2, \dots, n+2 \\ \boldsymbol{\lambda}(T) &= \mathbf{0} \end{aligned} \quad (11)$$

151 Eq. (11) cannot be solved numerically like an initial value problem as the final condition is given  
 152 here. In Ref [26], the canonical equations were solved based on the combination of discretization and  
 153 dynamic programming. Zhong and Yeung [27] derived the control sequence with the so-called  
 154 quadratic programming formulation. In our study, an iterative process is applied to calculate  $\boldsymbol{\lambda}$ . Firstly,  
 155 run the simulation with  $\beta(t) = 0$  to obtain the motions free of latching action by integrating Eq. (4)  
 156 forward from  $t = 0$  to  $t = T$ . Subsequently, determine  $\boldsymbol{\lambda}$  by integrating Eq. (11) backwards from  $t = T$  to  
 157  $t = 0$  ( $\boldsymbol{\lambda}(T) = \mathbf{0}$  is now an initial condition). Based on Eq. (10), the control sequence  $\beta(t)$  is derived.  
 158 Iterate the process with the updated control sequence until it converges.

### 159 3.2. Real-time control

160 The optimal command theory cannot be applied directly since it is impossible to know the wave  
 161 forces over the entire interval. Nevertheless, one can forecast the short-term wave forces over a  
 162 relatively short interval  $[t, t+\Delta t]$  so that the optimal command theory can be used within the prediction  
 163 interval. It is the basic idea of the real-time control strategy in this study.

164 Assuming that a wave force prediction model has been developed (the prediction model is the  
 165 neural network in the present study), the procedure of the real-time control is illustrated in Fig. 3. At  
 166 time instant  $t_i$ , collect the historical wave forces and perform the forecasting within  $[t_{i+1}, t_{i+1}+\Delta t]$ . Then,  
 167 the control sequence  $\beta(t)$ ,  $t \in [t_{i+1}, t_{i+1}+\Delta t]$  can be estimated with the optimal command theory. Please  
 168 note that only the predicted control sequence  $\beta(t_{i+1})$  is adopted. At time instant  $t_{i+1}$ , apply the control  
 169 action which has been predicted at the previous step and repeat the process again to predict the control  
 170 action at time instant  $t_{i+2}$ . By repeating this algorithm step by step, the real-time control is  
 171 implemented throughout the entire interval. Such a real-time control is also known as the receding  
 172 horizon control or the model predictive control. Since the energy absorption is maximized over the  
 173 predictive interval, the real-time control strategy is sub-optimal.

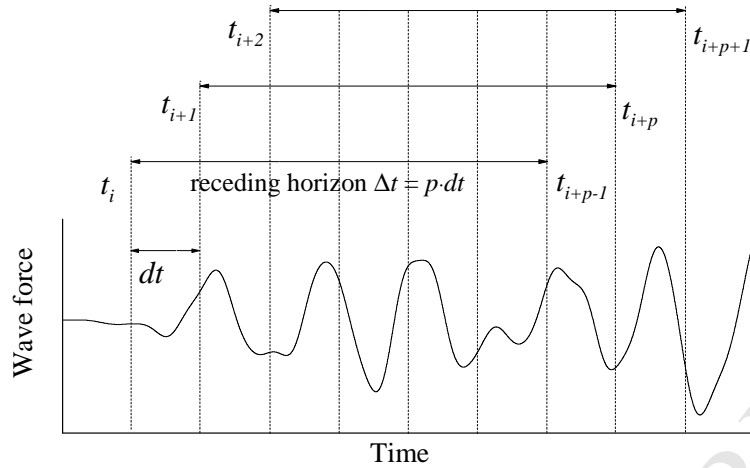


Fig. 3. Receding horizon control.

## 4. Machine learning algorithm

The machine learning algorithm is a member of the broad family of artificial intelligence. Its fundamental architecture is the artificial neural network, which is inspired by the biological neural network. The main idea of using machine learning to predict wave forces is that the neural network can learn and recognize the underlying relationships between the wave forces in the past and the coming wave forces through sufficient training examples.

### 4.1. Neural network

Fig. 4 illustrates the structure of an artificial neural network, which is composed of the input layer, the hidden layers, and the output layer. For the problem in this study, the input is namely the recorded wave forces in the past and the output is the prediction of wave forces over the predictive horizon. Several neurons are located in the layers to process the input signals. Two sets of parameters, weight  $\mathbf{w} = (w_{11}, w_{12}, \dots, w_{ji}, \dots)$  and threshold  $\mathbf{b} = (b_{11}, b_{12}, \dots, b_{ij}, \dots)$  are used to value the importance of the input signal.  $w_{ji}$  and  $b_{ji}$  are the parameters of the  $i$ -th neuron at the  $j$ -th layer. The estimated signal is subsequently normalized by the activation function before transferring to the next layer. The sigmoid function ( $\sigma(x) = 1/(1+e^{-x})$ ) is selected here as the activation function.

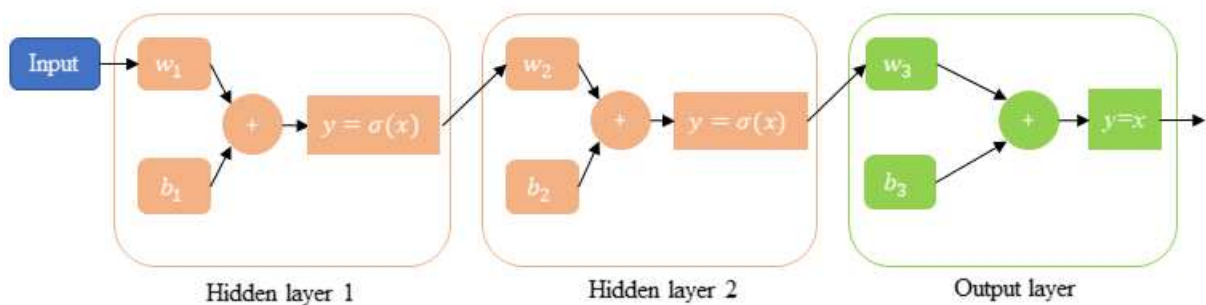


Fig. 4. Illustration of signal transformation between layers.



193 4.2. *Training algorithm*

194 Before the neural network can be used for prediction, one has to tune the parameters  $\mathbf{w}$  and  $\mathbf{b}$  make  
 195 it compatible with the problem concerned. This is fulfilled by training the network with a large  
 196 number of examples (big data) so that it can learn how to predict the wave forces according to  
 197 previous ‘experiences’. A cost function  $\Theta$  is introduced to estimate the performance of the neural  
 198 network.

$$199 \quad \Theta(\mathbf{w}, \mathbf{b}) = \frac{1}{2L} \sum_{k=1}^L \|y^k - a^k\|^2 \quad (12)$$

200  $L$  is the number of training examples.  $y^k$  and  $a^k$  are the prediction output and the prediction target  
 201 corresponding to the  $k$ -th training example. Given the examples, the objective of ‘training’ is to find  
 202 the optimal parameters  $\mathbf{w}$  and  $\mathbf{b}$ , which minimize the cost function  $\Theta$ . The optimal values are searched  
 203 using the gradient descent method.

$$204 \quad \begin{aligned} w_{ji}' &= w_{ji} - \kappa \frac{\partial \Theta}{\partial w_{ji}} \\ b_{ji}' &= b_{ji} - \kappa \frac{\partial \Theta}{\partial b_{ji}} \end{aligned} \quad (13)$$

205  $w_{ji}'$  and  $b_{ji}'$  are the updated weight and threshold after training.  $\kappa$  is the so-called learning rate,  
 206 representing the sensitivity of the network to training example. In the case of  $\kappa = 0$ , the network learns  
 207 nothing from the training examples. Now, the key point is to acquire the gradient of cost function  $\Theta$  at  
 208 various layers. The backpropagation algorithm is used here to estimate the gradient of cost function.  
 209 Start the backpropagation process by estimating the gradient at the output layer. Then the gradient at  
 210 previous layers can be estimated. Continue the backpropagation algorithm until the gradients at all  
 211 layer are obtained. Eventually, update parameters  $\mathbf{w}$  and  $\mathbf{b}$  with the acquired gradients.

212 We demonstrate the procedure of the training process with a simple case. Assume that the network  
 213 has two hidden layers and each layer has only one neuron (see Fig. 4). The neural network is trained  
 214 with a single example ( $L = 1$ ). Denote  $y_0$  the input to the network, the output of hidden layers is given  
 215 by  $y_1 = \sigma(w_1 y_0 + b_1)$  and  $y_2 = \sigma(w_2 y_1 + b_2)$ . The final output of the network is  $y_3 = w_3 y_2 + b_3$ . Then, the  
 216 deviation of cost function at the output layer is given by

$$217 \quad \delta_3 = \frac{\partial \Theta}{\partial y_3} \cdot \sigma'(w_2 y_1 + b_2) \quad (14)$$

218 The deviation of cost function at the hidden layer is obtained with the backpropagation algorithm

$$219 \quad \delta_i = w_{i+1} \delta_{i+1} \sigma'(w_i y_{i-1} + b_i), \quad i = 1, 2 \quad (15)$$

220 At this point, the deviation of cost function at all layers are acquired. Afterward, the gradient of  
 221 cost function at various layers is estimated with the following expressions



$$\begin{aligned} \frac{\partial \Theta}{\partial b_i} &= \delta_i \\ \frac{\partial \Theta}{\partial w_i} &= y_{i-1} \delta_i \end{aligned}, \quad i = 1, 2, 3 \quad (16)$$

222 With the gradient of cost function, the weights and thresholds are updated on the basis of Eq. (13).  
 223 Iterate the above process until the pre-defined termination criteria are satisfied. Please refer to [28] for  
 224 detailed procedures of the training process for more general cases.  
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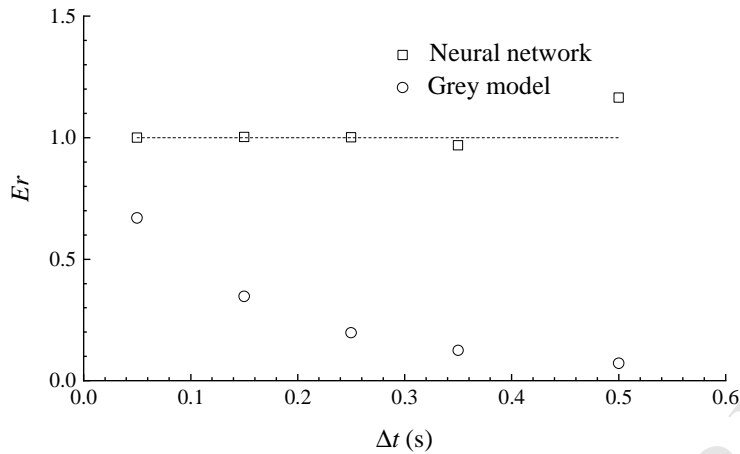
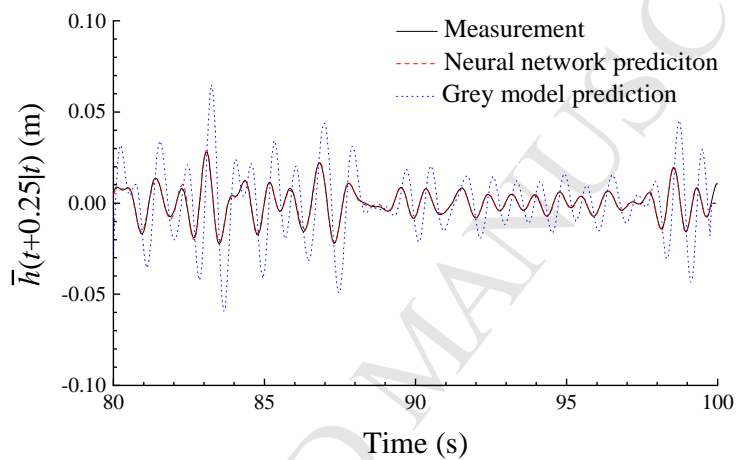
### 226 4.3. Estimation of prediction performance

227 It is well-known that the successful implementation of real-time control requires accurate  
 228 prediction of wave forces. Otherwise, the control performance may become bad [29]. The prediction  
 229 ability of the trained neural network is checked through comparison with the traditional prediction  
 230 methodology-grey model GM(1,1). The detailed procedure of using the grey model for prediction can  
 231 be found in Appendix A. The random wave elevations measured in the Ref [30] is used to examine  
 232 the prediction ability of the two models. The measured data were low-pass filtered to remove the  
 233 high-frequency wave noise. The significant wave height of the random wave elevations is 0.04 m and  
 234 the wave peak period is 1.13 s. Although the neural network forecasts wave forces in the present  
 235 research, it can be validated by the wave elevations since both variables are random signals by nature.  
 236 We select 100 s of wave elevation measurement. The first 50 s data are used to train the neural  
 237 network and the last 50 s are used for validation. Hong and Billings [31] proposed a simplified  
 238 prediction index for quantitative assessment of the prediction

$$Er(\Delta t) = \frac{\int_0^T h^2(\tau) d\tau}{\int_0^T \bar{h}^2(\tau + \Delta t | \tau) d\tau} \quad (17)$$

240 where  $h(t)$  is the measured wave elevation at time instant  $t$ ;  $\bar{h}(t + \Delta t | t)$  is the  $\Delta t$  time ahead wave  
 241 elevation predicted at time instant  $t$ .  $Er < 1$  indicates that the predicted values are larger than the target  
 242 and vice versa.  $Er$  close to 1 represents a good prediction capacity.

243 Fig. 5 compares the prediction performances of the two models. The neural network behaves better  
 244 than the grey model. The index  $Er$  estimated by the neural network is generally around 1. Fig. 7 plots  
 245 the predicted wave force histories. The force predicted by the neural network agrees well with the  
 246 measurement whereas the grey model overestimates the wave force substantially.

Fig. 5. Prediction index  $E_r$ .Fig. 6. Histories of predicted wave force with  $\Delta t = 0.25$  s.

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## 251 5. Validation

252 Two aspects of validation are performed, namely the WEC dynamics and the control sequence  
253 derivation.

### 254 5.1. WEC dynamics

255 Firstly, we validate the dynamic model in the absence of the latching control. The energy  
256 absorption in a set of unit regular waves with various oscillation frequencies is simulated. The results  
257 are compared with those estimated by frequency-domain hydrodynamic analysis programme Wadam  
258 [32]. Please note that the point-absorber is a linear system without the latching control so that Wadam  
259 is applicable here. The PTO system force is modeled with the ‘additional damping’ and ‘additional  
260 stiffness’ options provided in Wadam. As displayed in Fig. 7, the agreement between the two  
261 simulation tools are very good.

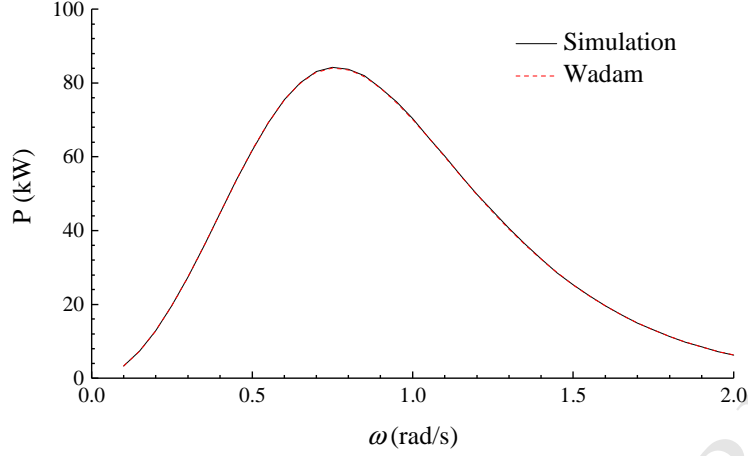


Fig. 7. Energy absorption in regular waves.  $A = 1$  m.

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## 264 5.2. Controller

265 Babarit and Clement [13] presented a semi-analytical solution of the optimal latched duration in  
266 regular waves. According to their study, the optimal latched duration  $t$  satisfies

$$\begin{aligned}
 & y_2 = 0 \\
 & \mathbf{Y} = [y_1, y_2, \dots, y_{n+1}, y_{n+2}] \\
 & = -(\mathbf{I} + e^{\boldsymbol{\gamma}' t}) \cdot (e^{\boldsymbol{\gamma}' t})^{-1} \times \text{Re} \left\{ \left[ (\mathbf{I} e^{i\omega \Delta} - e^{\boldsymbol{\gamma}' t}) (i\omega \mathbf{I} - \boldsymbol{\gamma})^{-1} \boldsymbol{\eta} \right] \times e^{i(\omega t + \varepsilon)} \right\} \\
 & \boldsymbol{\gamma}' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \bar{A} \end{bmatrix}, \Delta t = \pi / \omega - t
 \end{aligned} \tag{18}$$

268 where  $\mathbf{I}$  is the identity matrix with dimension  $(n+2) \times (n+2)$ .  $n$  is still the order of the system. The  
269 detailed derivation of the semi-analytic solution can be found in Ref [13].

270 The latched duration suggested by Eq. (18) and simulated by the present controller are compared  
271 in Fig. 8. As shown, the optimal latched duration increases with the wave period, implying that a  
272 stronger latching action is needed in the case of long waves to regulate the response. Some  
273 discrepancies are observed because the absolute latching control (locking the WEC instantaneously)  
274 was used in Ref [13] whereas the present simulation applies a large damping force to lock the floater.

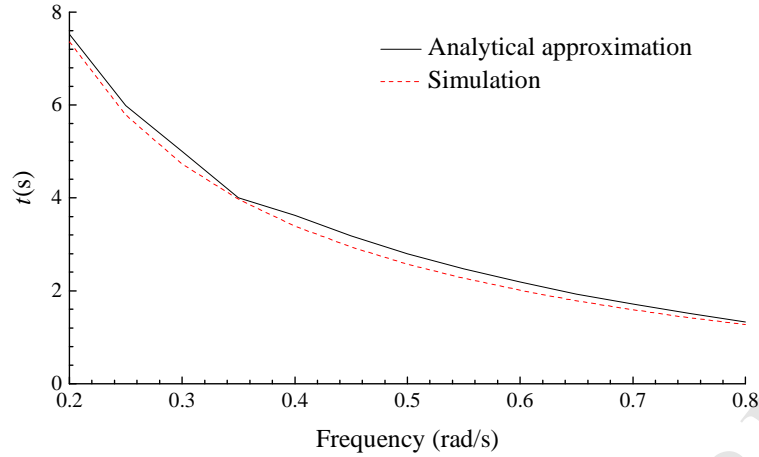


Fig. 8. Optimal latched durations in regular waves.  $A = 1$  m.

## 6. Energy absorption

The joint distribution model of stochastic waves proposed in Ref [33] is used to specify the random waves. The distribution model is based on the field measurement at Atlantic from 2001 to 2010. The marginal distribution of significant wave height  $H_s$  follows a hybrid lognormal and Weibull distribution

$$f_{H_s}(h) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma_{LHM}h} \cdot \exp\left[-\frac{1}{2}\left(\frac{\ln(h) - \mu_{LHM}}{\sigma_{LHM}}\right)^2\right] & h \leq h_0 \\ \frac{\alpha_{HM}}{\beta_{HM}} \left(\frac{h}{\beta_{HM}}\right)^{\alpha_{HM}-1} \cdot \exp\left[-\left(\frac{h}{\beta_{HM}}\right)^{\alpha_{HM}}\right] & h > h_0 \end{cases} \quad (19)$$

The conditional distribution of peak period  $T_p$  at a given significant wave height follows a lognormal distribution. Detailed values of these parameters can be found in [33].

$$f_{T_p|H_s}(t|h) = \frac{1}{\sqrt{2\pi}\sigma_{LTC}t} \cdot \exp\left[-\frac{1}{2}\left(\frac{\ln(t) - \mu_{LTC}}{\sigma_{LTC}}\right)^2\right]$$

$$\mu_{LTC} = c_1 + c_2 \cdot h^{c_3} \quad (20)$$

$$\sigma_{LTC}^2 = d_1 + d_2 \cdot \exp(d_3 \cdot h)$$

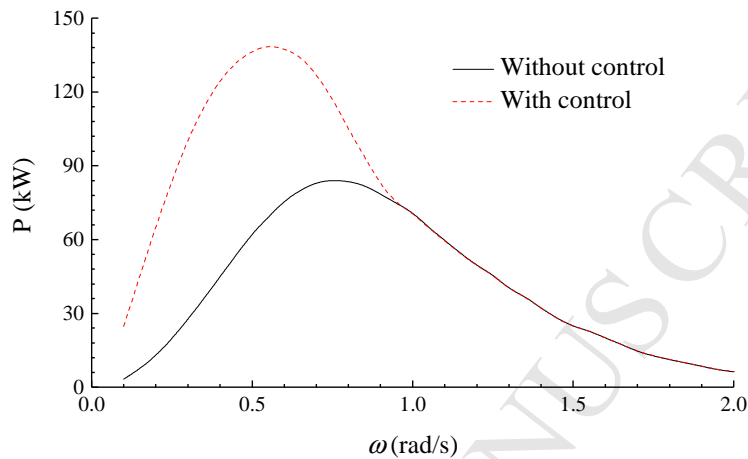
A set of significant wave heights are selected at first, and the most probable peak periods are subsequently determined based on the joint model. The selected random wave conditions are listed in Table 1. 4000 s of random wave elevations are generated in each simulation case and the response in the first 400 s is cut off to eliminate the transient effect arising in early simulation stage.

Table 1 Wave conditions

	Case 1	Case 2	Case 3
$H_s$ (m)	2	4	6
$T_p$ (s)	11.11	12.33	13.27

292 **6.1. Regular waves**

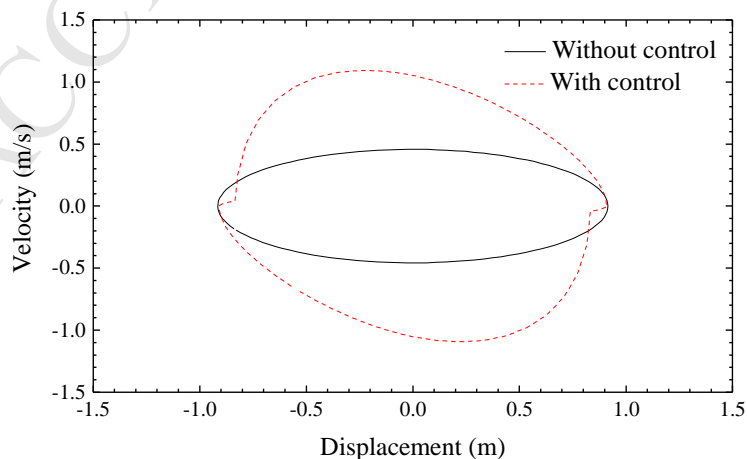
293 We first investigate the energy absorption in regular waves. Since the waves are regular, the wave  
 294 forces over the predictive horizon are given rather than predicted here. The predictive horizon is set to  
 295  $0.2 \cdot (2\pi/\omega)$ . The wave amplitude  $A$  is 1 m. Fig. 9 plots the sensitivity of energy absorption to wave  
 296 oscillation frequency. The control effect on the energy absorption is noticeable, especially within the  
 297 low-frequency range.



298  
 299

Fig. 9. Average energy absorption in regular waves.  $A = 1$  m.

300 Fig. 10 displays the phase portrait of the responses. It is obvious that the velocity phase is  
 301 regulated by the controller. The latching control is a kind of phase control by nature and it maximizes  
 302 energy absorption by tuning velocity phase. Fig. 11 displays the controlled motion and its relative  
 303 phase with respect to the wave forces. According to the velocity time-series, the point-absorber is  
 304 latched and released alternately so that the response is a succession of locked and ramp stages. Due to  
 305 the regulation of the controller, the velocity and the wave forces are in phase so that the wave forces  
 306 will always accelerate the floater and the floater carries more kinetic energy as a result. This property  
 307 has been widely used as the criterion to validate latching control since the work of Budal and Falnes  
 308 [12]. Therefore, Fig. 11 can be used to validate the present numerical model as well.



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 310

Fig. 10 Phase portrait in regular wave.  $A = 1$  m,  $\omega = 0.5$  rad/s.

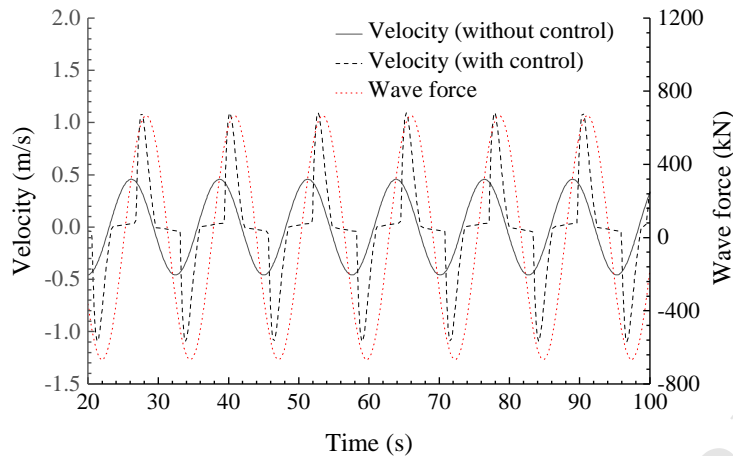


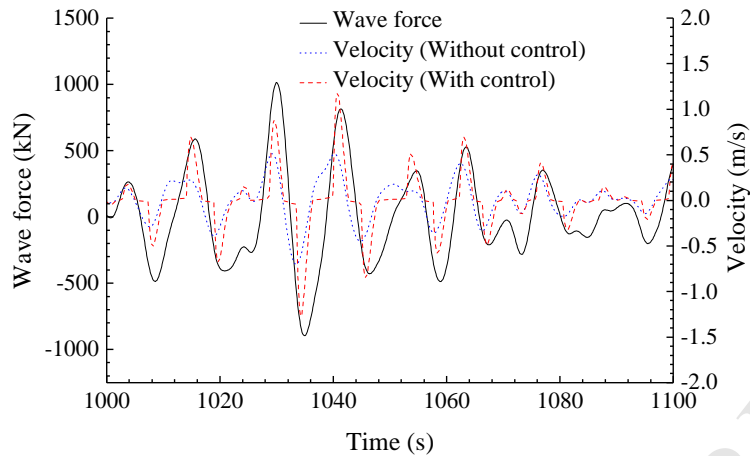
Fig. 11. Responses of the floater in regular wave.  $A = 1$  m,  $\omega = 0.5$  rad/s.

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313 Nevertheless, it seems that the controller is not effective at all within the high-frequency range.  
314 According to Fig. 8, the optimal latched duration reduces gradually when the wave frequency  
315 increases and reduces to nearly zero in the case of very high-frequency waves. It implies that it is  
316 unnecessary to regulate the point-absorber in high-frequency wave and thereby the energy absorption  
317 is hardly increased. As pointed out in Ref [34], the optimal duration of a single locked stage is close to  
318 half of the natural period of the WEC on condition that the PTO damping coefficient  $C$  is sufficiently  
319 small (weak damping). Therefore, the solution of the control sequence is only available when the  
320 wave period is sufficiently long. Although the damping coefficient  $C$  is very large in our model and  
321 the sub-optimal receding horizon control is used here, this property can still help to interpret why the  
322 latching control is merely effective in low-frequency waves.

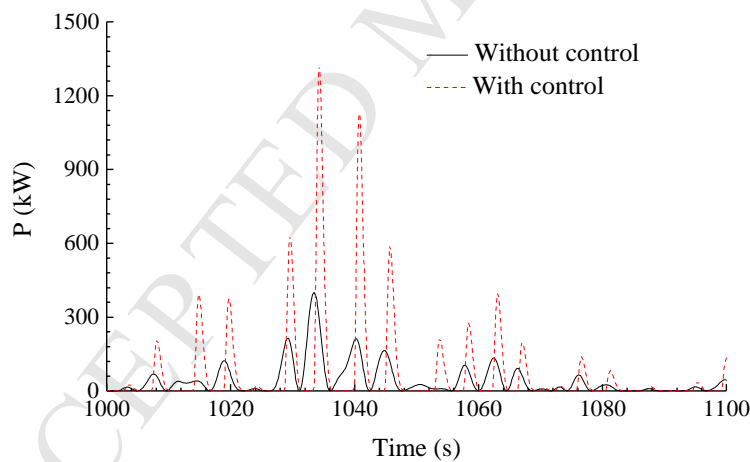
## 323 6.2. Irregular waves

324 In the real oceans, the waves are random and oscillate with multiple frequencies so that the wave  
325 forces are unknown. To examine the validity of the smart controller, we run simulations in regular  
326 waves where the short-term future wave forces are predicted by the neural network. Fig. 12 plots the  
327 histories of floater velocity and wave forces. Like regular wave case, the point-absorber is also locked  
328 frequently in random waves. Whenever the wave force and the velocity are reverse, the floater is  
329 locked to avoid the slowing down of velocity. Once the floater is released, the velocity builds up  
330 rapidly in a short time. Owing to the regulation of the controller, the velocity is generally in phase  
331 with the wave forces.



332  
333 Fig. 12. Responses of the point-absorber, Case 1.

334 The control effect on the energy absorption is illustrated in Fig. 13. Although the point-absorber is  
335 locked frequently, and the PTO systems stop working during the locked duration, the energy  
336 extraction ramps once the point-absorber is set free since the velocity runs up rapidly during the ramp  
337 stage. It leads to the enhancement of the average energy absorption. The 1-hr average energy  
338 harvesting under various wave conditions is presented in Fig. 14. Generally, the point-absorber  
339 produces 60%~80% more electrical energy with the smart controller. It manifests that the neural  
340 network can be successfully applied in the real-time control of WEC.



341  
342 Fig. 13. Power capture, Case 1.



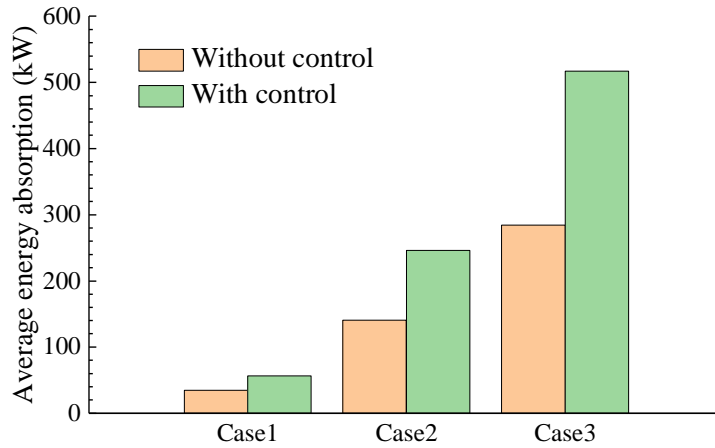


Fig. 14. Average energy harvesting.

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344

345 As discussed in Section 3.2, the model predictive control can be regarded as a kind of sub-optimal  
346 control. Consequently, the efficiency of the controller is evaluated by comparing with the optimal one.  
347 Please note that by implementing the optimal control it inherently implies that the wave forces over  
348 the entire interval are already known. Fig. 15 displays the control sequence using the two control  
349 schemes. The control sequence predicted by the neural network is close to the optimal one, indicating  
350 that the prediction accuracy of the neural network is satisfactory. Some discrepancies are observed  
351 since the prediction deviation is unavoidable. Also, the model predictive control is sub-optimal by  
352 nature and it cannot acquire the optimal sequence even in the absence of prediction deviation.

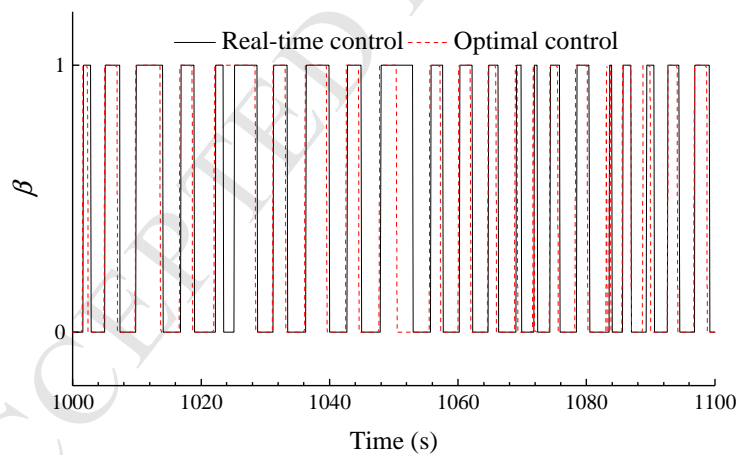


Fig. 15. Control sequence, Case 1.

353  
354

355 Table 2 lists the average energy absorption obtained using the model predictive control and the  
356 optimal control. Generally, the model predictive control underestimates the energy harvesting by no  
357 more than 9%. Although the real-time control is sub-optimal, the control efficiency is still satisfactory  
358 even if the wave forces are predicted.

359 Table 2 Average energy absorption estimated by real-time control and optimal control.

	Real-time control	Optimal control
Case1	56 kW	61 kW
Case2	246 kW	266 kW
Case3	517 kW	571 kW

360

361 *6.3. Energy absorption with different prediction models*

362 To demonstrate the advantage of the neural network against traditional prediction approach, the  
 363 neural network is compared with the grey model GM(1,1). The control efficiency will be evaluated to  
 364 show the accuracy of the two prediction models.

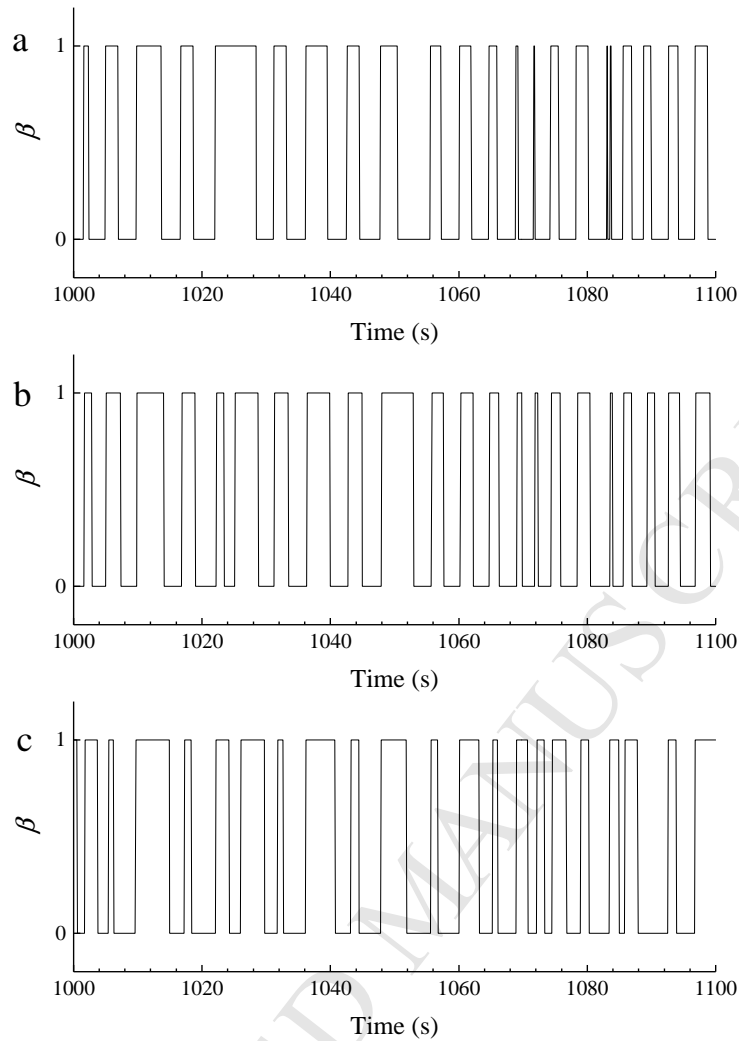
365 Table 3 lists the average power extraction of the point-absorber using different wave force  
 366 prediction models. As shown, the control efficiency is reduced substantially if the wave forces are  
 367 predicted by the grey model. Since the point-absorber is subject to identical wave forces, the  
 368 discrepancies on the control performance are completely caused by the prediction error. It thus  
 369 implies that the neural network is a more reliable prediction approach.

370 Table 3 Average power extraction with different prediction models

	Case1	Case2	Case3
Neural network	56 kW	246 kW	517 kW
GM (1,1)	50 kW	201 kW	429 kW

371

372 Fig. 16 plots the optimal control sequence and the predicted sequence with respect to the two  
 373 prediction models. Due to the unavoidable prediction deviation, the predicted control sequence is  
 374 somewhat different from the optimal one. Nevertheless, it is easy to find that the sequence predicted  
 375 by the neural network is closer to the optimal one, attributing to the better prediction capacity. A more  
 376 appropriate control sequence indicates the point-absorber will be released and locked at the right time  
 377 instants and thereby the point-absorber extracts more power with the neural network



378  
379 Fig. 16. Control sequence forecasted by different prediction models, Case1. (a) Optimal; (b) Neural network; (c) GM(1,1).

## 380 7. Conclusions

381 Wave force prediction is necessary for the implementation of WEC real-time control. As proved in  
382 previous studies, the prediction deviation will reduce the efficiency of real-time control and thereby  
383 an accurate wave force prediction method is essential. In our work, an artificial neural network is  
384 developed to predict the short-term wave forces. Based on the developed neural network, the real-time  
385 latching control is implemented to a heaving point-absorber to maximize the energy extraction in  
386 random waves.

387 The neural network is trained using the machine learning algorithm. The training process is based  
388 on the backpropagation algorithm, in which the gradient of cost function at various layers are  
389 estimated to update the weights and the thresholds. Based on a large number of training examples, the  
390 weights and thresholds identifying the neural network are optimized gradually until the outputs agree  
391 well the desired targets.

392 The point-absorber harvests more power with the real-time control, which is attributed to the  
393 tuning of velocity phase. The controller tunes the velocity by locking and releasing the floater

394 alternately so that the velocity and the wave forces are in phase. In that case, the wave forces always  
395 accelerate the point-absorber and the floater carries more kinetic energy. For the random wave  
396 conditions considered in this study, the energy absorption is increased by 60%~80% with the smart  
397 controller. Owing to the good prediction capacity of the neural network, the control efficiency is  
398 satisfactory, just slightly lower than the optimal level.

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480

## Appendix A

481 Step1: At time instant  $t_i$ , collect the raw data  $X$  over the past few seconds

482 
$$X = (x_1, x_2, \dots, x_n) \quad (\text{A.1})$$

483 Step2: Generate an accumulated series  $Y$ 

484 
$$Y = (y_1, y_2, \dots, y_n)$$

$$y_k = \sum_{i=1}^k x_i, k = 1, 2, \dots, n \quad (\text{A.2})$$

485 Step3: Generate the so-called background series  $Z$ 

486 
$$Z = (z_2, z_3, \dots, z_n)$$

$$z_k = (y_k + y_{k-1}) / 2 \quad (\text{A.3})$$

487 Step4: Set the grey differential formula

488 
$$x_k + az_k = b, k = 2, 3, \dots, n \quad (\text{A.4})$$

489 Step5: Estimate parameters  $a$  and  $b$  with the least square method

$$\begin{bmatrix} a \\ b \end{bmatrix} = (A^T A)^{-1} A^T B$$

$$A = \begin{bmatrix} -z_2 & 1 \\ -z_3 & 1 \\ \vdots & \vdots \\ -z_n & 1 \end{bmatrix}, B = \begin{bmatrix} x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} \quad (\text{A.5})$$

491 Step6: Establish the first order-one variable grey model GM(1,1) to predict the random signal  
492 within interval  $[t_{i+1}, t_{i+p}]$ 

493 
$$\hat{x}_{n+p} = \hat{y}_{n+p} - \hat{y}_{n+p-1}$$

$$\hat{y}_{n+p} = \left( y_1 - \frac{b}{a} \right) e^{-a(n+p-1)} + \frac{b}{a} \quad (\text{A.6})$$

494 where  $\hat{x}_{n+p}$  is the predicted data at time instant  $t_{i+p}$ .



A smart real-time controller is developed.

The smart controller uses artificial neural network to predict short-term wave forces.

The neural network is trained by the deep learning algorithm.

The prediction accuracy of the neural network is satisfactory.

The smart controller enhances the energy absorption substantially.

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